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A. Plastino *

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Article

Entropic Interpretation of Wave Function Collapse

A. Plastino

Universidad Nacional de La Plata, Argentina, 1900 La Plata, Argentina; angeloplastino@gmail.com

Abstract

We propose an information-geometric, entropic interpretation of wave function collapse based on the finite distinguishability between quantum states. By analyzing the second-order expansion of relative entropy, we show that collapse can be understood as a finite transition in state space governed by the Fisher information metric. This framework naturally assigns an energetic cost to collapse, identified with the canonical energy associated with the perturbation. The resulting picture provides a unified description linking quantum measurement, statistical distinguishability, and geometric structure.

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1. Introduction

The nature of wave function collapse remains one of the central open questions in the foundations of quantum mechanics. In the standard formulation, collapse is introduced as a non-unitary projection associated with measurement, without a clear dynamical or energetic description [1].

Information-theoretic approaches provide a complementary perspective. The relative entropy between quantum states quantifies statistical distinguishability and plays a central role in quantum information theory [2,3]. Its local expansion defines the Fisher information metric, which endows the space of states with a Riemannian structure [4].

Moreover, in holographic frameworks, the second-order variation of relative entropy has been identified with canonical energy in the bulk gravitational theory [6,7]. This relation establishes a deep link between information geometry and dynamical response.

In this work, we propose that wave function collapse can be interpreted as a finite transition in the information-geometric manifold, characterized by a non-zero relative entropy and governed by the Fisher metric. This interpretation provides a natural geometric and energetic description of collapse.

2. Relative Entropy and Local Geometry

Let $\rho(\theta)$ denote a smooth family of quantum states depending on a set of real parameters $\theta = (\theta^1, \theta^2, \dots)$. A natural measure of distinguishability between two states is provided by the quantum relative entropy

$$S(\rho\|\sigma) = \mathrm{Tr}(\rho \log \rho - \rho \log \sigma), \tag{1}$$

which is non-negative and vanishes if and only if $\rho = \sigma$ [2,3].

To extract the local geometric structure of the state space, we consider two infinitesimally close states $\rho(\theta)$ and $\rho(\theta + \delta\theta)$. Expanding the relative entropy to second order in $\delta\theta$, one obtains

$$S\big(\rho(\theta+\delta\theta)\|\rho(\theta)\big) = \frac{1}{2}\delta\theta^i \delta\theta^j F_{ij}(\theta) + \mathcal{O}(\delta\theta^3), \tag{2}$$

where the symmetric tensor

$$F_{ij}(\theta) = \mathrm{Tr}\big[\rho(\theta)\, \partial_i \log \rho(\theta)\, \partial_j \log \rho(\theta)\big] \tag{3}$$

is the quantum Fisher information matrix [5].

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This expansion shows that relative entropy induces a Riemannian metric on the manifold of quantum states. The associated line element

$$ds^2 = F_{ij}(\theta)\, d\theta^i d\theta^j \tag{4}$$

measures the infinitesimal statistical distinguishability between nearby states.

From a physical perspective, the Fisher metric quantifies the sensitivity of the state to variations in control parameters. In particular, it encodes the response of expectation values and fluctuations under small perturbations. For families of equilibrium states, this metric is directly related to thermodynamic susceptibilities, providing a bridge between information geometry and fluctuation theory.

Importantly, the metric structure arises solely from the convexity properties of relative entropy and is therefore independent of any particular dynamical assumption. This universality makes it a natural candidate for describing generic quantum state transitions.

3. Collapse as a Finite Relative Entropy Transition

We now apply the information-geometric framework to the problem of wave function collapse. Consider a transition between an initial state ρ_i and a final state ρ_f, associated with a measurement process,

$$\rho_i \longrightarrow \rho_f. \tag{5}$$

In the standard formulation of quantum mechanics, this transition is described as a projection and is not endowed with a dynamical or geometric structure. *Here we propose to characterize collapse in terms of the relative entropy between the two states,*

$$\Delta S = S(\rho_f \| \rho_i), \tag{6}$$

which measures their statistical distinguishability.

Unlike infinitesimal transformations, collapse typically involves a finite change in the state, so that ΔS is non-zero. In this sense, *collapse can be interpreted as a finite displacement in the information-geometric manifold* defined by the Fisher metric.

To connect this finite transition with the local geometry, we assume that the process can be approximated by a path $\rho(\theta(t))$ in parameter space, interpolating between ρ_i and ρ_f. For sufficiently smooth paths, the relative entropy between nearby points along the trajectory is controlled by the Fisher metric, leading to the approximation

$$\Delta S \approx \frac{1}{2} \int F_{ij}(\theta)\, \dot{\theta}^i \dot{\theta}^j \, dt. \tag{7}$$

This expression shows that the total distinguishability accumulated during the transition is determined by the geometric properties of the path in state space.

In this framework, collapse is not a singular or discontinuous event, but rather a process that can be described as a trajectory of finite length in the information manifold. The irreversibility of collapse is naturally encoded in the positivity of relative entropy, $\Delta S \geq 0$, which ensures that the transition is intrinsically directional.

This geometric interpretation provides a natural bridge between quantum measurement and nonequilibrium processes, where similar quadratic forms govern dissipation and entropy production. As will be discussed below, the same structure also admits an energetic interpretation in terms of canonical energy.

This expression anticipates the role of thermodynamic length as the geometric measure of the collapse trajectory.

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4. Energetic Interpretation and Canonical Energy

The geometric structure introduced above admits a natural dynamical interpretation. In particular, the quadratic form arising from the second-order expansion of relative entropy can be associated with an energetic cost for transitions between quantum states.

Consider again a parametric family $\rho(\theta)$ and a small variation $\delta\theta$. The second-order variation of relative entropy defines

$$\delta^2 S = \frac{1}{2}\delta\theta^i \delta\theta^j F_{ij}(\theta), \tag{8}$$

which is a positive semi-definite quadratic form. This quantity measures the leading contribution to the distinguishability between nearby states.

Remarkably, in holographic frameworks, this quadratic form has a direct physical interpretation. It has been shown that, for perturbations around a reference state, the second variation of relative entropy in the boundary theory is dual to the canonical energy of the corresponding perturbation in the bulk gravitational description [6,7]. That is,

$$E_c = \delta^2 S. \tag{9}$$

The canonical energy E_c is a conserved quantity associated with linear perturbations and plays a central role in stability analyses of gravitational systems. Its positivity is directly linked to the positivity of relative entropy, providing a nontrivial consistency condition.

Motivated by this correspondence, we extend the identification beyond strict holographic settings and interpret the quadratic form defined by the Fisher metric as an effective energy associated with transitions in the space of quantum states:

$$E_c = \delta\theta^i \delta\theta^j F_{ij}. \tag{10}$$

This expression assigns an intrinsic energetic cost to parameter variations, determined solely by the geometric structure of the state space. In particular, the energy depends on the direction of the displacement in parameter space, reflecting the anisotropic response of the system.

For finite processes, such as the collapse transition discussed in the previous section, one can generalize this local expression by considering a trajectory $\theta(t)$ and defining the total energetic cost as

$$E_c \sim \int_0^\tau F_{ij}(\theta)\, \dot{\theta}^i \dot{\theta}^j \, dt, \tag{11}$$

which is consistent with the quadratic form governing dissipation in nonequilibrium thermodynamics.

This formulation reveals a deep parallel between quantum state transitions and thermodynamic processes: in both cases, the metric structure determines the cost of driving the system away from equilibrium. In particular, the positivity of F_{ij} ensures that $E_c \geq 0$, reflecting the irreversible character of processes such as collapse.

From this perspective, wave function collapse acquires a natural energetic interpretation. Rather than being an instantaneous projection, it may be viewed as a finite process in state space, associated with a nonzero canonical energy determined by the Fisher metric. The magnitude of this energy is controlled by the distinguishability between the initial and final states, as quantified by relative entropy.

This viewpoint suggests that the geometry of quantum states encodes not only statistical properties, but also dynamical constraints on transitions, linking information, energy, and irreversibility within a unified framework.

5. Explicit Example: Two-Level System

To illustrate the general framework, we consider a two-level quantum system, which provides the simplest nontrivial setting where all relevant quantities can be computed explicitly.

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5.1. Hamiltonian and Thermal State

We take the Hamiltonian to be

$$H = \frac{\omega}{2}\sigma_z, \tag{12}$$

with eigenstates $|0\rangle$ and $|1\rangle$ satisfying

$$H|0\rangle = -\frac{\omega}{2}|0\rangle, \qquad H|1\rangle = +\frac{\omega}{2}|1\rangle. \tag{13}$$

The thermal (Gibbs) state at inverse temperature β is

$$\rho_i = \frac{e^{-\beta H}}{Z}, \tag{14}$$

where the partition function is

$$Z = \mathrm{Tr}\, e^{-\beta H} = e^{\beta\omega/2} + e^{-\beta\omega/2} = 2\cosh\left(\frac{\beta\omega}{2}\right). \tag{15}$$

In the energy eigenbasis, the density matrix is diagonal,

$$\rho_i = \begin{pmatrix} p_0 & 0 \\ 0 & p_1 \end{pmatrix}, \tag{16}$$

with occupation probabilities

$$p_0 = \frac{e^{\beta\omega/2}}{Z}, \qquad p_1 = \frac{e^{-\beta\omega/2}}{Z}. \tag{17}$$

5.2. Collapse Process

We consider a projective measurement in the energy basis that collapses the system into the excited state,

$$\rho_f = |1\rangle\langle 1| = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}. \tag{18}$$

This transition represents a maximal change in the state, from a mixed thermal distribution to a pure eigenstate.

5.3. Relative Entropy

The relative entropy between the final and initial states is

$$S(\rho_f \| \rho_i) = \mathrm{Tr}(\rho_f \log \rho_f - \rho_f \log \rho_i). \tag{19}$$

Since ρ_f is a projector onto $|1\rangle$, we have

$$\rho_f \log \rho_f = 0, \tag{20}$$

and therefore

$$S(\rho_f \| \rho_i) = -\langle 1|\log \rho_i|1\rangle. \tag{21}$$

Using the diagonal form of ρ_i,

$$\langle 1|\rho_i|1\rangle = p_1 = \frac{e^{-\beta\omega/2}}{Z}, \tag{22}$$

we obtain

$$S(\rho_f \| \rho_i) = -\log p_1 = \log Z + \frac{\beta\omega}{2}. \tag{23}$$

Substituting the partition function,

$$\boxed{S(\rho_f \| \rho_i) = \log\left[2\cosh\left(\frac{\beta\omega}{2}\right)\right] + \frac{\beta\omega}{2}} \tag{24}$$

This quantity is strictly positive and increases with β, reflecting the growing distinguishability between the thermal state and the excited state at low temperatures.

5.4. Fisher Information and Local Geometry

For the Gibbs family $\rho(\beta)$, the Fisher information is given by

$$F_{\beta\beta} = \langle (\Delta H)^2 \rangle. \tag{25}$$

The expectation value of the energy is

$$\langle H \rangle = -\frac{\partial}{\partial\beta} \ln Z = -\frac{\omega}{2} \tanh\left(\frac{\beta\omega}{2}\right), \tag{26}$$

and, as before,

$$\langle H^2 \rangle = \frac{\omega^2}{4}. \tag{27}$$

Thus,

$$F_{\beta\beta} = \frac{\omega^2}{4} \mathrm{sech}^2\left(\frac{\beta\omega}{2}\right). \tag{28}$$

This shows that the local geometry is controlled by energy fluctuations, which decrease as the system approaches a pure state.

5.5. Geometric and Energetic Interpretation

The collapse process $\rho_i \to \rho_f$ corresponds to a finite displacement in the information manifold. The relative entropy computed above measures the total distinguishability between the initial and final states, while the Fisher information governs the local structure along any interpolating path.

In particular, the quadratic form

$$E_c \sim \delta\beta^2 F_{\beta\beta} \tag{29}$$

defines the local energetic cost of variations in β, linking the collapse process to the canonical energy discussed in previous sections.

This example thus provides a concrete realization of the general framework: the collapse is characterized by a finite relative entropy, while its local geometric and energetic properties are determined by the Fisher metric.

5.6. Limiting Cases

It is instructive to consider two limiting regimes:

High-temperature limit ($\beta \to 0$): The state approaches the maximally mixed state, and

$$F_{\beta\beta} \to \frac{\omega^2}{4}, \qquad S(\rho_f \| \rho_i) \to \log 2. \tag{30}$$

Low-temperature limit ($\beta \to \infty$): The system approaches the ground state, and

$$F_{\beta\beta} \to 0, \qquad S(\rho_f \| \rho_i) \to \beta\omega. \tag{31}$$

These limits highlight the interplay between fluctuations and distinguishability: as fluctuations vanish, the Fisher metric becomes degenerate, while the global distinguishability between states increases.

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5.7. Discussion

This simple model captures the essential features of the proposed interpretation. The collapse from a thermal state to a pure eigenstate is associated with a finite relative entropy, while the Fisher metric encodes the local response properties and determines the energetic cost of variations.

The example demonstrates explicitly how distinguishability, fluctuations, and geometry are unified within a single framework, supporting the general interpretation of collapse as a finite process in the information manifold.

6. Example: Harmonic Oscillator and Gaussian States

We consider a quantum harmonic oscillator with Hamiltonian

$$H = \omega\left(a^{\dagger}a + \frac{1}{2}\right). \tag{32}$$

Let the system be initially in a thermal (Gaussian) state

$$\rho_i = \frac{e^{-\beta H}}{Z}, \tag{33}$$

with partition function $Z = (1 - e^{-\beta\omega})^{-1}$.

Suppose that a measurement induces a small change in inverse temperature, $\beta \to \beta + \delta\beta$, leading to a nearby state ρ_f.

6.1. Relative Entropy

For two nearby thermal states, the relative entropy admits the expansion

$$S(\rho_f \| \rho_i) \approx \frac{1}{2} F_{\beta\beta} (\delta\beta)^2, \tag{34}$$

where the Fisher information is given by

$$F_{\beta\beta} = \langle (\Delta H)^2 \rangle. \tag{35}$$

For the harmonic oscillator, one finds

$$\langle (\Delta H)^2 \rangle = \omega^2 \frac{e^{\beta\omega}}{(e^{\beta\omega} - 1)^2}. \tag{36}$$

6.2. Interpretation

Thus, the relative entropy between nearby Gaussian states is determined by energy fluctuations, as in the discrete case. The corresponding quadratic form defines the Fisher metric and governs the local geometry of the state space.

The associated canonical energy is therefore

$$E_c = F_{\beta\beta} (\delta\beta)^2, \tag{37}$$

which again reflects the fluctuation structure of the system.

6.3. Discussion

This example shows that the proposed framework applies equally to continuous-variable systems, where Gaussian states play a central role. The unification of distinguishability, fluctuations, and energy thus holds beyond finite-dimensional Hilbert spaces.

7. Thermodynamic Length and Collapse

A natural geometric quantity associated with the Fisher metric is the thermodynamic length, defined as

$$\mathcal{L} = \int_{\theta_i}^{\theta_f} \sqrt{F_{ij}(\theta)\, d\theta^i d\theta^j}. \tag{38}$$

For a single-parameter family, such as thermal states parametrized by β, this reduces to

$$\mathcal{L} = \int_{\beta_i}^{\beta_f} \sqrt{F_{\beta\beta}(\beta)}\, d\beta. \tag{39}$$

7.1. Explicit Computation: Two-Level System

We consider again a two-level system described by the Hamiltonian

$$H = \frac{\omega}{2}\sigma_z, \tag{40}$$

with eigenvalues $E_\pm = \pm\frac{\omega}{2}$. For section-completeness, we repeat here some formulas given already above.

The thermal (Gibbs) state at inverse temperature β is

$$\rho(\beta) = \frac{e^{-\beta H}}{Z}, \tag{41}$$

with partition function

$$Z = \mathrm{Tr}\, e^{-\beta H} = e^{-\beta\omega/2} + e^{\beta\omega/2} = 2\cosh\left(\frac{\beta\omega}{2}\right). \tag{42}$$

7.1.1. Fisher Information

For a Gibbs family parametrized by β, the Fisher information reduces to the energy variance,

$$F_{\beta\beta} = \langle (\Delta H)^2 \rangle. \tag{43}$$

The expectation value of the energy is

$$\langle H \rangle = -\frac{\partial}{\partial\beta}\ln Z = -\frac{\omega}{2}\tanh\left(\frac{\beta\omega}{2}\right). \tag{44}$$

The second moment is

$$\langle H^2 \rangle = \frac{\omega^2}{4}, \tag{45}$$

since $H^2 = \frac{\omega^2}{4}\mathbb{I}$.

Thus, the variance is

$$\langle (\Delta H)^2 \rangle = \langle H^2 \rangle - \langle H \rangle^2 = \frac{\omega^2}{4}\left[1 - \tanh^2\left(\frac{\beta\omega}{2}\right)\right]. \tag{46}$$

Using the identity $1 - \tanh^2 x = \mathrm{sech}^2 x$, we obtain

$$F_{\beta\beta} = \frac{\omega^2}{4}\,\mathrm{sech}^2\left(\frac{\beta\omega}{2}\right). \tag{47}$$

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7.1.2. Thermodynamic Length

The thermodynamic length between two states β_i and β_f is

$$\mathcal{L} = \int_{\beta_i}^{\beta_f} \sqrt{F_{\beta\beta}(\beta)}\, d\beta. \tag{48}$$

Substituting the explicit form of the Fisher information,

$$\mathcal{L} = \frac{\omega}{2} \int_{\beta_i}^{\beta_f} \operatorname{sech}\left(\frac{\beta\omega}{2}\right) d\beta. \tag{49}$$

Introducing the dimensionless variable

$$x = \frac{\beta\omega}{2}, \qquad d\beta = \frac{2}{\omega} dx, \tag{50}$$

we obtain

$$\mathcal{L} = \int_{x_i}^{x_f} \operatorname{sech}(x)\, dx. \tag{51}$$

The integral is elementary and yields

$$\int \operatorname{sech}(x)\, dx = 2 \arctan\left(\tanh \frac{x}{2}\right). \tag{52}$$

Thus, the thermodynamic length is

$$\mathcal{L} = 2\left[\arctan\left(\tanh \frac{x}{2}\right)\right]_{x_i}^{x_f}. \tag{53}$$

Returning to the original variable,

$$\boxed{\mathcal{L} = 2\left[\arctan\left(\tanh \frac{\beta\omega}{4}\right)\right]_{\beta_i}^{\beta_f}} \tag{54}$$

7.1.3. Physical Interpretation

This explicit result provides a closed-form expression for the geometric distance between thermal states of the two-level system. The dependence on β reflects the underlying fluctuation structure encoded in the Fisher metric.

In particular, the thermodynamic length remains finite for all finite temperature intervals, reflecting the bounded nature of the state space. Moreover, the integrand $\sqrt{F_{\beta\beta}}$ is proportional to the energy fluctuations, showing that the geometry is directly controlled by the susceptibility of the system.

Within the present framework, this length quantifies the geometric extent of the collapse trajectory, providing a concrete measure of the distance traversed in the information manifold during the transition.

8. Geodesics and Dissipation Bounds

8.1. Geodesic Equation

The Fisher information matrix defines a Riemannian metric on the parameter manifold,

$$ds^2 = F_{ij}(\theta)\, d\theta^i d\theta^j. \tag{55}$$

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The geodesics of this manifold are obtained by extremizing the thermodynamic length,

$$\mathcal{L} = \int \sqrt{F_{ij}\dot{\theta}^i\dot{\theta}^j}\, dt, \tag{56}$$

which leads to the Euler–Lagrange equations

$$\ddot{\theta}^k + \Gamma^k_{ij}\dot{\theta}^i\dot{\theta}^j = 0, \tag{57}$$

where the Christoffel symbols are

$$\Gamma^k_{ij} = \frac{1}{2}F^{kl}\Big(\partial_i F_{jl} + \partial_j F_{il} - \partial_l F_{ij}\Big). \tag{58}$$

These equations define the optimal paths in parameter space that minimize the thermodynamic length.

8.2. Single-Parameter Case

For a one-dimensional manifold parametrized by θ, the metric reduces to $ds^2 = F(\theta)\, d\theta^2$. In this case, the geodesic equation simplifies to

$$\ddot{\theta} + \frac{1}{2}\frac{d\ln F(\theta)}{d\theta}\,\dot{\theta}^2 = 0. \tag{59}$$

This equation admits a first integral:

$$\sqrt{F(\theta)}\,\dot{\theta} = \text{const.} \tag{60}$$

Thus, geodesics correspond to trajectories of constant thermodynamic speed.

8.3. Application: Two-Level System

For the two-level system, the Fisher information is

$$F(\beta) = \frac{\omega^2}{4}\,\text{sech}^2\!\left(\frac{\beta\omega}{2}\right). \tag{61}$$

The geodesic condition becomes

$$\frac{\omega}{2}\,\text{sech}\!\left(\frac{\beta\omega}{2}\right)\dot{\beta} = \text{const.} \tag{62}$$

This equation determines the optimal evolution of $\beta(t)$ that minimizes the thermodynamic length between two states.

8.4. Thermodynamic Length and Dissipation

A key result in nonequilibrium thermodynamics is that the thermodynamic length controls the minimal dissipation incurred during a finite-time process [8,9]. In particular, for slowly driven processes, the excess work satisfies the bound

$$W_{\text{diss}} \geq \frac{\mathcal{L}^2}{\tau}, \tag{63}$$

where τ is the duration of the protocol.

More precisely, the excess work can be expressed as a quadratic functional of the control parameters,

$$W_{\text{diss}} \sim \int_0^{\tau} F_{ij}(\theta)\,\dot{\theta}^i\dot{\theta}^j\, dt, \tag{64}$$

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which is minimized by geodesic paths.

8.5. Implications for Collapse

Within the present framework, wave function collapse can be interpreted as a finite-time process in the information manifold. The associated thermodynamic length then provides a lower bound on the energetic cost of the transition:

$$E_c \gtrsim \frac{\mathcal{L}^2}{\tau}. \tag{65}$$

This relation suggests that collapse is constrained by geometric bounds analogous to those governing nonequilibrium thermodynamic transformations.

In this sense, the Fisher metric not only determines distinguishability and canonical energy, but also sets fundamental limits on the dissipation associated with quantum state transitions.

Geodesic Optimality.

While generic quantum state transitions need not follow geodesic paths, the Fisher information metric singles out a distinguished class of trajectories that minimize thermodynamic length and the associated dissipation. These geodesics therefore provide a natural benchmark for collapse processes, representing the most efficient transitions between given initial and final states within the information-geometric framework.

While idealized collapse in standard quantum mechanics is instantaneous, any physical implementation of state reduction necessarily involves a finite-time process. In such cases, the information-geometric framework implies that geodesic trajectories minimize dissipation and therefore represent optimally efficient realizations of the transition. "'

Geodesic efficiency acquires operational significance whenever quantum state transitions are implemented as finite-time physical processes, providing a criterion for optimality in terms of minimal dissipation.

In the two-level system, the energetic cost of state transitions reflects a competition between global distinguishability, quantified by relative entropy, and local geometric cost, determined by the Fisher metric. This interplay suggests the existence of an intermediate temperature regime in which transitions can be implemented most efficiently.

9. General Theorem

We now formalize the central idea of this work.

Theorem. *Let $\rho(\theta)$ be a smooth parametric family of quantum states, and consider a finite transition between two states $\rho_i = \rho(\theta)$ and $\rho_f = \rho(\theta + \delta\theta)$. Then, to second order in $\delta\theta$, the relative entropy satisfies*

$$S(\rho_f \| \rho_i) = \frac{1}{2}\delta\theta^i \delta\theta^j F_{ij}(\theta) + \mathcal{O}(\delta\theta^3), \tag{66}$$

where F_{ij} is the quantum Fisher information matrix. Moreover, the associated quadratic form defines a positive semi-definite quantity that can be interpreted as the canonical energy of the transition:

$$E_c = \delta\theta^i \delta\theta^j F_{ij}. \tag{67}$$

Thus, any finite transition between nearby quantum states admits a geometric and energetic characterization in terms of the Fisher metric.

Proof (sketch). The result follows from the standard second-order expansion of relative entropy, which defines the Fisher information matrix as the Hessian of $S(\rho\|\sigma)$ evaluated at $\rho = \sigma$ [5]. Positivity follows from the convexity of relative entropy.

The identification with canonical energy follows from holographic results relating the second variation of relative entropy to energy functionals [6,7]. □

See the Appendicx for a complete proof.

10. Conclusion

We have proposed an information-geometric interpretation of wave function collapse as a finite relative entropy transition. The Fisher information metric governs the local structure of this transition, while the associated quadratic form defines an energetic cost identified with canonical energy.

This framework provides a unified perspective linking quantum measurement, statistical distinguishability, and geometric structure, and suggests that collapse is a process with intrinsic geometric and physical content.

Outlook

The information-geometric interpretation developed here suggests several intriguing extensions. First, since the Fisher metric bounds the speed of state evolution, the thermodynamic length associated with collapse may be related to quantum speed limits [10,11], indicating that collapse processes are subject to fundamental bounds on their duration. Second, the geometric structure underlying distinguishability is closely tied to entanglement properties of quantum states, suggesting that collapse may be viewed as a trajectory in an emergent entanglement geometry [12], where changes in correlations are encoded in the Fisher metric. Finally, given the established relation between relative entropy and gravitational dynamics in holographic settings [6,7], the energetic cost associated with collapse may reflect underlying gravitational constraints, hinting at a deeper connection between quantum measurement, spacetime geometry, and the thermodynamics of information. These perspectives open a route toward embedding collapse phenomena within a broader geometric and dynamical framework.

Appendix A. Details on the Connection Between Relative Entropy and Energy

To make the connection between relative entropy and energy more explicit, we consider a smooth trajectory $\rho(\theta(t))$ in parameter space, interpolating between an initial state ρ_i and a final state ρ_f over a finite interval $t \in [0, \tau]$. The parameters $\theta^i(t)$ are assumed to be differentiable functions of time.

For two infinitesimally close points along the trajectory, the relative entropy admits the expansion

$$S\big(\rho(\theta + d\theta) \| \rho(\theta)\big) = \frac{1}{2} F_{ij}(\theta)\, d\theta^i d\theta^j, \tag{A1}$$

where $d\theta^i = \dot{\theta}^i dt$.

Substituting this differential form, we obtain

$$dS = \frac{1}{2} F_{ij}(\theta)\, \dot{\theta}^i \dot{\theta}^j\, dt^2. \tag{A2}$$

Although this expression is second order in dt, it provides the local quadratic form that governs the accumulation of distinguishability along the path. Summing these contributions along the trajectory, one is led to consider the functional

$$\int_0^\tau F_{ij}(\theta)\, \dot{\theta}^i \dot{\theta}^j\, dt, \tag{A3}$$

which captures the total quadratic cost associated with the evolution.

This functional plays a dual role. On the one hand, it is directly related to the thermodynamic length,

$$\mathcal{L} = \int_0^\tau \sqrt{F_{ij}(\theta)\, \dot{\theta}^i \dot{\theta}^j}\, dt, \tag{A4}$$

which measures the geometric distance traversed in the information manifold. On the other hand, it has the structure of a kinetic term, suggesting an energetic interpretation.

To make this connection precise, we recall that for infinitesimal perturbations around a reference state, the second variation of relative entropy defines the canonical energy,

$$E_c = \delta^2 S. \tag{A5}$$

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Extending this identification to a continuous trajectory, we interpret the integrand

$$F_{ij}(\theta)\,\dot{\theta}^i\dot{\theta}^j \tag{A6}$$

as an instantaneous energy density associated with the motion in parameter space. The total energy cost of the process is then obtained by integrating over time, yielding

$$E_c \sim \int_0^\tau F_{ij}(\theta)\,\dot{\theta}^i\dot{\theta}^j\,dt. \tag{A7}$$

This expression coincides with the quadratic form that appears in the theory of finite-time thermodynamic transformations, where it governs the dissipation associated with driving the system along a protocol [8,9].

We thus arrive at a unified picture in which the same geometric object—the Fisher information matrix—simultaneously determines: (i) the local distinguishability between states, (ii) the thermodynamic length of a trajectory, and (iii) the energetic cost associated with the evolution.

In this framework, the transition between ρ_i and ρ_f can be viewed as a path in the information manifold whose geometric and energetic properties are controlled by F_{ij}.

Appendix B. Proving the Theorem of Section 9

We assume that $\rho(\theta)$ is full-rank and smoothly parametrized, ensuring that the logarithm and derivatives are well defined.

Proof. Let $\rho(\theta)$ be a smooth family of full-rank density operators. Consider a small variation

$$\rho(\theta+\delta\theta) = \rho + \delta\rho + \frac{1}{2}\delta^2\rho + \mathcal{O}(\delta\theta^3), \tag{A8}$$

where

$$\delta\rho = \partial_i\rho\,\delta\theta^i. \tag{A9}$$

The relative entropy is

$$S(\rho+\delta\rho\|\rho) = \mathrm{Tr}[(\rho+\delta\rho)(\log(\rho+\delta\rho) - \log\rho)]. \tag{A10}$$

We expand the logarithm using the operator identity

$$\log(\rho+\delta\rho) = \log\rho + \int_0^\infty \left[(\rho+s)^{-1}\delta\rho(\rho+s)^{-1}\right]ds + \mathcal{O}(\delta\rho^2). \tag{A11}$$

Substituting into the expression for S, the first-order term vanishes due to the normalization condition $\mathrm{Tr}(\delta\rho) = 0$.

The leading nonzero contribution is quadratic and yields

$$S(\rho+\delta\rho\|\rho) = \frac{1}{2}\mathrm{Tr}\left[\delta\rho\,\mathcal{L}_\rho^{-1}(\delta\rho)\right], \tag{A12}$$

where $\mathcal{L}_\rho$ is the symmetric logarithmic derivative (SLD) superoperator defined implicitly by

$$\partial_i\rho = \frac{1}{2}(L_i\rho + \rho L_i). \tag{A13}$$

One then obtains

$$S(\rho(\theta+\delta\theta)\|\rho(\theta)) = \frac{1}{2}\delta\theta^i\delta\theta^j\mathrm{Tr}\left[\rho L_i L_j\right]. \tag{A14}$$

The matrix

$$F_{ij} = \mathrm{Tr}\left[\rho L_i L_j\right] \tag{A15}$$

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is precisely the quantum Fisher information matrix.

Thus,

$$S(\rho(\theta + \delta\theta) \| \rho(\theta)) = \frac{1}{2}\delta\theta^i \delta\theta^j F_{ij} + \mathcal{O}(\delta\theta^3). \tag{A16}$$

This completes the proof. □

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