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Posted Date: 1 April 2025

doi: 10.20944/preprints202503.2057.v2

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Observer-Dependent Entropy Retrieval and Time-Adaptive Information Recovery in Black Hole Evolution

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Abstract: Background: The black hole information paradox arises from the tension between unitarity in quantum mechanics and apparent information loss in semiclassical gravity. **Methods:** We develop a relativistic framework in which entropy retrieval is an explicit function of an observer's proper time and trajectory. By defining observer-dependent density matrices and a continuous retrieval law, we derive testable expressions for Rényi entropy and second-order correlation functions. **Results:** Our model predicts measurable deviations in the entanglement structure of Hawking radiation across observer classes and modifies the Ryu–Takayanagi prescription to include frame-dependent surface contributions. **Conclusions:** The information paradox resolves as an observer-dependent phenomenon without requiring nonunitary dynamics or exotic physics. These experimentally accessible signatures offer a pathway toward empirical validation in analog black hole experiments, advancing the testability of quantum gravity frameworks.

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1. Introduction

The black hole information paradox remains a central challenge in theoretical physics: unitarity implies information preservation while semiclassical gravity suggests its destruction. Recent work has highlighted observer-dependence in entropic measures [3], yet a concrete retrieval mechanism remains elusive. Existing proposals—including Black Hole Complementarity [14], ER=EPR [9], and the Page curve [8,10]—do not quantify how information becomes accessible to different observers over time. Here, we introduce a relativistic, proper-time–dependent retrieval framework that resolves the paradox as an emergent, observer-dependent phenomenon, and we derive explicit, falsifiable predictions for entanglement signatures in Hawking radiation.

1.1. Observer-Dependent Frameworks: Context and Innovation

Observer-dependence in quantum gravity has precedents in the literature—from the complementarity principle [14] to gravitational quantum error correction [3] and algebraic quantum field theory in curved spacetime [4,15]. However, while these approaches recognize that different observers may access different information, they typically remain static or instantaneous in their formulation. Our contribution is threefold:

1. We establish entropy retrieval as explicitly time-adaptive, evolving with proper time along observer worldlines.
2. We provide a concrete mathematical framework that quantifies information accessibility across different reference frames.
3. We derive experimentally accessible predictions for analog systems that can validate the approach.

This dynamic formulation distinguishes our work from previous observer-dependent approaches that remain primarily conceptual or limited to specific regimes.

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1.2. Comparison with Existing Frameworks

Table 1 summarizes how our approach compares with existing frameworks.

Table 1. Comparative analysis of black hole information paradox frameworks.

Framework	Retrieval Mechanism	Observer-Dependence	Dynamics	Testability
Page Curve [8,10]	Global entropy evolution	No	Static	Indirect
BH Complementarity [14]	Event-horizon postulates	Partial	Conceptual	Not testable
Firewalls [1]	Entanglement breakdown	No	Abrupt	Speculative
ER=EPR [9]	Wormhole entanglement	No	Nonlocal	Unverifiable
This Work	τ-dependent retrieval	Yes	Continuous	Analog, measurable

Unlike these alternatives, our observer-dependent retrieval framework provides a continuous, dynamical description that preserves both unitarity and general covariance without invoking exotic structures. Moreover, it yields specific predictions testable in analog gravity systems.

2. Observer-Dependent Entropy in Curved Spacetime

2.1. Classification of Observers

Entropy retrieval depends on the observer's trajectory through spacetime. We consider three key observer classes:

Stationary Observer

Positioned at fixed spatial coordinates outside the event horizon ($r > 2M$), this observer detects Hawking radiation as thermal emission with gradually decreasing temperature. The retrieval rate follows $\gamma(\tau) \propto 1/r$, due to gravitational redshift, resulting in slower information acquisition at greater distances.

Freely Falling Observer

Crossing the event horizon along geodesics, this observer experiences no local discontinuity at the horizon but encounters different information accessibility. Inside the horizon, nonthermal corrections encode correlations with earlier emissions, providing access to information inaccessible to distant observers until later.

Accelerating Observer

Moving with proper acceleration a, this observer experiences Unruh radiation that modifies the perceived entropy. The effective retrieval rate includes contributions from both Hawking and Unruh effects:

$$\gamma_{\text{eff}}(\tau) := \gamma_{\text{Hawking}}(\tau) + \gamma_{\text{Unruh}}(\tau, a). \tag{1}$$

This superposition creates distinctive interference patterns in the correlation functions.

2.2. Definition of Observer-Dependent Entropy

We define the observer-dependent entropy as

$$S_{\text{obs}}(\tau) := -\operatorname{Tr}\Big[\rho_{\text{obs}}(\tau)\, \log\, \rho_{\text{obs}}(\tau)\Big], \tag{2}$$

where the observer's density matrix transforms under a Lorentz boost:

$$\rho_{\text{obs}} := U(\Lambda)\, \rho\, U^{\dagger}(\Lambda). \tag{3}$$

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This transformation reflects the genuine information-theoretic limitations on accessible quantum correlations.

2.3. Proper-Time Evolution of Information Retrieval

We propose a dynamical law for retrieval:

$$\frac{dS_{\text{retrieved}}}{d\tau} = \gamma(\tau)\left[S_{\max} - S_{\text{retrieved}}\right]\frac{1+\tanh\left(\frac{\tau}{\tau_{\text{Page}}}\right)}{2}. \quad (4)$$

The hyperbolic tangent transition smoothly interpolates across the Page time τ_{Page}, reflecting fundamental thermodynamic principles.

3. Quantum Information Correlations and Testable Predictions

The retrieval law in Equation (4) imprints on the statistical properties of radiation detected by observers. We derive expressions for Rényi entropy and second-order correlation functions.

3.1. Rényi Entropy and Second-Order Correlation Functions

The Rényi entropy is defined as

$$I(t) := \frac{1}{n-1}\log \operatorname{Tr}\left[\rho_A^n\right], \quad (5)$$

where ρ_A is the reduced density matrix for subsystem A. The second-order correlation function is given by

$$g^{(2)}(t_1,t_2) := \exp\left[-\frac{|t_2-t_1|}{\tau_{\text{retrieval}}}\right]\left[1+\frac{1+\tanh(t_1/\tau_{\text{Page}})}{2}\right]. \quad (6)$$

This function arises from the evolution of the density matrix and is related to the response function of an Unruh–DeWitt detector:

$$\mathcal{R}(\Omega) \propto \int dt_1\, dt_2\, e^{i\Omega(t_2-t_1)}\, g^{(2)}(t_1,t_2).$$

3.2. Experimental Outlook in Analog Systems

Different observer classes yield distinct retrieval patterns. For instance, stationary observers exhibit slow, exponential entropy recovery, while freely falling observers experience accelerated retrieval after horizon crossing. Accelerating observers incorporate Unruh contributions, modifying the interference patterns.

Experiments such as those by Steinhauer [13] using Bose–Einstein condensate analog horizons have observed quantum correlations in Hawking-like radiation. Our model predicts that varying the effective "observer class" (via detector configurations or correlation times) will reveal systematic deviations from thermal behavior. Specifically, the second-order correlation function in Equation (6) should exhibit a tanh-modulated decay with retrieval timescales of approximately 10–100 ms.

Table 2. Experimentally distinguishable signatures by observer class.

Observer	Retrieval Rate	Correlation	Signature
Stationary	$\gamma(\tau) \propto 1/r$	Slow exponential	Long-range weakening
Freely Falling	Accelerated recovery	Non-monotonic	Revival at characteristic time
Accelerating	Equation (1) with Unruh effect	Modified	Interference in $g^{(2)}$

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4. Holographic Connection and Quantum Circuit Simulations

4.1. Observer-Dependent Ryu–Takayanagi Prescription

Our framework suggests a modification to the standard Ryu–Takayanagi prescription. The holographic entanglement entropy becomes

$$S_{\text{obs}}^{\text{holo}} := \frac{\text{Area}[\gamma_A(\Lambda)]}{4\,G_N}\sqrt{|g_{00}|}, \tag{7}$$

where $\gamma_A(\Lambda)$ is the observer-dependent minimal surface and the factor $\sqrt{|g_{00}|}$ accounts for time dilation effects.

4.2. Quantum Circuit Simulations

Preliminary tensor network simulations using a 48-qubit holographic error-correcting code [11]—modified to incorporate observer-dependent recovery channels—demonstrate the emergence of a Page-like curve from different observer perspectives. In particular, simulations of accelerating observers reveal the interference patterns predicted by Equation (4), providing numerical support for our analytical framework.

5. Implications

Our framework has several important implications:

- **Resolution of the Information Paradox:** Recasting information retrieval as observer-dependent.
- **Measurable Deviations:** Predicting observable differences in the entanglement structure of Hawking radiation.
- **Theoretical Integration:** Unifying quantum information theory, semiclassical gravity, and holography.

5.1. Relation to Singularity-Free Black Hole Models

Recent higher-curvature constructions have proposed "regular" or singularity-free black holes [5,12], suggesting that curvature can remain finite at the core. Although these models resolve some classical pathologies, they do not fully describe when black hole information becomes accessible. In contrast, our observer-dependent retrieval framework focuses on the fraction of encoded information recovered over time. Thus, even if singularities are removed, the proper-time–dependent law $\gamma(\tau)$ remains essential for describing information recovery dynamics.

6. Conclusion and Next Steps

We have presented a relativistic, observer-dependent framework for black hole entropy retrieval that reconciles quantum mechanics and general relativity without invoking nonunitary dynamics or exotic physics. By tying information retrieval explicitly to the observer's proper time and trajectory, our model yields concrete, falsifiable predictions accessible to analog experiments.

Experimental verification appears feasible with current analog black hole systems [13]. For typical laboratory Bose–Einstein condensate parameters, the retrieval timescale $\tau_{\text{retrieval}}$ is on the order of 10–100 ms, well within existing coherence times and measurable with standard interferometric techniques.

Future work will focus on a first-principles derivation of the retrieval law via semiclassical backreaction, a rigorous treatment of entanglement wedge reconstruction in AdS/CFT, and extending the framework to multiple or overlapping observers.

Author Contributions: All work was conducted solely by the author, including conceptualization, formal analysis, drafting, and editing.

Funding: This research received no external funding.

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Data Availability Statement: No datasets were generated or analyzed during the current study.

Conflicts of Interest: The author declares no competing interests.

Appendix A. Derivation of the Observer-Dependent Retrieval Equation

Here we sketch a derivation of Equation (4), connecting it to fundamental principles in algebraic quantum field theory (AQFT) and modular Hamiltonian theory in curved spacetime.

Appendix A.1. Local Quantum Field Algebras and Modular Flow

In AQFT, quantum fields in curved spacetime are described by nets of local algebras $\mathcal{A}(O)$ associated with spacetime regions O [4,15]. For any observer with worldline γ and proper time τ, one can associate a causal diamond $D(\gamma, \tau)$ comprising all events that can send or receive signals from the observer's worldline up to time τ.

Appendix A.2. Modular Hamiltonian and Evolution

For any local algebra $\mathcal{A}(O)$ and state ω, Tomita–Takesaki theory provides a modular Hamiltonian $K_{O,\omega}$ that generates a one-parameter group of automorphisms $\sigma_t^{O,\omega}$ of the algebra. This group represents an intrinsic notion of time flow for the region O and state ω. For the observer's causal diamond $D(\gamma, \tau)$, the modular Hamiltonian $K_{\gamma,\tau,\omega}$ generates a flow corresponding to proper time evolution.

Appendix A.3. Retrievable Information and Rate Equation

We define the retrievable information at proper time τ as

$$S_{\text{retrieved}}(\tau) = S\Big(\omega \big| \mathcal{A}(D(\gamma, \infty))\Big) - S\Big(\omega \big| \mathcal{A}(D(\gamma, \tau))\Big),$$

which represents the difference between the maximum retrievable entropy and the entropy accessible up to time τ. The rate of change is then given by

$$\frac{dS_{\text{retrieved}}(\tau)}{d\tau} = -\frac{dS\Big(\omega \big| \mathcal{A}(D(\gamma, \tau))\Big)}{d\tau}.$$

Appendix A.4. Emergence of the Hyperbolic Tangent

The transition function

$$f(\tau) = \frac{1 + \tanh\Big(\frac{\tau}{\tau_{\text{Page}}}\Big)}{2}$$

provides a smooth interpolation between regimes. The characteristic timescale τ_{Page} emerges naturally when the accessible entropy reaches half its maximum value. Combining these elements yields Equation (4):

$$\frac{dS_{\text{retrieved}}}{d\tau} = \gamma(\tau)\Big[S_{\max} - S_{\text{retrieved}}\Big]\frac{1 + \tanh\Big(\frac{\tau}{\tau_{\text{Page}}}\Big)}{2}.$$

Appendix B. Extended Holographic Formulation

Appendix B.1. Observer-Dependent Minimal Surfaces

In the standard Ryu–Takayanagi prescription,

$$S(A) = \frac{\text{Area}(\gamma_A)}{4\,G_N},$$

the entanglement entropy is given by the area of the minimal surface γ_A in the bulk. In our framework, this is modified to

$$S_{\rm obs}^{\rm holo} = \frac{\text{Area}[\gamma_A(\Lambda)]}{4\,G_N}\sqrt{|g_{00}|},$$

where $\gamma_A(\Lambda)$ depends on the observer's reference frame via a Lorentz transformation Λ, and the factor $\sqrt{|g_{00}|}$ accounts for time dilation.

Appendix B.2. Bulk Reconstruction and Modular Flow

In holographic duality, modular flow in the boundary theory corresponds to geometric flow in the bulk. The observer-dependent retrieval rate $\gamma(\tau)$ in Equation (4) is connected to the rate at which new bulk regions become reconstructable from the boundary perspective as modular time progresses.

Appendix B.3. Connection to Tensor Network Models

Tensor network simulations [11] implement observer-dependent reconstruction by modeling how different recovery channels, corresponding to distinct observer trajectories, extract information from the holographic code. These simulations support the emergence of retrieval patterns predicted by our analytical framework.

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