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Posted Date: 14 February 2025

doi: 10.20944/preprints202501.1995.v2

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The Symmetry Number Structure about Line-1/2

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Abstract: In this paper, we discuss the symmetry number structure about line-1/2. We find that using the symmetry characters of those structures we can give proofs of the number Conjectures: Goldbach Conjecture, Twins Prime Conjecture and Polignac's conjecture and the Riemann Hypothesis. **In this paper, we also gave concise proofs of the Fermat' Last Theorem and the 3n+1 conjecture.**

Keywords: P/2n; Prime numbers Conjectures; Riemann Hypothesis

The Symmetry of P/2n and Prime Numbers Conjectures

We have

$$\frac{P}{2n} = \begin{cases} \frac{1}{2^{N+1}} & n = 2^N P \\ \frac{3}{4} & n = 2 \quad P = 3 \\ 1 & n = 1 \quad P = 2 \end{cases}$$

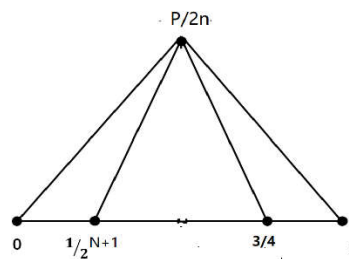


Figure 1. P/2n number structure with points [0 1/2^{N+1} 3/4 1].

$N \sim (0, 1, 2, 3, 4, \dots)$ All natural numbers

$n \sim (1, 2, 3, 4, \dots)$ All natural numbers excepted 0

$P \sim (2, 3, 5, 7, \dots)$ All prime numbers

And

$$\frac{P}{2n} = \begin{cases} \frac{1}{2} - \frac{1}{2n} & P = n - 1 \quad (n \geq 3) \\ \frac{1}{2} & P = n \\ \frac{1}{2} + \frac{1}{2n} & P = n + 1 \end{cases}$$

And We have

$$p_0 \in P \sim (0, n] \quad (n \geq 2)$$

And based on Bertrand -Chebyshev Theorem: when $n \geq 2$, there are at least a prime number between n and $2n$.

$$p_n \in P \sim [n, 2n) \quad (n \geq 2)$$

So we have:

$$0 < \frac{p_0}{2n} \leq \frac{1}{2}$$

$$\frac{1}{2} \leq \frac{p_n}{2n} < 1$$

So we have

$$\frac{p_0}{2n} = \frac{1}{2} - \frac{1}{2n} \quad (n \geq 3)$$

$$\frac{pn}{2n} = \frac{1}{2} + \frac{1}{2n}$$

So

$$\left(\frac{1}{2} - \frac{1}{2n}\right) + \left(\frac{1}{2} + \frac{1}{2n}\right) = \frac{p_0}{2n} + \frac{pn}{2n}$$

$$2n = p_0 + pn \quad (n \geq 3)$$

This is the proof of Goldbach conjecture.

And

$$\frac{pn}{2n} - \frac{p_0}{2n} = \left(\frac{1}{2} + \frac{1}{2n}\right) - \left(\frac{1}{2} - \frac{1}{2n}\right)$$

$$pn - p_0 = 2$$

This is the proof of Twin Primes Conjecture

And we also have

$$0 < \frac{p_0}{2n} = \frac{2k_1+1}{2n} \ll 1/2 \quad (n \geq 3)$$

$$0 < 2k_1 + 1 \ll n$$

$$0 < k_1 \ll \frac{n-1}{2}$$

k_1 is a positive integer

so $k_1 \sim 1, 2, 3, \dots, \left[\frac{n-1}{2}\right] \quad (n \geq 3)$

And

$$\frac{1}{2} \ll \frac{pn}{2n} = \frac{2k_2+1}{2n} < 1$$

$$n \ll 2k_2 + 1 < 2n$$

$$\frac{n-1}{2} \ll k_2 < \frac{2n-1}{2} \quad (n \geq 3)$$

k_2 is a positive integer

so $k_2 \sim \left[\frac{n-1}{2}\right], \left[\frac{n-1}{2}\right] + 1, \dots, \left[\frac{2n-1}{2}\right] \quad (n \geq 3)$

we have

$$\frac{pn}{2n} - \frac{p_0}{2n} = \frac{2k_2+1}{2n} - \frac{2k_1+1}{2n}$$

$$pn - p_0 = 2(k_2 - k_1)$$

$$(pn - p_0)_{max} = 2(k_{2_{max}} - k_{1_{min}}) = 2\left(\left[\frac{2n-1}{2}\right] - 1\right) = 2(n-2) \quad (n \geq 3)$$

This is the proof of Polignac’s conjecture.

So we get a symmetry structure of $P/2n$ as Figure 2

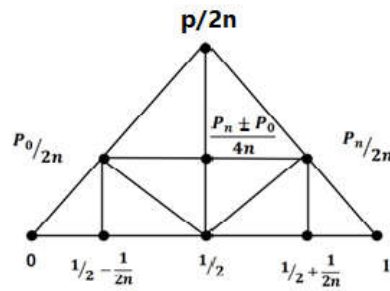


Figure 2. a symmetry structure of $P/2n$ about line- $1/2$.

$$\frac{p_0}{2n} = \frac{1}{2} - \frac{1}{2n} \quad (n \geq 3)$$

$$\frac{p_n}{2n} = \frac{1}{2} + \frac{1}{2n}$$

$$\frac{p_n \pm p_0}{4n} = \frac{1}{2}$$

A Concise Proof of The Fermat’ Last Theorem

The Fermat’ Last Theorem:

$$x^n + y^n = z^n \quad (x, y, z \in n, \quad xyz \neq 0 \quad n > 2) \text{ has no solution.}$$

$n \sim (1, 2, 3, 4, 5, 6, \dots)$ all the natural numbers excepted 0

The equivalent proposition of this conjecture is

$$\left(\frac{x}{z}\right)^n + \left(\frac{y}{z}\right)^n = 1$$

$(x, y, z \in n, \quad xyz \neq 0 \quad n > 2)$ has no solution.

We have

$$\left(\frac{x}{z}\right)^n + \left(\frac{y}{z}\right)^n = 1 = \sum \frac{1}{2^n}$$

$n \sim (1, 2, 3, 4, 5, 6, \dots)$ all the natural numbers excepted 0

$$\left(\frac{x}{z}\right)^n + \left(\frac{y}{z}\right)^n = 1$$

$$= \left(\frac{1}{2^n} - \frac{1}{2^n}\right) + 1$$

$$= \left(\frac{1}{2} - \frac{1}{2^n}\right) + \left(\frac{1}{2} + \frac{1}{2^n}\right)$$

$$= \frac{1}{2^n} + \left(1 - \frac{1}{2^n}\right)$$

Only When $n = 1$ we have

$$\left(\frac{1}{2^n} - \frac{1}{2^n}\right) = \left(\frac{1}{2} - \frac{1}{2^n}\right) = 0$$

$$1 = \left(\frac{1}{2} + \frac{1}{2^n}\right) = \left(\frac{1}{2} + \frac{1}{2}\right)$$

And Only When $n = 2$

$$\left(\frac{1}{2^n}\right) = \left(\frac{1}{2} - \frac{1}{2^n}\right) = \frac{1}{4}$$

$$\left(1 - \frac{1}{2^n}\right) = \left(\frac{1}{2} + \frac{1}{2^n}\right) = \frac{3}{4}$$

And We can get the figures as Figure 3.

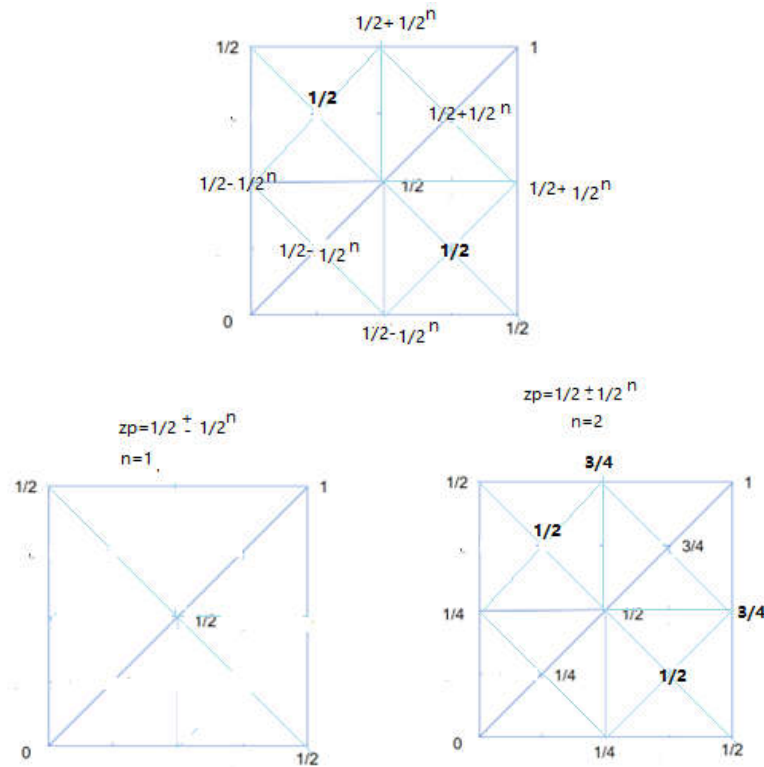


Figure 3. $D_{1/2 \pm 1/2^n}$ with points $1/2 - 1/2^n$ and $1/2 + 1/2^n$ $n \sim (1, 2, 3, 4, \dots)$.

$n = 1$ and $n = 2$

In fact we have

$$1 = \frac{1}{2^1} + \frac{1}{2^1} = \frac{1}{2^2} + \frac{3}{2^2} = \frac{1}{2^3} + \frac{7}{2^3} \text{ or } \frac{3}{2^3} + \frac{5}{2^3}$$

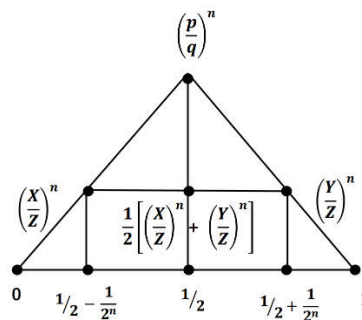


Figure 4. a symmetry structure of $\left(\frac{p}{q}\right)^n$ about line-1/2.

$(\frac{p}{q})^n$ p, q is relatively prime and $n \sim (1, 2, 3, 4, \dots)$

$$1/2[(\frac{x}{z})^n + (\frac{y}{z})^n] = 1/2 \leftrightarrow (\frac{x}{z})^n + (\frac{y}{z})^n = 1$$

$$(\frac{x}{z})^n = \frac{1}{2} - \frac{1}{2^n}$$

$$(\frac{y}{z})^n = \frac{1}{2} + \frac{1}{2^n}$$

3. a concise proof of Collatz Conjecture

Collatz Conjecture:

$$f(n) = \begin{cases} \frac{n}{2} & \text{if } n \equiv 0 \pmod{2} \\ 3n + 1 & \text{if } n \equiv 1 \pmod{2} \end{cases}$$

$k \in \mathbb{N} \rightarrow f^k(n) = 1$

$n \sim (1, 2, 3, 4, \dots)$ all the natural numbers excepted 0

$$\frac{\lfloor \frac{n+1}{2} \rfloor + \lfloor \frac{n-1}{2} \rfloor}{\lfloor \frac{n}{2} \rfloor} = \frac{2n+2}{n+1} = \frac{(n-1)+(3n+1)}{2n} = \frac{4n}{2n} = \frac{4}{2} = \frac{2}{1} = \frac{1}{\frac{1}{2}}$$

$$= \sum \frac{1}{2^N}$$

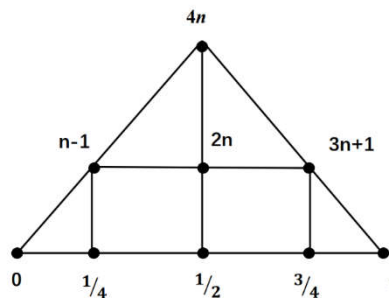


Figure 5. a symmetry structure of $4n$ about line-1/2

$$2n = 1/2[(n - 1) + (3n + 1)]$$

$$\lim_{n \rightarrow \infty} \left(\frac{n - 1}{4n} \right) = 1/4$$

$$\lim_{n \rightarrow \infty} \left(\frac{3n + 1}{4n} \right) = 3/4$$

4. The symmetry of $L^{1/2 \pm \epsilon}$ (0 1/2 1) and Riemann Hypothesis

Riemann Zeta-Function

$$\xi(s) = \sum_{n=1}^{\infty} \frac{1}{n^s} = \prod_p \frac{1}{1 - p^{-s}} \quad (s = a + bi)$$

$$s > 1 \quad \xi(s) \rightarrow \text{const}$$

The trivial zero-points of Riemann Zeta-Function is $-2n$ ($n \sim 1, 2, 3, \dots$)

Riemann Hypothesis: all the Non-trivial zero-point of Zeta-Function $Re(s) = 1/2$.

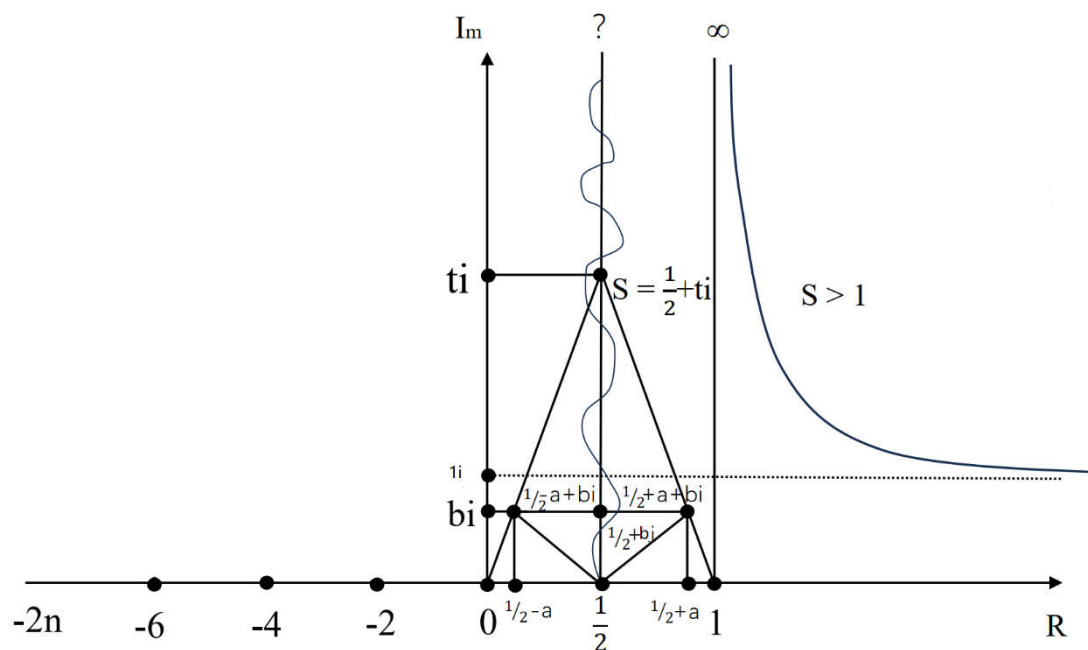


Figure 6. Riemann Hypothesis: all the non-trivial Zero points of Riemann zeta-function are on the 1/2 axis.

We can get a symmetry structure including all numbers about the line-1/2 as Figure 6

$$s = \frac{1}{2} + ti \quad t \in R$$

$$zp1 = \frac{1}{2} - a + bi \quad zp0 = \frac{1}{2} + bi \quad zp2 = \frac{1}{2} + a + bi$$

$$zp1 + zp2 = \left(\frac{1}{2} - a + bi\right) + \left(\frac{1}{2} + a + bi\right) = 1 + 2bi$$

$$zp2 - zp1 = \left(\frac{1}{2} + a + bi\right) - \left(\frac{1}{2} - a + bi\right) = 2a$$

$$a, b \in R \quad 0 \leq a \leq \frac{1}{2}$$

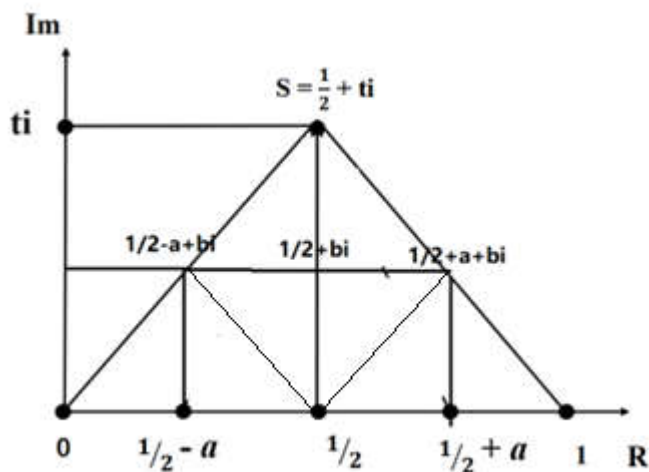


Figure 7. a symmetry structure about line $1/2 + a$ at the zero point $s = 1/2 + ti$.

As the Figure 7. If we have zero points of $\xi (s)$ as

$$zp1 = \frac{1}{2} - a + bi \quad zp2 = \frac{1}{2} + a + bi$$

And $s = \frac{1}{2} + ti \quad t \in R$ is the first zero point on line-1/2

We can get a zero point as

$$zp0 = \frac{1}{2} + bi \quad b < t \quad b, t \in R$$

It is contrary to that $s = \frac{1}{2} + ti \quad t \in R$ is the first zero point on line-1/2

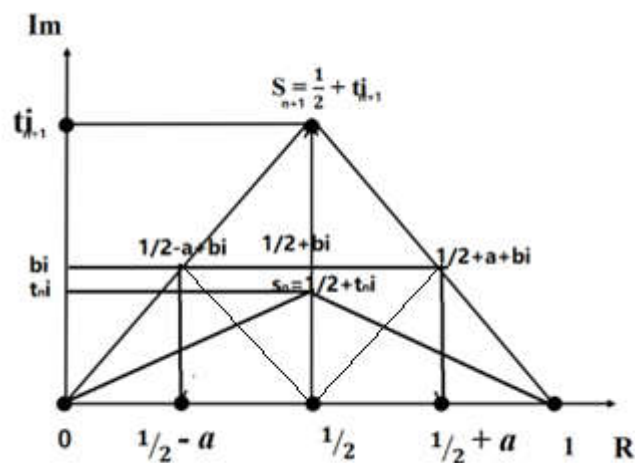


Figure 8. a symmetry structure about line-1/2 at the zero point $s_n=1/2+ti$ and $s_{n+1}=1/2+t_{n+1}i$.

As the Figure 8. If we have zero points of $\xi (s)$ as

$$zp1 = \frac{1}{2} - a + bi \quad zp2 = \frac{1}{2} + a + bi$$

And $s_n = \frac{1}{2} + t_n i \quad t \in R$ is the No. n zero point on line-1/2

$s_{n+1} = \frac{1}{2} + t_{n+1} i \quad t \in R$ is the No. n+1 zero point on line-1/2

We can get a zero point between s_n and s_{n+1} on line - 1/2 as

$$zp0 = \frac{1}{2} + bi \quad t_n < b < t_{n+1} \quad b, t \in R$$

It is contrary to that s_n and s_{n+1} are the adjacent zero points on line-1/2

So on complex plane, We can have the symmetry structure about the line-1/2 with $zp=1/2\pm a$ ($0 \leq a \leq \frac{1}{2} \quad a \in R$) show as on Figure 9.

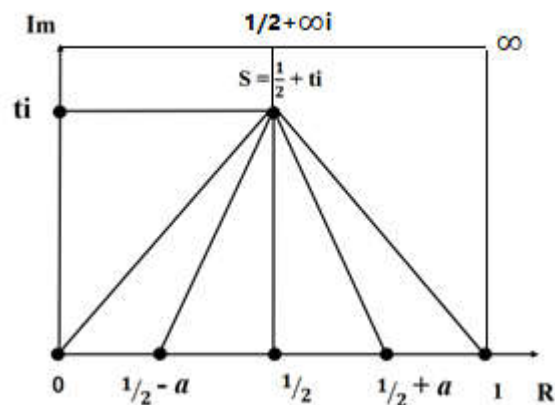


Figure 9. symmetry structure about the line-1/2 with $z_p=1/2\pm a$.

$$S = \frac{1}{2} + ti \quad (t \in \mathbf{R})$$

$$z_p = \frac{1}{2} \pm a \quad (0 \leq a \leq \frac{1}{2} \quad a \in \mathbf{R})$$

$$z_{p1} = \frac{1}{2} - a \quad z_{p0} = \frac{1}{2} \quad z_{p2} = \frac{1}{2} + a$$

$$z_{p1} + z_{p2} = \left(\frac{1}{2} - a\right) + \left(\frac{1}{2} + a\right) = 1$$

$$z_{p2} - z_{p1} = \left(\frac{1}{2} + a\right) - \left(\frac{1}{2} - a\right) = 2a$$

This is mean that there are no zero points on line- $1/2\pm a$ ($0 \leq a \leq \frac{1}{2}$ $a \in \mathbf{R}$).

Hardy and Littlewood give a proof that there are infinite zero points on line-1/2 (Hardy and Littlewood. 1914)

So we give a proof that all the non-trivial Zero points of Riemann zeta-function are on the Line-1/2. This is the proof of Riemann Hypothesis.

In fact, we have a symmetry number structure about line-1/2 as figure.10.

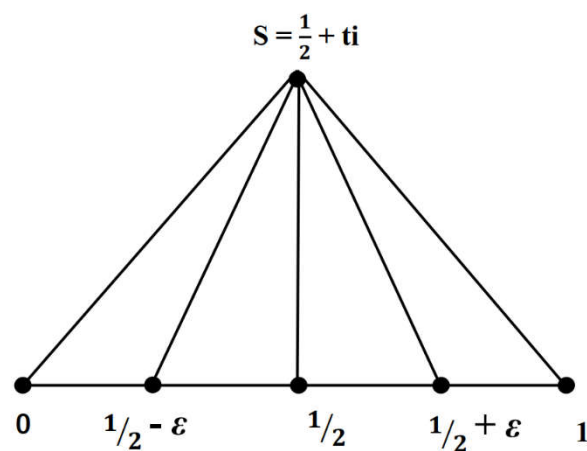


Figure 10. symmetry structure about the line-1/2 with $z_p=1/2\pm\epsilon$.

$$S = \frac{1}{2} + ti \quad (t \in \mathbf{R})$$

$$z_p = \frac{1}{2} \pm \varepsilon \quad (\varepsilon = a + bi \quad a, b \in \mathbb{R} \quad 0 \leq a \leq \frac{1}{2})$$

And we can get a symmetry number structure about line-1/2 as Figure 11. We should call it **Reimann dynamic space**.

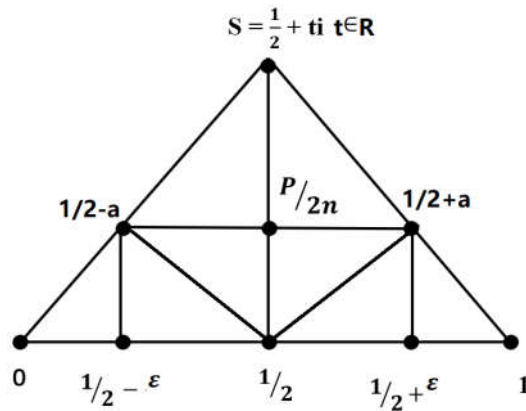


Figure 11. Reimann dynamic space.

$$1 + i^2 = 0$$

$$1 + \frac{1}{2} (i + 1) (i - 1) = 0$$

$$S = \frac{1}{2} + ti \quad (t \in \mathbb{R})$$

$$z_p = \frac{1}{2} \pm \varepsilon \quad (\varepsilon = a + bi \quad a, b \in \mathbb{R} \quad 0 \leq a \leq \frac{1}{2})$$

$$\frac{P}{2n} = \begin{cases} \frac{1}{2^{N+1}} & n = 2^N P \\ \frac{3}{4} & n = 2 \quad P = 3 \\ 1 & n = 1 \quad P = 2 \end{cases}$$

$N \sim (0, 1, 2, 3, 4, \dots)$ All natural numbers

$n \sim (1, 2, 3, 4, \dots)$ All natural numbers excepted 0

$P \sim (2, 3, 5, 7, \dots)$ All prime numbers

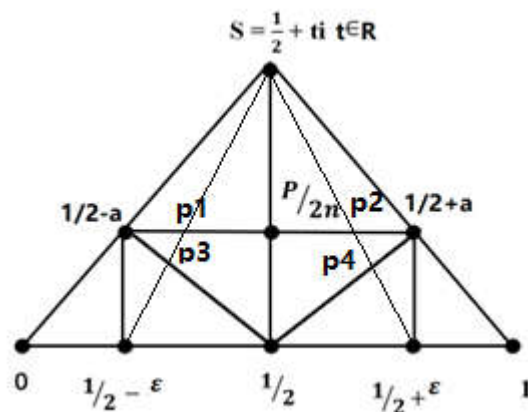


Figure 12. Reimann dynamic space with $p_1 \quad p_2 \quad p_3 \quad p_4$.

We can have point $p_1 \quad p_2 \quad p_3 \quad p_4$ and

$$\begin{aligned}
 p1 &\in \left(\frac{1}{2} - a, \frac{p}{2n}\right) \text{ and } p1 \in \left(\frac{1}{2} - \varepsilon, \frac{1}{2} + ti\right) \\
 p2 &\in \left(\frac{p}{2n}, \frac{1}{2} + a\right) \text{ and } p2 \in \left(\frac{1}{2} + \varepsilon, \frac{1}{2} + ti\right) \\
 p3 &\in \left(\frac{1}{2} - a, \frac{1}{2}\right) \text{ and } p3 \in \left(\frac{1}{2} - \varepsilon, \frac{1}{2} + ti\right) \\
 p4 &\in \left(\frac{1}{2}, \frac{1}{2} + a\right) \text{ and } p4 \in \left(\frac{1}{2} + \varepsilon, \frac{1}{2} + ti\right)
 \end{aligned}$$

And we can find that

$$\begin{aligned}
 \left(\frac{1}{2} - a, \frac{p}{2n}\right) \cap \left(\frac{1}{2} - \varepsilon, \frac{1}{2} + ti\right) &= \emptyset \\
 \left(\frac{p}{2n}, \frac{1}{2} + a\right) \cap \left(\frac{1}{2} + \varepsilon, \frac{1}{2} + ti\right) &= \emptyset \\
 \left(\frac{1}{2} - a, \frac{1}{2}\right) \cap \left(\frac{1}{2} - \varepsilon, \frac{1}{2} + ti\right) &= \emptyset \\
 \left(\frac{1}{2}, \frac{1}{2} + a\right) \cap \left(\frac{1}{2} + \varepsilon, \frac{1}{2} + ti\right) &= \emptyset
 \end{aligned}$$

This means that there are no zero points on line $-1/2 \pm \varepsilon$.

So we can get Figure 13.

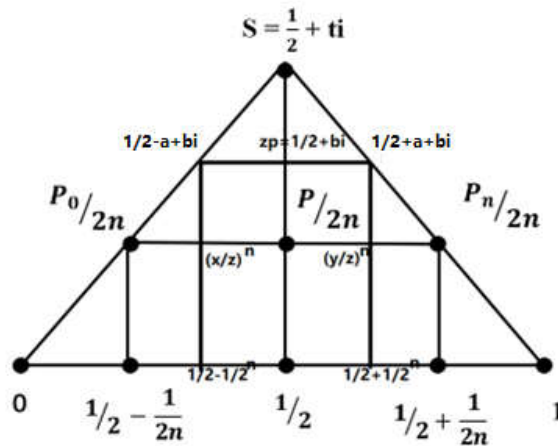


Figure 13. Reimann dynamic space and number conjectures.

1. $zp = \frac{1}{2} + bi \quad 0 < b < t \quad b, t \in R \quad (\text{the proof of RH})$

2. $\left(\frac{x}{z}\right)^n + \left(\frac{y}{z}\right)^n = 1$

$$\left(\frac{x}{z}\right)^n \leftrightarrow \frac{1}{2} - \frac{1}{2^n}$$

$$\left(\frac{y}{z}\right)^n \leftrightarrow \frac{1}{2} + \frac{1}{2^n} \quad (\text{the proof of F.L.T})$$

3. $\frac{p_0}{2n} \leftrightarrow \frac{1}{2} - \frac{1}{2n}$

$$\frac{p_n}{2n} \leftrightarrow \frac{1}{2} + \frac{1}{2n} \quad (\text{the proof of GC/BC/TPC})$$

The Symmetry Number Structure about Line-1/2 including all numbers

And we have

$$1/2 = 1/2 \quad 0 = 1/2 - 1/2 \quad 1 = 1/2 + 1/2$$

$$1 + (\pm i)^2 = 0$$

$$1 + 1/2(i + 1)(i - 1) = 0$$

$$\infty = 1 + 1 + 1 + 1 + \dots$$

We called it $L^{1/2 \pm \epsilon}$ $[0 \ 1/2 \ 1]$ and analytic continuation to $\begin{bmatrix} +\infty & -\infty \\ -\infty & +\infty \end{bmatrix}$ we can get Figure 14.

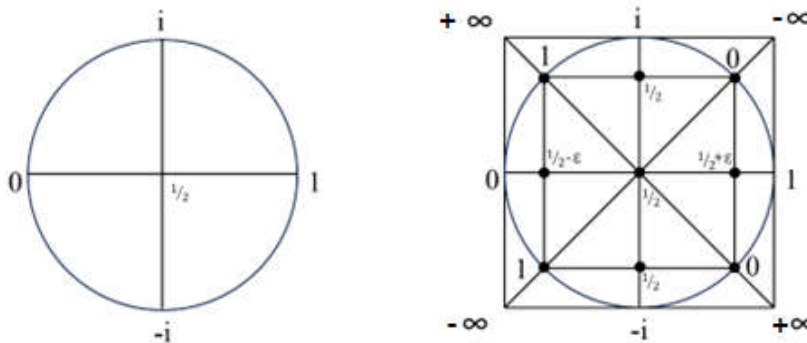


Figure 14. The Symmetry of $L^{1/2 \pm \epsilon}$ $[0 \ 1/2 \ 1]$ with $zp = \frac{1}{2} \pm \epsilon$.

So we have:

$$1 + \begin{bmatrix} +\infty & i & -\infty \\ 0 & 1/2 & 1 \\ -\infty & -i & +\infty \end{bmatrix} \begin{bmatrix} 1 & 1/2 & 0 \\ \frac{1}{2} - \epsilon & 1/2 & \frac{1}{2} + \epsilon \\ 1 & 1/2 & 0 \end{bmatrix}^{-1} = 0$$

$$zp = \frac{1}{2} \pm \epsilon$$

$$\epsilon = a + bi \quad (a, b \in \mathbb{R} \quad 0 \leq a \leq \frac{1}{2})$$

$$zp1 = \frac{1}{2} - \epsilon \quad zp2 = \frac{1}{2} + \epsilon$$

We have

$$zp1 + zp2 = \left(\frac{1}{2} - \epsilon\right) + \left(\frac{1}{2} + \epsilon\right) = 1$$

$$zp2 - zp1 = \left(\frac{1}{2} + \epsilon\right) - \left(\frac{1}{2} - \epsilon\right) = 2\epsilon = 2(a + bi)$$

And we have

$$n^2 = \frac{1}{2} \cdot n \cdot 2n = \sum_{n=1}^N \frac{1}{2} \sum_{n=1}^N \frac{1}{2^n} \left[\left(\frac{1}{2} - \epsilon\right) + \left(\frac{1}{2} + \epsilon\right) \right]$$

$N \sim (0, 1, 2, 3, 4, \dots)$ All natural numbers

$n \sim (1, 2, 3, 4, \dots)$ All natural numbers excepted 0

We can get a matrix $(n \times n)$

$$\begin{bmatrix} 1/2 & \dots\dots & \frac{1}{2^n}(1/2+\epsilon) \\ \dots\dots & 1/2 & \dots\dots \\ \frac{1}{2^n}(1/2-\epsilon) & \dots\dots & 1/2 \end{bmatrix} (n \times n)$$

The $\text{tr}(A)=1/2 \cdot n$

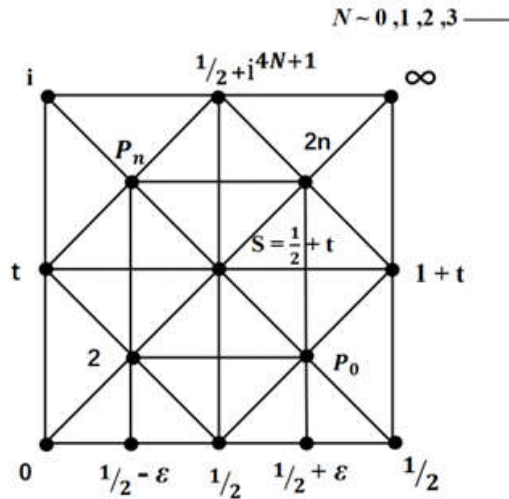


Figure 15. The Symmetry of $S_{\infty+i}$.

We have

$$0 = \frac{1}{2} - \frac{1}{2} \quad 1 = \frac{1}{2} + \frac{1}{2} \quad 2 = 1 + 1$$

$$1 + i^2 = 0 \quad 1/2 + i^{4N+1} = 1/2 + i$$

$$\infty = 1 + 1 + 1 + 1 + \dots$$

$$p_0 \in P \leq 2n \quad p_n \in P \geq 2n$$

$N \sim (0, 1, 2, 3, 4, \dots)$ all the natural numbers.

$n \sim (1, 2, 3, 4, \dots)$ All natural numbers excepted 0

$P \sim (2, 3, 5, 7, \dots)$ All odd prime number

$$S = \frac{1}{2} + t \quad (t \in \mathbf{R})$$

$$z_p = \frac{1}{2} \pm \epsilon \quad (\epsilon = a + bi \quad a, b \in \mathbf{R} \quad 0 \leq a \leq \frac{1}{2})$$

And we find that

1. $1 + e^{\pi i} = 0$ (Euler's Formula)

$$1 + i^2 = 0 \quad 1 + \frac{1}{2}(i+1)(i-1) = 0 \quad (1+i)(1-i) = \sum \frac{1}{2^N}$$

$$1 + e^{\pi i} = 0 \quad 1 + \frac{1}{2}(e^{ip\pi} - e^{i2N\pi}) = 0$$

$N \sim (0, 1, 2, 3, 4, \dots)$ all the natural numbers.

$p \in \{3, 5, 7, \dots\}$ All odd prime number

$$2. \quad 2(n \pm 1) = pn \pm p_0$$

$$pn - 2n + p_0 = 2$$

And

$$2n - pn + p_0 = 2$$

It is like the **Euler's Polyhedron Formula**

We can get Figure 15. This is a symmetry number structure about line-1/2 including all numbers.

Competing Interests statement: The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data Availability statement: No datasets were generated or analyzed during the current study.

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