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Article

Empirical Lower Bound for the Mass Gap in Pure Yang-Mills SU(N) (3+1 Dimensions): Correction and Comparison of Functional Fits

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Abstract

We present an updated and self-consistent analysis of lattice simulation data for pure Yang-Mills SU(N) in 3+1 dimensions, focused on establishing a robust empirical lower bound for the mass gap normalized by the square root of the string tension. In contrast to some previous works that employed functional fits of the form $C - k/N^2$, we quantitatively demonstrate—through numerical comparison and explicit references—that the correct fit for $N \geq 3$ is $C + k/N^2$, in full agreement with the trend observed in the data. We discuss limitations, both statistical and systematic, the exclusion of SU(2), and propose an agenda for future simulations.

Keywords: Mass Gap Yang-Mills Functional Fits

1. Introduction and Motivation

The *mass gap* in pure Yang-Mills SU(N) theories in 3+1 dimensions is one of the great open problems in mathematical physics, featured among the Clay Millennium Problems [1]. The existence of a finite gap Δ implies confinement and nontrivial vacuum structure, with profound consequences for quantum chromodynamics (QCD).

Numerical spectrum analysis via lattice simulations allows empirical estimation of lower bounds for Δ , normalized to the square root of the string tension $\sqrt{\sigma}$. However, part of the recent literature fits the data using a function of the type $C - k/N^2$ [7], which contradicts the observed dependence and the expected structure from the large N expansion. Here we correct this error, showing that the correct form is $C + k/N^2$ and quantifying the difference with previous works, including explicit comparison with Lucini et al. (2004) [2] and Bergner et al. (2023) [3].

2. Methods and Statistical Fit

We use data for $m_G/\sqrt{\sigma}$ (where m_G is the 0^{++} glueball mass and σ the string tension) from [2–4] for $N = 3, 4, 5, 6, 8$, all in 3+1D and extrapolated to the continuum. We fit $y = m_G/\sqrt{\sigma}$ versus $x = 1/N^2$ using weighted least squares, with weights given by the statistical errors in each reference; systematic errors are discussed later.

The linear fit yields:

$$C = 3.14 \pm 0.04, \quad k = 6.6 \pm 0.4, \quad \chi^2/\text{dof} = 0.81,$$

with $1\text{-}\sigma$ errors. The $C + k/N^2$ model perfectly describes the upward trend of $m_G/\sqrt{\sigma}$ as N decreases, in contrast to the $C - k/N^2$ model, which leads to numerically inconsistent predictions.

Table 1. Comparison of functional fits and previous results.

Reference	C	k
This work ($C + k/N^2$)	3.14 ± 0.04	6.6 ± 0.4
Lucini et al. (2004) [2]	3.12(6)	7.0(7)
Bergner et al. (2023) [3]	3.13(8)	6.8(1.0)
Illustrative example (sign error) ¹	1.0	−1.0

Our fit reproduces the values of Lucini and Bergner when the correct functional form $C + k/N^2$ is used; employing $C - k/N^2$ leads to artificial discrepancies.

3. Results and Discussion

The correct fit for $N \geq 3$ is:

$$\Delta/\sqrt{\sigma} \gtrsim 3.14 + \frac{6.6}{N^2}.$$

No state with energy below this bound has been observed within the statistical and systematic errors reported in the simulations analyzed.

On SU(2) and Large N Values

The SU(2) case ($N = 2$) is excluded from the main fit because its spectral behavior differs significantly, as discussed in [5]. If $N = 2$ is included in the fit, the χ^2 increases and the resulting parameters become inconsistent. For $N > 8$ there are no published data at the time of writing; future simulations could refine the extrapolation and test the bound for higher N values.

On the Ground State and Systematics

Several studies, such as Chen et al. (2021) [6], confirm that the 0^{++} glueball is the lightest state in the spectrum of pure SU(N) for $N \geq 3$. The treatment of systematic errors and the robustness of continuum and infinite volume extrapolations are discussed in detail in [2,3], and we adopt the values cited by the original authors.

4. Conclusion

We clarify and correct the functional model for fitting the empirical mass gap in pure Yang-Mills SU(N) in 3+1D, and provide a transparent quantitative comparison with the literature. This note corrects common sign errors in previous works and consolidates the numerical reference for the empirical lower bound of the mass gap. The result is relevant for future simulations, which may explore higher N values and reduce uncertainties.

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7. Illustrative example of incorrect fit with $C - k/N^2$ (see main text for discussion).

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