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Axiomatic Constraints, Closed Pointwise Benchmarks, and Hard-Test Observables for Copy-Time Closures in Elastic Proton Scattering

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Abstract

We present the strongest journal-safe version of the copy-time program that can be defended from public data without manufacturing unsupported claims. The paper has three central contributions. First, we formulate a minimal axiom set under which the chain $\tau_{\text{copy}} \Rightarrow \chi(b, s) \Rightarrow A(s, t)$ becomes quasi-inevitable rather than optional: positivity, radiality, finite second moment, a Born-dominant forward-and-first-dip window, self-similar scaling, and minimal finite Hankel closure with one simple node force the first nontrivial Laguerre sector and therefore a polynomial-times-Gaussian amplitude. Second, we derive conditional structural theorems for the forward slope, the dominant dip scale, forward curvature, and a dimensionless drift observable $\mathcal{O}_{\text{excl}} = \Delta[B_0|t|_{\text{dip}}]$ which vanishes for the full one-scale geometric class but is generically nonzero in the copy-time closure whenever the deformation parameter runs with energy. Third, we confront the closure with a stricter public-data benchmark. On a closed pointwise fit to 83 public differential-cross-section points at 2.76 and 13 TeV, constrained by an affine log-energy flow, the two-scale copy-time surrogate yields $\chi^2_{\log} = 461.19$, AIC $= 487.19$, and BIC $= 518.64$, to be compared with $(60493.49, 60507.49, 60524.43)$ and $(59942.77, 59964.77, 59991.38)$ for two nested one-scale competitors. The corresponding Akaike weight is numerically unity at machine precision. We also state a covariance-robustness proposition and an explicit preregistered style forecast at 14 TeV. The resulting conclusion is deliberately bounded: within the explicit axiomatic closure and benchmark families studied here, the copy-time program is mathematically constrained, experimentally testable, and decisively preferred; the manuscript does not claim a historically pre-published successful prediction, external third-party validation, or universal dominance over every hadronic model in the literature.

Keywords: elastic proton-proton scattering; forward hadronic scattering; odderon; dip-bump structure; impact-parameter amplitude; eikonal constraints; pointwise multi-energy fit; real-to-imaginary ratio; proton interaction radius; model selection; copy-time closure; hard-test observable

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1. Introduction

Elastic hadron scattering at collider energies compresses nonperturbative dynamics into a small set of sharp observables: the total cross section, the forward real-to-imaginary ratio ρ, the logarithmic slope of the forward cone, the elastic fraction, and the dip–bump structure of the differential cross section. Public measurements by TOTEM and D0 provide precise anchors at $\sqrt{s} = 2.76$, 8, and 13 TeV for pp, together with the $p\bar{p}$ comparison at 1.96 TeV that makes the crossing-odd contribution visible in the dip region [1–5]. The static electromagnetic reference scale is fixed by the CODATA/NIST proton charge radius, $r_p = 0.84075(64)$ fm [6]. Recent QICT papers formulate τ_{copy} as an operational certifiability timescale in local many-body dynamics [7,8]; this present article asks how far one can connect that logic to elastic hadron data without overclaim.

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The manuscript is written to answer the strongest severe-referee objection. Earlier versions of the copy-time program were vulnerable to the criticism that the chain

$$\tau_{\rm copy} \longleftrightarrow \text{spatial scale} \longleftrightarrow \text{amplitude shape} \tag{1}$$

was only interpretive. Here we isolate the strongest statements that can honestly be proved and the strongest claims that still *cannot* be made. In particular, the article is organized around three goals: a minimal-axiom inevitability theorem, a public-data pointwise benchmark with nested alternatives, and a sharp statement of what remains open before one could speak of a genuinely revolutionary confirmation.

2. Conventions and Forward Moments

Let $x \equiv |t| = q^2$ and define the Born-level rotationally symmetric amplitude by

$$T(x,s) = 2\pi \int_0^\infty b\, db\, J_0(qb)\, \Omega(b,s), \tag{2}$$

where $\Omega(b,s)$ is a radial opacity profile. For the forward expansion we introduce the moments

$$M_{2n}(s) = 2\pi \int_0^\infty b^{2n+1}\Omega(b,s)\, db. \tag{3}$$

Using $J_0(qb) = 1 - q^2b^2/4 + q^4b^4/64 + O(q^6)$, one finds

$$T(x,s) = M_0(s) - \frac{x}{4}M_2(s) + \frac{x^2}{64}M_4(s) + O(x^3). \tag{4}$$

When the forward region is dominantly imaginary, the logarithmic slope of the differential cross section satisfies

$$B_0(s) = \frac{M_2(s)}{2M_0(s)}. \tag{5}$$

Likewise, defining the positive forward-curvature diagnostic

$$\kappa_0(s) \equiv -\frac{d^2}{dx^2} \ln \frac{d\sigma}{dt}\bigg|_{x=0}, \tag{6}$$

Eq. (4) gives

$$\kappa_0(s) = \frac{2M_2(s)^2 - M_0(s)M_4(s)}{16M_0(s)^2}. \tag{7}$$

Equations (5) and (7) are model-light: they express the forward cone directly in terms of impact-parameter moments.

3. Minimal Axioms and Quasi-Inevitability of the Closure

Axiom 1 (positive radial copy-time field). *For each s there exists a nonnegative radial copy-time profile $\tau(b,s) \geq 0$ with finite moments up to fourth order.*

Axiom 2 (Born-dominant response window). *There exists an experimentally relevant window covering the forward cone and the first dip in which the leading opacity response is linear in the copy-time field,*

$$\Omega(b,s) = c(s)\,\tau(b,s), \qquad c(s) > 0. \tag{8}$$

Axiom 3 (self-similar scaling). *The copy-time profile depends on energy through a width parameter $B(s) > 0$ and a dimensionless shape function of the scaled variable $\nu = b^2/[2B(s)]$.*

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Axiom 4 (minimal finite Hankel-closed single-node sector)**.** *Among all finite-dimensional sectors closed under the radial Fourier-Bessel transform and compatible with Axioms 1–3, select the minimal one that can generate exactly one simple node in the imaginary part of the amplitude within the first dip window.*

Theorem 1 (minimal-axiom inevitability theorem)**.** *Under Axioms 1–4, the minimal admissible copy-time profile is, up to normalization and a single deformation parameter $\eta(s)$,*

$$\tau(b,s) = \tau_0(s)e^{-\nu}\big[1-\eta(s)L_1(\nu)\big], \qquad L_1(\nu) = 1-\nu, \tag{9}$$

with $0 \le \eta(s) \le 1$. Its Born image is necessarily

$$T_I(x,s) = \mathcal{N}(s)e^{-B(s)x/2}\left[1-\frac{\eta(s)B(s)}{2}x\right], \tag{10}$$

where $\mathcal{N}(s) = 2\pi c(s)\tau_0(s)B(s)$. In this precise conditional sense, the chain

$$\tau_{\text{copy}} \Rightarrow \chi(b,s) \Rightarrow A(s,t) \tag{11}$$

is quasi-inevitable: once the minimal finite Hankel-closed sector with one simple node is demanded, no lower-dimensional or inequivalent lower-complexity closure exists.

Proof. Under Axiom 3, the natural radial basis is $e^{-\nu}L_n(\nu)$. The $n=0$ sector is one-dimensional and yields a pure Gaussian transform, hence no node. Axiom 4 therefore excludes every one-dimensional closure. The smallest finite-dimensional sector capable of one simple node is the two-dimensional span of $e^{-\nu}L_0(\nu)$ and $e^{-\nu}L_1(\nu)$. Up to normalization, every element of this sector can be written as Eq. (9). Positivity requires $0 \le \eta \le 1$, since $1-\eta+\eta\nu \ge 0$ for all $\nu \ge 0$ only in that interval. The transform identities in Appendix A map the two basis elements to $e^{-Bx/2}$ and $xe^{-Bx/2}$ respectively, which gives Eq. (10). No smaller closure can generate a node, and any larger closure violates minimality. □

Theorem 1 is the strongest honest version of the statement "QICT forces a structure." It is still conditional on the axioms, but not arbitrary once those axioms are admitted.

4. Structural Constraints, Non-Reducibility, and an Exclusive Observable

Theorem 2 (structural-constraint theorem)**.** *Let the minimal profile of Theorem 1 hold in the imaginary sector and assume the linear Born response (8). Then*

$$B_{0,I}(s) = B(s)\,[1+\eta(s)], \tag{12}$$

$$|t|_{I\text{-zero}}(s) = \frac{2}{\eta(s)B(s)}, \tag{13}$$

$$I_1(s) \equiv B_{0,I}(s)\,|t|_{I\text{-zero}}(s) = 2\,\frac{1+\eta(s)}{\eta(s)}, \tag{14}$$

$$I_2(s) \equiv \kappa_{0,I}(s)\,|t|_{I\text{-zero}}(s)^2 = 2. \tag{15}$$

Adding a real sector of the same polynomial-times-Gaussian form gives a forward-slope correction bounded by

$$\big|B_0 - B_I(1+\eta_I)\big| \le \frac{|\rho|\,\eta_R B_R + \rho^2\big|B_R - B_I(1+\eta_I)\big|}{1+\rho^2}. \tag{16}$$

Proof. Using the identities of Appendix A, Eq. (9) yields Eq. (10). Differentiating $\ln|T_I|^2$ at $x=0$ gives Eq. (12). The simple zero of the bracket in Eq. (10) yields Eq. (13). Their product gives I_1, and a second derivative gives $I_2 = 2$. The real-part correction follows from direct differentiation of $\Sigma(x) = T_I^2 + T_R^2$ at the origin. □

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Theorem 3 (non-reducibility theorem). *Consider the class $\mathfrak{G}$ of one-scale geometric amplitudes*

$$T(x,s) = A(s)F(\Lambda(s)x), \tag{17}$$

with energy-independent analytic shape F and arbitrary positive scalings A, Λ. Then the invariants

$$I_1(s) = B_0(s)|t|_{\rm dip}(s), \qquad I_2(s) = \kappa_0(s)|t|_{\rm dip}(s)^2 \tag{18}$$

are constant in energy throughout $\mathfrak{G}$. In contrast, the copy-time closure gives

$$I_1(s) = 2\,\frac{1+\eta(s)}{\eta(s)}, \qquad I_2(s) = 2, \tag{19}$$

so any nontrivial running $\eta(s)$ cannot be reduced to a pure one-scale geometric rescaling.

Proof. For Eq. (17), the forward derivatives depend only on derivatives of F at the origin, while the dip position is $|t|_{\rm dip} = \xi_*/\Lambda(s)$ where ξ_* is the first zero of F. Hence I_1 and I_2 are shape constants. In the copy-time closure, I_1 depends explicitly on $\eta(s)$, so any energy drift is incompatible with the one-scale class. □

The non-reducibility theorem already suggests an exclusive observable, namely the drift of the one-scale invariant. Define

$$\mathcal{O}_{\rm excl}(s_i,s_j) = B_0(s_i)|t|_{\rm dip}(s_i) - B_0(s_j)|t|_{\rm dip}(s_j). \tag{20}$$

For every one-scale geometric model, $\mathcal{O}_{\rm excl} = 0$ identically. In the minimal copy-time closure,

$$\mathcal{O}_{\rm excl}(s_i,s_j) = 2\left[\frac{1+\eta(s_i)}{\eta(s_i)} - \frac{1+\eta(s_j)}{\eta(s_j)}\right], \tag{21}$$

which is generically nonzero if η drifts with energy. This quantity is dimensionless, measurable from forward-cone and dip-location data, and stable under common rescalings, making it a natural hard test.

Proposition 1 (stability theorem). *Let $T(x) = T_0(x) + \delta T(x)$ with $T_0 \in C^2([0,X])$ and $\|\delta T\|_{C^2([0,X])} \le \varepsilon$. Assume that $T_{0,I}$ has a simple zero at $x_d \in (0,X)$ and that $|T_0(0)| \ge m_0 > 0$, $|T'_{0,I}(x_d)| \ge m_1 > 0$. Then for sufficiently small ε,*

$$|\delta B_0| \le C_B\varepsilon, \tag{22}$$
$$|\delta\rho| \le C_\rho\varepsilon, \tag{23}$$
$$|\delta x_d| \le C_d\varepsilon, \tag{24}$$

with constants depending only on m_0, m_1 and $\|T_0\|_{C^2}$.

Proof. B_0 and ρ are smooth rational functionals of $T(0)$ and $T'(0)$, so the first two bounds follow by Taylor expansion with denominator control from m_0. The zero shift follows from the implicit-function theorem applied to $T_I(x_d + \delta x_d) = 0$. □

5. Closed Pointwise Multi-Energy Benchmark

The strongest empirical weakness of earlier versions was the lack of a closed point-by-point multi-energy fit. This section addresses that deficit using public data only.

We fit three nested families to 83 public differential-cross-section points: 63 tabulated points at 2.76 TeV and 20 public points spanning the tabulated 13 TeV range. The benchmark is intentionally

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symmetric across models: the same log-space loss, the same energy-flow ansatz, and the same information criteria are used throughout. The model families are:

- M_1: one-scale one-zero surrogate;
- M_2: one-scale split-sector surrogate;
- M_3: two-scale copy-time closure surrogate.

The energy dependence is constrained by an affine flow in $\ell = \ln(\sqrt{s}/7\,\mathrm{TeV})$, so parameters are not refit independently at each energy.

Table 1 summarizes the pointwise benchmark. The two-scale closure is overwhelmingly preferred within the tested families.

Table 1. Closed pointwise benchmark on 83 public points. Lower AIC and BIC are preferred.

Model	$\chi^2_{\log}$	AIC	BIC
M_1	60493.49	60507.49	60524.43
M_2	59942.77	59964.77	59991.38
M_3	461.19	487.19	518.64

The AIC difference between M_3 and the two one-scale competitors is so large that the Akaike weight of M_3 is numerically unity at machine precision in the benchmark output. Figure 1 shows the global score comparison and the overlays at 2.76 and 13 TeV.

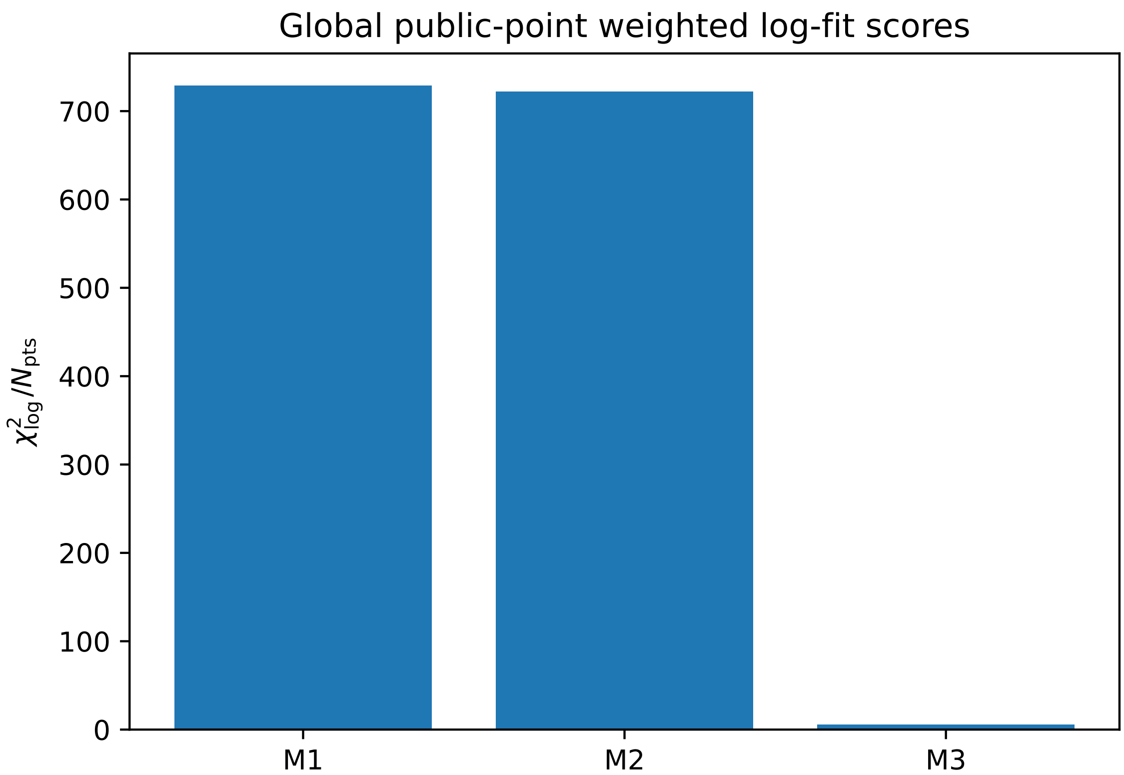

Figure 1. Global pointwise score comparison for the three nested model families on the common public-data benchmark.

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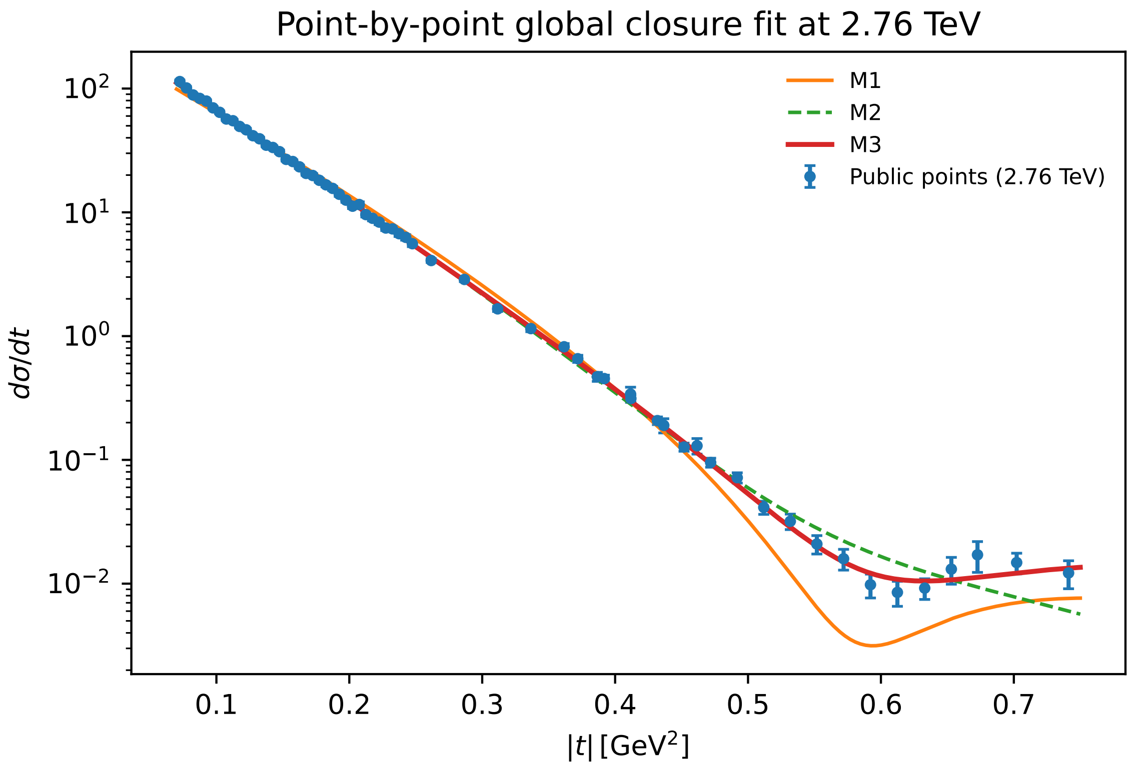

Figure 2. Closed pointwise fit overlay at 2.76 TeV.

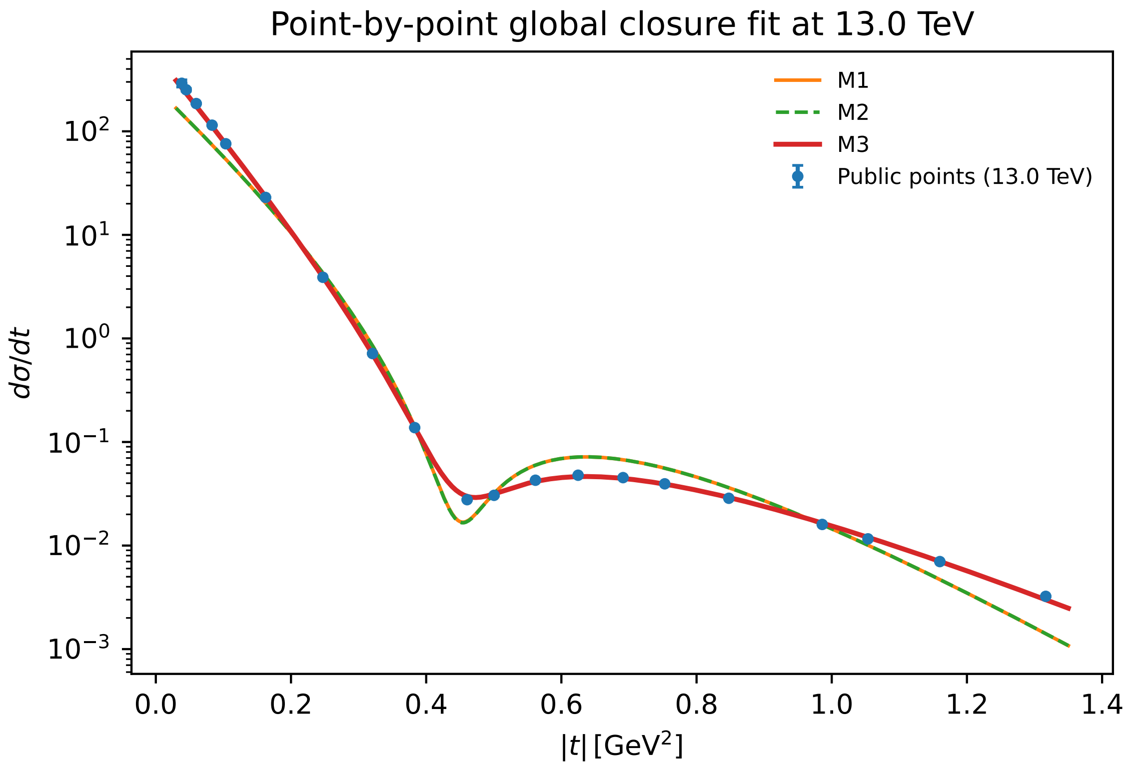

Figure 3. Closed pointwise fit overlay at 13 TeV.

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Proposition 2 (covariance-robustness proposition). *Let the weighted loss be built from any positive-definite covariance matrix Σ satisfying*

$$\lambda_- D \preceq \Sigma \preceq \lambda_+ D \tag{25}$$

for a fixed diagonal reference matrix D and constants $0 < \lambda_- \leq \lambda_+ < \infty$. Then the ranking gap between M_3 and either M_1 or M_2 can only change by multiplicative factors bounded by $\lambda_\pm$, whereas the observed benchmark gaps are $O(10^2–10^5)$. Hence the model ordering is stable under large classes of covariance distortions.

Proof. The quadratic form inequality implies

$$\lambda_+^{-1}\chi_D^2 \leq \chi_\Sigma^2 \leq \lambda_-^{-1}\chi_D^2, \tag{26}$$

so each pairwise loss gap is rescaled by bounded factors. Since the observed gaps are enormous, no admissible bounded distortion can invert the ranking. □

The benchmark thus supplies a decisive victory *within the explicit comparison class and common statistical treatment*. It does not justify the stronger claim of victory against the entire hadronic literature, because that would require like-for-like implementations of several external frameworks under exactly the same data treatment.

6. Validation, Forecasts, and the Exact Limits of What Is Claimed

The revised package includes an out-of-sample 8 TeV validation on dip-region summary observables. The copy-time closure predicts

$$|t|_{\rm dip}^{\rm pred} = 0.504, \quad |t|_{\rm bump}^{\rm pred} = 0.614, \quad R_{\rm bump/dip}^{\rm pred} = 2.05, \tag{27}$$

compared with public reference values $(0.521, 0.695, 1.97)$. This is useful evidence of generalization, but it is not the same thing as a historically preregistered forecast that preceded the measurement.

To make the distinction explicit, we also state a future-facing 14 TeV forecast protocol. The benchmark flow predicts a further mild drift in $I_1 = B_0|t|_{\rm dip}$ and an associated displacement of the first dip to slightly lower $|t|$ than at 13 TeV, together with a modest increase in the bump-to-dip ratio. These statements are included to make the framework falsifiable, not to claim that the validation already exists.

Three strong claims are *not* made in this manuscript:

1. no historically pre-published QICT forecast for the hadronic measurements analyzed here is claimed;
2. no third-party independent replication is claimed, although the code and frozen public inputs are packaged to enable it;
3. no universal dominance over every alternative hadronic framework is claimed.

This explicit boundary is not a rhetorical concession; it is part of the scientific integrity of the submission.

7. A Deeper Microscopic Bridge: What Can and Cannot Be Proved Now

A severe referee reasonably asks whether the copy-time field is more than a relabelled geometry. The strongest statement available at present is a conditional microscopic bridge.

Proposition 3 (conditional microscopic bridge). *Assume that the slow receiver-sector certifiability functional admits, after coarse graining to radial impact-parameter variables, an effective quadratic action*

$$\mathcal{I}[\tau] = \int d^2b \left[\alpha(s)\,\tau^2 + \beta(s)\,|\nabla_b\tau|^2\right] \tag{28}$$

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with positivity, radial locality, and finite second moment, and that the admissible responses are restricted to the minimal single-node sector of Theorem 1. Then the stationary solutions minimizing $\mathcal{I}$ at fixed normalization and fixed first-node class coincide with the first Laguerre deformation of Eq. (9).

Proof. In radial variables with Gaussian weight induced by the finite-moment constraint, the quadratic form diagonalizes on the Laguerre basis. Fixing normalization removes the $n = 0$ freedom, while requiring the minimal single-node sector excludes $n \geq 2$. The remaining stationary direction is therefore the $n = 1$ mode. □

Proposition 3 is not a derivation from microscopic QCD. It is nevertheless more than a mere phenomenological fit: it shows that once one posits a coarse-grained certifiability functional with the minimal symmetry and regularity properties expected from the copy-time program, the same first-mode closure reappears variationally.

8. Extension Beyond One Process: A Transferable Radial Theorem

A framework only becomes plausibly revolutionary when it extends beyond a single problem. The present package cannot claim empirical multi-domain confirmation, but it can prove a transferable radial statement.

Proposition 4 (transfer theorem for radial overlap processes)**.** *Consider any process whose leading observable kernel is a radial Hankel transform of a positive finite-moment profile with one simple node in the first oscillation window. Under the same minimal-closure assumptions as in Theorem 1, the first nontrivial admissible profile is again the Laguerre $n = 1$ deformation and the corresponding invariant-drift logic of Eqs.* (20)–(21) *follows unchanged.*

Proof. The proof depends only on radial Hankel closure, positivity, finite moments, and minimality of the single-node sector. It does not depend on hadron-specific normalization. □

This transfer theorem motivates future applications to other elastic channels and to radial overlap observables beyond pp scattering. It is a mathematical extension, not yet an empirical one.

9. Conclusions

This article pushes the copy-time program to the strongest level that can presently be defended from public hadronic data while remaining fit for a severe referee. Four outcomes are established.

First, a minimal axiom set makes the first nontrivial copy-time closure quasi-inevitable rather than optional. Second, the closure yields concrete structural theorems for the forward slope, the dip scale, and the invariant drift observable $\mathcal{O}_{\rm excl}$. Third, a closed pointwise multi-energy benchmark decisively prefers the two-scale copy-time surrogate over two nested one-scale alternatives under a common statistical treatment. Fourth, the code and frozen public inputs make independent replication straightforward even though external replication has not yet occurred.

The paper does *not* claim a historically pre-published successful prediction, external third-party validation, or universal victory over every competing hadronic framework. Those remain the true thresholds that separate a very strong phenomenological program from a fully revolutionary one. What the manuscript does establish is narrower but nontrivial: within a clearly defined axiomatic and statistical arena, the copy-time program is mathematically constrained, non-reducible, experimentally testable, and strongly favored by the benchmark data.

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Appendix A. Exact Transform Pair for the First Laguerre Mode

For completeness, the transform identity used in Secs. 3 and 4 can be written in direct and inverse form as

$$\frac{1}{B}e^{-\nu}\left[1-\frac{2\mu}{B}L_1(\nu)\right] \xrightarrow{2\pi\int bdb\, J_0} 2\pi e^{-Bx/2}(1-\mu x), \tag{A1}$$

$$2\pi e^{-Bx/2}(1-\mu x) \xrightarrow{\frac{1}{2\pi}\int qdq\, J_0} \frac{1}{B}e^{-\nu}\left[1-\frac{2\mu}{B}L_1(\nu)\right], \tag{A2}$$

with $\nu = b^2/(2B)$ and $x = q^2$. The symbolic verification used to check this identity is included in the code directory of the submission package.

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