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Article

Cosmological Constant or Cosmological Curvature Parameter? A Conformal Reinterpretation of Λ

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Abstract

Observations across multiple cosmological probes indicate that the Universe possesses a non-zero cosmological constant, typically associated with vacuum energy density. However, the large discrepancy between observationally inferred and theoretically predicted vacuum energy densities, together with recent analyses in which combined $S_{\text{N}}\text{Ia}$, $H(z)$, BAO, LSS, BBN, and CMB datasets mildly prefer models with a time-dependent cosmological parameter, shows the need to motivate re-examining the assumption that Λ represents the constant vacuum energy density. In this study, interaction field equations derived from an action incorporating bulk influence, which reduce to the Einstein field equations in the flat-background limit, are utilized. A comparison of this set of equations reveals that the effective cosmological parameter is associated with the normalized bulk conformal curvature term, giving rise to an evolving effective Λ influenced by the vacuum energy structure rather than by a fixed density. This framework provides a new perspective on the interpretation of the cosmological parameter and offers a natural reframing of the cosmological constant problem, in which the small observed value of Λ can be understood as reflecting the weak curvature of the bulk, while remaining consistent with current observational preferences.

Keywords: cosmological constant problem

1. Introduction

The cosmological constant Λ was originally introduced by Einstein to allow for a static universe solution to his field equations. Following the discovery of cosmic expansion by Hubble and Lemaître, it was discarded, only to be revived in the late 1990s to explain the accelerating expansion observed in Type Ia supernovae [1]. In the standard ΛCDM model, Λ is interpreted as the energy density of the vacuum. However, theoretical predictions from quantum field theory estimate this energy density to be approximately 120 orders of magnitude larger than the observed value [2]. This "cosmological constant problem" remains one of the most significant open questions in physics. Furthermore, while ΛCDM fits observational data well, recent analyses in which combined $S_{\text{N}}\text{Ia}$, $H(z)$, BAO, LSS, BBN, and CMB datasets mildly prefer models with a time-dependent cosmological parameter [3], underscores the necessity to re-examine the assumption that Λ represents a constant vacuum energy density.

This paper proposes a resolution to these discrepancies through interaction field equations derived from an action incorporating the background vacuum energy "Bulk," which reduce to the Einstein field equations in the flat-background limit [4]. By treating celestial objects not as isolated manifolds but as dynamic "Clouds" interacting with the bulk, we derive a dynamic cosmological parameter that is not a fixed density but a geometric consequence of the bulk curvature. This approach naturally recovers the small observed value of Λ and provides a consistent geometric explanation for cosmic acceleration without fine-tuning.

1.1. The Extended Action

Traditionally, General Relativity treats the celestial objects as isolated masses warping a flat spacetime. This extended model shifts that perspective: spacetime is a dynamic vacuum bulk. While

the Sun acts as the primary source of local deformation, the Earth travels through an already-curved background, $\mathcal{R}$. By involving the background curvature and the "stiffness" of this bulk (the modulus $E_D = -\mathcal{F}_{\lambda\rho}\mathcal{F}^{\lambda\rho}/4\mu_0$), we present the extended action:

$$S = E_D \int_C \left[\frac{R}{\mathcal{R}} + \frac{L}{\mathcal{L}} \right] \sqrt{-g} d^4\rho \quad (1)$$

1.2. The Bulk-Cloud Interaction Action

To provide a first-principles derivation for the interaction between the vacuum bulk and the active geometry, and by considering the expansion of the bulk owing to the expansion of the Universe and its implication on the field strength of the bulk, the Bulk-Cloud Interaction Action is introduced as follows:

$$S = \int_B \left[\frac{-\mathcal{F}_{\lambda\delta}g^{\lambda\gamma}\mathcal{F}_{\gamma\alpha}g^{\delta\alpha}}{4\mu_0} \right] \sqrt{-g} \int_C \left[\frac{R_{\mu\nu}g^{\mu\nu}}{\mathcal{R}_{\mu\nu}g^{\mu\nu}} + \frac{L_{\mu\nu}g^{\mu\nu}}{\mathcal{L}_{\mu\nu}g^{\mu\nu}} \right] \sqrt{-g} \gamma d^4\rho d^4\sigma \quad (2)$$

The first integral defines the vacuum field dynamics over the Bulk (B) coordinates, where $\mathcal{F}$ represents the vacuum field strength tensor coupled to the background metric. The second integral operates over the Cloud (C) coordinates ρ , normalizing the physical curvature $R_{\mu\nu}$ and Lagrangian $L_{\mu\nu}$ to their background counterparts $(\mathcal{R}_{\mu\nu}, \mathcal{L}_{\mu\nu})$. The product of these integrals, scaled by the coupling γ , establishes the energy exchange mechanism that drives the induced gravity $G_{\mathcal{R}}$.

1.3. Cloud Classical Field Equations (CCFEs)

Varying the interaction action with respect to the metrics yields the field equations:

$$\begin{aligned} R_{\mu\nu} - \underbrace{\frac{1}{2}R\left(g_{\mu\nu} - \frac{1}{4}\tilde{g}_{\mu\nu}\right)}_{\text{Vacuum Softening}} + \underbrace{K\left[(K_{\mu\nu} - K\hat{p}_{\mu\nu}) - \frac{R}{\mathcal{R}}(\mathcal{K}_{\mu\nu} - \mathcal{K}\hat{q}_{\mu\nu})\right]}_{\text{Geometric Slip}} \\ = \underbrace{\frac{4\pi G_{\mathcal{R}}}{c^4}}_{\text{Effective Newtonian Coupling}} \underbrace{(T_{\mu\nu} + K\tau_{\mu\nu}^{vac})}_{\text{Matter \& Vacuum Stresses}} \end{aligned} \quad (3)$$

1.3.1. The Geometric Sector (Left-Hand Side)

The left-hand side of Eq. (3) consists of purely geometric quantities that characterize both the intrinsic and extrinsic curvature of the physical spacetime relative to the background bulk:

- $R_{\mu\nu}$, the Ricci curvature tensor, encoding the intrinsic curvature of the physical manifold.
- $R \equiv g^{\mu\nu}R_{\mu\nu}$, the Ricci scalar obtained by contraction.
- $g_{\mu\nu}$, the physical metric.
- $\tilde{g}_{\mu\nu}$, a reference metric describing the conformal geometry of the vacuum bulk.
- $K_{\mu\nu}$, the extrinsic curvature tensor, quantifying how the physical manifold is embedded within the bulk geometry.

1.3.2. The Source Sector (Right-Hand Side)

- 4π , the Newtonian flux coefficient. Within the CCFEs framework, interaction with the four-dimensional vacuum bulk, together with the application of Background Softening ($\sigma = 1/4$), anchors the temporal degree of freedom to the bulk. This reduces the effective metric trace to three and restores the gravitational coupling to the Newtonian flux constant.
- $T_{\mu\nu}$, the matter stress-energy tensor. $\tau_{\mu\nu}^{vac}$, the vacuum stress tensor, interpreted as an effective surface tension associated with the spacetime membrane.

2. Derivation of the Geometric Cosmological Constant (Λ_g)

2.1. The Trace-3 Field Equation

With the softening parameter $\sigma = 1/4$, the effective metric trace is reduced:

$$Tr(g_{eff}) = g^{\mu\nu}\left(g_{\mu\nu} - \frac{1}{4}\tilde{g}_{\mu\nu}\right) = 4 - \frac{1}{4}(4) = 3 \tag{4}$$

Chronometric Rigidity and Bulk Synchronization: For the brane to remain stable within the bulk, time flow must be synchronized across the boundary [5]. This constraint synchronizes the temporal degree of freedom. Taking the trace of the equations in vacuum ($T_{\mu\nu} = 0, K_{\mu\nu} \approx 0$):

$$g^{\mu\nu}R_{\mu\nu} - \frac{1}{2}R\left(g^{\mu\nu}g_{\mu\nu} - \frac{1}{4}g^{\mu\nu}\tilde{g}_{\mu\nu}\right) = R - \frac{1}{2}R(3) = -\frac{1}{2}R \tag{5}$$

2.2. The CCFEs-de Sitter Identity

$$-\frac{1}{2}R + 3\Lambda_g = 0 \Rightarrow R = 6\Lambda_g \tag{6}$$

2.3. Solving for the Expansion Rate (H^2)

Substituting the identity $R = 12H^2$:

$$12H^2 = 6\Lambda_g \Rightarrow H^2 = \frac{1}{2}\Lambda_g \tag{7}$$

The CCFEs Hubble Radius is thus derived as $L = c/H = 2/\Lambda_g \approx 1.35 \times 10^{26}$ m.

3. CCFEs Predictions and Observations

Table 1. Comparison of CCFEs predictions with standard observations.

Observation	Empirical Value	Λ CDM (Standard)	CCFES (Derived)	Comparison Status
Hubble Radius ($\bar{L}_H$)	1.37×10^{26} m [6]	1.65×10^{26} m ($L = 3/\Lambda$)	1.35×10^{26} m ($L = \sqrt{2}/\Lambda$)	98.5%
Growth Rate ($f\sigma_8$)	0.76 ± 0.04	0.83 (Planck Tension) [6]	$0.75 - 0.77$	Alleviates Tension
CMB Power Shift / ISW	Phase Consistent	Requires DE Fit [6]	Phase Anchored to $\Pi_{\mu\nu}$	Consistent
BAO Scale Evolution	150 Mpc	150 Mpc (Numerical)	148 Mpc	Compatible
Cosmic Event Horizon	$\sim 1.3 \times 10^{26}$ m	Depends on Ω_Λ Fit [6]	1.35×10^{26} m	Consistent
CMB 1st Acoustic Peak	$l \approx 220$	$l \approx 220$ (Numerical) [6]	$l \approx 215$	Approximate match
Horizon Temp.	$\sim 2.7 \times 10^{-30}$ K	Input Parameter	2.7×10^{-30} K [8]	Theoretical agreement

4. Covariant Exchange and Bianchi-Identity Modification

In Special Relativity (SR), global energy conservation is guaranteed by Noether’s theorem because the fixed background spacetime (Minkowski space) possesses global time and space translation invariance. In contrast, standard General Relativity (GR) is background independent; the metric is dynamic, and there are no global symmetries to define a global energy conservation law, leading only to the local covariant conservation $\nabla^\mu T_{\mu\nu} = 0$. CCFEs, however, reintroduces a reference background metric $\tilde{g}_{\mu\nu}$. This structure establishes a covariant exchange mechanism between the localized matter

sector and the geometric background tension. Open-System Description: The theory functions as an open-system description. This entails three key physical consequences:

1. Exchange with Vacuum Geometry: Apparent non-conservation of the matter sector ($\nabla^\mu T_{\mu\nu} \neq 0$) is physically interpreted as a covariant exchange of energy-momentum with the background vacuum geometry.
2. Covariant Balance: While matter is not conserved in isolation ($\nabla^\mu T_{\mu\nu} \neq 0$), the total system (Matter + Vacuum Stress + Geometric Slip) satisfies a strict covariant balance equation.
3. Local Suppression: Violations of standard conservation are suppressed locally by factors of r/L_c ensuring that the theory remains indistinguishable from standard physics at solar system scales.

4.1. Divergence of the CCFEs

To ensure mechanical consistency, we derive the associated balance condition by taking the covariant divergence ∇^μ . Applying ∇^μ to Eq. (3) yields the generalized balance condition:

$$\nabla^\mu \left[\frac{G_{\mu\nu}}{\mathcal{R}} - \frac{R\mathcal{R}_{\mu\nu}}{\mathcal{R}^2} + K\tau_{\mu\nu}^{geo} \right] = \nabla^\mu \left[\frac{T_{\mu\nu} + K\tau_{\mu\nu}^{vac}}{\mathcal{T}} \right] \quad (8)$$

Here $\mathcal{R}$ and $\mathcal{T}$ are scalar fields, encoding the exchange between Cloud-localized and bulk degrees of freedom.

4.2. Constitutive Exchange Equation

Using the identity $\nabla^\mu G_{\mu\nu} = 0$, the remaining terms must satisfy a dynamical equilibrium relation:

$$\nabla^\mu (K\tau_{\mu\nu}^{geo}) - \nabla^\mu \left(\frac{R\mathcal{R}_{\mu\nu}}{\mathcal{R}^2} \right) = \frac{1}{\mathcal{T}} [\nabla^\mu T_{\mu\nu} + \nabla^\mu (K\tau_{\mu\nu}^{vac})] \quad (9)$$

This equation defines the generalized exchange dynamics. The matter stress-energy tensor is not conserved in isolation, $\nabla^\mu T_{\mu\nu} \neq 0$, but only when combined with the vacuum contribution.

The mechanical Interpretation: within this framework, inertia admits a mechanical interpretation as a back-reaction of the vacuum fabric. Accelerated matter induces changes in the vacuum's extrinsic surface tension, providing resistance to acceleration.

4.3. Proof of Local Lorentz Invariance in Trace-3 Cosmology

To investigate that the Trace-3 condition preserves Local Lorentz Invariance (LLI) and avoids preferred-frame effects:

1. Covariance of the Trace Constraint: The effective trace is defined as:

$$\text{Tr}(g_{eff}) = g^{\mu\nu}(g_{\mu\nu} - \sigma\tilde{g}_{\mu\nu}) = 4 - \sigma\phi(x) = 3 \quad (10)$$

where $\phi(x) = g^{\mu\nu}\tilde{g}_{\mu\nu}$. Since the contraction of two tensors $g^{\mu\nu}\tilde{g}_{\mu\nu}$ yields a scalar, the constraint $\phi(x) = 4$ is coordinate-independent.

2. Local Flatness (Equivalence Principle): At any spacetime point P , the Equivalence Principle allows the choice of a Local Inertial Frame (LIF) where $g_{\mu\nu} \rightarrow \eta_{\mu\nu}$ and $\partial g \rightarrow 0$. In this frame:

$$\eta^{\mu\nu} \left(\eta_{\mu\nu} - \frac{1}{4}\tilde{g}_{\mu\nu} \right) = 3 \implies \eta^{\mu\nu}\tilde{g}_{\mu\nu} = 4 \quad (11)$$

This requires the background metric to be locally conformal to the physical metric.

3. Boost Invariance: The "anchoring" of the temporal degree applies to the metric trace, which is a Lorentz invariant:

$$\text{Tr}(g') = \text{Tr}(\Lambda g \Lambda^{-1}) = \text{Tr}(g) \quad (12)$$

Therefore, the condition $\text{Tr}(g_{eff}) = 3$ holds for all inertial observers, ensuring no preferred reference frame exists.

4.4. Absence of Bimetric Instabilities (The "Ghost" Problem)

Linearity of the Kinetic Term (Ostrogradsky Stability): The extended action depends linearly on the Ricci scalar R : $S_{geo} \propto \int \sqrt{-g} R d^4\rho$. This guarantees that the equations remain second-order, satisfying the Ostrogradsky theorem. **The Trace Constraint as a Ghost-Killing Mechanism:** The BD ghost is eliminated by the Trace-3 condition $\mathcal{C}(g, \tilde{g}) = g^{\mu\nu} \tilde{g}_{\mu\nu} - 4 = 0$, which explicitly eliminates the scalar degree of freedom (breathing mode) that would otherwise appear as a ghost. **Hamiltonian Linearity:** In the ADM decomposition, the background softening ($\sigma = 1/4$) effectively decouples the lapse N from the interaction. The Hamiltonian constraint $\mathcal{H}_0$ remains linear in the lapse, removing the non-physical longitudinal mode.

4.5. Resolution of the Trace-3: Recovering standard GR Locally

1. **The Vainshtein Radius (r_V):** The transition is governed by $r_V = (r_s L^2)^{1/3}$. For the Sun, $r_V \approx 100$ parsecs. The Solar System resides deep within this radius ($r \ll r_V$).
2. **Non-Linear Kinetic Suppression:** The interaction term K introduces non-linear derivative interactions for the scalar mode π :

$$\mathcal{E}_\pi = 3\Box\pi + \frac{1}{\Lambda_g^3} \left[(\Box\pi)^2 - (\nabla_\mu \nabla_\nu \pi)^2 \right] = \frac{T}{M_{pl}^2} \quad (13)$$

3. **Recovery of Trace-4 (GR) Potentials:** The magnitude scales as:

$$\frac{\delta\Phi_\pi}{\Phi_N} \approx \left(\frac{r}{r_V} \right)^{3/2} \quad (14)$$

Inside the Solar System, this ratio is $\sim 10^{-12}$, rendering the modification negligible.

4.6. Gravitational Wave Speed (Compliance with GW170817)

1. **Perturbation Analysis:** Consider tensor perturbations $h_{\mu\nu}$. In TT gauge:

$$h = \tilde{g}^{\mu\nu} h_{\mu\nu} = 0, \quad \nabla^\mu h_{\mu\nu} = 0 \quad (15)$$

2. **Decoupling:** Linearized Ricci scalar perturbation δR vanishes for tensor modes. In the high-frequency limit:

$$\delta Q_{\mu\nu} \approx \frac{1}{2}(\delta R)(\dots) + \frac{1}{2}\tilde{R}(\delta g_{\mu\nu} \dots) \approx 0 \quad (16)$$

3. **Standard Dispersion:** The equation reduces to $\delta R_{\mu\nu} \approx -\frac{1}{2}\Box h_{\mu\nu} = 0$, yielding:

$$c_{gw} = \frac{\omega}{k} = c \quad (17)$$

4.7. Derivation of Scalar Perturbations

1. **Decomposition:** $\gamma \equiv 1/L_p^4$ and $g_{\mu\nu} = e^{2\pi(x)} \tilde{g}_{\mu\nu} \approx (1 + 2\pi) \tilde{g}_{\mu\nu}$. This connects to Newtonian gauge via $\Phi = \Phi_{GR} + \pi$ and $\Psi = \Psi_{GR} - \pi$.
2. **Variation:**

$$S_{geo}^{(2)} = -\frac{3}{\mathcal{R}} \int d^4x \sqrt{-\tilde{g}} (\tilde{\nabla}_\mu \pi \tilde{\nabla}^\mu \pi) \quad (18)$$

Variation yields $\frac{\delta S_{geo}}{\delta \pi} = \frac{6}{\mathcal{R}} \sqrt{-\tilde{g}} \Box \pi$.

3. **Matter Coupling:** $\delta S_m = \int d^4x \sqrt{-g} T \delta \pi$ where T is the trace.

4. Equation of Motion:

$$\tilde{\square}\pi = -\frac{\gamma\mathcal{R}}{6}T \quad (19)$$

Feeding into Poisson: $\nabla^2\Phi = 4\pi G\rho + \frac{\gamma\mathcal{R}}{6}\rho = 4\pi G_{eff}\rho$. Slip: $\Psi - \Phi = -2\pi$.

5. Analysis: Vacuum Energy Density Modulus (E_D)

The unified coupling constant yields:

$$G_{\mathcal{R}} = \left(\frac{c^4}{4\pi}\right)\frac{\mathcal{R}}{E_D} \quad (20)$$

Using the experimental G range [7], the discrepancy could represent a residual metric anisotropy.

6. Summary

The CCFEs framework alleviates the Hubble tension by applying Background Softening ($\sigma = 1/4$), identifying a Trace-3 manifold where $R = 6\Lambda_g$ and $H^2 = \Lambda/2$. This alignment recovers observed cosmological data with 98.5% accuracy [1, 6].

Appendix A. Derivation of Geometric Horizon Temperature (T_g)

$$T_g = \frac{\hbar c}{2\pi k_B L} \approx 2.70 \times 10^{-30} K \quad (A1)$$

Appendix B. Cosmological Perturbation Calculations

The growth index $\gamma_{cloud} \approx 0.55 + 0.125 = 0.675$ yields $S_8 \approx 0.77$ aligning with observations [6].

$$\Omega_{\Lambda} = \frac{\Lambda_g}{3(H^2)} = \frac{2}{3} \approx 0.667; \quad \Omega_m = 1 - \frac{2}{3} = \frac{1}{3} \approx 0.333 \quad (B1)$$

Appendix C. The Acceleration Transition

Acceleration begins when $\rho_m = 2\rho_{\Lambda}$

$$\Omega_{m0}(1+z)^3 = 2\Omega_{\Lambda0} \Rightarrow (1+z)^3 = 4.0 \Rightarrow z_t \approx 0.587 \quad (C1)$$

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