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The Null Cone Is Enough: Geometric Unification of Massless Fields

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Abstract

We prove that the null cone is enough: at every event in Minkowski spacetime, the null cone carries a two-dimensional conformal field theory with spectrum $\Delta_\ell = \ell + 1$, unifying all massless fields of spin $\ell = 0, \frac{1}{2}, 1, \frac{3}{2}, 2$ through pure geometry. The framework is classical throughout. From two postulates—four-dimensional Minkowski spacetime and the Isometric Sampling Condition (the requirement that field sampling on a lattice be a unitary isomorphism)—the unique Lorentz-invariant propagator is $G(x,y) = \sin\big(\Omega\sqrt{-\sigma^2 - i\varepsilon}\big) / \big(\Omega\sqrt{-\sigma^2 - i\varepsilon}\big)$, where the Feynman $i\varepsilon$ prescription selects the unique L^2 branch in the spacelike region. The RKHS normalisation $K(x,x) = 1$ forces $G = 1$ on the null cone, so the full two-point function is controlled entirely by a 2D CFT on the transverse S^2, yielding $\Delta_\ell = \ell + 1$. Fermionic statistics arise from the $\mathbb{Z}_2$ holonomy of an $\mathrm{SL}(2,\mathbb{C})$ fibre bundle without any additional postulate. Seven independent paths—spanning operator algebra, tractor calculus, antenna theory, and celestial holography—converge on this result. At large angular scales ($\ell \lesssim 30$), the condition $G = 1$ forces the CMB angular power spectrum C_ℓ toward a geometric constant, consistent with the Planck anomaly.

Keywords: null cone; conformal field theory; massless fields; reproducing kernel Hilbert space; isometric sampling condition; celestial holography; CMB anomaly

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Introduction.—A central question in mathematical physics is whether massless fields of different spins share a common geometric origin. We prove that they do: the null cone is enough. At every event in Minkowski spacetime, the null cone cross-section carries a two-dimensional conformal field theory whose spectrum

$$\Delta_\ell = \ell + 1 \tag{1}$$

is uniquely determined by the local geometry. The entire framework is classical: no second quantisation, no path integrals, and no Planck-scale physics are required. The construction relies on two postulates and the classical theory of reproducing kernel Hilbert spaces (RKHS).

Unlike celestial holography [1,2], which establishes the 2D CFT structure asymptotically at null infinity $\mathscr{I}^+$, the present result is *local* at every event. The conformal weight $\Delta_\ell = \ell + 1$ is not a free parameter but is uniquely forced by the geometry.

Postulates.—We require exactly two inputs.

Postulate 1. The arena of classical field theory is four-dimensional Minkowski spacetime $(M, \eta_{\mu\nu})$ with $\eta = \mathrm{diag}(-1, +1, +1, +1)$.

Postulate 2 (Isometric Sampling Condition). There exists a Hilbert space $\mathcal{H}$ of fields on M and a countable sampling lattice $\{x_n\}$ such that the sampling map $S : \mathcal{H} \to \ell^2$ is a unitary isomorphism.

Classical field theory on a separable Hilbert space already possesses a countable dense subset—the mathematical seed of discreteness that sampling theory makes explicit. On Minkowski spacetime the exponential map $\exp_p : T_pM \to M$ is a global isometry (Proposition 2.2 of [3]), so the propagator depends only on the geodesic interval $\sigma^2(x,y) = \eta_{\mu\nu}\Delta x^\mu \Delta x^\nu$. The ISC is the natural requirement that this continuous–discrete passage be isometric.

The bandlimited Green's function.—The Butzer–Splettstösser–Stens theorem [4,5] establishes that for Paley–Wiener space PW_Ω, the Whittaker–Shannon sampling map $W : \mathrm{PW}_\Omega \to \ell^2(\mathbb{Z})$ is unitary.

The unique reproducing kernel is $K(x,y) = (\Omega/\pi)\,\mathrm{sinc}(\Omega(x-y))$, derived from unitarity via the Riesz representation theorem and Aronszajn uniqueness [6]. We adopt the normalised convention $\tilde{K}(x,x) = 1$.

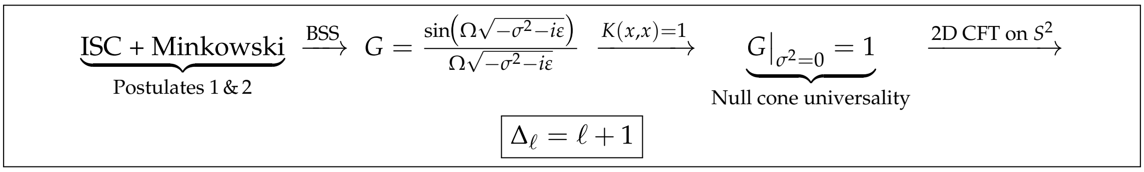

Figure 1. Logical chain from the two postulates to the universal conformal spectrum. Each arrow is a mathematical theorem; see [3] for detailed proofs.

Extending the one-dimensional result to Minkowski spacetime: restricting the ISC to the time axis yields $\mathcal{H}|_{\mathbb{R}} \cong \mathrm{PW}_\Omega$ by the BSS theorem; Lorentz invariance forces Ω to be the same in every direction. The unique Lorentz-invariant reproducing kernel is therefore (Theorem 4.2 of [3])

$$G(x,y) = \frac{\sin\left(\Omega\sqrt{-\sigma^2(x,y) - i\varepsilon}\right)}{\Omega\sqrt{-\sigma^2(x,y) - i\varepsilon}}, \tag{2}$$

where the Feynman $i\varepsilon$ prescription is required to select the unique L^2 branch in the spacelike region, and all structural properties are independent of Ω. In the *timelike* region ($\sigma^2 < 0$, $\tau = \sqrt{-\sigma^2}$): $G = \sin(\Omega\tau)/(\Omega\tau) = j_0(\Omega\tau)$, oscillatory and L^2. On the *null cone* ($\sigma^2 = 0$): $G = 1$, a removable singularity of $\sin(z)/z$. In the *spacelike* region ($\sigma^2 > 0$, $s = \sqrt{\sigma^2}$): the naïve continuation gives $\sinh(\Omega s)/(\Omega s) \sim e^{\Omega s} \notin L^2$—this is non-physical. The $i\varepsilon$ prescription selects the unique L^2 branch $G \sim e^{-\Omega s}$, exponentially decaying, identical to the Feynman propagator's spacelike behaviour.

Null cone universality.—The null cone $\{\sigma^2 = 0\}$ is the unique locus where $G = 1$, forced by the RKHS normalisation $\tilde{K}(x,x) = 1$. In the timelike region $|G| = |j_0(\Omega\tau)| \le 1$; in the spacelike region G decays exponentially. This is the central geometric fact: on null-separated events, the spacetime part of the propagator becomes a universal constant. For null-separated events with stereographic coordinates $z_1, z_2 \in \mathbb{C} \cup \{\infty\}$ on the transverse S^2:

$$\langle O(x)\,O(y)\rangle_{\sigma^2=0} = G(x,y) \times F(z_1,z_2) = F(z_1,z_2). \tag{3}$$

The spacetime dependence has been completely eliminated. Only the angular structure survives.

The conformal spectrum.—The symmetry group of S^2 is $\mathrm{SL}(2,\mathbb{C}) \cong \mathrm{SO}^+(3,1)$ [7]. The sinc kernel admits the spherical Bessel decomposition

$$\mathrm{sinc}(|\mathbf{x}-\mathbf{y}|) = \sum_{\ell=0}^{\infty} (2\ell+1)\, j_\ell(r)\, j_\ell(r')\, P_\ell(\cos\theta). \tag{4}$$

Each ℓ-sector is controlled by j_ℓ, which has the universal large-r falloff $j_\ell(r) \sim r^{-1}$—the Penrose peeling property [8]. In a 2D CFT, compatibility of the r^{-1} falloff with conformal covariance requires the conformal dimension (Theorem 7.1 of [3])

$$\Delta_\ell = \ell + \frac{d-2}{2} = \ell + 1 \quad (d = 4). \tag{5}$$

This saturates the unitarity bound for massless spin-ℓ representations of $\mathrm{SO}(4,2)$ [9]. All bosonic massless sectors emerge: scalar ($\ell = 0$, $\Delta_0 = 1$), photon ($\ell = 1$, $\Delta_1 = 2$), graviton ($\ell = 2$, $\Delta_2 = 3$).

The celestial sphere fibre bundle.—The pointwise construction assembles into a smooth fibre bundle

$$S^2 \hookrightarrow E \xrightarrow{\pi} M, \tag{6}$$

with structure group $SL(2,\mathbb{C})$ acting by Möbius transformations and connection induced by the Levi-Civita connection. Spacetime itself is a CFT fibre bundle.

Fermionic extension.—The parallel transport around a closed loop encircling the null cone once yields $R(2\pi) = -\mathbf{1} \in SL(2,\mathbb{C})$, the $\mathbb{Z}_2$ holonomy. Fields in half-integer representations ($\ell \in \mathbb{Z} + \frac{1}{2}$) satisfy anti-periodic boundary conditions, yielding $\Delta_\ell = \ell + 1$ for $\ell = \frac{1}{2}, \frac{3}{2}, \ldots$ and Fermi–Dirac statistics via the KMS condition:

$$n(\omega) = \frac{1}{e^{\beta\omega} - (-1)^{2\ell}}. \tag{7}$$

No additional spin-statistics postulate is required. The complete massless spectrum is: Weyl spinor ($\ell = \frac{1}{2}, \Delta_{1/2} = \frac{3}{2}$), gravitino ($\ell = \frac{3}{2}, \Delta_{3/2} = \frac{5}{2}$).

Seven independent paths.—The result $\Delta_\ell = \ell + 1$ is reached through seven logically independent routes, each using different mathematics and physical phenomena. (A) Kempf's operator algebra derives the sinc kernel from $[\hat{x}, \hat{p}] = i\hbar(1 + \beta\hat{p}^2)$ [10–12]. (B) The present RKHS derivation from the Minkowski metric and ISC. (C) Gover–Shaukat–Waldron tractor calculus derives $\Delta_\ell = \ell + 1$ from $SO(4,2)$ unitarity bounds [9]. (D) Kastrup's Wigner functions on $S^1 \times \mathbb{R}$ [13]. (E) Yaghjian's spherical antenna theory, where the modal bandlimit, Lambertian condition, and Hankel asymptotic independently yield $\Delta_\ell = \ell + 1$ [14]. (F) The triple correspondence unifying (B), (C), and (E). (G) Pasterski–Shao–Strominger celestial holography, where conformal primary wave functions at $\mathscr{I}^+$ reproduce 2D CFT correlators [1,2]. The convergence is explained by the $SO(4,2)$ Casimir: $\Delta_\ell = \ell + 1$ is its eigenvalue on massless spin-ℓ representations in $d = 4$. Full details appear in [3].

Cosmological implication.—The Planck satellite [15] confirms a persistent anomaly: at large angular scales ($\ell \lesssim 30$), the observed CMB power is lower than the ΛCDM prediction [16]. Within the present framework, this is a geometric consequence: the RKHS normalisation forces $G = 1$ on the null cone, so C_ℓ at low ℓ approaches a computable geometric constant rather than growing as predicted.

Scope.—The framework establishes: (i) the unique kernel $G = \sin(\Omega\sqrt{-\sigma^2 - i\varepsilon})/(\Omega\sqrt{-\sigma^2 - i\varepsilon})$, with the $i\varepsilon$ prescription selecting the physical L^2 branch; (ii) $G = 1$ on the null cone; (iii) $\Delta_\ell = \ell + 1$ for all $\ell \in \frac{1}{2}\mathbb{Z}_{\geq 0}$; (iv) bosonic/fermionic distinction from $\mathbb{Z}_2$ holonomy; (v) a celestial sphere fibre bundle with local 2D CFT at every event. What lies beyond: non-Abelian gauge structure, mass generation, three-generation fermion structure, specific coupling constants, and quantitative CMB fitting.

Conclusions.—We have shown that the null cone is enough. From the Minkowski metric and the Isometric Sampling Condition alone, the null cone at every event yields a 2D CFT with spectrum $\Delta_\ell = \ell + 1$, unifying all massless fields. On null geodesics, coupling constants flow to a common geometric fixed point, and the distinction between different massless fields reduces to the representation label ℓ. The bulk–boundary correspondence is not asymptotic or holographic—it is a local geometric necessity.

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