
On the Cross-Scale Prospects of the Logarithmically Corrected Gravitational Potential: From Black Hole Singularities to Galactic Rotation

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Article

On the Cross-Scale Prospects of the Logarithmically Corrected Gravitational Potential: From Black Hole Singularities to Galactic Rotation

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Abstract

Aiming at the black hole "singularity" puzzle in general relativity and the dark matter dilemma of the flattening of galaxy rotation curves, traditional theories have core limitations of cross-scale fragmentation and reliance on unobservable entities (such as higher-dimensional space, dark matter particles, etc.). Starting from the universal logarithmic asymptotic behavior of mainstream dark matter halo models, this paper proposes a minimalist gravitational potential framework with a logarithmic correction term (endowed with the core mechanism of "near-repulsion and far-attraction"). Combined with quantum vortices and nested AdS/CFT correspondence, we present a microscopic physical picture, and construct a modified Poisson equation and Schwarzschild metric. This framework makes a parameter-free a priori prediction of the black hole shadows of Sgr A* and M87* at the black hole scale, which is consistent with the Event Horizon Telescope (EHT) observations. The a priori calculation of the periastron velocities of high-velocity stars S4714 and S62 yields errors within a reasonable range. At the galaxy scale, it fits the rotation curves of various types of galaxies including the Milky Way and the Andromeda Galaxy, which is in high agreement with observations, and physically resolves the singularity through the short-range repulsive potential. This paper presents rigid a priori predictions for the shadows of 6 unobserved black holes, among which NGC315 can be used as a crucial experimental source to distinguish this theory from the standard Kerr paradigm. Without invoking dark matter or higher-dimensional hypotheses, this framework, only using the logarithmically corrected gravitational potential, makes it possible to uniformly describe the cross-scale gravitational mechanism with only the mass of ordinary matter. It provides an observable and falsifiable empirical path for quantum gravity research, and makes it possible to pull the research paradigm of pure mathematical modeling of quantum gravity (which requires conditions far from contemporary observations such as the Planck energy scale) back to the empirical research of contemporary physics.

Keywords: black hole; singularity; general relativity; gravitational potential; quantum gravity; event horizon telescope (EHT); metric; geodesic; adS/CFT correspondence; quantum vortex; logarithm; einstein field equations; black hole shadow; sagittarius A* (sgr A*); messier 87* (m87*); ngc 315; high-velocity star; s62; s4714; dark matter; galaxy rotation curve; milky way galaxy; andromeda galaxy; ngc 2974 galaxy; gravitational lensing; equivalence principle; modified newtonian dynamics (MOND); modified gravity

1. Introduction

Modern astrophysics and gravitational theory have long faced two major cross-scale challenges: at the microscopic scale of black holes, singularities predicted by classical general relativity exhibit infinite curvature, violating the finiteness requirement of physical quantities in quantum mechanics, and the "information paradox" triggered by Hawking radiation remains unresolved; at the macroscopic scale of galaxies, the observed rotational velocities of peripheral stars and gas are much higher than the limit sustainable by the gravity of visible matter. The mainstream Λ CDM model relies

on the hypothesis of unobserved dark matter halos, and there is tension between small-scale predictions and observations.

Traditional theories explain these two major problems in an isolated manner: black hole physics relies on the Kerr metric (requiring post-hoc fitting of spin and inclination), while galaxy dynamics relies on the dark matter hypothesis, both lacking a unified physical core. More critically, these theories either suffer from inherent incompleteness (such as singularities) or lack direct physical carriers (such as dark matter particles).

By analyzing the dynamics of various dark matter halo models, this paper discovers an overlooked and universal logarithmic asymptotic behavior: a simple logarithmic correction term is generated in the gravitational potential ($\Phi_{halo}(r) \sim -\frac{\ln r + 1}{r}$), so can it solve both the black hole singularity and galaxy rotation curve problems? Subsequently, we will show a possible physical picture: at the black hole singularity distance ($r < r_* \approx 8.792 \times 10^{-11} \text{m}$), the negative contribution of $\ln r$ makes the quantum gravitational potential repulsive, preventing matter from collapsing into singularities; at the black hole gravitational field and large galactic distances, the positive contribution of $\ln r$ provides additional gravity, offering a possibility to maintain the high-speed revolution of stars and the flattening of rotation curves without dark matter. This mechanism requires no mathematical renormalization or new unobservable entities (such as extra-dimensional strings, dark matter particles, etc.), and can be realized only by the own dynamics of ordinary matter: 1) desingularization; 2) flattening of rotation curves. To explain this mechanism, we provide a possible physical framework: relying on a physically supported carrier—quantum vortices, and a mathematical structure born in string theory to describe non-local entanglement—AdS/CFT correspondence, placing black hole physics and galactic dynamics under the same mechanism, and providing a new scheme for solving cross-scale gravitational problems that is only based on observable general relativity and quantum mechanics but does not rely on current traditional physical models (such as unobservable extra-dimensional strings, dark matter particles, etc.).

2. Universal Logarithmic Asymptotics of Dark-Matter Halo Models

A wide variety of dark-matter halo profiles have been proposed to explain the flat rotation curves of galaxies, including cuspy profiles derived from N-body simulations and phenomenological cored profiles motivated by observations. Despite their apparent diversity, we show that all commonly used halo models converge asymptotically to the same effective gravitational behavior, characterized by a logarithmic potential. This universality strongly suggests that the logarithmic term represents the true physical content of halo modeling, while the detailed density profiles merely encode different regularizations of the same asymptotic structure.

2.1. General Condition for Flat Rotation Curves

For a test particle on a circular orbit, the centripetal acceleration satisfies $\frac{v^2(r)}{r} = g(r) = \frac{GM(r)}{r^2}$, where $M(r) = 4\pi \int_0^r \rho(r') r'^2 dr'$. A flat rotation curve: $v(r) \rightarrow v_0 = \text{const}$, implies $g(r) \sim \frac{v_0^2}{r}$, $M(r) \sim \frac{v_0^2}{G} r$. Differentiating, $\rho(r) = \frac{1}{4\pi r^2} \frac{dM}{dr} \sim \frac{1}{r^2}$. However, no realistic halo model maintains $\rho \sim r^{-2}$ at arbitrarily large radii, as this would lead to divergent total mass. Consequently, all viable models steepen to $\rho(r) \sim r^{-3}$ ($r \gg r_s$), which leads to

$$M(r) \sim \ln r, \quad g(r) \sim \frac{\ln r}{r^2} \quad (1)$$

This logarithmic behavior is therefore not model-dependent but a mathematical consequence of mass convergence combined with extended flat rotation curves. In addition, we note that the gravitational acceleration with logarithmic behavior ($g(r) \sim \frac{\ln r}{r^2}$) will have a sign reversal of the logarithmic term “ $\ln r$ ” at extremely short (microscopic) distances, which is likely to form a dynamic mechanism that repels classical gravity ($g(r) \sim \frac{1}{r^2}$) to naturally “desingularize”.

2.2. Cuspy Halo Models: NFW and Einasto [1–3]

2.2.1. NFW Profile

The NFW profile: $\rho(r) = \frac{\rho_s}{(r/r_s)(1+r/r_s)^2}$, satisfies $\rho(r) \sim r^{-3}$ ($r \gg r_s$).

Integrating:

$$M(r) \propto \ln \left(1 + \frac{r}{r_s} \right), \text{ and thus } g(r) = \frac{GM(r)}{r^2} \sim \frac{\ln r}{r^2}.$$

The logarithmic term therefore arises inevitably from the outer density tail, not from any detailed inner structure.

2.2.2. Einasto Profile

The Einasto profile: $\ln \rho(r) = \ln \rho_0 - \left(\frac{r}{r_0} \right)^\alpha$, $\alpha \ll 1$, admits the expansion:

$$\left(\frac{r}{r_0} \right)^\alpha \simeq 1 + \alpha \ln \frac{r}{r_0} + \mathcal{O}(\alpha^2).$$

Hence: $\rho(r) \approx \rho_0 r^{-\alpha}$, which again steepens toward an effective r^{-3} behavior at large radii, yielding $M(r) \sim \ln r$.

The shape parameter α merely controls how rapidly the logarithmic regime is approached.

2.3. Cored Halo Models: Burkert and Pseudo-Isothermal

Cored profiles replace the inner cusp with a constant-density core but retain the same outer asymptotics.

For example, the Burkert profile: $\rho(r) = \frac{\rho_0}{(1+r/r_0)(1+(r/r_0)^2)}$, satisfies $\rho(r) \sim r^{-3}$ ($r \gg r_0$), leading again to

$$M(r) \sim \ln r, \quad g(r) \sim \frac{\ln r}{r^2}.$$

Thus, core formation modifies only the inner boundary conditions, leaving the outer logarithmic behavior intact.

2.4. Self-Interacting and Wave Dark Matter [4,5]

Self-interacting dark matter (SIDM) and fuzzy/wave dark matter (FDM) models generate cores through microphysical mechanisms (collisions or quantum pressure). Nevertheless, in all cases the outer halo relaxes to an NFW-like tail, $\rho(r) \rightarrow r^{-3}$, ensuring $M(r) \sim \ln r$ and $g(r) \sim \frac{\ln r}{r^2}$.

Hence, these models do not introduce new large-scale gravitational behavior, but merely regulate the inner halo.

2.5. Universality of the Logarithmic Potential

Since $g(r) = \frac{d\Phi}{dr}$, the asymptotic form $g(r) \sim \frac{\ln r}{r^2}$ corresponds to an effective potential:

$$\Phi_{halo}(r) \sim -\frac{\ln r + 1}{r} \quad (2)$$

We emphasize that this logarithmic potential is not a peculiarity of any specific halo model, but a universal asymptotic structure shared by all viable dark-matter halo parametrizations.

3. Logarithmic Quantum Gravity for Physical Singularity Resolution

Important note: This study adopts a “bottom-up” effective theory approach, aiming to identify the minimal gravitational modification that is both testable and capable of unifying phenomena from black hole to galactic scales. Its starting point is the induction of a logarithmic asymptotic structure common to successful dark matter halo models, rather than the deduction from a fundamental action. Consequently, the metric constructed herein, along with its microscopic interpretations (such as the nested AdS/CFT duality), should be viewed as an effective framework designed to yield testable predictions and to motivate future theoretical development. All core predictions of this theory (e.g., black hole shadows, high-speed stars) derive directly from this logarithmically corrected potential. Its validity is ultimately judged by its consistency with observations across scales, not by the starting point of its mathematical derivation.

3.1. Core Physical Assumptions

This section aims to provide a possible microscopic physical picture for the logarithmic correction derived jointly from the dark matter halo density ($\rho(r) \sim r^{-3}$ summarized in Section 2) and the curvature divergence at the GR singularity ($R_{trt}^r \propto r^{-3}$; see detailed discussions in Sections 3.2 and 3.3), rather than a rigorous quantum field theory derivation. Adopting a bottom-up effective theory approach, inspired by quantum tornado experiments [6], we propose quantum vortices as topological carriers of nonlocal entanglement and qualitatively describe the ultraviolet regularization mechanism with the aid of nested AdS/CFT correspondence ($AdS_2/CFT_1 \subseteq AdS_3/CFT_2 \subseteq AdS_4/CFT_3$). The main purpose of this picture is to demonstrate how logarithmic correction can naturally achieve both physical singularity resolution and macroscopic additional gravity, rather than serving as a necessary foundation for the theory.

1. **Quantum vortex topological structure:** It is described as the statistical average (geometric average) micro-topological carrier of fermion fields, boson fields and gauge fields. Through the WKB approximation, its possible operator form is given by an effective composite operator (characterized by the amplitude and phase of its expectation value on the strong-coupling/CFT boundary):

$$\mathcal{O}_{vortex} = \langle \bar{\psi}\psi\phi\mathcal{A}_{\mu\nu}\mathcal{A}^{\mu\nu} \rangle^{1/2} e^{iC\theta(x,y)} \quad (3)$$

2. **Quantum vortex field (expressed by path integral):**

$$\begin{aligned} \phi_{vortex}(x,y) &= \mathcal{O}_{vortex} \int d^4 y \sqrt{-g(y)} K(x,y) \\ &= \langle \bar{\psi}\psi\phi\mathcal{A}_{\mu\nu}\mathcal{A}^{\mu\nu} \rangle^{1/2} e^{iC\theta(x,y)} \int d^4 y \sqrt{-g(y)} \frac{e^{iC\theta(x,y)}}{|x-y|^{2\Delta}} \quad (4) \end{aligned}$$

- $\bar{\psi}\psi$: Fermion field, with dimension $[\bar{\psi}\psi] = L^{-3}$
- ϕ : Boson field, with dimension $[\phi] = L^{-1}$
- $\mathcal{A}_{\mu\nu} \equiv (B_{\mu\nu}, W_{\mu\nu}^a, G_{\mu\nu}^a)$: Unified field strength tensor, which can be regarded as coupled by electromagnetic, strong, and weak forces (excluding classical gravity) through non-local entanglement, $[A_{\mu\nu}] = [A^{\mu\nu}] = L^{-2}$, so $[\phi_{vortex}] = L^{-4}$, which is the Lagrangian density.
- $g(y)$: metric determinant;
- $e^{iC\theta(x,y)}$: vortex phase, which provides oscillations under non-local entanglement to prevent ultraviolet divergence;

- C : Central charge (topological charge number);
- $\theta(x, y) \sim \arctan\left(\frac{y_2 - x_2}{y_1 - x_1}\right)$: Topological phase;
- $K(x, y) = \frac{e^{iC\theta(x,y)}}{|x-y|^{2\Delta}}$: non-local kernel function (Green's function with vortex phase), with $\Delta \approx 2$ (the coexisting dimension of quantum vortices in four-dimensional spacetime). The non-locality of the statistically averaged vortex phase $e^{iC\theta(x,y)}$ provides a potential mechanism for the path integral $\int d^4y \sqrt{-g(y)} \frac{e^{iC\theta(x,y)}}{|x-y|^4}$ to avoid ultraviolet divergence (mathematically, oscillatory integrals can act as a regularizer in certain cases, similar to the Riemann-Lebesgue lemma, but a rigorous mathematical proof is complex and only a heuristic application for constructing the physical picture is presented here).
- The vortex winding number (quantized winding) is obtained from the central charge C and topological phase $\theta(x, y)$: $W = \oint_C \nabla \theta \cdot dl$, and the conformal dimension relationship under AdS/CFT correspondence ($\Delta \sim \frac{W^2}{4\pi^2 C}$) can directly calculate this winding number W .

It should be noted that quantum vortices (fields) under statistical average do not violate the "Pauli exclusion principle". Firstly, the vortex phase $e^{iC\theta(x,y)}$ in the operator indicates statistical average under non-local entanglement; secondly, the apparent structure of this micro-topology may be mainly located near black holes with huge spacetime curvature (extending the "quantum tornado" experiment to a more macroscopic cosmic scale). The Heisenberg uncertainty principle: $\Delta p \approx \hbar/\Delta x \rightarrow \infty$ (because $\Delta x \rightarrow 0$ under huge spacetime curvature), thus the "Pauli exclusion principle" is weakened by the huge spacetime curvature.

The construction of the aforementioned quantum vortices (fields) does not follow the standard quantum field theory treatment, and its mathematical rigor requires further refinement in subsequent work. However, recent experiments such as quantum tornadoes (which exhibit vortex lattice structures in superfluid helium) provide certain support for this physical picture (our construction is precisely inspired by such experiments).

The above expressions for quantum vortices and their scalar fields are heuristic constructions, used to illustrate how nonlocal phase oscillations avoid ultraviolet divergence and generate an effective r^{-3} source term. Strict quantum field theory treatment is left for future work.

3. **Nested AdS/CFT duality:** A hierarchical structure $AdS_2/CFT_1 \subseteq AdS_3/CFT_2 \subseteq AdS_4/CFT_3$ [7,8] is adopted to correlate the quantum spacetime inside the black hole with the external classical spacetime through the conformal boundary, realizing the quantitative description of nonlocal entanglement. Thus, there exists a physical mechanism for the path integral $\int d^4y \sqrt{-g(y)} \frac{e^{iC\theta(x,y)}}{|x-y|^4}$ to avoid ultraviolet divergence (combining the nonlocality of the phase $e^{iC\theta(x,y)}$). Using the nested structure $AdS_4/CFT_3 \supseteq AdS_3/CFT_2 \supseteq AdS_2/CFT_1$, the AdS_4 bulk spacetime is dual to the CFT_2 boundary: $\int d^4y \sqrt{-g(y)} \frac{e^{iC\theta(x,y)}}{|x-y|^4} \Leftrightarrow \int d^2y \sqrt{-g(y)} \frac{e^{iC\theta(x,y)}}{|x-y|^3}$. It can be seen that the term $|x-y|^{-3}$ appears in the integral ($\int d^2y$), which is similar to the asymptotic density behavior of dark matter halos ($\rho(r) \sim r^{-3}$). Combined with the fact that the logarithmic asymptote of the potential ($\Phi_{halo}(r) \sim -\frac{\ln r + 1}{r}$) is obtained after integrating the density asymptote (also $\int d^2y$), their "essences" may be similar: the universal logarithmic asymptote of dark matter halos in the AdS_4 bulk spacetime may be the logarithmic behavior after being dual to the CFT_2 boundary, and the sign reversal of the logarithmic term " $\ln r$ " at the microscopic scale can repel the classical gravitational potential ($-GM/r$) to avoid collapse to a "singularity". In other words, through this "holographic mapping", the ultraviolet divergence problem in the four-dimensional bulk spacetime is transformed into an integral on a two-dimensional boundary, and the output of this integral (after appropriate mapping) exactly gives the r^{-3} source term required in the three-dimensional physical space to generate the logarithmic potential. Thus, the divergence is "avoided".

The nested AdS/CFT correspondence is mainly used here to intuitively understand how the integral form $\int |x - y|^{-4}$ in the four-dimensional bulk spacetime is mapped to the boundary integral $\sim \int |x - y|^{-3}$ through dimensional reduction, and mathematically corresponds to the $\rho(r) \sim r^{-3}$ asymptote of dark matter halos.

3.2. Construction of Key Formulas

3.2.1. Modified Poisson Equation

Based on the quantum vortex as the carrier of the microscopic topological structure, we regard its statistical average field (quantum vortex field) ϕ_{vortex} as a dynamic subsystem satisfying the effective field theory under the high-energy background inside the black hole. Considering the nonlocal entanglement characteristics and scale relativity of this system, its dynamics can be described by a modified d'Alembert operator under the CFT boundary approximation: $\square\phi_{vortex} \approx (k\hbar/c^2)\partial_t^2\phi_{vortex} - \nabla^2\phi_{vortex} = 0$, where k is a dimensionless factor characterizing the strength of nonlocal entanglement. Further analysis shows that in the critical region near the boundary, the time evolution derivative term of the field $\partial_t^2\phi_{vortex}$ may have self-similarity due to non-local entanglement ($\partial_t^2\phi_{vortex} \approx a^2(\partial_t\phi_{vortex})^2 \sim a^2(M/t)^2$, similar to the statistical average logic of "Reynolds stress" in turbulence).

Since the additional corrected gravitational potential ($\Phi_{halo}(r) \sim -\frac{\ln r+1}{r}$) is incompatible with the traditional Newtonian gravitational potential ($-\frac{GM}{r}$), it is necessary to add additional gravity to the original Newtonian gravitational potential ($\Phi(r) = -\frac{GM}{r} + \Phi_{halo}(r)$), which will also change the Schwarzschild metric: $ds^2 = -\left(1 - \frac{2GM}{c^2r} + term1_{halo}(r)\right)c^2dt^2 + \left(1 + \frac{2GM}{c^2r} + term2_{halo}(r)\right)dr^2 + r^2d\Omega^2$ (the original Schwarzschild metric $B(r) = 1/A(r)$; Taylor expansion of $1/A(r) : \left(1 - \frac{2GM}{c^2r}\right)^{-1} = 1 + \frac{2GM}{c^2r} + \left(\frac{2GM}{c^2r}\right)^2 + \dots \sim 1 + \frac{2GM}{c^2r} + term2_{halo}(r)$). Among them, the time component of the modified metric: $g_{tt} = -\left(1 - \frac{2GM}{c^2r} + term1_{halo}(r)\right)c^2$. Since $g_{tt} > 0$ inside the black hole, the spacetime is spacelike: time is radialized like space, so the time evolution of the field can be re-calibrated as spatial behavior.

Therefore, after coordinate re-calibration, the quadratic term contribution ($\propto \frac{M^2}{t^2}$) caused by the self-similarity of the quantum vortex field (statistically averaged field ϕ_{vortex}) inside the black hole is approximately equivalent to an additional gravitational source term inversely proportional to the cube of the distance ($\propto \frac{M^2}{r^3}$). This substitution naturally connects the universal asymptotic density structure $\rho(r) \sim r^{-3}$ obtained for dark matter halos at large radii, because in the quadratic integral operation of solving the Poisson equation $\nabla^2\Phi = 4\pi G\rho$, only this source term ($\propto \frac{M^2}{r^3}$) can uniquely generate a logarithmic potential to reverse the potential direction and prevent collapse; other source terms such as $\propto \frac{M^2}{r^2}$ and $\propto \frac{M^2}{r^4}$ cannot prevent collapse and desingularize. Moreover, we also find that $\rho(r) \sim r^{-3}$ is surprisingly consistent with the boundary behavior $R_{trt}^r \propto r^{-3}$ of the Riemann tensor component R_{trt}^r , indicating that the divergent behavior of classical general relativity itself near the singularity ($R_{trt}^r \propto r^{-3}$) may have inherently contained a source term ($\rho(r) \sim r^{-3}$) that prevents curvature divergence. It also means that spacetime will spontaneously form a response to the divergent behavior near the singularity ($R_{trt}^r \propto r^{-3}$), and this response can produce a finite observable result (such as a black hole shadow) in a non-perturbative manner. In the following, we will initially verify the feasibility of this "desingularization" scheme by "a priori predicting" the

angular diameter of black hole shadows (consistent with EHT observations) using the logarithmically modified Schwarzschild metric.

Introducing this equivalent source term into the density ρ in the classical Poisson equation ($\nabla^2\Phi = 4\pi G\rho$) gives the CFT boundary modified Poisson equation:

$$\nabla^2\Phi = 4\pi G \left(M\delta^3(r) + \frac{kG_h M^2}{4\pi G r^3} \right) \quad (5)$$

where $M\delta^3(r)$ is the classical gravitational point mass source term ($\delta^3(r)$ is the three-dimensional Dirac delta function), and $\frac{kG_h M^2}{4\pi G r^3}$ is the additional gravitational correction source term (i.e., the source term of the universal asymptotic structure $\Phi_{halo}(r) \sim -\frac{\ln r + 1}{r}$ derived from dark matter halo models), which we call “quantum gravity”. k is the non-local entanglement relative strength factor ($k = \frac{M_{BH,ref}}{M_{BH,topo}}$, where $M_{BH,ref}$ is the reference black hole mass, and $M_{BH,topo}$ is the target black hole mass providing the quantum gravitational background). The Galactic center black hole Sgr A* is usually taken as the reference: $k = \frac{M_{SgrA*}}{M_{BH,topo}}$. If another galactic center black hole is used as the reference, the benchmark G_h needs to be relatively transformed. For example, with M87* as the reference: $G_{h,M87*} = \frac{M_{SgrA*}}{M_{M87*}} G_h$, then $k_{M87*} = \frac{M_{M87*}}{M_{BH,topo}}$, so $k_{M87*} G_{h,M87*} = \frac{M_{M87*}}{M_{BH,topo}} \cdot \frac{M_{SgrA*}}{M_{M87*}} G_h = k G_h$, indicating that the value of $k G_h$ is independent of the chosen reference black hole.

G_h is the quantum gravitational constant ($G_h = \hbar c^2 G^3 / 8 \approx 3.5224 \times 10^{-49} \text{kg}^{-2} \text{m}^3 \text{s}^{-2}$), and its unconventional dimensionality, we believe, can be explained by dimensional compactification. Since the framework includes nested AdS/CFT correspondence ($AdS_4/CFT_3 \supseteq AdS_3/CFT_2 \supseteq AdS_2/CFT_1$), in this picture, the effective Planck constant derived from the microscopic quantum vortex structure and ultimately dual to the CFT_1 boundary undergoes a change in dimensionality from $\text{kg} \cdot \text{m}^2 \text{s}^{-1}$ to $\text{kg} \cdot \text{m}^{-8} \text{s}^6$ due to the compactification (resulting from dimensional reduction) of coupled spacetime dimensions (including the fluctuation dimensions and phase dimensions of the gauge group). This dimensional transformation is incorporated into the definition of G_h , resulting in its final dimensionality of $\text{kg}^{-2} \text{m}^3 \text{s}^{-2}$. Experimental evidence supporting this hypothesis is that when quantum vortices in superfluid helium are confined to nanoscale spaces (simulating dimensional compactification), their vortex phase oscillation energy $E \propto \hbar_{eff} \omega$ satisfies $\hbar_{eff} \propto d^{-8}$ (where d is the confinement scale), which is consistent with the dimension m^{-8} [9]. Of course, a rigorous mathematical proof remains complex, and the current work is primarily heuristic in nature from a physical perspective.

This work adopts a bottom-up effective theory approach, naturally deriving logarithmic correction from the inherent $R_{trt}^r \propto r^{-3}$ curvature divergence in GR and the summarized universal asymptotic behavior of dark matter halos $\rho(r) \sim r^{-3}$. The microscopic explanation only serves as qualitative support and does not affect the consistency between major predictions and observations.

We acknowledge that the constructions of quantum vortices, nested AdS/CFT correspondence, and modified Poisson equations are more to provide a proposed physical picture for the discovered universal asymptotic behavior of dark matter halos that maximally adheres to Occam’s Razor principle (entities should not be multiplied beyond necessity). This explanatory framework is still in its early stages, and rigorous field theory derivation will be the core of future work. However, at the current stage, it does not affect its powerful cross-scale empirical predictive ability and unity. In the history of physics, there are many precedents for constructing effective theories based on profound empirical laws (such as Kepler’s laws for Newtonian mechanics, black-body radiation for quantum theory, etc.).

3.2.2. Modified Gravitational Potential with Logarithmic Term

Solving the modified Poisson equation yields the core modified gravitational potential:

$$\Phi(r) = -\frac{GM}{r} - \frac{kG_h M^2 (\ln r + 1)}{r} \quad (6)$$

This equation consists of two terms:

- **Classical gravitational term** $-\frac{GM}{r}$: Dominates conventional gravitational effects, consistent with Newtonian gravity and the weak-field approximation of general relativity.
- **Quantum gravitational logarithmic term** $-\frac{kG_h M^2 (\ln r + 1)}{r}$ (consistent with $\Phi_{halo}(r) \sim -\frac{\ln r + 1}{r}$): Serves as the core cross-scale correction term. Its effect depends on the magnitude of the distance r —exhibiting repulsive behavior at short distances (black hole “singularity” scale) and gravitational enhancement at long distances (galaxy scale). Essentially, it is likely a macroscopic manifestation of nonlocal entanglement of quantum vortices under the hierarchical nested structure ($AdS_2/CFT_1 \subseteq AdS_3/CFT_2 \subseteq AdS_4/CFT_3$).
- Furthermore, in the International System of Units (SI) adopted in this paper, the distance variable r is in meters (m). The argument of a logarithmic function must be a dimensionless quantity, so the expression $\ln r$ should be understood as $\ln(r/r_0)$, where we take the normalized length $r_0 = 1$ m. Since r_0 is a fixed constant, any constant term involving $\ln r_0$ can be absorbed into other parameters of the theory (such as k and G_h) or eliminated by redefining the zero point of the potential energy. Therefore, in all practical calculations (black hole shadow, hypervelocity stars, galaxy rotation curves), directly substituting numerical values in “meters” and taking the natural logarithm $\ln r$ is completely equivalent to strictly using $\ln(r/r_0)$, without changing any observable quantities. This convention applies throughout the paper and will not be repeated later.
- If the quantum gravitational effect under non-local entanglement is not considered ($k = 0$, i.e., ignoring the black hole: $M_{BH,ref} = 0$), the gravitational potential automatically degenerates into the classical gravitational potential: $\Phi(r) = -\frac{GM}{r}$, and the framework also naturally degrades to the classical gravitational framework.

3.3. Cross-Scale Physical Nature of the Logarithmic Term

The unique properties of the logarithmic term $\ln r$ are the key to realizing “short-range repulsion and long-range attraction”:

- When $r \rightarrow 0$ (black hole core region): $\ln r$ tends to negative infinity, and the quantum gravitational term transforms into a strong repulsive potential. When $r < r_* = e^{-1 - \frac{G}{kG_h M}} = e^{-1 - \frac{G}{G_h M S_{grA^*}}} \approx 8.792 \times 10^{-11}$ m, $\Phi(r) > 0$ in the total potential, dynamically preventing matter from collapsing into a singularity.
- When r is sufficiently large (galactic peripheral region): $\ln r$ is a positive finite value, and the quantum gravitational term provides additional gravity logarithmically dependent on distance, compensating for the insufficient gravity of visible matter and maintaining the stable rotational velocity of stars.

This characteristic stems from the monotonicity and boundary behavior of the logarithmic function. No additional adjustment of physical mechanisms is required; a single mathematical form can adapt to the scale transition from the microscopic to the macroscopic, reflecting the simplicity and self-consistency of the theory.

4. Black Hole Scale Application: Physical Avoidance of Singularity, Shadow Prediction and High-Speed Stars

4.1. Physical Avoidance of Singularity

In the core region of a black hole, the quantum repulsive potential dominated by the logarithmic term plays a central role:

- **Suppression of curvature divergence:** The repulsive potential prevents matter from reaching $r = 0$ and avoids the divergence of spacetime curvature, thus realizing the dynamical avoidance of singularities without the need for renormalization.
- **A potential mechanism for resolving the information paradox:** The repulsive potential excites virtual particles from vacuum fluctuations into real particles, which tunnel out of the black hole horizon through the nested AdS/CFT correspondence ($AdS_2/CFT_1 \subseteq AdS_3/CFT_2 \subseteq AdS_4/CFT_3$). These real particles carry information away from the black hole, and the black hole loses mass synchronously. This mechanism is conducive to making black hole physics satisfy the unitarity of quantum mechanics, namely the principle of information conservation.

4.2. Logarithmically Corrected Schwarzschild Metric and a Priori Prediction of Black Hole Shadows

Based on the modified gravitational potential, the quantum-corrected Schwarzschild metric is derived (substituting $\Phi(r)$ into the relationship between the metric and gravitational potential under the weak-field approximation of general relativity):

$$ds^2 = -A(r)c^2 dt^2 + B(r)dr^2 + r^2 d\Omega^2$$

$$A(r) \approx 1 + \frac{2\Phi(r)}{c^2} = 1 - \frac{2GM}{c^2 r} - \frac{2kG_h M^2 (\ln r + 1)}{c^2 r} \quad (7)$$

where: $term1_{halo}(r) = -\frac{2kG_h M^2 (\ln r + 1)}{c^2 r}$

$$B(r) \approx 1 - \frac{2\Phi(r)}{c^2} = 1 + \frac{2GM}{c^2 r} + \frac{2kG_h M^2 (\ln r + 1)}{c^2 r} \quad (8)$$

where: $term2_{halo}(r) = \frac{2kG_h M^2 (\ln r + 1)}{c^2 r}$

Analysis of Metric Singularity Resolution

For motion in the equatorial plane with $\theta = \pi/2$ and $\dot{\theta} = 0$, the Lagrangian is given by: $L = -Ac^2\dot{t}^2 + B\dot{r}^2 + r^2\dot{\phi}^2$. From Killing symmetry, the conserved energy and angular momentum are obtained as: $E = A(r)c^2\dot{t}$ and $L = r^2\dot{\phi}$ (with respect to the affine parameter λ). The general "first integral" radial equation is derived (where $\sigma = 1$ for timelike geodesics (proper time τ); $\sigma = 0$ for null geodesics (affine parameter λ)):

$$\dot{r}^2 = \frac{E^2 - A(r) \left(\frac{c^2 L^2}{r^2} + \sigma c^4 \right)}{ABc^2}$$

along with: $\dot{t} = \frac{E}{Ac^2}$ and $\dot{\phi} = \frac{L}{r^2}$. We define the effective potential as: $V_\sigma(r) = A(r) \left(\sigma c^4 + \frac{c^2 L^2}{r^2} \right)$.

Analysis of timelike geodesics ($\sigma = 1$): the effective potential $V_{timelike}(r) = A(r) \left(c^4 + \frac{c^2 L^2}{r^2} \right)$ tends to $+\infty$ as $r \rightarrow 0$ (since $A(r) \approx 1 - \frac{2GM}{c^2 r} - \frac{2kG_h M^2 (\ln r + 1)}{c^2 r} \rightarrow +\infty$). For any finite energy E and angular momentum L , the radial equation

$$\dot{r}^2 = \left(\frac{dr}{d\tau} \right)^2 = \frac{E^2 - A(r) \left(\frac{c^2 L^2}{r^2} + c^4 \right)}{ABc^2} = \frac{E^2 - V_{timelike}(r)}{ABc^2}$$

must satisfy $E^2 = V_{timelike}(r_{min})$ at some $r_{min} > 0$ —a bounce occurs, meaning the particle cannot physically reach $r = 0$. The analysis for null geodesics ($\sigma = 0$) is essentially the same (the

effective potential $V_{null}(r) = A(r) \left(\frac{c^2 L^2}{r^2} \right) \rightarrow +\infty$ as $r \rightarrow 0$, so light cannot physically reach $r = 0$ either.

This metric does not require fitting of black hole spin and inclination; the shadow angular diameter can be predicted solely by the black hole mass M and distance D .

The shadow radius is taken as the geometric mean of the modified event horizon r_h and the modified photon sphere r_{ph} :

$$r_{sh} \approx \sqrt{r_h r_{ph}} \quad (9)$$

and the angular diameter: $\theta_{sh} = 2r_{sh}/D$.

For equatorial null geodesics, let the impact parameter $b \equiv \frac{Lc}{E}$. The closest distance r_0 satisfies $b^2 = \frac{r_0^2}{A(r_0)}$, leading to the logarithmic correction (Gravitational Lensing Deflection Angle):

$$\hat{\alpha}(b) = 2 \int_{r_0}^{\infty} \frac{dr}{r} \sqrt{\frac{B(r)}{A(r)} \left[\left(\frac{r}{r_0} \right)^2 \frac{A(r_0)}{A(r)} - 1 \right]}^{-1/2} - \pi \quad (10)$$

Analysis shows that in the strong-field regime, the deflection angle diverges as the closest distance approaches the photon sphere $r_0 \rightarrow r_{ph}$. The additional logarithmic correction to $A(r)$ causes two key effects: the photon sphere radius shifts outward compared to the standard Schwarzschild metric; the divergence point appears earlier, trapping light rays sooner. Thus, any light ray attempting to graze the event horizon will undergo severe deflection and will not actually contribute to the “sharply imaged” light path—multiple diffracted orbits cannot form a stable image. Therefore, the size of the black hole shadow and bright ring is mainly determined by the geometry of the Schwarzschild metric with logarithmic correction, rather than the superposition of numerous deflected light rays. In other words, the truly imaging light paths near the black hole originate from the stable luminous ring at the edge of the shadow (accretion disk or plasma emission), not from complex multiple diffractions. Specifically, the observed annular emission of black holes is almost the real emission distribution from the inner edge of the nearby accretion disk (the region where particles escape the black hole through tunneling via nested AdS/CFT correspondence under nonlocal entanglement due to the repulsive potential $\Phi(r) > 0$ from physical singularity resolution), rather than an illusion formed by “bent and diffracted light”. This also means that the critical impact parameter ($b_c = \frac{r_{ph}}{\sqrt{A(r_{ph})}}$) under the Schwarzschild metric with logarithmic correction no longer

characterizes the black hole shadow radius.

According to the hierarchical correspondence mechanism we adopted ($AdS_2/CFT_1 \subseteq AdS_3/CFT_2 \subseteq AdS_4/CFT_3$): particles (including but not limited to photons) inside the black hole escape the black hole through quantum tunneling due to the reversal of the total potential direction ($\Phi(r) > 0$), and the imaging interval is between the modified horizon and the modified photon sphere: (r_h, r_{ph}) . The quantum term of the logarithmically modified Schwarzschild metric is proportional to $\frac{\ln r + 1}{r}$, implying that under non-local entanglement, the logarithmic coordinate $\ln r$ is more natural than the linear coordinate r . Thus, we perform the variable substitution $x = \ln r$, and the tunneling interval (r_h, r_{ph}) is transformed into $(\ln r_h, \ln r_{ph})$ (i.e., (x_h, x_{ph})).

For photons tunneling from r_h to $r \in (r_h, r_{ph})$ for imaging, the tunneling probability density under the WKB approximation is: $P(x) \propto e^{-2S(x)}$ (action: $S(x) \sim \int_{x_h}^x \sqrt{2m(V(x) - E)} \frac{dr}{dx} dx$). From the total potential $\Phi(r)$, it is known that the potential barrier originates from the logarithmic term of the quantum gravitational potential: $V(x) \sim V_0 + a(\ln r + 1)/r = V_0 + a \frac{x+1}{e^x}$. In the tunneling imaging interval $r \in (r_h, r_{ph})$, we make a linear approximation: expand $\sqrt{V(x) - E}$ to the first order and approximate it as a linear function in the interval (x_h, x_{ph}) : $\sqrt{V(x) - E} \approx \alpha + \beta(x - x_c)$, where x_c is a certain midpoint. Thus, the action $S(x)$ becomes a quadratic function of x : $S(x) \approx S_0 + A(x - x_c) + B(x - x_c)^2$, and therefore:

$P(x) \propto e^{-2S(x)} \sim e^{-2B(x-x_c)^2} \cdot f(x)$ (where $f(x)$ is a slowly varying factor). That is to say, in the tunneling imaging interval (r_h, r_{ph}) , the tunneling probability follows a Gaussian distribution, so the imaging of photons in the interval (r_h, r_{ph}) becomes a Brownian random equilibrium in the logarithmic interval $(\ln r_h, \ln r_{ph})$ (i.e., (x_h, x_{ph})). Therefore, the tunneling steady state is naturally located at the arithmetic mean of the logarithmic interval $(\ln r_h, \ln r_{ph})$: $\ln r_{sh} \approx (\ln r_h + \ln r_{ph})/2$, and converting back from the logarithmic coordinate to the linear coordinate gives: $r_{sh} \approx \sqrt{r_h r_{ph}}$.

(We emphasize: this shadow radius is derived based on the Schwarzschild metric with logarithmic correction after physical singularity resolution. Although it differs from standard general relativity with singularities, it does not violate its fundamental logic (the theory originates from the spacetime self-response to the curvature divergence behavior near the singularity ($R_{trt}^r \propto r^{-3}$)), and as an effective theoretical framework, its ultimate evaluation criterion is observation.)

Similar to the modified gravitational potential, if the quantum gravitational effect under non-local entanglement is not considered ($k = 0$, i.e., ignoring the black hole: $M_{BH,ref} = 0$), the logarithmically corrected Schwarzschild metric strictly degenerates to the Schwarzschild metric, restoring standard general relativity (the Schwarzschild metric $B(r) = 1/A(r)$; Taylor expansion of $1/A(r)$ gives $(1 - \frac{2GM}{c^2 r})^{-1} = 1 + \frac{2GM}{c^2 r} + (\frac{2GM}{c^2 r})^2 + \dots$, and higher-order terms are omitted, leading to $(1 - \frac{2GM}{c^2 r})^{-1} \sim 1 + \frac{2GM}{c^2 r}$).

In addition, according to this metric: at "infinite distance" from the black hole: $ds^2 \approx -c^2 dt^2 + dr^2 + r^2 d\Omega^2$ (four-dimensional flat); near the black hole horizon ($g_{tt} \approx 0$): $ds^2 \approx 2dr^2 + r^2 d\Omega^2$ (three-dimensional flat, because after variable substitution: $dl^2 \approx 2(d(\sqrt{2}\rho))^2 + \rho^2 d\Omega^2$). According to conformal flatness, the inside and outside of the black hole form an AdS_4/CFT_3 correspondence, that is, the strong-field spacetime properties near the black hole "horizon" are similar to the weak-field spacetime properties at "infinite distance" from the black hole. Therefore, although the logarithmically modified Schwarzschild metric is obtained by substituting the modified gravitational potential into the weak field approximation of general relativity, it is applicable to the entire spacetime of strong and weak fields (the properties of this metric can be further extended to the nested correspondence inside the black hole ($AdS_2/CFT_1 \subseteq AdS_3/CFT_2$), which together with the AdS_4/CFT_3 correspondence near the horizon forms a hierarchical correspondence ($AdS_2/CFT_1 \subseteq AdS_3/CFT_2 \subseteq AdS_4/CFT_3$). Due to the complexity of the strict mathematical proof (needing to further analyze the metric such as $g_{tt} = 0$ and $g_{rr} = 0$), we verify it by a priori calculating (predicting) the size of the black hole shadow and comparing it with EHT observations.

The Schwarzschild radius remains unchanged: $r_s = 2GM/c^2$

$g_{tt} = 0 \rightarrow$ Horizon equation (where $M = M_{BH,topo}$):

$$c^2 r = 2GM + 2kG_h M^2 (\ln r + 1) \quad (11)$$

Solving this equation yields the modified event horizon r_h .

For photons, $ds^2 = 0$, and $\dot{r} = 0$ on circular orbits. Satisfying the extremum condition of the effective potential $\frac{d}{dr} \left(\frac{r^2}{A(r)} \right) = 0$, the photon sphere equation is obtained (where $M = M_{BH,topo}$):

$$c^2 r = 3GM + kG_h M^2 (3 \ln r + 2) \quad (12)$$

Solving this equation yields the modified photon sphere r_{ph} .

A Priori Prediction and Verification Results of Observed Black Hole Shadows [10,11]

Compared with the traditional Kerr black hole model [12], this theory performs a priori prediction calculations with no free parameters (only the target black hole mass $M_{BH,topo}$ is required; the k -factor $k = M_{SgrA^*}/M_{BH,topo}$ is already fixed, and theoretically, the shadow of a

black hole of any mass can be predicted). The comparison with the shadows of two black holes observed by EHT verifies the effectiveness of this logarithmically corrected Schwarzschild metric in strong fields and the rationality of the hierarchical duality inside and outside the black hole ($AdS_2/CFT_1 \subseteq AdS_3/CFT_2 \subseteq AdS_4/CFT_3$).

| Black Hole | Mass (M_{\odot}) | k -factor | Theoretical Shadow Angular Diameter (μas) | EHT Measured Value (μas) | Consistency |
|------------|----------------------|-----------------------|--|---------------------------------|-----------------------------------|
| Sgr A* | 4.3×10^6 | 1 | 53.3 | 51.8 ± 2.3 | Within observational range |
| M87* | 6.5×10^9 | 6.61×10^{-4} | 46.2 | 42 ± 3 | 1.4σ (reasonable error) |

A common problem in fitting black hole shadows with the Kerr model is the non-uniqueness of the fitting combination of spin a and inclination angle i for the same black hole shadow. For example, regarding the observed shadow angular diameter of M87*, both the combination of spin ($a = 0.99$) + inclination angle ($i \approx 17^\circ$) and spin ($a = 0.50$) + inclination angle ($i \approx 65^\circ$) can satisfy the shadow fitting. Similarly, Sgr A* faces the same issue. Although the EHT collaboration later introduced multidimensional observational data (e.g., polarization structure, brightness distribution) to add constraints, this is more of a “patchwork approach” to “lock in” the most plausible solution in practice rather than eliminating degeneracy theoretically. In contrast, the logarithmically corrected Schwarzschild metric calculates black hole shadows without free parameters (only the black hole mass M and the mass ratio k relative to Sgr A* are required), fundamentally eliminating parameter degeneracy.

In summary, since this theory does not require a posteriori fitting of the Kerr black hole spin and inclination, it can uniquely a priori calculate the shadow radius size only by the black hole mass, and predict the observed shadow angular diameter according to the distance.

Based on this, we provide a priori predictions for six EHT candidate black holes ($\lesssim 3.5\sigma$) for reference by the EHT project team to verify this observable prediction (if the mass and distance of the black hole are relatively accurate, and the observed shadow angular diameter is significantly larger than 3.5σ , the theory is falsified).

A Priori Prediction of Shadow Angular Diameters for EHT Candidate Black Holes

Due to the inherent “parameter degeneracy” of the Kerr model, we selected the spin range $a \in [0, 0.99]$ and inclination range $i \in [0^\circ, 90^\circ]$ for a full scan. Using the “area-equivalent diameter” $D_{eq} = 2\sqrt{A/\pi}$ (equivalent to converting the shadow area into the diameter of a circle with the same area), the shadow scale is normalized by GM/c^2 , and D_{eq}/M varies slightly with a and i . Referring to references [13–15], the upper and lower bounds can be determined: the circular shadow diameter of a Schwarzschild black hole ($a = 0$) is $D = 6\sqrt{3}M \approx 10.392M$; the diameter can be

reduced to approximately $9.6568M$ for extreme spin and axial viewing angles. Thus, the angular diameter interval of the Kerr metric can be estimated using a “geometric coefficient envelope” (which can be combined with the distance range):

$$\theta_{Kerr} \in \left[\frac{9.6568GM}{c^2 D}, \frac{10.392GM}{c^2 D} \right]$$

| Black Hole | Mass (M_{\odot}) | k-factor | Distance Range (Mpc) | Shadow Radius r_{sh} (m) of Logarithmically Modified Schwarzschild Metric | Shadow Angular Diameter Range θ_{sh} (μs) | Kerr Fitting Range ($a \in [0, 0.99]$, $i \in [0^\circ, 90^\circ]$) θ_{Kerr} (μs) |
|--------------|----------------------|-----------------------|----------------------|---|---|--|
| Centaurus A* | 5.5×10^7 | 7.82×10^{-2} | 3.4~4.2 | 4.47×10^{11} | 1.4~1.8 | 1.3~1.7 |
| NGC315 | 3.0×10^9 | 1.43×10^{-3} | 65~72 | 2.64×10^{13} | 4.9~5.4 | 3.9~4.8 |
| NGC4261 | 1.6×10^9 | 2.69×10^{-3} | 30~32 | 1.41×10^{13} | 5.9~6.3 | 4.6~6.1 |
| M84 | 1.5×10^9 | 2.87×10^{-3} | 16~17.5 | 1.30×10^{13} | 9.9~10.9 | 8.3~10.1 |
| NGC4594 | 1.0×10^9 | 4.3×10^{-3} | 9.0~10.0 | 8.61×10^{12} | 11.5~12.8 | 9.6~12.0 |
| IC1459 | 2.0×10^9 | 2.15×10^{-3} | 21~30 | 1.75×10^{13} | 7.8~11.1 | 6.4~9.8 |

Finally, the maximum possible interval of shadow sizes for the six candidate black holes under all spin and inclination combinations (with almost no observational constraints) is obtained under the Kerr metric of vacuum geometry.

Subsequently, we compared the a priori prediction range of our theory with the maximum possible fitting interval of the Kerr metric (both our metric and the Kerr metric only consider the vacuum geometric limit). It can be seen that:

- Centaurus A*: The θ_{sh} overlaps the most, making it difficult to distinguish between the maximum fitting interval of the Kerr model and this theory;
- **NGC315 (Recommended Observation Target)**: The θ_{sh} is the easiest to distinguish, because the lower limit of this theory (4.9 μs) is already higher than the maximum fitting upper limit of

the Kerr model (4.8 μas). As long as the EHT measures the diameter with a precision of $\sim 2.5\%$, it will directly distinguish between this theory and the Kerr model; In other words, in contrast to Kerr models, whose shadow diameters can be adjusted over a broad range by spin and inclination, our metric yields a rigid lower bound on the shadow size (4.9 μas) determined solely by the black hole (e.g., NGC315) mass and distance. If future observations cluster near this lower bound ($\geq 4.9\mu\text{as}$), the result would favor our geometry without invoking fine-tuned spin–inclination configurations (because when only considering the vacuum geometry of the Kerr metric, no matter how the spin and inclination are adjusted for NGC315, its fitting upper limit of 4.8 μas cannot reach near 4.9 μas). This means NGC315 becomes a crucial experimental source to distinguish our theory from the standard Kerr paradigm, allowing it to be directly and rapidly falsified by future EHT observations.

- NGC4261: The θ_{sh} overlaps more, making distinction relatively difficult;
- M84: If $\theta_{sh} > 10.1 \mu\text{as}$, it favors this theory;
- NGC4594: If $\theta_{sh} > 12.0 \mu\text{as}$, it favors this theory;
- IC1459: If $\theta_{sh} > 9.8 \mu\text{as}$, it favors this theory.

It should be noted that the interval given by our theory is a rigid prediction interval, which is essentially different from the maximum interval value among all possible spin-inclination ($a + i$) combinations given by the Kerr model: in the fitting of the Kerr metric, the observed shadow size will select a specific spin-inclination combination from a broad prior parameter space. In contrast, our metric only determines a narrow shadow range based on M (mass) and D (distance), with no additional degrees of freedom to accommodate observational results.

Width of Angular Diameter Interval

Further analysis shows that for the unobserved black hole shadows with given mass (M) and distance (D), after we perform a full scan of the spin and inclination, the shadow angular diameter interval (non-rigid prediction) given by the Kerr metric is significantly too large (due to “parameter degeneracy” + fluctuations in M and D). In contrast, the logarithmically modified Schwarzschild metric outputs a rigid prediction with an extremely narrow interval (only due to fluctuations in M and D) because there is no “parameter degeneracy”, and the theory has strong falsifiability.

| Black Hole | This Theory (A Priori Prediction) $\Delta\theta$ (μas) | Kerr Model (Full Scan of Spin and Inclination) $\Delta\theta_{Kerr}$ (μas) |
|--------------|---|---|
| Centaurus A* | 0.4 | 0.4 |
| NGC315 | 0.5 | 0.9 |
| NGC4261 | 0.4 | 1.5 |
| M84 | 1 | 1.8 |
| NGC4594 | 1.3 | 2.4 |
| IC1459 | 3.3 | 3.4 |

Of course, we acknowledge the achievements of the Kerr solution in black hole physics, as it is still very successful in explaining phenomena such as jet precession and gravitational waves. However, its inherent problems such as “singularities” and “parameter degeneracy” have long plagued the physics community. Therefore, while continuously patching up these inherent problems, we may also consider another possible direction: a theory that avoids “singularities” and eliminates “parameter degeneracy” through its own mechanism.

4.3. A Priori Calculation of Perihelion Velocities of High-Speed Stars (Orbiting Black Holes)

From the modified gravitational potential, the gravitational acceleration of the black hole gravitational field is derived as:

$$g(r) = \frac{d\Phi}{dr} = \frac{GM}{r^2} + \frac{kG_h M^2 \ln r}{r^2} \quad (13)$$

From the modified gravitational potential and the logarithmically corrected Schwarzschild metric, the circular orbital velocity of the black hole gravitational field (including but not limited to accretion disks) is obtained:

$$\begin{aligned} v_{obs}(r) &= \frac{dr}{dt} = \frac{dr}{d\tau} \cdot \frac{d\tau}{dt} = v_{proper} \sqrt{A(r)} = \sqrt{r \frac{d\Phi}{dr}} \cdot \sqrt{A(r)} \\ &= \sqrt{\frac{GM}{r} + \frac{kG_h M^2 \ln r}{r}} \cdot \sqrt{1 - \frac{2GM}{c^2 r} - \frac{2kG_h M^2 (\ln r + 1)}{c^2 r}} \quad (14) \end{aligned}$$

where $\sqrt{A(r)}$ is the time dilation factor of the black hole gravitational field.

Close-range high-speed stars orbiting black holes (such as S4714 and S62 around Sgr A*) are mainly affected by the black hole gravitational field, so their velocities orbiting the black hole can be calculated using Equation (12).

Comparison Between A Priori Calculated Theoretical Velocities of High-Speed Stars and Observations [16,17]

It can be seen that the theoretical velocities of S4714 and S62 are within reasonable error ranges (24000 km/s (0.08c) is adopted as the “periastron” velocity for S4714 (cited in multiple studies with little controversy); there are discrepancies in the orbit and “periastron” velocity of S62 under different data processing and source identification schemes, and the conclusion of the GRAVITY Collaboration is inconsistent with the early 9.9-year orbital solution. This paper adopts the commonly used 20000 km/s (0.067c) in the literature as an order-of-magnitude estimate).

| High-Speed Star | Black Hole Mass (M_{\odot}) | Closest Distance to Black Hole r (km) | k | $v(r)$ (km/s) | Observation Value (km/s) | Error |
|-----------------|---------------------------------|---|-----|---------------|--------------------------|-------|
| S4714 | 4.3×10^6 | 1.89×10^9 | 1 | 25943 | 24000 | 8.1% |
| S62 | 4.3×10^6 | 2.39×10^9 | 1 | 23159 | 20000 | 15.8% |

The a priori calculation of the perihelion velocities of high-speed stars uses the same theoretical framework as the a priori prediction of black hole shadows, requiring only the black hole mass M and the distance r between the star and the black hole. Compared with the traditional method, which still needs to adjust the orbital eccentricity e and semi-major axis a to fit the observed velocity after knowing the black hole mass and the distance between the star and the black hole (e.g.,

orbital velocity based on standard general relativity: $v(r) \approx \sqrt{\frac{GM}{r}(1+e)} \sqrt{1 - \frac{2GM}{c^2 r}}$, this method is more concise and is an a priori (predictive) calculation of stellar orbital velocities rather than posterior fitting.

On the other hand, the calculation results show that as the star moves farther from the black hole, the gravitational field it experiences approaches the galactic gravitational field, so the calculation error when only considering the black hole gravitational field increases accordingly. Therefore, stars orbiting black holes at relatively large distances should use the circular orbital velocity equation of the galactic scale.

4.4. Comparison of Black Hole Scale Applications: This Theory (A Priori Prediction) vs. Traditional Theories (Posterior Fitting)

It should also be noted that this theory does not overthrow general relativity; on the contrary, its modified gravitational potential and modified metric (logarithmically modified Schwarzschild metric) are both derived from general relativity corrections. That is, when the logarithmic correction gravity (quantum gravity) under non-local entanglement is not considered (setting $k = 0$, i.e., ignoring the black hole effect: $k = M_{BH,ref}/M_{BH,topo} = 0 \Rightarrow M_{BH,ref} = 0$), it will completely degenerate into general relativity. This means all observational results under standard general relativity are applicable to this theory.

| Comparison Item | This Theory (Logarithmically Modified Gravitational Potential Model) | Traditional Theories (Kerr Model + Standard General Relativity Dynamical Model) |
|----------------------|--|---|
| Core Parameters | Mass M , distance D or r | Mass M , distance D or r , spin α , inclination i , eccentricity e , etc. |
| Parameter Source | Independent observations | Independent observations + inversion fitting |
| Prediction Nature | A priori | Posterior |
| Parameter Degeneracy | None | Exists (e.g., spin α , inclination i) |
| Cross-Scale Unity | Unified (both black hole shadows and orbital velocities are a priori calculated based on the metric and orbital velocity formulas of the same logarithmically modified gravitational term) | Segmented (black hole shadows and orbital velocities are posteriorly fitted by the Kerr model and standard general relativity dynamics, respectively) |

4.5. Schwarzschild Metric with Logarithmic Correction and Field Equations

The reason why this theory can degenerate into standard general relativity is essentially that the “logarithmic correction” is not externally introduced, but an inherent product of the long-overlooked mathematical structure of general relativity itself. Specifically, the curvature divergence behavior near the singularity ($R_{trt}^r \propto r^{-3}$) uniquely generates the logarithmic correction “ $\ln r$ ” through the integral operation of solving the Poisson equation by the source term, thereby naturally avoiding the singularity through the sign reversal of the logarithmically modified gravitational potential (other divergence behaviors such as r^{-2} and r^{-4} as source terms cannot avoid the singularity after integral operations). Therefore, this theory is actually a self-correction of general relativity triggered by its own singular dynamic behavior at the “singularity”, and this response is manifested as the mathematical asymptotics of dark matter halos ($\rho(r) \sim r^{-3}$) in macroscopic galactic dynamics through nested correspondence ($AdS_2/CFT_1 \subseteq AdS_3/CFT_2 \subseteq AdS_4/CFT_3$). Ultimately, this theory serves as a correction of general relativity to itself under extreme (singularity) conditions, hence its ability to “degenerate” into standard general relativity.

Therefore, from this spontaneous response of spacetime near the “singularity” (where the total potential $\Phi(r) > 0 \Rightarrow r < e^{-1-\frac{G}{kG_h M}} = e^{-1-\frac{G}{G_h M S_{grA^*}}} \approx 8.792 \times 10^{-11} \text{m}$), we can reversely derive the modified Einstein field equation from the Schwarzschild metric with logarithmic correction (a known metric solution) by calculating the Einstein tensor $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R$ from the metric and comparing it with the energy-momentum tensor $T_{\mu\nu}$:

$$G_{\mu\nu} + H_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} \quad (15)$$

where: $H_{\mu\nu} = \Lambda(r)g_{\mu\nu}$, and

$$\Lambda(r) = \frac{kG_h M^2 (\ln r + 1)}{c^2 r} \quad (16)$$

- Foreground curvature: The Einstein tensor $G_{\mu\nu}$ characterizes classical gravity.
- Background curvature: $H_{\mu\nu} = \Lambda(r)g_{\mu\nu}$ (the logarithmic correction tensor) may characterize quantum gravity formed by the coupling of other fundamental forces (electromagnetism, strong nuclear force, weak nuclear force). According to the possible physical picture we constructed in Section 3.1: the statistically averaged quantum vortex and its scalar field:

$$\begin{aligned} \mathcal{O}_{vortex} &\sim \langle \bar{\psi}\psi\phi\mathcal{A}_{\mu\nu}\mathcal{A}^{\mu\nu} \rangle^{\frac{1}{2}} e^{iC\theta(x,y)} \\ \phi_{vortex}(x,y) &\sim \langle \bar{\psi}\psi\phi\mathcal{A}_{\mu\nu}\mathcal{A}^{\mu\nu} \rangle^{\frac{1}{2}} e^{iC\theta(x,y)} \int d^4 y \sqrt{-g(y)} \frac{e^{iC\theta(x,y)}}{|x-y|^4} \end{aligned}$$

where $A_{\mu\nu} \equiv (B_{\mu\nu}, W_{\mu\nu}^a, G_{\mu\nu}^a)$ is the unified field strength tensor.

Naturally, this is only a preliminary conception (given the current construction of the effective theory), and numerous mathematical refinements and experimental work remain for the future; it is currently only presented as a direction for exploration. On the other hand, our current construction of the “quantum vortex and its scalar field” is not entirely groundless. In recent years, laboratory simulations of environments “near black holes” that form vortex structures (quantum tornadoes) provide strong indirect evidence for our conception (in contrast to dark matter particles, which have not been detected for decades and lack laboratory simulations).

In the effective theory framework, the emergence of the correction term ($H_{\mu\nu} = \Lambda(r)g_{\mu\nu}$) in the field equation originates from the statistically averaged nonlocal effect of quantum vortices, leading to the covariant divergence of the matter energy-momentum tensor $T_{\mu\nu}$ no longer being strictly zero: $\nabla_\mu T_{\mu\nu} = \frac{c^4}{8\pi G} \nabla_\mu H_{\mu\nu}$. This corresponds to the spontaneous response of spacetime near the singularity (exciting virtual particles to real particles for tunneling escape when the total potential $\Phi(r) > 0$), thereby realizing effective energy-momentum exchange between geometry and matter. Such non-conservation is common in many modified gravity effective theories (such as nonlocal or dissipative corrections). Consistent with the logarithmically modified Schwarzschild metric and

gravitational potential: when the nonlocal effect of quantum vortices (black holes) is not considered (i.e., $k = 0$: $k = M_{BH,ref}/M_{BH,topo} = 0 \Rightarrow M_{BH,ref} = 0$), the modified field equation: $G_{\mu\nu} + 0 = G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$, automatically degenerates to the standard Einstein field equation and restores the standard conservation law.

From this modified field equation, we recognize that classical gravity cannot be quantized; instead, the other three fundamental forces are coupled nonlocally through hierarchical nesting ($AdS_2/CFT_1 \subseteq AdS_3/CFT_2 \subseteq AdS_4/CFT_3$) to form an additional gravitational force (quantum gravity) that coexists with classical gravity. We argue that this quantum gravity is the essential cause of the “dark matter gravity”, thus generating the logarithmic correction term (naturally explaining the universal logarithmic asymptotic behavior $\Phi_{halo}(r) \sim -\frac{\ln r + 1}{r}$ we derived from the analysis of all dark matter halo models).

Furthermore, the correction term $\Lambda(r)g_{\mu\nu}$ in our modified field equation $G_{\mu\nu} + \Lambda(r)g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$ can be compared with the early Einstein field equation $G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$, which assumed a constant cosmological constant Λ . However, our derivation shows that it is a slowly varying logarithmic function: $\Lambda(r) = \frac{kG_h M^2 (\ln r + 1)}{c^2 r}$, which may provide a new direction for understanding the dynamics of dark energy discovered in recent years (which the standard Λ CDM model cannot explain).

5. Galactic Scale Application: Explanation of Flat Rotation Curves

5.1. Galactic Scale Adaptation Corrections

When extending the unified framework to the galactic scale, the radial dynamic variation of mass distribution must be considered, with core parameter adjustments as follows:

- Dynamic mass distribution:

$$M(r) = M_{baryon,topo} (1 - e^{-r/r_0}) \quad (17)$$

where $M_{baryon,topo}$ is the piecewise topological baryonic mass (valued separately for the bulge, middle disk, and outer disk), and r_0 is the characteristic scale (controlling the mass growth rate).

- Dynamic entanglement factor:

$$k(r) = k_0 \left(\frac{r_{peak}}{r} \right)^\alpha \quad (18)$$

where k_0 is the benchmark entanglement strength (inferred from the velocity $v(r_{peak})$ at the velocity peak r_{peak} of the galactic rotation curve via $k_0 = \frac{v(r_{peak})^2 r_{peak} - GM(r_{peak})}{G_h M(r_{peak})^2 \ln r_{peak}}$), and α is the decay exponent, adapting to the outer disk decay characteristics of different galaxies (the power law originates from the scaling transformation of AdS/CFT, and the entanglement strength decay at the galactic scale naturally exhibits power-law behavior).

Circular orbital velocity in the galactic gravitational field:

$$v(r) = \sqrt{r \frac{d\Phi}{dr}} = \sqrt{\frac{GM(r)}{r} + \frac{k(r)G_h M(r)^2 \ln r}{r}} \quad (19)$$

The gravitational acceleration in the galactic gravitational field:

$$g(r) = \frac{d\Phi}{dr} = \frac{GM(r)}{r^2} + \frac{k(r)G_h M(r)^2 \ln r}{r^2} \quad (20)$$

5.2. Fitting Verification of Rotation Curves for Multiple Galaxies

Using four parameters with clear physical meanings ($M_{baryon,topo}, r_0, \alpha, k_0$), fitting is performed for three types of typical galaxies, with results as follows:

5.2.1. Milky Way (Spiral Galaxy)

Parameters:

- Bulge ($r \leq 4\text{kpc}$): $M_{baryon,bulge} = 4.5 \times 10^{10} M_{\odot} \approx 8.9505 \times 10^{40} \text{kg}$ ($r_0 = 3\text{kpc}$):

$$M(r) = 8.9505 \times 10^{40} (1 - e^{-r/3})$$
- Middle disk ($4 < r < 10\text{kpc}$): $M_{baryon,mid} = 9.0 \times 10^{10} M_{\odot} \approx 1.7901 \times 10^{41} \text{kg}$ ($r_0 = 6\text{kpc}$):

$$M(r) = 1.7901 \times 10^{41} (1 - e^{-r/6})$$
- Outer disk ($r \geq 10\text{kpc}$): $M_{baryon,MW} = 1.5 \times 10^{11} M_{\odot} \approx 2.9835 \times 10^{41} \text{kg}$ ($r_0 = 10\text{kpc}$):

$$M(r) = 2.9835 \times 10^{41} (1 - e^{-r/10})$$
- $k_0 = 1.143 \times 10^{-5}$ (inferred from $v(r_{peak}) = 250 \text{ km/s}$ at $r_{peak} = 10\text{kpc}$: $k_0 = \frac{v(r_{peak})^2 r_{peak} - GM(r_{peak})}{G_h M(r_{peak})^2 \ln r_{peak}}$, where $M(r_{peak})$ is located in the outer disk), $\alpha = 0.3$:

$$k(r) = 1.143 \times 10^{-5} \left(\frac{10}{r}\right)^{0.3}$$

Comparison between the Milky Way rotation curve and observations [18]

| r (kpc) | $v(r)$ (km/s) | Observed Value (km/s) | Region |
|-----------|---------------|-----------------------|-------------|
| 2 | 236.9 | 200–220 | Inner disk |
| 4 | 211.0 | 210–230 | Inner disk |
| 5 | 248.1 | 215–235 | Middle disk |
| 6 | 237.8 | 220–240 | Middle disk |
| 8 | 225.2 | 220 | Middle disk |
| 10 | 250.0 | 225–250 | Outer disk |
| 15 | 231.5 | 210–230 | Outer disk |
| 20 | 212.4 | 200–220 | Outer disk |

Fitting effect: Except for the maximum error at 5 kpc (13–33 km/s), the errors at other points are within ± 10 km/s. Inner disk: Dominated by the bulge, low mass, increasing velocity; Middle disk: Transition region, moderate mass, smoothly connecting the inner and outer disks; Outer disk: Full disk mass, velocity flattens and then slowly decreases.

5.2.2. Andromeda Galaxy (Spiral Galaxy)

Parameters:

- Bulge ($r \leq 4\text{kpc}$): $M_{baryon,bulge} = 5.0 \times 10^9 M_{\odot} \approx 9.945 \times 10^{39} \text{kg}$ ($r_0 = 3\text{kpc}$):

$$M(r) = 9.945 \times 10^{39} (1 - e^{-r/3})$$

- Middle disk ($4 < r < 15\text{kpc}$): $M_{baryon,mid} = 6.0 \times 10^{10} M_{\odot} \approx 1.1934 \times 10^{41} \text{kg}$ ($r_0 = 5\text{kpc}$):

$$M(r) = 1.1934 \times 10^{41} (1 - e^{-r/5})$$
- Outer disk ($r \geq 15\text{kpc}$): $M_{baryon,M31} = 1.2 \times 10^{11} M_{\odot} \approx 2.3868 \times 10^{41} \text{kg}$ ($r_0 = 15\text{kpc}$):

$$M(r) = 2.3868 \times 10^{41} (1 - e^{-r/15})$$
- $k_0 = 4.911 \times 10^{-4}$ (inferred from $v(r_{peak}) = 250 \text{ km/s}$ at $r_{peak} = 15\text{kpc}$: $k_0 = \frac{v(r_{peak})^2 r_{peak} - GM(r_{peak})}{G_h M(r_{peak})^2 \ln r_{peak}}$, where $M(r_{peak})$ is located in the outer disk), $\alpha = 1.5$ (reflecting the rapid decay of the Andromeda outer disk):

$$k(r) = 4.911 \times 10^{-4} \left(\frac{15}{r} \right)^{1.5}$$

Comparison between the Andromeda Galaxy rotation curve and observations [19]

| r (kpc) | $v(r)$ (km/s) | Observed Value (km/s) | Region | Error Analysis |
|-----------|---------------|-----------------------|-------------|---------------------------------------|
| 2 | 248.8 | 200–250 | Inner disk | Error ~1.2% |
| 10 | 261.0 | 225–250 | Middle disk | ~9.8–34.8 km/s higher (4%–15%) |
| 15 | 250.0 | 250 | Peak | Perfect consistency (inferred k_0) |
| 20 | 234.8 | 200–225 | Outer disk | ~9.8–34.8 km/s higher (4%–15%) |

Fitting effect: The inner disk velocity (248.8 km/s) falls within the observational range (200–250 km/s), with errors of 5%–15% in the middle and outer disks, consistent with its mass concentration and rapid outer disk decay characteristics.

5.2.3. NGC 2974 (Elliptical Galaxy)

Parameters:

- Bulge ($r \leq 3\text{kpc}$): $M_{baryon,bulge} = 6.0 \times 10^{10} M_{\odot} \approx 1.19 \times 10^{41} \text{kg}$ ($r_0 = 2\text{kpc}$):

$$M(r) = 1.19 \times 10^{41} (1 - e^{-r/2})$$
- Middle disk ($3\text{kpc} < r \leq 4\text{kpc}$): $M_{baryon,mid} = 8.0 \times 10^{10} M_{\odot} \approx 1.59 \times 10^{41} \text{kg}$ ($r_0 = 3\text{kpc}$):

$$M(r) = 1.59 \times 10^{41} (1 - e^{-r/3})$$
- Outer disk ($r > 4\text{kpc}$): $M_{baryon,NGC2974} = 1.2 \times 10^{11} M_{\odot} \approx 2.39 \times 10^{41} \text{kg}$ ($r_0 = 5\text{kpc}$):

$$M(r) = 2.39 \times 10^{41} (1 - e^{-r/5})$$

- $k_0 = 2.96 \times 10^{-4}$ (inferred from $v(r_{peak})$ at $r_{peak} = 5\text{kpc}$: $k_0 = \frac{v(r_{peak})^2 r_{peak}^{-GM(r_{peak})}}{G_h M(r_{peak})^2 \ln r_{peak}}$, where $M(r_{peak})$ is located in the outer disk), $\alpha = 0.3$ (reflecting the approximately flat, slow decay characteristics of elliptical galaxies, similar to the Milky Way):

$$k(r) = 2.96 \times 10^{-4} \left(\frac{5}{r}\right)^{0.3}$$

Comparison between the NGC 2974 rotation curve and observations [20]

| r (kpc) | $v(r)$ (km/s) | Observed Value (km/s) | Region |
|-----------|---------------|---|-------------|
| 1 | 318.7 | Ionized gas + drift correction $\approx 320 \pm 20$ | Inner disk |
| 2 | 300.6 | — | Inner disk |
| 4 | 283.8 | Inner region decline $\approx 310 \pm 20$ | Middle disk |
| 5 | 300.0 | H I + gas combination, start of flat curve $\approx 300 \pm 10$ | Outer disk |
| 6 | 294.4 | H I flat segment extension $\approx 300 \pm 10$ | Outer disk |
| 8 | 281.4 | Middle of H I flat segment $\approx 300 \pm 10$ | Outer disk |
| 10 | 267.5 | Outer edge of H I flat segment $\approx 300 \pm 10$ | Outer disk |
| 20 | 208.9 | — | Outer disk |

Fitting effect: The maximum error is only $3.2\sigma (< 5\sigma)$, and the outer disk flat segment (300 ± 10 km/s) is highly consistent with observations, demonstrating the universality of the model for elliptical galaxies.

5.3. Logarithmic Asymptotics of Gravitational Lensing by Dark Matter Halos

Many commonly used profiles of “dark matter halos” correspond to the gravitational lensing deflection angle of the projected enclosed mass $M_{2D}(< b)$ within certain radial ranges (especially the outer halo/weak lensing-dominated regions): $\hat{\alpha}(b) = \frac{4GM_{2D}(< b)}{c^2 b}$, where $M_{2D}(< b) = 2\pi \int_0^b \Sigma(b) b db$ is the enclosed mass of the projected surface density, and b is the impact parameter. As long as the extra gravity produces an external asymptotics of $g_{extra}(r) \sim \frac{\ln r}{r^2}$ in 3D, the projected form naturally emerges: $\hat{\alpha}_{extra}(b) \propto \frac{\ln b}{b}$.

The gravitational lensing deflection angle under the weak-field approximation of general relativity (in the scalar potential form) is: $\hat{\alpha}(b) = \frac{2}{c^2} \int_{-\infty}^{+\infty} \nabla_{\perp} \Phi(\sqrt{b^2 + z^2}) dz$. Substitute $\Phi(r)$ solved from the modified Poisson equation, combined with the $1/r$ -order expansion of the logarithmically corrected Schwarzschild metric ($\frac{2GM}{c^2 r}, \frac{2kG_h M^2(\ln r + 1)}{c^2 r} \ll 1$), and adopt the “thin-lens” paraxial approximation:

$$\hat{\alpha}(b) \approx \frac{4GM(r)}{c^2 b} + \frac{4k(r)G_h M(r)^2 \ln b}{c^2 b} \quad (21)$$

It is evident that $\hat{\alpha}_{extra}(b) \approx \frac{4k(r)G_h M(r)^2 \ln b}{c^2 b} \propto \frac{\ln b}{b}$, which is consistent with the logarithmic term appearing after the projection of the aforementioned “dark matter halo”. When quantum gravitational effects (dark matter halo) are not considered ($k(r) = 0$), the gravitational lensing formula naturally reduces to the general relativity form: $\hat{\alpha}(b) \approx \frac{4GM}{c^2 b}$.

5.4. Role of the Logarithmic Term at the Galactic Scale

In the peripheral regions of galaxies, the positive contribution of the logarithmic term $\ln r$ enables the quantum gravitational term to provide stable additional gravity, which is equivalent to the gravitational effect of the traditional dark matter halo but without the need to hypothesize unknown particles:

- Physical nature: The statistical average effect of non-local entanglement of quantum vortices at the galactic scale, transmitted as macroscopic gravity enhancement through AdS/CFT duality.
- Advantage: All parameters are correlatable with observations (e.g., $M_{baryon, topo}$ corresponds to stellar luminosity and gas distribution), avoiding the theoretical flaw of dark matter being “undetectable”.
- Final summary: All standard halo models can be interpreted as effective parameterizations of this logarithmic term ($\Phi_{halo}(r) \sim -\frac{\ln r + 1}{r}$), and their apparent diversity is essentially a reflection of different regularizations of the same asymptotic behavior

6. Cross-Scale Consistency and Theoretical Advantages

6.1. Consistency of Dual-Scale Mechanisms

Although the effects of the logarithmic term at the black hole and galactic scales seem opposite, they originate from the same physical nature:

- Scale correlation: Both the repulsive potential at the black hole scale and the additional gravity at the galactic (black hole gravitational field) scale are macroscopic manifestations of the topological structure and non-local entanglement of quantum vortices, with only changes in the sign and magnitude of $\ln r$ caused by distance r .
- Parameter unification: Core parameters such as the k -factor and G_h have consistent definitions across dual scales; only dynamic adjustments of $M(r)$ and $k(r)$ are made to adapt to scale differences, with no additional hypotheses.

6.2. Comparative Advantages over Traditional Theories

| Comparison Dimension | This Theory (Quantum Gravitational Correction with Logarithmic Term) | Traditional Theories (Kerr Black Hole + Dark Matter) |
|----------------------------|--|---|
| Singularity problem | Physically resolved, satisfying information conservation | Unresolved, with curvature divergence |
| Free parameters | None (black holes) / 4 physical parameters (galaxies) | Black holes require fitting of spin and inclination; galaxies rely on dark matter distribution hypotheses |
| Cross-scale unification | Covers microscopic to macroscopic scales under a single framework | Black hole and galactic dynamics are fragmented |
| Observational verification | Multiple verifications including black hole shadows, high-speed stars, galaxy rotation curves, and mathematical asymptotic behavior of dark matter halos | Dark matter particles not directly detected; black hole spin lacks independent verification |
| Physical picture | Clear image of quantum vortices + AdS/CFT duality | Dark matter nature unknown; Kerr black hole lacks microscopic physical support |

6.3 Advantages over Other Modified Gravity Theories

Compared with most modified gravity theories such as Modified Newtonian Dynamics (MOND), a prominent advantage of this framework is that it fully preserves the structure of metric theory, thereby strictly satisfying the strong equivalence principle. The logarithmic correction term constitutes a holistic modification to spacetime geometry, rather than introducing a fifth force dependent on mass or composition. Therefore, the acceleration exerted on all celestial bodies in the Solar System by the Galactic Center (GC) gravitational field (including the correction term) depends solely on their distance to the GC (Sgr A*), and is independent of the intrinsic properties of the celestial bodies themselves.

Furthermore, the correction term of this model at the galactic scale ($g(r) = \frac{GM(r)}{r^2} + \frac{k(r)G_h M(r)^2 \ln r}{r^2}$) manifests as a uniform background acceleration field at the Solar System scale. As indicated by the Milky Way rotation curve described earlier, the Solar System is located in the mid-disk of the Milky Way ($R_0 \approx 8 \text{ kpc} \approx 2.53 \times 10^{20} \text{ m}$), where the galactic topological mass is $M(8) \approx 1.318 \times 10^{41} \text{ kg}$ and the dynamic entanglement factor is $k(8) \approx 1.222 \times 10^{-5}$. Substituting these values into $g(r)$ yields $g(8) \approx 2.02 \times 10^{-10} \text{ m/s}^2$. The spatial scale of the Solar System is $\Delta r \approx 30 \text{ AU} \approx 4.5 \times 10^{12} \text{ m}$ (the orbit of Neptune, the outermost planet), thus the relative tidal acceleration difference between the two ends of the Solar System (e.g., from the Sun to Neptune) is:

$$\Delta g \approx \frac{dg}{dr} \cdot \Delta r \sim \frac{g(r)}{R_0} \cdot \Delta r \approx 3.6 \times 10^{-18} \text{ m/s}^2$$

Current high-precision experiments in the Solar System (such as Lunar Laser Ranging, the Cassini spacecraft, and LISA Pathfinder) place constraints on anomalous acceleration at the level of 10^{-13} to 10^{-15} m/s^2 , with even tighter constraints on tidal effects. The value of $3.6 \times 10^{-18} \text{ m/s}^2$ is at least 3 orders of magnitude lower than these detection limits. Therefore, all relative dynamical behaviors within the Solar System under this framework are completely consistent with standard general relativity, the strong equivalence principle holds strictly in local inertial frames, and no screening mechanism is required.

7. Conclusions and Outlook

Through a unified non-perturbative quantum gravity framework, this paper reveals the cross-scale universality of the quantum gravitational correction term containing a logarithmic term—its extremely simple mathematical form not only prevents collapse to the singularity through a repulsive potential at the black hole core and may resolve the information paradox but also maintains the velocity of hypervelocity stars and flattens rotation curves through additional gravity in the black hole gravitational field and galactic periphery. It provides a possible solution to two major cross-scale problems (singularity resolution + rotation curve flattening) without relying on assumptions such as extra dimensions or dark matter particles. Starting from the analysis of the mathematical asymptotic behavior of dark matter halo dynamics, this framework conducts preliminary cross-scale multiple verifications of quantum gravity with a logarithmic term through black hole shadows (EHT observations), hypervelocity stars, rotation curves of multiple galaxies (astronomical measurements), and the mathematical asymptote of gravitational lensing. It enables a potentially non-fragmented scheme for the unified description of gravity from the microscopic to the macroscopic scale and provides an observable, repeatable empirical framework for quantum gravity theory, which is different from current mainstream paths (such as string theory, loop quantum gravity, etc.). Moreover, the six rigid predictions of black hole shadows presented in this paper make the theory one of the few frameworks that can provide clear, specific, and falsifiable (unable to adjust spin α and inclination i) targets for the next generation of EHT observations.

Future research can focus on: 1) Further improving the consistency of the quantum vortex picture and field equations through more rigorous field theory calculations or numerical simulations (though the current framework already has sufficient logical consistency and testability); 2) Attempting to apply the theoretical framework to the a priori prediction of multi-messenger astronomical research such as black hole thermodynamics, radio bursts, and astroparticle physics, and using telescopes such as JWST, CTA, H.E.S.S, Fermi-LAT, IceCube, and KM3NeT to test the universal boundary of the application of logarithmically corrected gravity (quantum gravity); 3) Studying the impact of this correction term on the Friedmann equations to explore possible insights into cosmological problems such as dark energy dynamics and Hubble tension; 4) Directly verifying the nonlocal entanglement (quantum entanglement) effect corresponding to the logarithmic term through laboratory simulations (such as superfluid helium quantum vortex systems), providing a more solid microscopic experimental foundation for the theory.

This study indicates that the gravitational behavior of the universe, from black holes to galaxies, may be governed by the same quantum gravitational mechanism, with the logarithmic term serving as the core carrier of this mechanism. It also strongly suggests that black holes and galaxies may share a common topological origin, which we interpret as follows: the overall dynamics of galaxy disks may be the holographic manifestation of the quantum topological structure of their central black holes on the macrocosmic scale through hierarchical nesting ($AdS_2/CFT_1 \subseteq AdS_3/CFT_2 \subseteq AdS_4/CFT_3$). This idea resonates with multiple cutting-edge physical concepts such as quantum fluid cosmology, fractal cosmology, and recursive structures. With its simplicity and powerful cross-scale adaptability, this model may pave a brand-new path for the unified description of gravity in astrophysics.

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References

1. Navarro, J. F., Frenk, C. S., White, S. D. M. "A Universal Density Profile from Hierarchical Clustering." *Astrophys. J.* 490, 493 (1997)
2. Burkert, A. "The Structure of Dark Matter Halos in Dwarf Galaxies." *Astrophys. J. Lett.* 447, L25–L28 (1995)
3. McGaugh, S. S. "The Baryonic Tully-Fisher Relation of Galaxies." *Astrophys. J.* 632, 859–871 (2005)
4. Rocha, M., Peter, A. H. G., Bullock, J. S., et al., Cosmological simulations with self-interacting dark matter – I. Constant-density cores and substructure, *Monthly Notices of the Royal Astronomical Society*, 430, 81–104 (2013). doi:10.1093/mnras/sts514
5. Schive, H.-Y., Chiueh, T., & Broadhurst, T., Cosmic structure as the quantum interference of a coherent dark wave, *Physical Review Letters*, 113, 261302 (2014). doi:10.1103/PhysRevLett.113.261302
6. Abo-Shaer, J. R., Raman, C., Vogels, J. M., & Ketterle, W., Observation of vortex lattices in Bose–Einstein condensates, *Science*, 292, 476–479 (2001). doi:10.1126/science.1060182
7. Skenderis, K., & Taylor, M. "The fuzzball proposal for black holes." *Phys. Rep.*, 467, 117–171 (2008)
8. Aharony, O., Bergman, O., Jafferis, D. L., & Maldacena, J. "N=6 superconformal Chern–Simons–matter theory and its gravity dual." *JHEP*, 2008(10), 091
9. Hall, D. S. et al. Tying quantum knots. *Nat. Phys.* 12, 478–483 (2016)
10. Event Horizon Telescope Collaboration. First M87 Event Horizon Telescope Results. I. The Shadow of the Supermassive Black Hole. *Astrophys. J. Lett* 875, L1 (2019)
11. Event Horizon Telescope Collaboration. First Sagittarius A* Event Horizon Telescope Results. I. The Shadow of the Supermassive Black Hole in the Center of the Milky Way. *Astrophys. J. Lett* 930, L12 (2022)
12. Kerr RP. Gravitational Field of a Spinning Mass as an Example of Algebraically Special Metrics. *Phys. Rev. Lett* 11, 237 (1963)
13. Hioki, K., & Maeda, K. Measurement of the Kerr spin parameter by observation of a compact object's shadow *Phys. Rev. D* 80, 024042 (2009)
14. Tsukamoto, N. Black hole shadow in various spacetime geometries *Phys. Rev. D* 97, 064021 (2018)
15. Kraav, K. Bounding the size of the shadow of a Kerr black hole *Phys. Rev. D* 101, 024039 (2020)
16. Peiřker, F., Eckart, A., Zajaček, M., et al. S62 on a 9.9 yr Orbit around Sgr A*. *The Astrophysical Journal*, 889, 61 (2020)
17. Peiřker, F., Eckart, A., Zajaček, M., et al. S62 and S4711: Indications of a Population of Faint Fast-moving Stars inside the S2 Orbit. *The Astrophysical Journal*, 899, 50 (2020)
18. Sofue, Y., Grand rotation curve and dark matter halo in the Milky Way Galaxy, *Publ. Astron. Soc. Japan*, 64, 75 (2012). doi:10.1093/pasj/64.4.75
19. Chemin, L., Carignan, C., Foster, T. "HI Kinematics and Dynamics of Messier 31." *Astrophys. J.* 705, 1395–1415 (2009)

20. Noordermeer, E., van der Hulst, J. M., Sancisi, R. A., Swaters, R. A., & van Albada, T. S., The mass distribution in early-type disc galaxies: declining rotation curves and correlations with optical properties, *Mon. Not. R. Astron. Soc.*, 376, 1513–1546 (2007). doi:10.1111/j.1365-2966.2007.11533.x

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