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Article

Hubble Entropic States and Hubble Radiation Pressure from the Hubble Sphere Energy-Gap (Mass-Gap)

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Abstract: We will demonstrate that the number of states, based on entropy, is related to the Hubble sphere radiation energy gap, which is directly linked to the Hubble sphere mass gap (energy gap) recently introduced by Haug [1–3]. This energy gap represents the minimum energy a photon can possess inside a black hole Hubble sphere. Moreover, this minimum energy leads to a minimum acceleration that has been detected, suggesting that the Hubble pressure that also can be called entropic pressure may have already been observed.

Keywords: Entropic states; energy gap; mass gap; Hubble pressure; entropic temperature; minimum temperature; entropic pressure

1. Entropic States and the Hubble Sphere Energy Gap (Mass Gap)

Entropic gravity is an interesting and evolving theory; see [4,5]. The number of entropic states plays a central role in entropic gravity and is normally given as:

$$N = \frac{A}{l_p^2} = \frac{4\pi R_s^2}{l_p^2} \quad (1)$$

where R is the black hole event horizon, that is the Schwarzschild radius if we limit ourselves to Schwarzschild black holes. This is essentially the Bekenstein-Hawking [6–8] entropy for a black hole with a surface area equal to $A = 4\pi R^2$, where $l_p = \sqrt{\frac{\hbar G}{c^3}}$ is the Planck length [9,10]. In the case above, this is simply the surface area of the black hole divided by the Planck area. However, we introduce a minor modification: since l_p^2 represents the surface area of a Planck square, we will instead compare the spherical surface area of the black hole with the spherical Planck surface area. This gives:

$$N = S_{BH} = \frac{A}{4\pi l_p^2} = \frac{4\pi R^2}{4\pi l_p^2} = \frac{R^2}{l_p^2} \quad (2)$$

In a Schwarzschild black hole, the Hubble radius is equal to the Schwarzschild radius. This can be seen from the critical mass, which is given by $M_{c,t} = \frac{c^2 R_{H,t}}{2G}$. We are here working in a $R_{H,t} = ct$ cosmological model so the critical mass will vary over time. Solving for $R_{H,t}$, we obtain:

$$R_{H,t} = \frac{2GM_{c,t}}{c^2} \quad (3)$$

which is identical to the Schwarzschild radius for a black hole with mass M_c . This means for the Hubble sphere we have:

$$N_t = S_{BH}(t) = \frac{A_{H_t}}{4\pi l_p^2} = \frac{4\pi R_{H_t}^2}{4\pi l_p^2} = \frac{R_{H_t}^2}{l_p^2} \quad (4)$$

where $R_{H_t} = \frac{c}{H_t}$ is the Hubble radius at time t .

Haug [1] has recently suggested that the Hubble sphere has an energy gap or an energy-equivalent mass gap; see also [3,11]. The idea is very simple: a photon cannot have a wavelength longer than the

diameter of the Hubble sphere (or potentially the radius). No photon can have a longer wavelength than this, particularly in a black hole universe. The idea of black hole universe was introduced by at least 1972 by Pathria (1972) [12] and is actively discussed to this day, see [13–22]. The energy gap is simply given by:

$$E_{min} = E_g = \hbar f = \hbar \frac{c}{2R_{H_t}} \quad (5)$$

where E_{min} represents the minimum possible observable energy in the Hubble sphere, corresponding to the photon with the longest possible wavelength. This is the same as the Hubble sphere energy gap E_g , just two different terms for the same concept. Furthermore, since we have Einstein's equation $E = mc^2$, the minimum equivalent mass above zero, that we have called the Hubble sphere mass gap, must be:

$$m_{min} = m_g = \frac{E_{min}}{c^2} = \frac{\hbar}{2R_{H_t}} \frac{1}{c} \quad (6)$$

The number of entropic states in the Hubble sphere is given by:

$$N_t = S_{BH}(t) = \frac{A_{H_t}}{4\pi l_p^2} = \frac{M_{c,t}}{m_{min}} = \frac{E_{c,t}}{E_{min}} = \frac{\frac{c^4 R_{H_t}}{2G}}{\frac{\hbar}{2R_{H_t}} c} = \frac{R_{H_t}^2}{\frac{G\hbar}{c^3}} = \frac{R_{H_t}^2}{l_p^2} \approx 7.3 \times 10^{121} \quad (7)$$

the number of stats clculated here is given for when $t = 0$ as then $R_{H,t} = \frac{c}{H_0}$. Haug [3] has also linked the minimum energy in the Hubble sphere to a minimum temperature, given simply as:

$$T_{min} = \frac{m_{min}c^2}{k_b} = \frac{E_{min}}{k_b} \quad (8)$$

The entropic states (number of degrees of freedom) can therefore also be described as:

$$N_t = \frac{E_{c,t}}{E_{min}} = \frac{E_{c,t}}{k_b T_{min}} \quad (9)$$

Note that $\frac{E_{c,t}}{E_{min}} = \frac{\frac{c^2 R_{H_t}}{2G}}{\frac{\hbar}{2R_{H_t}} \frac{1}{c}} = \frac{c^3 R_{H_t}^2}{4G\hbar} = \frac{R_{H_t}^2}{l_p^2}$, and since we assume $R_H = ct$, this means N_t is a deterministic function dependent on $R_{H_t}^2 = c^2 t^2$ over time, as illustrated in Figure 1.

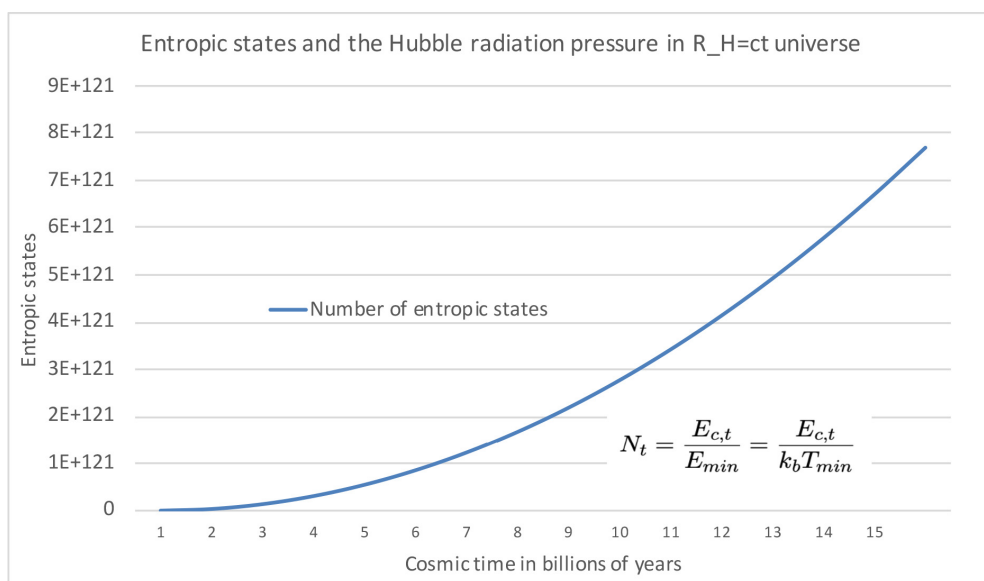


Figure 1. This figure shows how the number of entropic states vary with cosmic time.

This means the entropic force can be described as (see also Tatum and Haug [23] who have pointed out the entropic force in their $R_{Ht} = ct$ model as $F_e = \frac{E_{c,t}}{R_{Ht}}$, this can be re-written as:

$$F_{e,t} = \frac{E_{c,t}}{R_{Ht}} = \frac{Nk_b T_{min}}{R_{Ht}} \quad (10)$$

where $E_{c,t} = \frac{c^4 R_{Ht}^2}{2G}$ is the entropic energy. Be aware also $T_{min} = \hbar \frac{c}{2R_{Ht}}$ is time dependent as we have $R_{Ht} = ct$. The meaning of entropy is often defied as a measure of the unavailable energy in a closed thermodynamic system. In a black hole universe the Hubble sphere is indeed a closed thermodynamic system.

This is also fully consistent with [24], where Haug and Wojnow recently derived the Hubble Radiation Pressure Law:

$$\begin{aligned} P_{c,t} V_{Ht} &= Nk_b T_{min} \\ E_{c,t} &= Nk_b T_{min} \\ \frac{E_{c,t}}{R_{Ht}} &= \frac{Nk_b T_{min}}{R_{Ht}} = F_{e,t} \end{aligned} \quad (11)$$

where Haug and Wojnow describe the Hubble radiation pressure as:

$$P_{c,t} = \frac{Nk_b T_{min}}{\frac{4}{3}\pi R_{Ht}^3} = \frac{E_{c,t}}{\frac{4}{3}\pi R_{Ht}^3} = \frac{3H_t c^2}{8\pi G} \quad (12)$$

As they describe it is identical to the critical energy (density), but link it to the CMB temperature. Further $V_{Ht} = \frac{4}{3}\pi R_{Ht}^3$ is the Hubble volume. The suggested Hubble radiation pressure and its corresponding entropic force is causing the black hole universe not to collapse into a singularity.

2. The Hubble Radiation Pressure Hindering the Energy and Mass Inside the Black Hole Universe to Collapse into a Center Singularity

The Hubble radiation pressure [24]: $P_{c,t} = \frac{E_{c,t}}{\frac{4}{3}\pi R_{Ht}^3} = \frac{3H_t c^2}{8\pi G}$, when assuming $R_{Ht} = ct$ in black hole cosmology, will vary over time, as shown in Figure 2, and prevent the mass and energy of the universe from collapsing into a central singularity.

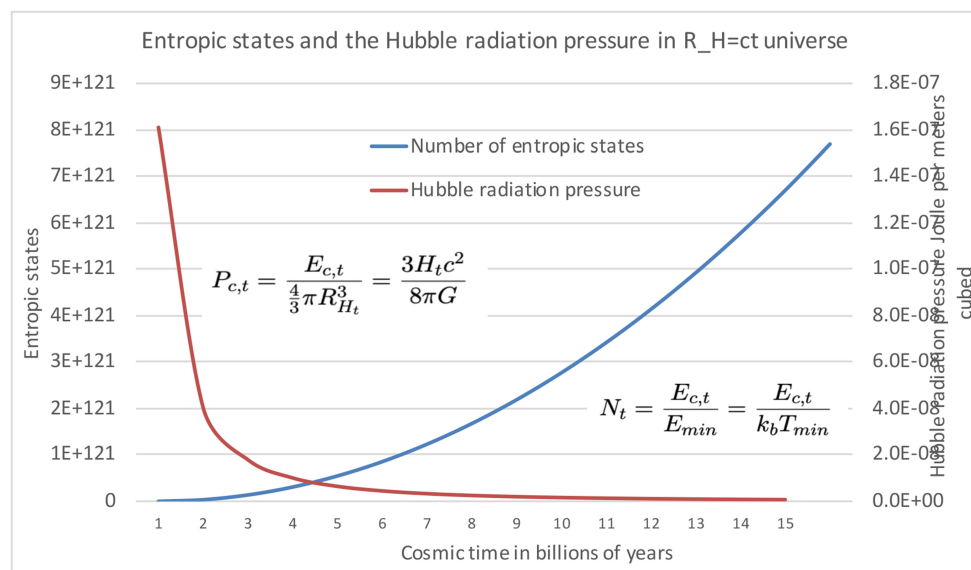


Figure 2. This figure shows how the number of entropic states as well as the Hubble radiation pressure vary with cosmic time in a $R_{Ht} = ct$ model.

We can see that as the entropic states increase with cosmic time as it expectedly should, the Haug and Wojnow Hubble radiation pressure (energy density) decreases. The reason for this is that the energy becomes more dispersed. Tatum and Haug [23] later refer and discuss the same radiation pressure as entropic energy density and interestingly link it to a entropic force $F_e = \frac{E_{e,t}}{R_{H,t}}$. The interpretation may vary depending on whether one also considers it in terms of entropy. Therefore, it is important to highlight this counteracting force to gravity from different perspectives and papers throwing light at the topic from different angles is what science is about.

3. The Geometric Mean Temperature Is the CMB Temperature and It Is Linked to the Minimum and Maximum Temperature

Since the minimum temperature in the Hubble sphere seems to play an important role in cosmic entropy, it is also worth mentioning that it plays a significant role in the CMB temperature as well. Haug and Tatum [11] have demonstrated how the CMB temperature is simply related to the geometric mean of basically the minimum and maximum physical temperatures. Haug [3] has elaborated further on this and provided the elegant formula:

$$T_{\text{geometric mean}} = T_{\text{cmb}} = \frac{\sqrt{T_{\text{min}} T_p}}{4\pi} \quad (13)$$

This is, at a deeper level, identical to the CMB formula that Haug and Wojnow [25,26] have derived from the Stefan-Boltzmann law [27,28], which, in turn, is consistent with the CMB prediction formula first heuristically suggested by Tatum et al. [29]. It is important to be aware the CMB is considered a almost perfect black body, as described by for example Muller et al. [30] that states :

“Observations with the COBE satellite have demonstrated that the CMB corresponds to a nearly perfect black body characterized by a temperature T_0 at $z = 0$, which is measured with very high accuracy, $T_0 = 2.72548 \pm 0.00057\text{K}$.”

Be aware that while the Planck temperature is a constant, $T_p = \frac{1}{k_b} \sqrt{\frac{\hbar c^5}{G}} = \frac{m_p c^2}{k_b} = \frac{E_p}{k_b}$, the minimum temperature varies as we travel back in time in the $R_{H_t} = ct$ universe, as it is given by $T_{\text{min}} = \frac{E_{\text{min}}}{k_b} = \hbar \frac{c}{2R_{H_t}} \frac{1}{k_b}$.

While the geometric mean temperature (energy) is relatively easy to detect and is likely the most precisely measured cosmological parameter [31–34], the minimum temperature is hard to detect because it is so small, approximately $\approx 8.3 \times 10^{-30}\text{K}$. However, the minimum temperature is likely detectable indirectly as a minimum acceleration caused by radiation acceleration of about the order of $a_0 = \frac{1}{2}H_0c$, something we will get back to in section 4.

The maximum temperature, which is the Planck temperature is likely linked to quantum gravity and lasts only about the Planck time, making it also nearly impossible to detect directly. However, it may be detected indirectly as gravity itself; see [35,36]. Haug [37–39] has recently demonstrated that Planck units can be measured from a series of gravitational observations without any knowledge of G .

4. Could It Be That the Entropy Force Is Detectable as a Minimum Acceleration or Indirectly Through also the CMB Temperature?

The entropic force, in our view, could be detectable, as it predicts a minimum acceleration of approximately:

$$g_{\text{min}} = \frac{Gm_{\text{min}}}{l_p^2} = \frac{GN_t \frac{k_b T_{\text{min}}}{c^2}}{R_{H_t}^2} = \frac{GM_{c,t}}{R_{H_t}^2} \approx 3.25 \times 10^{-10} \text{ m/s}^2 \quad (14)$$

That is for the case when $t = 0$ (so $z = 0$) now.

This value is close to the minimum acceleration proposed in Modified Newtonian Dynamics (MOND) [40,41]. Also, be aware that:

$$g_{min} = \frac{GM_c}{R_H^2} = \frac{1}{2}H_0c \quad (15)$$

A minimum acceleration close to H_0c plays an important role in MOND, as discussed by Milgrom [41], and in Verlinde's Emergent Gravity model [42].

Our minimum temperature from the energy gap also clearly corresponds, at least mathematically, to the Unruh temperature. The Unruh [43] temperature is given by:

$$T = \frac{\hbar a}{2\pi c k_b} \quad (16)$$

where a is the Unruh acceleration. If we instead input our minimum acceleration, we obtain:

$$T = \frac{\hbar a}{2\pi c k_b} = \frac{\hbar \frac{GM_{min}}{l_p^2}}{2\pi c k_b} = \frac{\hbar c}{4\pi k_b R_H} \quad (17)$$

which is identical to our Hubble sphere minimum temperature, differing only by a factor of $\frac{1}{2\pi}$. The minimum temperature is also closely connected to the Hawking [44] temperature, as for the Hubble sphere, it is given by:

$$T_{Haw} = \frac{\hbar g}{2\pi c k_b} \quad (18)$$

where g is the gravitational acceleration. For the Hubble sphere, we can set $g = \frac{GM_c}{R_H^2}$, and we obtain:

$$T_{Haw} = \frac{\hbar \frac{GM_c}{R_H^2}}{2\pi c k_b} = \frac{\hbar c}{4\pi R_H} = \frac{\hbar c^3}{8\pi k_b GM_c} \quad (19)$$

The minimum temperature T_{min} is so small it is unlikely to be easy to detect. But it can be detected indirectly through the CMB temperature as the CMB temperature is given by the formula recently given by Hayg

$$T_{cmb} = \frac{\sqrt{T_{min} T_{max}}}{4\pi} = \frac{\sqrt{T_{min} T_p}}{4\pi} \quad (20)$$

So solved for T_{min} we get:

$$T_{min} = \frac{T_{cmb}^2}{T_p} 16\pi^2 \quad (21)$$

And T_{cmb} can be easily measured, and since the Planck temperature is a constant, it can be calculated. This allows us to indirectly detect the entropic temperature, as it is linked to the CMB temperature. The CMB temperature is the geometric mean of the minimum and maximum physically possible temperatures within the Hubble sphere.

Since the Planck temperature is constant, we can calculate it using the traditional method, or it can even be extracted from the Hubble sphere using:

$$T_p = 8\pi T_{cmb} \sqrt{\frac{R_H}{2l_p}} \quad (22)$$

Thus, to determine the Planck temperature, all we need is the Hubble constant and the CMB temperature, even though it is more precisely calculated using the traditional Max Planck approach:

$T_p = \frac{1}{k_b} \sqrt{\frac{\hbar c}{G}} = \frac{m_p c^2}{k_b}$. The important point here is we can extract the Planck temperature from observable cosmology parameters. And the reason for this is in our view that the Planck temperature is linked to gravity, and gravity naturally affect even the cosmological scale.

Table 1 presents these three different temperatures and their relationships. Therefore, one could say that detecting the CMB temperature is equivalent to indirectly detecting both the maximum and minimum temperatures in the universe, as the CMB temperature is simply the geometric mean of the two. This is fully consistent with a subgroup of $R_{H_t} = ct$ cosmology.

Table 1. This table show the relation between the CMB temperature the Planck temperature and a minimum temperature that we also can call the entropic temperature, or radiation pressure temperature. They are all related as the CMB temperature is simply the geometric mean of the two other temperatures divided by 4π . The relation holds at any cosmic epoch in a $R_{H_t} = ct$ universe and is fully consistent also with the observed relation $T_t = T_0(1+z)$.

Type of temperature:	Mathematical expression:
CMB temperature, geometric mean temperature:	$T_{cmb} = \frac{\sqrt{T_{min}T_{max}}}{4\pi} = \frac{\sqrt{T_{min}T_p}}{4\pi}$
Entropic temperature (minimum temperature):	$T_{min} = \hbar \frac{c}{2R_{H_t}} \frac{1}{k_b} = \frac{T_{cmb}^2}{T_{max}} 16\pi^2 = \frac{T_{cmb}^2}{T_p} 16\pi^2$
Planck temperature, related to gravity:	$T_{max} = T_p = \frac{1}{k_b} \sqrt{\frac{\hbar c^5}{G}} = \hbar \frac{c}{l_p} \frac{1}{k_b} = \frac{T_{cmb}^2}{T_{min}} 16\pi^2$

If this entropic temperature (Hubble radiation pressure) can be indirectly observed both in the minimum observed acceleration of galaxy rotation arms and through the CMB temperature, then the term “Dark Energy”, as Tatum and Haug [23] have compared this radiation pressure to, may be somewhat of a misnomer. On the other hand, what one calls this radiation pressure is largely a matter of wording and labeling. The important point is that we have built a very solid mathematical framework for entropic states in the universe and linked it to the minimum (entropic temperature), the maximum (Planck temperature), and even the directly detectable CMB temperature. This is fully consistent with recent developments in $R_{H_t} = ct$ cosmology (see [45,46]). Further if we should term it dark energy as Tatum and Haug has linked it to, then it is clear from the rest of their model that this dark energy is not needed to get a perfect match with the full distance ladder of SN Ia (in the HTC model). It is only needed indirectly to avoid the black hole universe to collapse into a singularity and to keep it expanding or alternatively to keep the Hubble sphere stay steady state. Steady state models should not be excluded fully yet at this point something we will come back to in a future paper.

An interesting question is the CMB temperature could be signature of two opposite forces, the gravity force that we claim must be linked to the Planck temperature and the Planck scale and the entropic force that we link to the minimum temperature (entropic temperature). The geometric mean bind these two extreme temperatures that both are not possible to detect directly but only indirectly together in the easily observable CMB temperature [3]. The CMB temperature is then a type of equilibrium temperature between two forces, the gravity force and the entropic force.

5. Other Ways to Express the Numbers of Entropic States

If we take the critical mass in the Hubble sphere and divide it by the mass gap, or the critical energy and divide it by the mass gap, Haug [25] has demonstrated:

$$N_t = \frac{A_{H_t}}{4\pi l_p^2} = \frac{4\pi R_{H_t}^2}{4\pi l_p^2} = \frac{4l_p^2}{\bar{\lambda}_{c,t}^2} \approx 7.3 \times 10^{121} \quad (23)$$

where $\bar{\lambda}_{c,t}$ is the reduced Compton wavelength of the critical mass, $\bar{\lambda}_{c,t} = \frac{\hbar}{M_{c,t}c} = \frac{\hbar 2GH_t}{c^3}$. This is the same as estimated as given by [47]: $\frac{c^2 t^2}{l_p^2} = \frac{t_H^2}{l_p^2}$ which is identical to $N = \frac{4l_p^2}{\bar{\lambda}_c^2}$ and also in a $R_{H_t} = ct$ universe it is clear that $c^2 t^2 = R_{H_t}^2$, so this is also identical to $N_t = \frac{R_{H_t}^2}{l_p^2}$ (the Bekenstein-Hawking entropy), see also [48].

Haug [25] has also demonstrated that one must have:

$$N_t = S_{BH}(t) = \frac{A_{H_t}}{4\pi l_p^2} = 4 \frac{T_{cmb,t}^4}{T_{Haw,t}^4} = \frac{4l_p^2}{\lambda_{c,t}^2} \quad (24)$$

This even means the entropic force can be described as:

$$F_e = \frac{E_{c,t}}{R_{H_t}} = \frac{N_t k_b T_{min}}{R_{H_t}} = \frac{\frac{T_{cmb}^4}{T_{Haw}^4} k_b T_{min}}{R_{H_t}} = 8\pi \frac{k_b T_{cmb}^4}{T_{Haw}^3 R_{H,t}} \quad (25)$$

And the entropic force can then also be described as:

$$F_e = \frac{2\pi N_t k_b T_{Haw}}{R_{H_t}} \quad (26)$$

6. Conclusions

We have demonstrated that the entropic states in the universe correspond simply to the critical 'Friedmann' energy divided by the Hubble energy gap: $N_t = \frac{E_{c,t}}{E_{min}} = \frac{R_H^2}{l_p^2}$. It is therefore possible that the entropic force is actually detectable, as this energy gap also seems lead to a minimum acceleration, similar to what has been observed in galaxy rotation curves.

In addition, this minimum energy is related to a minimum temperature that we also have called entropic temperature. This entropic temperature is linked to the CMB temperature and the Planck temperature by $T_{min} = \frac{T_{cmb}^2}{T_p} 16\pi^2$. Since the Planck temperature T_p is known and the CMB temperature is easily measurable, we can say that this minimum Hubble sphere temperature (the entropic temperature) is easily and indirectly detectable. The Hubble radiation pressure is what keeps a black hole Hubble sphere universe from collapsing into a singularity over time. Entropy is related to the counterforce of gravity on a cosmic scale and this force is indirectly detectable through both minimum acceleration and the CMB temperature.

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