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Victor M. Valenzuela-Alcaraz , Efrain Ruiz-y-Ruiz * , A. Danisa Romero-Ocaño , Pamela Chiñas-Sánchez , Cecilia Mota-Gutiérrez

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A Matheuristic for the Distance Constrained Inventory Routing Problem

Víctor M. Valenzuela-Alcaraz [1], Efraín Ruiz-y-Ruiz [2,*], A. Danisa Romero-Ocaño [1], Pamela Chiñas-Sánchez [2] and Cecilia Mota-Gutiérrez [2]

[1] TECNM Tecnológico Nacional de México, Instituto Tecnológico de Agua Prieta, Sonora, 84260, México
[2] TECNM Tecnológico Nacional de México, Instituto Tecnológico de Saltillo, Saltillo, Coahuila, 25280, México
* Correspondence: hector.ry@saltillo.tecnm.mx

Abstract

This paper addresses the Distance-Constrained Inventory Routing Problem (DCIRP), a complex problem that combines inventory management and vehicle routing in a logistics context. The problem arises in the context of a specialty gas delivery company that maintains a specialty gas holding facility at each customer's site and uses several trucks to deliver specialty gas, with the additional constraint that drivers are limited to the number of kilometers they can drive each day. A Mixed Integer Linear Programming (MILP) formulation is proposed to model the DCIRP. The DCIRP is a variant of the Inventory Routing Problem (IRP), and a combinatorial optimization problem of type NP-hard. The main objective of this research is to improve the efficiency and effectiveness of DCIRP resolution, while accounting for vehicle capacity constraints, customer inventory levels, and delivery route distance constraints. By optimizing routes and inventory management, the company's operations become more sustainable. To solve the problem, three solution approaches are proposed. The first is an exact method based on the MILP formulation. The second is a matheuristic that uses an inventory-first, route-second (IFRS) approach, including a minimum route cost for each cost and a local search procedure. The results show that the matheuristic solution algorithm produces high-quality solutions with a reasonable computational effort.

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1. Introduction

The coordination of vehicle routing and inventory management decisions is a fundamental aspect of the design and operation of distribution systems. In many real-world logistics contexts, decisions regarding when to replenish customers and how to construct delivery routes are strongly interdependent. Optimizing these decisions separately may therefore lead to inefficient or even infeasible solutions, particularly in systems with recurring demand, multi-period planning horizons, and capacity constraints. This interaction has been widely recognized in the literature, where integrated approaches have been shown to be essential for achieving efficient and reliable distribution performance by jointly considering transportation and inventory holding costs [1–3].

The modeling framework that explicitly integrates inventory and routing decisions is known as the Inventory Routing Problem (IRP). This problem has been studied in a variety of distribution settings, including vendor-managed inventory systems and environments with continuous consumption, where the synchronization between delivery schedules and inventory levels is critical to prevent both stockouts and excessive inventory. As logistics systems become more complex, research on the IRP has progressively incorporated additional operational features, such as vehicle capacity constraints, multi-period decision structures, and considerations related to operational efficiency and sustainability. These extensions have motivated the development of more sophisticated models and solution approaches that better reflect practical distribution conditions [4,5].

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Beyond classical inventory and routing decisions, many distribution systems are subject to operational restrictions on the total length of delivery routes or the maximum distance vehicles are allowed to travel within a planning period. Such limitations typically arise from driving-hour regulations, internal safety policies, or practical operational considerations associated with fleet management. Within the IRP context, explicit route-distance limits further strengthen the interdependence between inventory and routing decisions, as route feasibility is determined not only by vehicle capacity but also by allowable travel distance. This observation motivates the extension of the traditional IRP framework toward formulations that explicitly account for route-length constraints as an additional operational restriction, in line with the integrated logistics principles discussed in prior IRP studies [1–3].

The explicit incorporation of route-length constraints into the Inventory Routing Problem leads to the Distance-Constrained Inventory Routing Problem (DCIRP). In this variant, delivery routes must satisfy inventory balance and vehicle capacity requirements while also respecting a maximum allowable travel distance within each planning period. These distance limits are treated as hard operational constraints that directly affect route feasibility and, consequently, the timing and quantity of customer replenishments. As a result, routing and inventory decisions become even more tightly coupled than in the classical IRP [1–3].

Distance-constrained operations are particularly relevant in distribution systems where driving-hour regulations, safety requirements, or internal operational policies impose strict limits on vehicle usage. In such contexts, routing solutions that are feasible when only capacity considerations are taken into account may become infeasible once distance limits are enforced. Consequently, inventory replenishment plans must be carefully coordinated with routing feasibility to ensure uninterrupted service and avoid operational disruptions. This setting highlights the need for integrated decision-making frameworks capable of simultaneously addressing inventory dynamics and distance-limited routing, in real-world applications like the following:

1. Specialty gas product distribution systems.
2. Fuel distribution in rural areas.
3. L.P. gas distribution in rural areas.
4. Raw milk recollection in rural areas.
5. Produce recollection in rural areas.
6. Drug distribution systems for pharmacies and hospitals.

From a computational perspective, adding explicit route-length constraints further increases the complexity of the integrated inventory-routing problem. The feasible solution space is significantly reduced, and classical solution approaches developed for the standard IRP may experience difficulties when strict distance limits are imposed. These challenges affect both exact optimization formulations and heuristic solution strategies, motivating the development of models and algorithms specifically designed to handle the additional structure introduced by distance constraints [4,5].

In response to these challenges, recent research has explored matheuristic approaches that combine mathematical programming techniques with heuristic or iterative solution strategies. These methods aim to achieve a balance between solution quality and computational efficiency by decomposing the integrated problem into more tractable subproblems while preserving key interactions between inventory and routing decisions [5]. However, there is still limited evidence on the effectiveness of such approaches when strict route-length constraints are explicitly embedded within the inventory-routing framework.

Motivated by these gaps, this paper addresses the DCIRP arising in the context of a specialty gas distribution company, where vehicles must comply with strict daily travel limits due to safety regulations. The objective is to jointly optimize inventory replenishment decisions and delivery routes while respecting vehicle capacity, storage limitations, and maximum route-distance constraints. The objective is to jointly optimize inventory replenishment decisions and delivery routes while respecting vehicle capacity, storage limitations, and maximum route-distance constraints. The main contribution of this paper is the development of an integrated mathematical optimization approach for the DCIRP. To this end, the paper makes the following contributions:

1. A new MILP formulation for the DCIRP, capturing inventory balance, storage limits, vehicle capacities, and maximum travel distance per route.
2. An IMM that integrates an inventory first allocation phase that includes route-cost and route-length approximations with a routing optimization phase enhanced by local search.
3. An extensive computational study comparing the proposed IMM with a classical exact method, demonstrating that the IMM achieves higher quality solutions for medium and large instances while requiring substantially lower computation time.
4. Experimental evidence on the scalability limitations of exact MILP methods and insights into the structural characteristics of DCIRP solutions.

The remainder of this paper is organized as follows. Section 2 presents a relevant review of the literature related to the DCIRP. Section 3 describes the problem and proposes a mixed-integer programming formulation. Three solution approaches are presented in Section 4. Computational experiments and their results are presented in Section 5. Finally, the most relevant findings and potential future research are presented in Section 6.

2. Literature Review

IRP has attracted increasing attention in recent decades due to its ability to combine vehicle routing and inventory management decisions, which were previously optimized separately; this integration directly addresses a key challenge in supply chain management: coordinating mutually influential decisions such as inventory levels, replenishment quantities, routing plans, and delivery schedules [6]. IRP was first used in maritime logistics and then expanded to gas distribution by tanker, automotive supply chains, and, especially, to the distribution of perishable products [5].

The study presented in [7] investigates the integration of production, inventory, and distribution decisions through the Inventory and Production Routing Problem (IRRP), providing an important contribution to the development of integrated logistics models. Their work laid the foundation for the joint optimization of distribution, inventory, and production decisions, thus facilitating the development of an extensive family of IRP variants. Subsequent research expanded these formulations to incorporate more realistic operational constraints, supplier-managed inventory systems, and multi-period planning horizons. IRP is a computationally hard NP problem, which significantly restricts the applicability of exact optimization methods in small instances; one of the first exact solution methods used a branch-and-cut algorithm, which was implemented in the Vendor-Managed Inventory Routing Problem (VMIRP) proposed in [8,9].

Although exact methods have been used successfully in specific variants, such as green IRP [10,11] and IRP for blood distribution under uncertainty [12,13], their high computational cost limits their application in realistic, large-scale contexts. Consequently, most IRP research has shifted toward heuristic and metaheuristic approaches, which sacrifice the certainty of optimality for the benefit of computational feasibility [5,14,15]. These methods have proven useful for managing larger networks, longer planning horizons, and complex sets of constraints [15].

2.1. IRP Variants and Operational Extensions

Over time, the literature on IRP has diversified considerably, giving rise to numerous variants that more accurately reflect real-world logistics circumstances [10,16]. Initial contributions were typically based on deterministic demand assumptions, simple replenishment strategies, and periodic delivery schedules [16]. The Inventory Routing Problem with Time Windows (IRPTW) was developed through more sophisticated formulations that included additional operational constraints, such as time windows for delivery. Representative studies in this area are [17–19].

Other extensions of IRP include heterogeneous vehicle fleets [11], multiple products [20–22], multiple depots [23], and stochastic travel times [24–26]. Similarly, some variants address the distribution of perishable products [27–29] and uncertain demand [30–32]. These developments highlight the flexibility of the IRP framework, but also reinforce its increasing computational complexity as additional constraints are incorporated.

2.2. Routing with Distance Constraints and Its Interaction with Inventory Decisions

A related line of research focuses on routing difficulties with explicit constraints on route length or distance, often investigated under the Distance-Constrained Vehicle Routing Problem (DCVRP) [33]. These constraints are usually motivated by legal regulations on driving hours, fuel range limitations, safety requirements, or internal company policies [34]. Although DCVRP algorithms perform well in pure routing problems, they do not capture the interrelationships between inventory and routing decisions, which are fundamental in IRP scenarios [35,36]. The challenge that arises when distance constraints are incorporated into an inventory routing framework is known as the Distance-Constrained Inventory Routing Problem (DCIRP). In this integrated context, route length constraints not only impact routing feasibility but also directly affect inventory levels, delivery frequencies, and the risk of stockouts.

Solution methodologies for the IRP and DCIRP are typically divided into two main categories: hybrid heuristic frameworks and exact approaches based on mixed-integer linear programming (MILP) formulations [15]. Although exact models can generate optimal solutions, they quickly become computationally intractable as instance size increases [14,37]. As a result, metaheuristics, which are hybrid methods that fuse mathematical programming with heuristic or metaheuristic search, have emerged as a predominant practical option [37,38]. Iterative methods that alternately improve inventory decisions and routing plans, as well as the Inventory First, Routing Second (IFRS) framework [39–41], are among the most outstanding metaheuristic strategies.

Current research indicates that, in large multi-period instances, iterative metaheuristics are capable of reaching near-optimal solutions with considerably reduced computational effort [20–22]. Simultaneously, other research has investigated versions of IRP with an environmental focus, including fuel consumption, carbon emissions [11,42,43], and sustainability goals [11,44].

2.3. Research Gaps and Contribution Positioning

Research on Inventory Routing Problems (IRP) has gradually progressed from deterministic models to more practical, computationally challenging ones. Initially, studies focused on improving mathematical formulations and techniques for finding exact solutions, highlighting the complexity involved in simultaneously optimizing inventory and routing decisions [5,14]. Extensions to multiple-product and multi-period contexts further demonstrated the benefits of integrated logistics planning in reducing total supply chain costs [20,21].

Since then, the operational and ecological perspective has become a primary focus of studies. Various contributions have integrated fuel use, CO_2 emissions, and optimization into multiple objectives, demonstrating that environmentally friendly IRP models can achieve significant reductions in emissions without compromising economic efficiency [27,43–45]. More recent works also addressed vendor-managed inventory and risk-averse decision-making under demand ambiguity, further extending the scope of IRP applications [30,34]. At the same time, the incorporation of time intervals and continuous-time formulations has increased the usefulness of IRP models in real distribution systems with rigorous service requirements [17,18,31]. In parallel, humanitarian and pre-disaster logistics applications demonstrated the relevance of IRP-based models in complex, uncertain, and large-scale environments [32].

Recent research highlights uncertainty and random factors, such as fluctuating demand, product deterioration, and transport times that depend on conditions. These analyses show that explicitly addressing uncertainty enables more robust and effective logistics decisions in areas such as agribusiness, healthcare, and drug distribution [19,22,24,25,28]. However, they also indicate the increased computational complexity associated with realistic, large-scale Inventory Routing Planning scenarios. To address these challenges, the literature is increasingly turning to combinations of hybrid and mathematical methods; although exact techniques such as Branch and Cut remain effective for small to medium-sized problems [5,11,46], more complex and extensive variants of IRP are often handled with decomposition methods, evolutionary algorithms, and combined metaheuristics [15,23,47]. These methods strike a balance between solution quality and computational efficiency, especially in situations involving split deliveries, multiple warehouses, and hazardous material constraints. Despite extensive research on inventory resource planning (IRP), challenges related to DCIRP have received less attention. Many studies focus on traditional IRP formulations or address distance implicitly through criteria based on costs, fuel consumption, or emissions, rather than treating it as an explicit operational constraint. As a result, the relationship between inventory decisions, routing feasibility, and strict distance constraints has not yet been extensively researched, particularly in large-scale environments and realistic operational situations.

This lack of research drives the present analysis, which examines DCIRP by explicitly incorporating distance constraints into an integrated inventory management and routing framework. By merging its methodological approach with its solution strategy, this study enriches the literature by adapting existing IRP models to a more operationally robust formulation, as well as offering effective solution methods that are suitable for large-scale applications [48].

In addition to optimization-based approaches, recent applied studies have analyzed inventory and distribution decisions from an operational and sustainability-oriented perspective. For instance, [49] examines real-world inventory and distribution practices and highlights structural and operational challenges in supply chain decision-making. While such contributions provide valuable contextual and empirical insights, they do not address the problem through an integrated mathematical optimization framework that jointly considers inventory dynamics and distance-constrained routing decisions. This gap motivates the approach proposed in this paper.

As summarized in Tables 1 and 2, the existing literature on inventory routing problems has progressively incorporated environmental, stochastic, and operational considerations, as well as hybrid and matheuristic solution approaches. However, explicit distance-constrained inventory routing formulations remain scarce, motivating the present DCIRP study.

Table 1. Overview of Key Contributions in IRP, DCIRP, and Matheuristic-based Approaches (part 1)

Reference	Problem Variant Method	Main Objective	Distance Constraints	Instances	Main Findings
Coelho et al. (2014) [5]	IRP	Inventory management, vehicle routing and delivery scheduling	NO	Large-scale synthetic	Introduce new valid inequalities to the IRP and some extensions and solve it using a Branch and Cut algorithm on the IRP classical instances.
Archetti et al.(2014) [14]	IRP	Survey and complexity analysis	NO	Integrated and Exact and heuristic review	Literature based
Mirzapour et al. (2014) [21]	Multi-product and period IRP	Minimizes the total cost of the supply chain, inventory holding cost, and transportation cost	NO	Real small-sized test problem	The model could present more constructive solutions from a green logistics point of view.
Cordeau, J. F. et al. (2015) [20]	IRP	the cost savings associated with integrated models for logistics systems	NO	instances of [39]	Show that heuristic is effective on instances with up to 50 customers and 5 products

Table 1. *Cont.*

Reference	Problem Variant Method	Main Objective	Distance Constraints	Instances	Main Findings
Cheng and Rousseau (2018) [43]	multiperiod inventory routing problem (MIRP)	Variable transportation cost by fuel consumption	NO	Generate instances	Incorporating fuel consumption into inventory routing reduces costs and emissions compared to distance-based approaches.
Zapata et al. (2018) [44]	IRP	Optimizing logistics activities and approaches the CO2 emissions	NO	Archetti Instances [39]	Observed that the multiobjective models allow finding an adequate combination between logistics costs and emissions, which is attractive to companies and society
Chekoubi, et al. (2022) [49]	Integrated Production-Inventory- Routing Problem with Remanufacturing (IPIRP-R)	Identification of operational and structural challenges in inventory and distribution decision-making from a sustainability perspective	NO	Generate instances	Studied the Distance-Constrained Inventory Routing Problem (DCIRP), an NP-hard extension of the classical IRP. A MILP formulation and an exact solution method were proposed, performing well on small instances, while an iterative matheuristic proved more effective for medium and large instances, achieving high-quality solutions with lower computational effort.
Demantova et al.(2023) [17]	IRPTW	Minimize the total distance travelled	NO	Instances created by Lappas, Kritikos, and Ioannou (2017) [50] based on the benchmark VRPTW instances of Solomon(1987) [51]	Show how our solution algorithm provides promising results
Ortega and Doerner (2023) [18]	IRPTW	Reduce the routing, consistency, inventory, and lost sales costs	NO	Set of instances introduced in Alarcon et al. (2020) [52]	The Continuous Time Stochastic IRPTW is formulated, and efficient ALNS-based algorithms are proposed to solve it under stochastic demand.
Liu and Zuo (2024) [19]	Single period Inventory Routing problem with split delivery and variable time windows for Small customers	Minimize the sum of transportation and stockout costs	NO	Some problem instances and a real world gasoline delivery case	Experimental results show that it can effectively solve SIRSC, and outperforms CPLEX and other metaheuristics.
Zhang and Li (2024) [27]	Closed-loop inventory routing problem (CIRP)	Maximise the total profit of the holistic	NO	Generate instances	Improving the obtained profit by 80.03% on average under the same computational time

Table 1. *Cont.*

Reference	Problem Variant Method	Main Objective	Distance Constraints	Instances	Main Findings
Violi and Fattoruso (2024) [28]	IRP with stochastic demands	Optimal distribution and inventory planning over a short to medium horizon	NO	Real data related to an agrifood company	Computational experiments on test cases based on a real-life agrifood company located in Italy show the effectiveness of the proposed approach for evaluating different risk attitudes.
Feng and Tian, (2024) [30]	IRP-VMI	Optimal timing and quantity of products to be delivered as well as the optimal vehicle delivery routes from the vendor to retailers	NO	Generate instances	The article introduces a distributionally robust IRP model that integrates the worst case M-CVaR criterion to manage demand ambiguity.
Hasturk et al. (2024). [31]	stochastic cyclic inventory routing problem (SCIRP)	The tactical level inventory decision, one for the operational level purchasing decision, and one for the joint optimization of both levels	NO	Enabling the efficient coordination of tactical and operational decisions in green hydrogen logistics.	
Karakostas and Sifaleras, (2024) [34]	IRP	Total cost of the supply chain network	NO	Ten small and medium sized problem instances were randomly selected by the works of [53]	Introduces two new IRP variants with realistic operational constraints and shows, through MILP models, the advantages of flexible replenishment policies under a vendor-managed inventory strategy.

Table 2. Overview of Key Contributions in IRP, DCIRP, and Matheuristic-based Approaches (part 2)

Reference	Problem Variant Method	Main Objective	Distance Constraints	Instances	Main Findings
Martino, (2024) [15]	IRP	Minimum cost solution that addresses both inventory and transportation problems simultaneously	NO	Generates instances	Proposes exact formulations, an iterative algorithm, and a split-based metaheuristic capable of generating efficient solutions for large-scale instances.
Mundim et al. (2025) [11]	Green IRP with heterogeneous fleet and multiple fuel types	Exact biobjective model minimizing total costs and CO_2 emissions using augmented e-constrained and Branch and Cut	NO	Large scale adapted instances	Proposes a biobjective green IRP with an explicit CO_2 emission model and solves it optimally via augmented e-constrained Branch and Cut.
Haseltalab and Karim (2025) [22]	Deteriorating IRP (DIRP) formulation for PSRP using MINLP and metaheuristics (GATS, GAG)	Minimize routing costs, evaporation losses, and inventory levels	NO	Small and large scale DIRP instances	This research proposes a DIRP approach to solve the PSRP

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Table 2. *Cont.*

Reference	Problem Variant Method	Main Objective	Distance Constraints	Instances	Main Findings
Jiang et al. (2025) [23]	The Multi-depot Vehicle Routing Problem with Uncertain Demand (MD-VRPUD)	Optimization of the assigning customers to different depots and determining the routes for servicing customers	NO	large-scale instances	this paper tailors a surrogate-assisted bi-level evolutionary algorithm (SABLEA) to achieve highly efficient nested algorithms tailored for solving the MD-VRPUD
Baghdadi et al. (2025) [24]	Time dependent pharmaceutical IRP with perishable products on a multi graph network under uncertainty	Minimize total supply chain costs integrating routing, inventory, and disruption management	NO	Realistic urban based pharmaceutical network instances	Shows that time dependent travel times and delivery horizon constraints may lead to longer but more efficient routes
Rave et al. (2025) [25]	Two stage stochastic multiproduct two echelon IRP with instant replenishment and drone/van delivery	Minimize total inventory and distribution costs under stochastic demand	NO	Real world case study based on a German healthcare network	Integrated stochastic IRP with drones significantly reduces healthcare supply chain costs
Temiz et al. (2025) [32]	Pre - disaster humanitarian logistics with cross-docking, heterogeneous fleet, and fuel consumption estimation	Minimize total logistics cost and total travel time under disaster uncertainty	NO	Case study and large-scale humanitarian logistics instances	The proposed probabilistic biobjective model effectively supports pre-disaster planning by integrating location, routing, and uncertainty, with a clustering-based heuristic enabling large-scale solutions
Sharma et al. (2025) [45]	e-IRP framework for green and flexible supply chains	Enhance supply chain flexibility and sustainable performance under low carbon transition	NO	Real world case study in Indian petroleum refinery sector	Green innovation, digitalization, and institutional pressure enhance supply chain flexibility and sustainability in the petroleum refinery sector
Maia et al. (2026) [48]	Review and extended taxonomy of stochastic dynamic IRP (SDIRP)	Analyze existing SDIRP models, policies, and research gaps	NO	NO	Highlights research gaps and emerging trends in SDIRP, emphasizing uncertainty modeling and real-time, large-scale decision methods.
Yu et al. (2026) [46]	Integrated IRP combining OVRP and inventory replenishment solved via hybrid Branch and Cut	Minimize total logistics and inventory costs over multiple periods	NO	Small and large scale test instances (72 instances)	The proposed algorithm efficiently solves the integrated IRP, significantly outperforming a commercial solver on large instances
Avishan et al. (2026) [47]	IRP with split delivery and split backhauling for multi-commodity distribution, solved via Branch and Cut and a two-phase decomposition matheuristic	Minimize delivery, backhauling, and inventory holding costs under hazardous-material separation constraints	NO	Synthetic large-scale instances and real-world case study (Hydro-Québec)	Integrating split delivery and backhauling reduces routing costs and enables efficient solutions for large-scale IRP instances

3. Problem Description and Formulation

The Distance Constrained Inventory Routing Problem (DCIRP) focuses on optimizing the inventory and distribution route costs for a commodity over a given planning horizon. The goal is to

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minimize the total cost (inventory and routing). In this context, operational decisions must be made regarding when and how much of the commodity should be sent to each of the clients. In this problem, a set of customers is considered. Each customer has a capacity-limited installation for storing the commodity, which consumes at a certain rate (units of the commodity over a period of time). The commodity is produced at a given facility located at the depot, which also has a limited-capacity storage installation to maintain a stock of the commodity. There are costs associated with storing the commodity at each client and at the supplier's facility. To deliver the commodity, a fleet of homogeneous vehicles with limited capacity is used. There are also costs associated with travelling between the depot and the client's locations.

The problem is formally defined with a complete directed graph, $G = (V, A)$, where V is the set of vertices and $A = (i, j)|i \in V, j \in V|i \neq j$ is the set of arcs. The set V contains two subsets of vertices: i) $V^0 \subset V$, the depot, and ii) $V^+ \subset V$, the subset of clients. Therefore, $V = V^0 \cup V^+$. For every arc $(i, j) \in A$, there is an associated cost, c_{ij}, that represents the cost of traveling between vertices $i \in V$ and $j \in V$. There is also a set of available homogeneous vehicles K with capacity Q and a set of periods in the planning horizon T.

Formulation

A mixed-integer linear programming (MILP) formulation is proposed and presented in this section to model the DCIRP. This MILP formulation uses binary variables and continuous variables. Before formulating the problem, the following parameters are defined:

n	the total number of vertices in graph G, with $n = |V|$;
$\{0\}$	the depot;
t	the number of periods in the planning horizon $t = |T|$.
h_i	the cost of storage per unit at the storage facility at vertex $i \in V$
p_t	production rate at the depot at period $t \in T$;
r_{jt}	consumption rate of customer $j \in V^+$ at period t;
c_{ij}	the cost of travelling using the arc $(i, j) \in A$;
dt_{ij}	the distance to travel from vertex $i \in V$ to vertex $j \in V$;
U_i	the limited capacity of the storage facility at client $i \in V^+$;
U_0	the limited capacity of the storage facility at the depot $\{0\}$;
Ip_i	the initial inventory in the storage facility at client $i \in V^+$;
Ip_0	the initial inventory in the storage facility at the depot $\{0\}$;
$Ireq_i$	the minimum inventory level required in the storage facility at client $i \in V^+$;
Q	the capacity of vehicles;
k	the number of available vehicles at each period $k = |K|$;
L	the maximum route duration;

Also, the following variables are defined:

$$z_{it} = \begin{cases} 1, & \text{if client } i \in V^+ \text{ is visited in period } t \in T \\ 0, & \text{otherwise.} \end{cases}$$

$$x_{ijtk} = \begin{cases} 1, & \text{if arc } (i,j) \text{ is used by a vehicle at period } t \in T \\ 0, & \text{otherwise.} \end{cases}$$

$$I_{it} = \text{the inventory at vertex } i \in V \text{ at the end of period } t \in T$$

$$q_{it} = \text{the quantity to be sent to client } i \in V \text{ in period } t \in T$$

$$w_{it} = \text{load on the vehicle after visiting client } i \in V^+ \text{ at period } t \in T$$

$$y_{it} = \text{accumulated distance used by the vehicle after visiting client } i \text{ at period } t \in T$$

Using the previously defined parameters and variables, the following optimization formulation arises:

$$\text{Min} \quad \sum_{t\in T}\sum_{i\in V}\sum_{j\in V} c_{ij}x_{ijt} + \sum_{i\in V}\sum_{t\in T} h_i I_{it} \tag{1}$$

subject to

$$I_{0t} = Ip_0 + r_{0t} - \sum_{i\in V^+}\sum_{k\in K} ql_{itk} \qquad t = 1 \tag{2}$$
$$I_{0t} = I_{0t-1} + r_{0t} - \sum_{i\in V^+}\sum_{k\in K} ql_{itk} \qquad \forall t\in T, t>1 \tag{3}$$
$$I_{it} = Ip_i - r_{it} + \sum_{k\in K} ql_{itk} \qquad \forall i\in V^+, t=1 \tag{4}$$
$$I_{it} = I_{it-1} - r_{it} + \sum_{k\in K} q_{itk} \qquad \forall i\in V^+, t\in T, t>1 \tag{5}$$
$$q_{it} \le U_i - Ip_i \qquad \forall i\in V^+, \forall t\in T, t=1 \tag{6}$$
$$q_{it} \le U_i - I_{it-1} \qquad \forall i\in V^+, \forall t\in T, t>1 \tag{7}$$
$$q_{it} \le Qz_{it} \qquad \forall i\in V^+, \forall t\in T \tag{8}$$
$$\sum_{j\in V} x_{0jt} \le |K| \qquad \forall t\in T \tag{9}$$
$$\sum_{i\in V} x_{ijt} = z_{jt} \qquad \forall j\in V^+, \forall t\in T \tag{10}$$
$$\sum_{j\in V} x_{ijt} - \sum_{j\in V} x_{ijt} = 0 \qquad \forall i\in V^+, \forall t\in T \tag{11}$$
$$w_{it} \ge q_{it} \qquad \forall i\in V^+, \forall t\in T \tag{12}$$
$$w_{it} \le Q \qquad \forall i\in V^+, \forall t\in T \tag{13}$$
$$w_{jt} \ge w_{it} + q_{jt} - Q + Qx_{ijt} \qquad \forall j\in V^+, \forall i\in V^+, \forall t\in T \tag{14}$$
$$y_{jt} \ge y_{it} + dt_{ij} - L + Lx_{ijt} \qquad \forall j\in V^+, \forall i\in V^+, \forall t\in T \tag{15}$$
$$y_{it} \le L \qquad \forall i\in V^+, \forall t\in T \tag{16}$$
$$z_{it} \in \{0,1\} \qquad \forall i\in V^+, \forall t\in T \tag{17}$$
$$x_{ijt} \in \{0,1\} \qquad \forall i\in V, \forall j\in V, \forall t\in T \tag{18}$$
$$I_{it} \ge Ireq_i \qquad \forall i\in V^+, \forall t\in T \tag{19}$$
$$q_{it} \ge 0 \qquad \forall i\in V^+, \forall t\in T \tag{20}$$
$$w_{it} \ge 0 \qquad \forall i\in V^+, \forall t\in T \tag{21}$$
$$y_{it} \ge 0 \qquad \forall i\in V^+, \forall t\in T \tag{22}$$

The objective function (1) minimizes the total cost, which includes routing costs for customer deliveries (first term) and inventory maintenance costs (second term). Constraints 2 and 3 are inventory conservation constraints for the depot. The term on the left of the equation represents the inventory at the end of the given period $t \in T$. On the right side of the equation, the first term represents the inventory level at the previous period $t-1$, the second the production rate at the period $t \in T$, and the third term the quantity sent for distribution during the period $t \in T$. Constraints 4 and 5 are inventory conservation constraints for the clients. The first is for the initial period $t = 1$, while the second is for the subsequent periods $t > 1$. For these constraints, the term on the left of the equation represents the inventory at the end of the given period $t \in T$, while the first term on the right side represents the inventory level at the previous period $t-1$, the second the consumption rate at the period $t \in T$, and the third term the quantity sent to the client during the period $t \in T$. Constraints 6-8 limit the amount of product that can be sent to a customer. The first two, 6 and 7, limit the amount based on storage capacity, while constraints **??** limit the amount sent to the customer based on the vehicle's capacity.

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Constraint 9 limits the number of vehicles that can depart in the same period. Constraints 10 state that a client i can be distributed in period t only if it is visited by a vehicle. Constraints 11 state that if a client is visited by a vehicle in period T, the vehicle should depart from the client at such period. Constraints 12 to 14 are an adaptation of the subcircuit breaking constraints known as MTZ constraints proposed by [54]. The adaptation is made considering the vehicle load and the variables x_{ijt} used in the formulation. In this case, constraints 12 and 13 establish, respectively, lower and upper bounds on the vehicle's load after visiting a client. Constraints 14 increase the load at the vehicle after departing client j, if arc (i, j) is used to arrive at j. Distance (Route length) constraints 15 and 16 are used to control the route length. These constraints are also adapted from the previously mentioned MTZ constraints. With constraints 15, the distance traveled after leaving vertex j is updated considering the distance dt_{ij} if the arc (i, j) is used, while constraints 16 guarantee that routes do not exceed the maximum traveling distance. Finally, constraints 17 to 22 are variable domain constraints.

This type of MILP model can be solved using optimization methods such as the simplex, combined with integer optimization techniques, to obtain solutions for the DCIRP. For this research, the EM combines a dual-simplex algorithm with branch-and-bound techniques to solve DCIRP instances. An example of a valid solution for the DCIRP is presented in Figure 1.

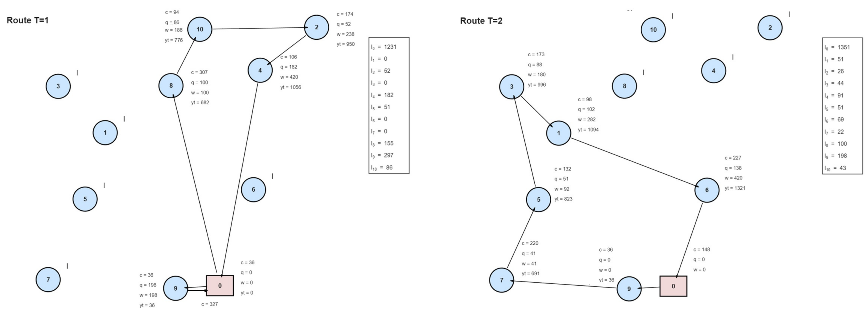

Figure 1. DCIRP solution example

4. Iterative Metaheuristic with Local Search

Instances of the DCIRP can be solved using exact, metaheuristic, and matheuristic methods. Exact methods are based on mathematical programming formulations. On the other hand, metaheuristic methods are inspired by natural phenomena, physics phenomena, and animal behavior, among other sources. Finally, matheuristic methods combine mathematical programming formulations with metaheuristics. In this section, two solution approaches are presented. The first is an exact method (EM) based on the formulation presented in Section 3. The EM uses a branch-and-bound algorithm over the DCIRP formulation to obtain solutions. A good description of the branch-and-bound algorithm can be found in [55]. Since the DCIRP is NP-hard, solution methods such as EMs are non-polynomial and require significant computational resources. In other words, the complexity of the EM grows exponentially as the size of the instances increases, and so does the computational effort.

Therefore, an Iterative Matheuristic Method (IMM) was designed and developed to solve DCIRP instances. The IMM is a heuristic decomposition method that divides the problem into two subproblems. The first subproblem ($Sub1Inv$) addresses the inventory component of the DCIRP, while the second subproblem ($Sub2VRP$) addresses the routing component.

4.1. Inventory Subproblem

As mentioned before, the DCIRP is divided into two subproblems, and the first one, $Sub1Inv$ is explained in this section. The $Sub1Inv$ subproblem considers an objective function that includes inventory costs and an approximation of routing costs. For this subproblem a k index related to vehicles is added to some of the variables to help approximate routing and inventory costs. The model also includes inventory flow conservation constraints and constraints related to vehicle capacity and route length. To model the $Sub1Inv$ subproblem, the following parameters and variables were introduced.

- Parameters:
  mtc_i the minimum travelling cost to arrive to vertex $i \in V$;
  mdt_i the minimum required time to arrive to vertex $i \in V$;
- Variables:

$$zl_{itk} = \begin{cases} 1, & \text{if client } i \in V^+ \text{ is visited by vehicle } k \in K \text{ in period} t \in T \\ 0, & \text{otherwise.} \end{cases}$$
$$ql_{itk} = \text{the quantity to delivered to client } i \in V^+ \text{ using vehicle } k \in K \text{ in period } t \in T$$

The minimum travelling cost to arrive at vertex i is computed using:

$$mtc_i = minargc(c_{ji}|j \in V)$$

, while the minimum distance required to arrive at vertex i is computed using:

$$mdt_i = minargc(tt_{ji}|j \in V)$$

.

Using the previous and new parameters and variables, the $Sub1Inv$ subproblem is formulated as follows:

$$\text{Min} \quad \sum_{i\in V}\sum_{t\in T} h_i I_{it} + \sum_{t\in T}\sum_{i\in V}\sum_{k\in K} mtc_i zl_{itk} \tag{23}$$

subject to

$$I_{0t} = Ip_0 + r_{0t} - \sum_{i\in V^+}\sum_{k\in K} ql_{itk} \qquad t = 1 \tag{24}$$
$$I_{0t} = I_{0t-1} + r_{0t} - \sum_{i\in V^+}\sum_{k\in K} ql_{itk} \qquad \forall t \in T, t > 1 \tag{25}$$
$$I_{it} = Ip_i - r_{it} + \sum_{k\in K} ql_{itk} \qquad \forall i \in V^+, t = 1 \tag{26}$$
$$I_{it} = I_{it-1} - r_{it} + \sum_{k\in K} q_{itk} \qquad \forall i \in V^+, t \in T, t > 1 \tag{27}$$
$$\sum_{k\in K} ql_{itk} \le U_i - Ip_i \qquad \forall i \in V^+, t \in T, t = 1 \tag{28}$$
$$\sum_{k\in K} ql_{itk} \le U_i - I_{it-1} \qquad \forall i \in V^+, t \in T, t > 1 \tag{29}$$
$$q_{itk} \le Qzl_{itk} \qquad \forall k \in K, \forall \in V^+, \forall t \in T \tag{30}$$
$$\sum_{i\in V^+} ql_{itk} \le Q \qquad \forall k \in K, t \in T \tag{31}$$
$$\sum_{i\in V^+} mdt_i zl_{itk} \ge Q \qquad \forall k \in K, t \in T \tag{32}$$
$$zl_{itk} \in \{0,1\} \qquad \forall i \in V^+, \forall t \in T \tag{33}$$
$$I_{it} \ge Ireq_i \qquad \forall i \in V^+, \forall t \in T \tag{34}$$
$$ql_{itk} \ge 0 \qquad \forall i \in V^+, \forall t \in T \tag{35}$$
$$w_{it} \ge 0 \qquad \forall i \in V^+, \forall t \in T \tag{36}$$

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The objective function 23 includes two terms: the first computes the inventory cost, while the second term is an approximation of the routing costs. Constraints 24 to 30 are inventory-related constraints from the DCIRP model updated using the q_{itk} variables. The group of constraints 31 is capacity-related, while constraints 32 are an approximation of route length constraints. Let zl^* and ql^* represent the values for variables zl^*_{itk} and ql^*_{itk} in the optimal solution of subproblem $Sub1Inv$. The values of these variables indicate the period in which a client will be visited, and the quantity they will receive. The solutions to subproblem $Sub1Inv$ are inventory-feasible but may be route-infeasible, as shown in the Section 4.2.

4.2. Routing Subproblem

The routing subproblem $Sub2VRP$ obtains routes for every period $t \in T$ using zl^* and ql^* from the solution of subproblem $Sub1Inv$. In this case, the following parameters are defined:

- Parameters:
  $fq_{it} = \sum k \in Kql^*_{itk}$ demand to be delivered to the client $i|inV^+$ in period $t \in T$;
  $vt_{it} = \sum k \in Kzl^*_{itk}$ if the client $i \in V$ is visited in period $t \in T$;

Note that parameters vt_{it} and fq_{it} for $Sub2VRP$ are computed using the solution of variables zl^*_{itk} and ql^*_{ikt} of subproplem $Sub1Inv$. In other words, the quantity and the period in which a client $i|inV^+$ will receive its required demand are fixed. Therefore, to obtain routes, a multiperiod VRP has to be solved. Using the previous and new parameters and variables, the $Sub2VRP$ subproblem is formulated as follows:

$$\text{Min} \quad \sum_{t\in T}\sum_{i\in V}\sum_{j\in V} c_{ij}x_{ijt} \tag{37}$$

subject to

$$\sum_{i\in V} x_{ijt} = vt_{it} \qquad \forall j \in V^+, t \in T \tag{38}$$

$$\sum_{j\in V} x_{ijt} - \sum_{j\in V} x_{ijt} = 0 \qquad \forall i \in V^+, t \in T \tag{39}$$

$$w_{it} \geq fq_{it} \qquad \forall i \in V^+, t \in T \tag{40}$$

$$w_{it} \leq Q \qquad \forall i \in V^+, t \in T \tag{41}$$

$$w_{jt} \geq w_{it} + fq_{jt} - Q + Qx_{ijt} \qquad \forall j \in V^+, i \in V^+, t \in T \tag{42}$$

$$yt_{jt} \geq yt_{it} + dt_{ij} - L + Lx_{ijt} \qquad \forall j \in V^+, i \in V^+, t \in T \tag{43}$$

$$yt_{it} \leq L \qquad \forall i \in V^+, t \in T \tag{44}$$

$$x_{ijt} \in \{0,1\} \qquad \forall i \in V, \forall j \in V, \forall t \in T \tag{45}$$

$$w_{it} \geq 0 \qquad \forall i \in V^+, \forall t \in T \tag{46}$$

$$y_{it} \geq 0 \qquad \forall i \in V^+, \forall t \in T \tag{47}$$

Constraints 38 state that a client i should be visited in period t. Constraints 39 state that if a client is visited by a vehicle in period T, the vehicle should depart from the client at such period. Constraints 40 to 42 are the already mentioned MTZ subcircuit breaking constraints. Contraints 43 and 44 are the previously mentioned MTZ constraints for route length control. Finally, constraints 45 to 47 are variable domain constraints.

Note that it is possible that solutions from subproblem $Sub2VRP$ could be infeasible, because of route length constraints. The latter, since clients are assigned when solving subproblem $Sub1Inv$ that uses an approximation to determine if the route length constraints will be respected when solving subproblem $Sub2VRP$. The solution of subproblem $Sub2VRP$, (if there is one) will be denoted as x^*.

mtc_i the minimum travelling cost to arrive to vertex $i \in V$;
mdt_i the minimum required time to arrive to vertex $i \in V$;

4.3. Iterative Process

The iterative process starts by solving the first subproblem $Sub1Inv$. Using the results from subproblem $Sub1Inv$, the second subproblem $Sub2VRP$ is solved. Variables I_{it} are fixed after solving subproblem $Sub1Inv$. Subproblem $Sub2VRP$ receives the values of variables zl^*_{itk} and ql^*_{itk} and uses them to define parameters vt_{it} and fq_{it}, defining in which period a clients $i \in V^+$ is visited and the product quantity it receives. After solving $Sub2VRP$, if the solution is feasible, the values of variables x_{ijt} define a set of valid routes. In Table 3, the inputs, outputs, and variables fixed after solving each subproblem are presented.

Table 3. Inputs and outputs of subproblems

Subproble	Input	Output	Fixed variables
$Sub1Inv$	h_i, mtc_i, mdt_i	$I_{it}, zl^*_{itk}, ql^*_{ikt}$	I_{it}
$Sub2VRP$	$c_ij, dt_{ij}, zl^*_{itk}, ql^*_{itk}$	Objective function value, x_{ijt}, w_{it}, y_{it}	x_{ijt}

Therefore, delivery quantities, routing nodes per period, and inventory levels are defined by subproblem $Sub1Inv$. Using the delivery quantities and routing nodes, subproblem $Sub2VRP$ is solved. If the solution is feasible, then a solution of the problem is obtained. Using the results of the solution of subproblem $Sub2VRP$, coefficients in the objective function of subproblem $Sub1Inv$ are modified. This process is iteratively repeated until specific criteria are met. The stop criteria are met when either a specified number of iterations without improvement is reached or a time limit is exceeded. Let be zl^{iter*}, ql^{iter*}, I^{iter*}, x^{iter*} and OF^{iter} the solutions for variables zl^*_{itk}, ql^*_{ikt}, I^*_{it} and x^*_{ijt}, and the value of the objective function at each iteration fo the IMM. Also, let $BestI$ and $Bestx$ the values the values for variables I^*_{it} and x^*_{ijt} with the best objective function $BestOF$. Note that OF^{iter} and $BestOF$ are computed using equation (1). After each iteration Using the results of $Sub2VRP$, parameters mtc_i and mdt_i for clients $i \in V^+$ are disturbed using:

$$mtc_i = \alpha \cdot rand(0,1) \cdot \sum_{j \in V} \sum_{t \in t} c_{ji} x^*_{ijt} \tag{48}$$

$$mdt_i = \alpha \cdot rand(0,1) \cdot \sum_{j \in V} \sum_{t \in t} tt_{ji} x^*_{ijt} \tag{49}$$

where $rand(0,1)$ is a random number in the interval $[0,1]$ and $alpha$ is a number in the interval $[1,10]$. The parameter $alpha$ is increased when subproblem $Sub1Inv$ yields infeasible solutions in subproblem $Sub2VRP$, and decreased after no improvement is the objective function that is computed using equation (1). The pseudocode and the flow chart for the IMM are presented in Algorithm 1 and in Figure 2.

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Algorithm 1: Pseudo-code for the IMM

Iterative Matheuristic Method

Input: $V, T, K, h, p, r, c, tt, U, Ip, Q, L, d,$
Output: $Bestql^*$,$Bestx^*$
Initialize subproblem $Sub1Inv$;
Initialize subproblem $Sub2VRP$;
while *Stop criteria not met* **do**
  Solve $Sub1Inv$ an obtain zl^{iter*} and ql^{iter*};
  Solve $Sub2VRP$ using zl^{iter*} and ql^{iter*}, and obtain x^{iter*};
  if x^* *is feasible* **then**
    Compute OF^{iter} using ql^{iter*} and x^{iter*};
    if $OF^{iter} < BestOF$ **then**
      $BestOF \leftarrow OF^{iter}$;
      $BestI \leftarrow I^{iter}$;
      $Bestx \leftarrow x^{iter*}$;
    end
  end
  Update parameters mtc_i and mdt_i for clients $i \in V^+$;
end
return

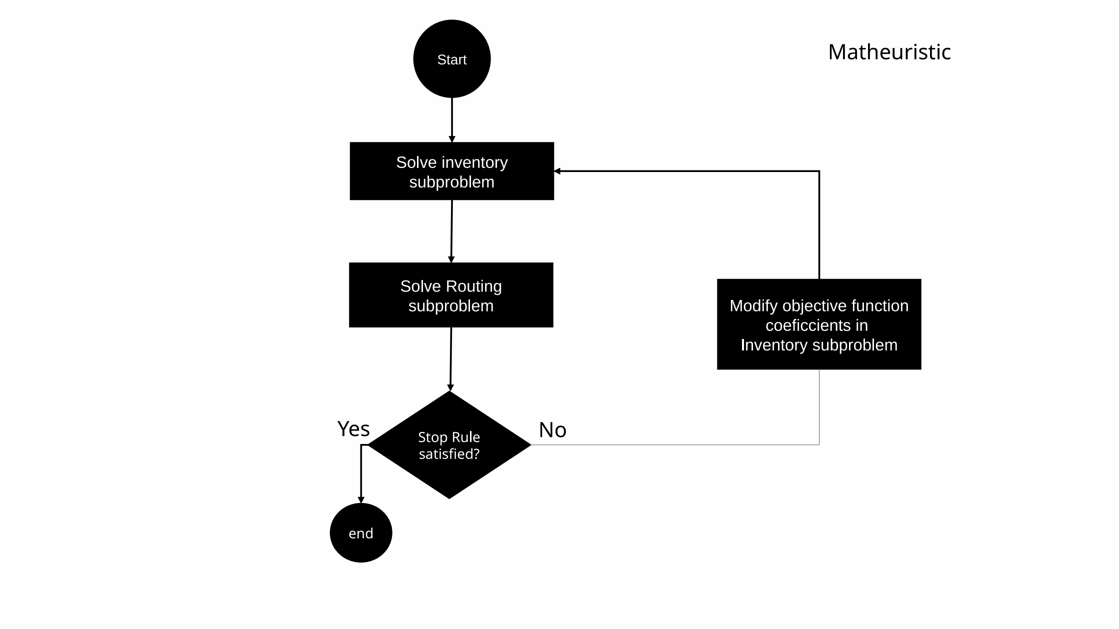

Figure 2. Matheuristic algorithm flow chart.

4.4. Local Search

At the end of each iteration, a local search procedure is performed to improve the solution obtained. In this case, a two-opt neighborhood was chosen. In the two-opt local search, two client vertices belonging to different routes are selected. It is worth noting that the vertices can belong to routes at different time periods. For example, vertex i belonging to route $R1$ in $t = 1$ can be exchanged with vertex j in route $R2$ in $t = 3$. The selected vertices must respect the vehicle capacity constraints if they are traded; if not, they are disposed of, and a new pair of vertices is selected. When the pair of vertices satisfies the capacity constraints related to vehicle and storage capacity, an exchange function is computed to determine whether the resulting routes reduce the routing cost. If the routes do not reduce the routing cost, they are disposed of, and a new pair of vertices is selected. If the chosen

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vertices reduce the routing cost and belong to different time periods, an inventory cost change function must be computed. The latter, because inventory costs differ from client to client. Therefore, before any exchange is made, capacity is checked, and the exchange cost is evaluated considering inventory and routing costs. If the exchange reduces the objective function value, it is performed, and the routes are updated.

To describe the local search, the following notation is introduced:

- p_i the vertex visited before vertex $i \in V^+$ (predecessor).
- s_i the vertex visited after vertex $i \in V^+$ (succcesor).
- t_i the time period in which vertex $i \in V^+$ is visited.
- dq_i the quantity delivered to vertex $i \in V^+$
- k_i the vehicle visitingto vertex $i \in V^+$
- AvQ_{k_i} the available capacity of the vehicle visiting vertex $i \in V^+$
- ΔQ_{k_i} the capacity change for the vehicle visiting vertex $i \in V^+$
- OF^{iter} the objective function value at the current iteration

In Algorithm 2, the pseudocode for the local search procedure is presented. In lines 3 and 4, information regarding the vertices involved in the exchange is extracted from variables x^{iter*} and ql^{iter*}. In line 5, the route-cost change $\Delta Rcost$ is computed. If $\Delta Rcost$ is negative (line 6), changes in the vehicle capacity and storage capacity constraints are computed (lines 7-8). The feasibility of the changes in capacity-related constraints is checked at line 9. If the routes belong to the same period, the inventory cost $\Delta Icost$ is set to zero (line 10). In case the vertices belong to routes in different time periods, $\Delta Icost$ is computed either in line 12 or 15. In line, the total cost change $\Delta Tcost$ is computed using $\Delta Rcost$ and $\Delta Icost$. If $\Delta Tcost$ is negative, the change is made, and the values of the objective function and the variables are updated in lines 20 to 26

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Algorithm 2: Pseudo-code for the 2-opt local search

seudo-code for the 2-opt local search

Input: $V, T, K, h, p, r, c, tt, U, Ip, Q, L, d, I^{iter*}, zl^{iter*}, ql^{iter*}, x^{iter*}, OF^{iter}$
Output: $OF^{iter}, I^{iter*}, zl^{iter*}, ql^{iter*}, x^{iter*}$

for $i \in V^+$ **do**
  for $i \in V^+, j \neq i$ **do**
    Obtain p_i, s_i, p_j, s_j, t_i and t_j using x^{iter*}
    Obtain dq_i and dq_j using ql^{iter*}
    Compute $\Delta Rcost := c_{p_j,i} + c_{i,s_j} + c_{p_j,i} + c_{i,s_j} - c_{p_i,i} - c_{i,s_i} - c_{p_j,j} - c_{j,s_j}$
    if $\Delta Rcost < 0$ **then**
      Compute $\Delta Q_{k_i} := dq_j - dq_i$
      Compute $\Delta Q_{k_j} := dq_i - dq_j$
      if $AvQ_{k_i} \geq \Delta Q_{k_i}$ *and* $AvQ_{k_j} \geq \Delta Q_{k_j}$*and* $U_i \leq Iiter*_{i,t_j}$ *and* $U_j \leq Iiter{**}_{j,t_i}$ **then**
        $\Delta Icost := 0$
        if $t_i \geq t_j$ **then**
          Compute $\Delta Icost := (t_i - t_j) \cdot h_i dq_i - (t_i - t_j) \cdot h_j dq_j + h_0 \cdot (dq_i - dq_j)$
        end
        if $t_j \geq t_i$ **then**
          Compute $\Delta Icost := (t_j - t_i) \cdot h_i dq_j - (t_j - t_i) \cdot h_i dq_i + h_0 \cdot (dq_j - dq_i)$
        end
      end
      $\Delta Tcost := \Delta Rcost + \Delta Icost$
      if $\Delta Tcost < 0$ **then**
        Update $OF := OF - \Delta Tcost$ Update variables x^{iter*}
        if $t_i \neq t_j$ **then**
          Update variables $Iiter*$ using dq_i, dq_j
          Update variables zl^{iter*}
          Update variables ql^{iter*}
          Update $AvQ_{k_i} := AvQ_{k_i} + \Delta Q_{k_i}$
          Update $AvQ_{k_j} := AvQ_{k_j} + \Delta Q_{k_j}$
        end
      end
    end
  end
end
return

An example of the two-opt local search is presented in Figure 3. In the example, two routes: route 1 (0-2-1-3-5) and route 2(7-6-4) are part of the solution. In this case, vertex 4 from Route 1 is exchanged with vertex 5 from Route 2, leading to the new Route 1 (0-2-1-3-4) and Route 2 (7-6-5).

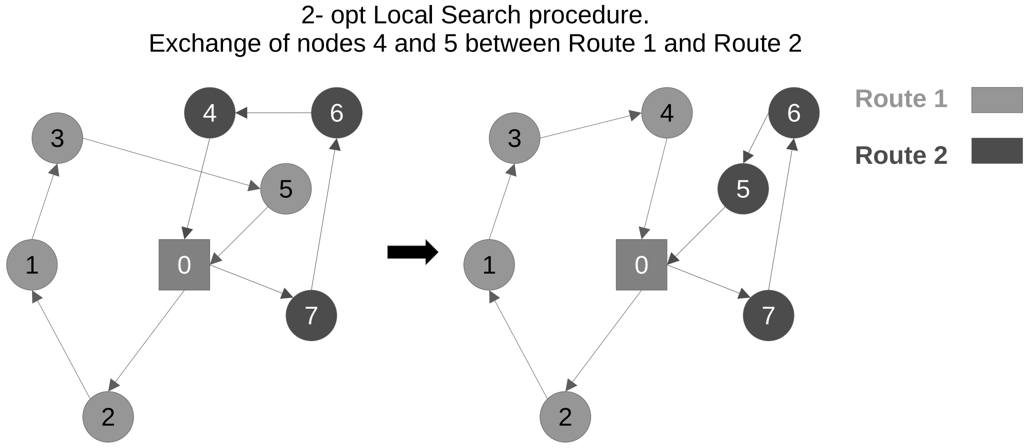

Figure 3. 2-opt Local Search example. Exchange of nodes 4 and 5

5. Computational Experiments

To test the performance of the proposed EM and IMM solution algorithms, 50 test instances from the literature proposed by [56] for the IRP with priorities. To capture the spirit of the real-world problem that inspired this research, the original test instances were modified to include route-length constraints. The modifications included adding the maximum route length parameter L, modifying the capacity parameter Q, and eliminating the priority parameter. Let $Q_{ne}w$ and Q_{orig} be the modified and original parameters of Q. The parameter Q_{new} was modified using $Q_{new} = 10 \cdot \lceil \frac{Q_{orig}}{1.8+rand(0,1)} \rceil$. To establish the L parameter, each updated instance (with $Q_{ne}w$) was solved using the formulation by [56], without the priority parameter. Since the IRP is NP-hard, a time limit of 10,800 seconds was set for the solver. Using the best solution for each instance, a parameter L_{orig}, defined as the length of the largest route, was computed. To define parameter L the following formula was used $L = 0.75 \cdot L_{orig}$. It is important to note that this type of definition of L could lead to infeasible instances. To assess the feasibility of the instance, the DCIRP formulation presented in Section 3 and the solver's presolve function were used in preliminary experiments. The presolve function of the solver can determine whether the test instance is infeasible. Using the presolve function, each instance was tested for feasibility. If the presolver led to an infeasible result, the L value was updated using $L = L + 10$, until the presolver did not obtain an infeasible solution result. The benchmark instances for the DCIRP are available at https://www.saltillo.tecnm.mx/posgrado/linktobedefined.html. The test instances ranged in size from 5 to 50 clients. The names of the test instances reflect the number of clients. The EM and IMM solution algorithms were developed in the MOSEL language within FICO Xpress 9.01.

The experimental setup was designed to assess both solution quality and scalability. The number of clients was varied from 5 to 50 to represent small to moderately large instances commonly used in IRP studies. Route-length constraints were modeled as hard operational limits that reflect practical distribution constraints. A maximum computation time of 3,600 seconds was imposed for each instance to ensure fair comparisons between solution approaches.

For the IMM, the following parameters were used:

- Stopping criterion 1: $MaxT = 3600$, the maximum execution time;
- Stopping criterion 2: $ItStop = 350$, the number of iterations without improvement in the BRKGA
- $alpha = 1$, the initial value for α

All experiments were run on a PC with an AMD Ryzen 9 3950 processor at 3.5 GHz. with 16 cores and 64 GB of RAM. Tables 4–5 show, respectively, the results of EM (Formulation) and IMM using the

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benchmark test instances. The EM was run once for each test instance, since this solution method is deterministic (The solution is the same no matter how many times it is run). For EM, a maximum solution time of 3,600 s was defined. For the IMM, five runs were performed. For each run a different seed was used. To define the seed the Mosel command ***setrandseed(currenttime)*** was used.

The results obtained by each solution method are presented in Tables 4 and 5. A comparison of the best solutions obtained by each solution algorithm and the time required to find them is presented in Table 6.

For Tables 4 and 5, column **Instance** gives the name of the test instance, while column **UB** shows the upper bound obtained by the EM. The columns **IC** and **RC** show, respectively, the Inventory and Routing costs for each instance, and *Time* shows the elapsed time used by the EM. For Table 5, columns **Best UB**, **Mean UB**, and **Median UB** present, respectively, the best, mean, and median values of the objective function after five runs of the IMM. Columns **Avg. IC**, **Avg. RC**, and **Avg. Time** present, respectively, the average Inventory Cost, Routing Cost, and solution time. Finally, column **Itbest** presents the average iteration in which the IMM found the best solution, and σ presents the standard deviation between the five solutions obtained by the IMM.

Table 4. Exact method's results.

Instances	**UB**	**IC**	**RC**	**Time**
InsC5P0T3-1.dat	1,775.49	471.49	1,304	0.73
InsC5P0T3-2.dat	1,899.02	657.02	1,242	0.20
InsC5P0T3-3.dat	4,110.76	980.76	3,130	0.85
InsC5P0T3-4.dat	2,381.27	476.27	1,905	0.92
InsC5P0T3-5.dat	2,237.60	928.60	1,309	1.40
InsC10P0T3-1.dat	4,828.95	2,251.95	2,577	222.92
InsC10P0T3-2.dat	4,698.02	1,885.02	2,813	606.73
InsC10P0T3-3.dat	3,744.11	1,759.11	1,985	8.26
InsC10P0T3-4.dat	3,870.99	1,756.99	2,114	31.87
InsC10P0T3-5.dat	4,040.98	2,293.98	1,747	4.45
InsC15P0T3-1.dat	5,195.63	2,799.63	2,396	115.96
InsC15P0T3-2.dat	5,007.31	2,687.31	2,320	1237.41
InsC15P0T3-3.dat	5,864.45	3,197.45	2,667	3600.00
InsC15P0T3-4.dat	4,827.26	2,262.26	2,565	3600.00
InsC15P0T3-5.dat	4,712.60	2,213.60	2,499	3600.00
InsC20P0T3-1.dat	7,297.67	3,781.67	3,516	3600.00
InsC20P0T3-2.dat	6,251.49	3,737.49	2,514	3600.00
InsC20P0T3-3.dat	6,880.18	3,879.18	3,001	3600.00
InsC20P0T3-4.dat	7,676.71	3,124.71	4,552	3600.00
InsC20P0T3-5.dat	8,278.08	4,123.08	4,155	3600.00
InsC25P0T3-1.dat	7,974.01	4,300.01	3,674	3600.00
InsC25P0T3-2.dat	9,660.05	4,688.05	4,972	3600.00
InsC25P0T3-3.dat	10,611.84	5,152.84	5,459	3600.00
InsC25P0T3-4.dat	7,809.74	4,393.74	3,416	3600.00
InsC25P0T3-5.dat	12,142.33	5,875.33	6,267	3600.00
InsC30P0T3-1.dat	14,504.28	6,941.28	7,563	3600.00
InsC30P0T3-2.dat	10,672.61	6,195.61	4,477	3600.00
InsC30P0T3-3.dat	10,618.15	7,067.15	3,551	3600.00
InsC30P0T3-4.dat	9,223.39	5,123.39	4,100	3600.00
InsC30P0T3-5.dat	9,561.75	5,304.75	4,257	3600.00
InsC35P0T3-1.dat	12,207.68	6,661.68	5,546	3600.00
InsC35P0T3-2.dat	10,235.58	5,623.58	4,612	3600.00
InsC35P0T3-3.dat	15,418.24	8,069.24	7,349	3600.00
InsC35P0T3-4.dat	10,988.42	6,136.42	4,852	3600.00
InsC35P0T3-5.dat	10,834.69	6,081.69	4,753	3600.00
InsC40P0T3-1.dat	15,922.32	7,802.32	8,120	3600.00
InsC40P0T3-2.dat	11,119.67	6,133.67	4,986	3600.00
InsC40P0T3-3.dat	11,779.37	7,816.37	3,963	3600.00
InsC40P0T3-4.dat	10,225.01	6,311.01	3,914	3600.00
InsC40P0T3-5.dat	12,195.05	7,668.05	4,527	3600.00
InsC45P0T3-1.dat	12,341.69	8,236.69	4,105	3600.00
InsC45P0T3-2.dat	15,313.19	7,786.19	7,527	3600.00
InsC45P0T3-3.dat	13,686.17	8,829.17	4,857	3600.00
InsC45P0T3-4.dat	14,926.33	7,645.33	7,281	3600.00
InsC45P0T3-5.dat	11,893.88	8,076.88	3,817	3600.00
InsC50P0T3-1.dat	16,875.41	8,632.41	8,243	3600.00
InsC50P0T3-2.dat	18,069.74	8,457.74	9,612	3600.00
InsC50P0T3-3.dat	16,597.47	8,747.47	7,850	3600.00
InsC50P0T3-4.dat	18,575.31	9,801.31	8,774	3600.00
InsC50P0T3-5.dat	16,972.31	9,309.31	7,663	3600.00
Average				2780.60

The results in Table 4 show that the exact algorithm could find optimal solutions for instances with up to 15 clients. However, for instances with 20 or more clients, the algorithm was unable to find optimal solutions. This was expected since exact algorithms struggle with large test instances because the number of possible solutions grows exponentially. The average solution time for all test instances was 2780.60 s. For instances with 10 or fewer clients, the exact method found the optimal solution in less than 5 minutes, except for "InsC10P0T3-2.dat", which required 606.73 s (10.11 minutes). For benchmark instances with 20 or more clients, the EM could not find optimal solutions within 3600.00 s, although it found near-optimal solutions for instances with up to 20 clients. The EM found a feasible solution for all test instances. Note that test instances with the **Time** column showing less than 3600 seconds indicate that the EM found the optimal solution.

The results for the IMM are presented in Table 5 and show that the IMM took less time on average than the EM. The latter is due to the stop criteria set for the IMM, which requires 150 iterations without change. The stop criterion was established based on preliminary experiments, in which the IMM results barely improved after 100 iterations and showed no further improvement. Therefore, there was no reason to let the algorithm run longer. The IMM found solutions for all test instances every time it was run. As mentioned before, the IMM could reach infeasible solutions for the subproblem $Sub2VRP$. The average percentage of infeasible solutions across all test instances was 15.08 %. The standard deviation σ varied on the test instances in the interval $[0, 556.38]$, with an overall average of 160.20. The value $\sigma = 0$ means that the IMM obtained the same solution in every run. On the other hand, large values of $sigma$ indicate that the algorithm produced solutions with large differences across the five runs. To find out why some of the test instances had large values for $sigma$, the differences between solutions for the same instances were analyzed. The analysis showed that the parameters $mtci$ (the minimum traveling cost) and $mtti$ (the minimum required traveling) significantly affect the quantity sent to a client in a given period. At the same time, the quantity sent to a client significantly affects the routing process, since sending too much can lead to more costly routes. For instance "InsC40P0T3-3.dat", which obtained the largest σ value, the IMM struggled in three of the five runs, because it tends to create routes in which isolated clients with low inventory costs and high storage capacity are visited in more than one period. The IMM also struggles in instances where the vehicle's capacity parameter Q is very large, because capacity constraints are less relevant.

The boxplots for the results of the five runs of the IMM for all the test instances are presented in Figures 4 to 8 The boxplot analysis shows highly stable behavior across all customer sets. The medians remain consistently constant within each customer group; that is, there are no significant variations in the central value across instances. Most customer groups exhibit uniform distributions without outliers. Outliers appear only in isolated cases, such as the group of 35 customers; however, this does not affect the overall trend. The results show that each customer group exhibits a consistent pattern, with no abrupt fluctuations across instances and dispersion consistent with its scale.

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Table 5. IMM with local search results.

Instances	Best UB	Mean UB	Median UB	Avg. IC	Avg. RC	Avg. Time	Itbest	σ
InsC5P0T3-1.dat	1,783.41	1,784.91	1,785.25	480.91	1,304	2.84	39.17	0.84
InsC5P0T3-2.dat	1,964.35	1,964.35	1,964.35	640.35	1,324	1.60	7.67	0.00
InsC5P0T3-3.dat	4,194.88	4,202.26	4,202.34	988.26	3,244	3.10	3.50	5.43
InsC5P0T3-4.dat	2,422.07	2,475.72	2,432.67	488.56	1,987	2.63	33.67	68.13
InsC5P0T3-5.dat	2,249.84	2,249.84	2,249.84	928.45	1,325	2.92	0.00	0.00
InsC10P0T3-1.dat	4,860.97	4,895.29	4,897.25	2,259.95	2,635	14.38	0.17	20.65
InsC10P0T3-2.dat	4,724.14	4,724.14	4,724.14	1,868.14	2,856	12.91	4.67	0.00
InsC10P0T3-3.dat	3,770.04	3,779.77	3,774.36	1,785.77	1,994	6.43	24.83	13.02
InsC10P0T3-4.dat	3,905.18	4,123.97	4,179.24	1,748.97	2,375	8.63	0.00	149.92
InsC10P0T3-5.dat	4,109.95	4,113.27	4,114.12	2,266.27	1,847	6.91	23.00	1.93
InsC15P0T3-1.dat	5,305.97	5,414.62	5,346.02	2,807.45	2,607	43.22	32.50	114.22
InsC15P0T3-2.dat	5,018.67	5,081.69	5,044.11	2,698.36	2,383	58.06	1.00	64.44
InsC15P0T3-3.dat	6,001.59	6,194.04	6,170.57	3,144.04	3,050	37.60	41.50	165.58
InsC15P0T3-4.dat	4,844.76	4,844.76	4,844.76	2,279.76	2,565	223.91	0.00	0.00
InsC15P0T3-5.dat	4,725.30	4,767.96	4,745.83	2,220.29	2,548	274.59	9.67	57.86
InsC20P0T3-1.dat	7,441.19	7,441.19	7,441.19	3,671.19	3,770	335.49	0.00	0.00
InsC20P0T3-2.dat	6,306.84	6,409.48	6,437.02	3,760.65	2,649	175.44	27.50	59.37
InsC20P0T3-3.dat	7,116.66	7,212.74	7,199.24	3,928.74	3,284	160.55	2.17	66.99
InsC20P0T3-4.dat	7,782.68	7,791.22	7,790.66	3,095.88	4,695	403.81	27.50	6.66
InsC20P0T3-5.dat	8,285.93	8,769.13	8,695.39	4,163.13	4,606	1799.50	20.33	334.23
InsC25P0T3-1.dat	8,070.71	8,158.63	8,124.77	4,393.80	3,765	332.42	15.83	88.10
InsC25P0T3-2.dat	9,672.18	9,693.65	9,687.35	4,723.15	4,971	1400.19	26.33	23.03
InsC25P0T3-3.dat	10,422.07	10,718.47	10,777.12	5,185.97	5,533	1923.06	9.00	167.16
InsC25P0T3-4.dat	7,863.91	7,950.79	7,893.89	4,311.93	3,639	759.19	5.00	95.22
InsC25P0T3-5.dat	12,322.28	12,893.81	12,882.61	5,817.14	7,077	1928.96	16.33	402.29
InsC30P0T3-1.dat	13,848.55	13,905.62	13,919.10	7,031.79	6,874	1993.37	4.83	35.96
InsC30P0T3-2.dat	10,659.49	11,080.86	11,231.40	6,185.71	4,895	2206.91	8.00	338.90
InsC30P0T3-3.dat	10,932.93	11,150.36	10,973.51	7,042.65	4,108	1080.49	39.57	283.17
InsC30P0T3-4.dat	9,429.23	9,546.95	9,468.24	5,137.67	4,409	1445.00	20.43	136.37
InsC30P0T3-5.dat	9,414.22	9,505.33	9,459.87	5,280.67	4,225	1920.90	4.00	97.84
InsC35P0T3-1.dat	10,942.18	11,570.09	11,563.14	6,783.43	4,787	1925.01	3.17	393.72
InsC35P0T3-2.dat	10,165.66	10,236.53	10,173.56	5,694.10	4,632	2122.68	12.14	145.17
InsC35P0T3-3.dat	14,194.89	14,399.34	14,303.64	8,251.67	6,148	1935.07	2.67	217.82
InsC35P0T3-4.dat	10,609.81	10,914.83	10,641.12	5,739.33	5,176	2045.62	2.17	417.01
InsC35P0T3-5.dat	10,466.88	11,019.63	10,991.88	6,166.46	4,853	2021.93	2.67	454.87
InsC40P0T3-1.dat	14,678.15	15,091.93	14,831.54	8,018.77	7,073	1918.90	3.33	464.82
InsC40P0T3-2.dat	10,781.71	11,240.93	11,120.53	6,216.43	5,025	2154.12	0.83	474.74
InsC40P0T3-3.dat	10,783.07	11,241.01	10,901.35	7,919.01	3,322	1965.99	4.83	556.38
InsC40P0T3-4.dat	10,048.39	10,446.39	10,493.58	6,331.39	4,115	2153.48	0.57	277.74
InsC40P0T3-5.dat	11,878.29	12,200.44	12,316.29	7,812.73	4,388	2353.37	1.57	293.61
InsC45P0T3-1.dat	11,854.50	12,075.94	12,176.08	6,817.94	5,258	1927.12	7.17	181.00
InsC45P0T3-2.dat	13,938.99	14,111.59	14,112.60	7,714.59	6,397	1966.89	1.17	153.39
InsC45P0T3-3.dat	13,359.05	13,428.56	13,409.86	8,898.56	4,530	2138.65	3.00	61.13
InsC45P0T3-4.dat	13,891.21	14,200.86	14,359.47	7,755.86	6,445	1928.69	2.00	260.21
InsC45P0T3-5.dat	11,860.39	12,042.80	11,929.68	8,097.30	3,946	1877.93	4.83	239.19
InsC50P0T3-1.dat	15,218.15	15,339.04	15,361.76	8,621.37	6,518	1921.46	1.67	92.20
InsC50P0T3-2.dat	16,055.49	16,381.98	16,461.38	8,460.48	7,922	1888.39	0.67	193.82
InsC50P0T3-3.dat	15,070.16	15,273.73	15,320.10	8,741.40	6,532	1960.31	1.00	171.60
InsC50P0T3-4.dat	17,528.02	17,718.11	17,772.00	9,744.11	7,974	1992.65	1.33	158.28
InsC50P0T3-5.dat	15,736.43	15,742.69	15,739.64	9,259.86	6,483	2147.48	0.83	6.19
Average						1178.21	10.12	160.20

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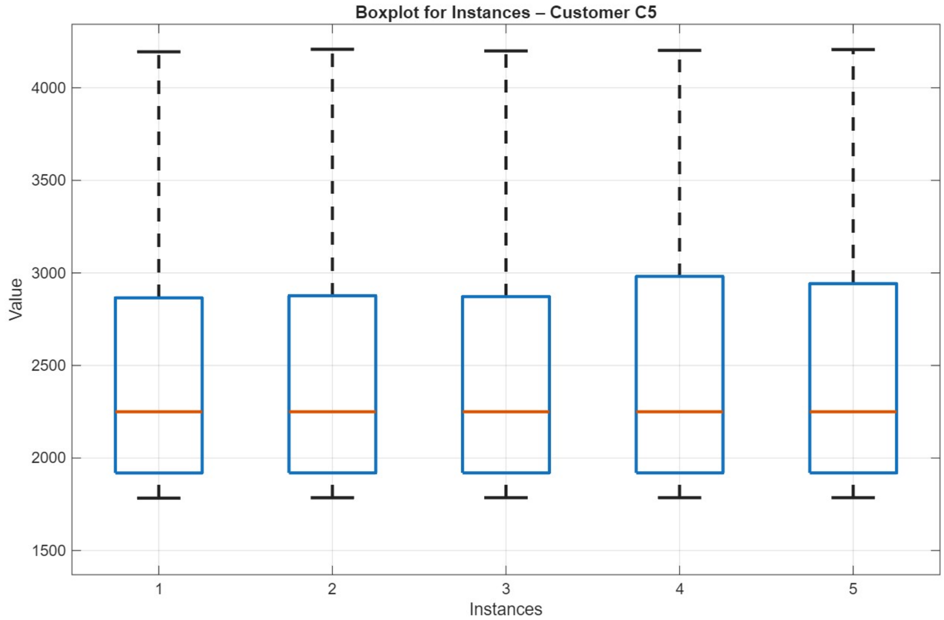

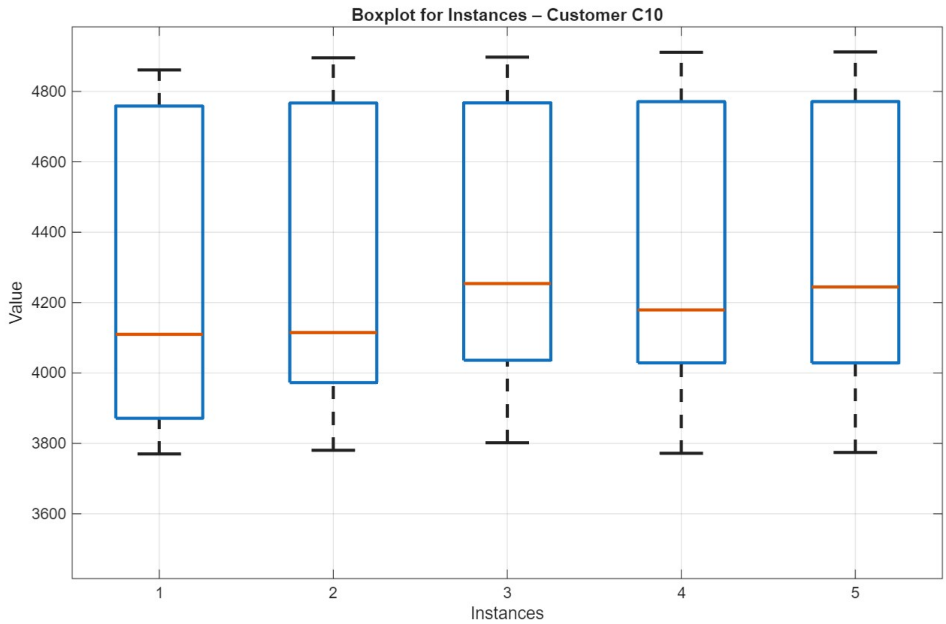

Figure 4. Boxplot graphs for instances with 5 and 10 clients

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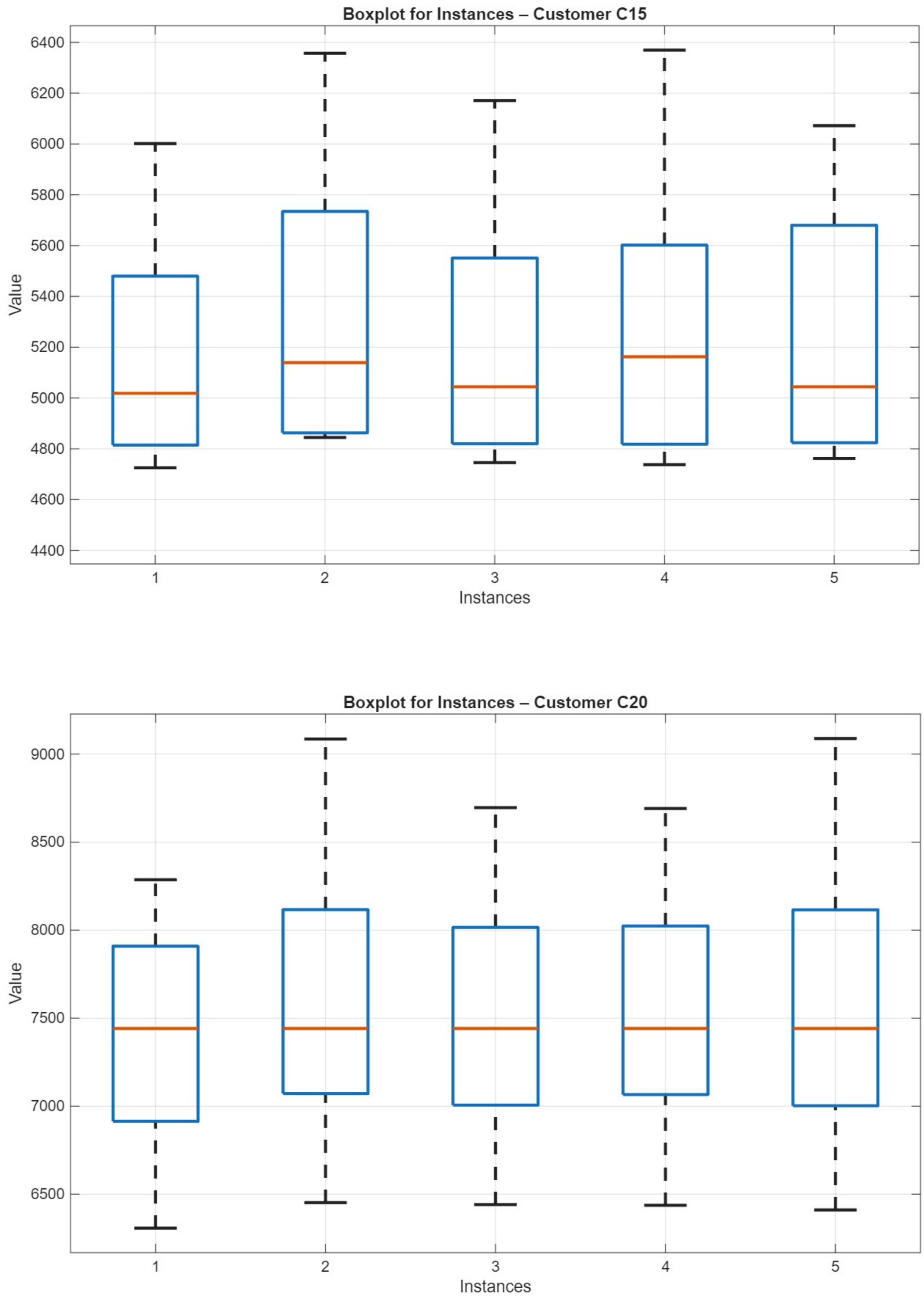

Figure 5. Boxplot graphs for instances with 15 and 20 clients

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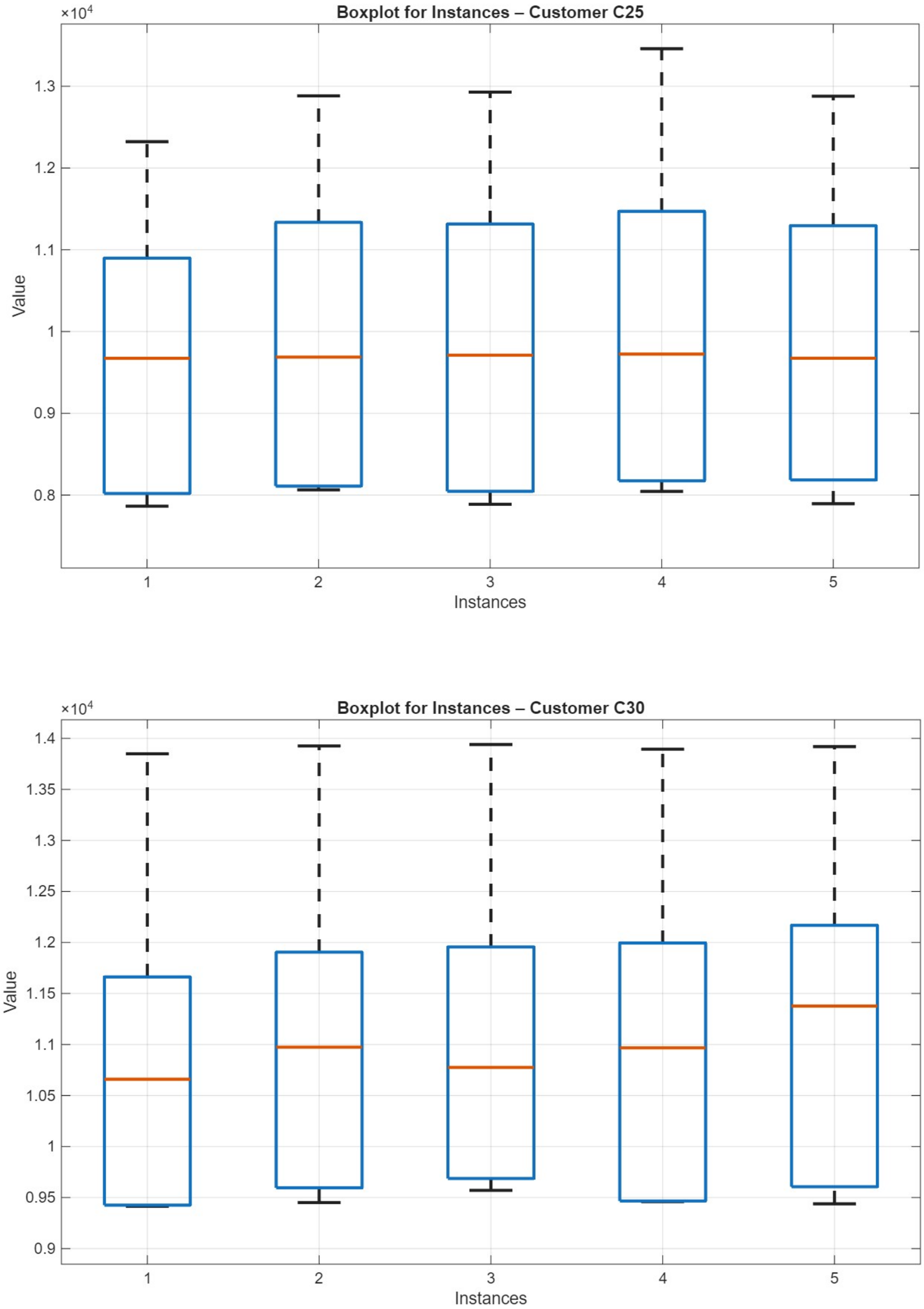

Figure 6. Boxplot graphs for instances with 25 and 30 clients

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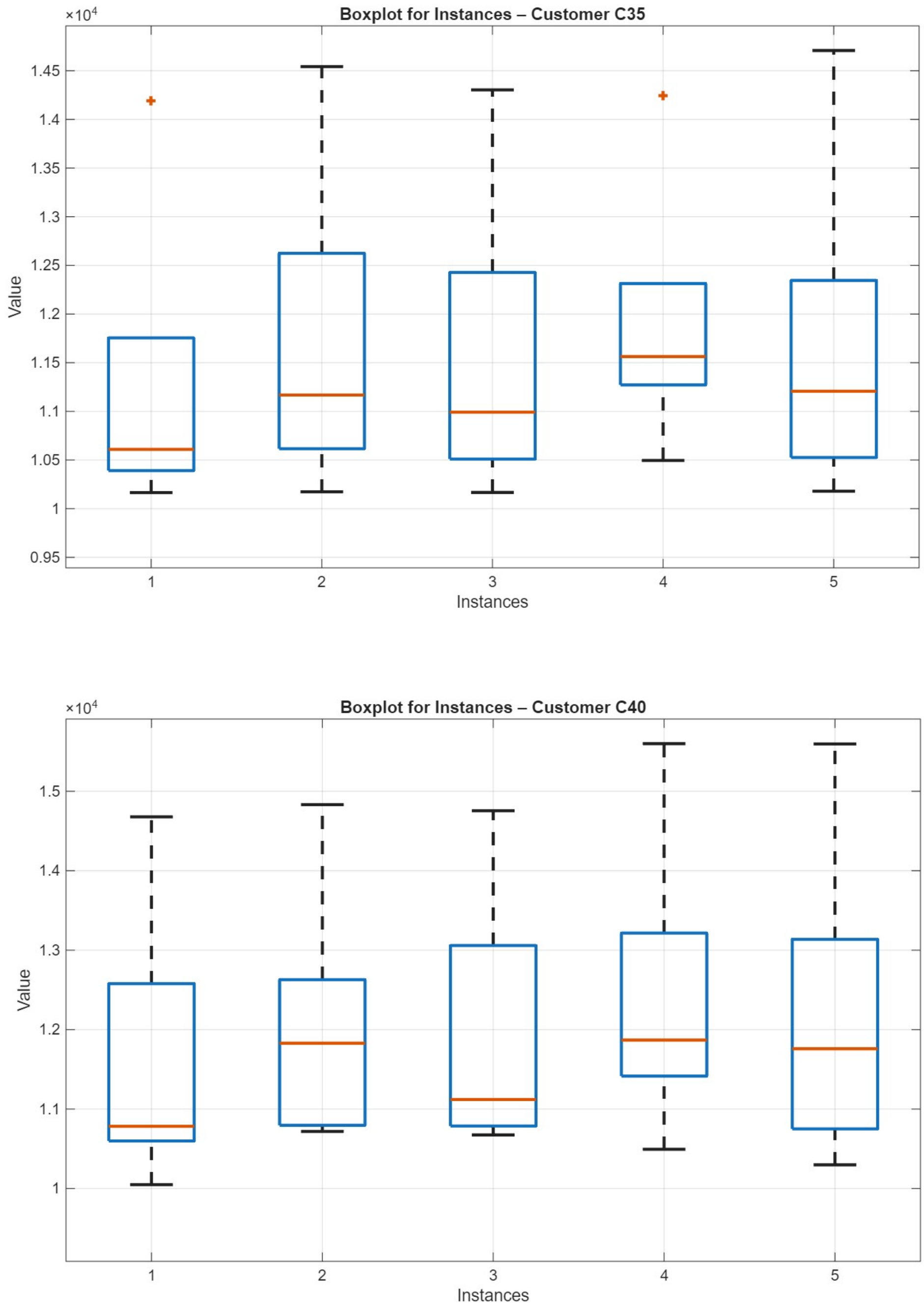

Figure 7. Boxplot graphs for instances with 35 and 40 clients

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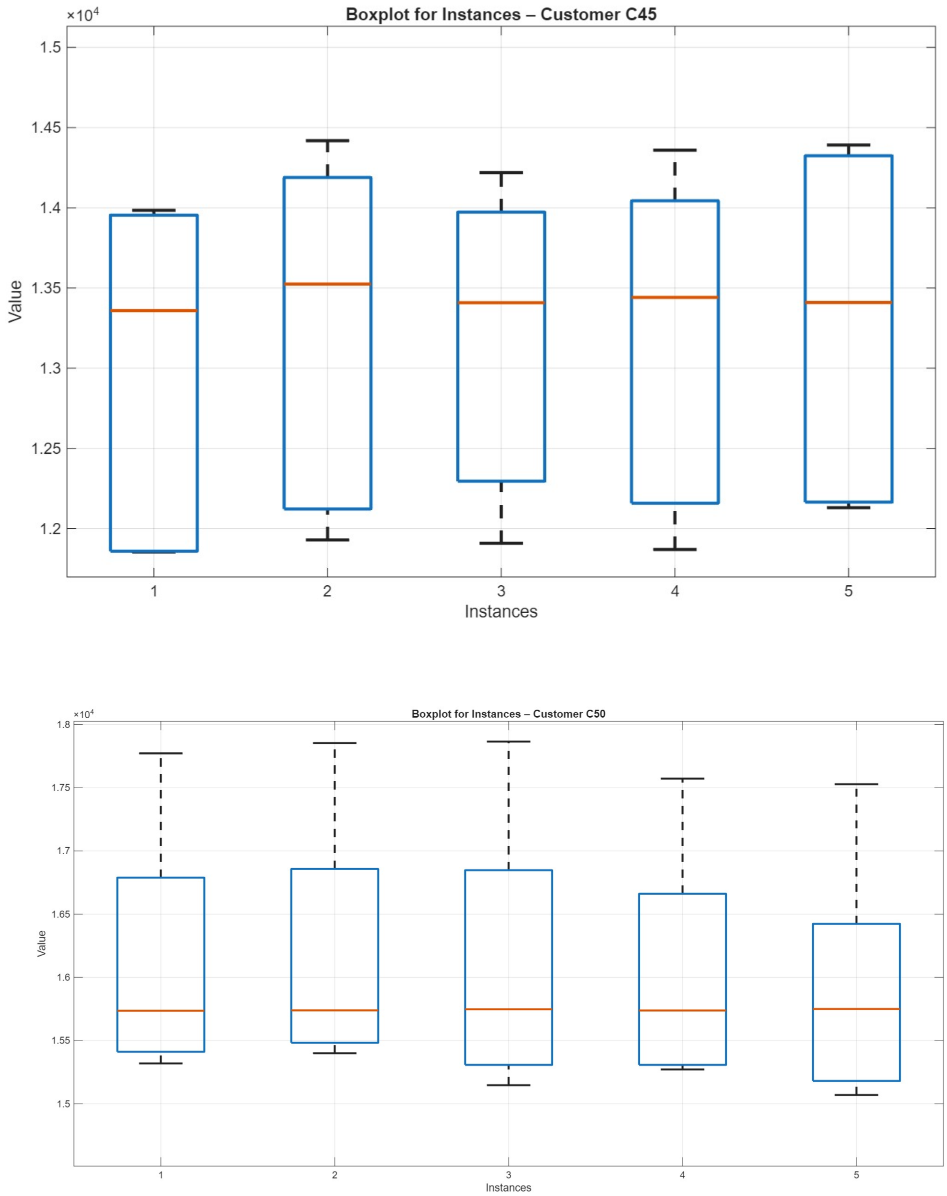

Figure 8. Boxplot graphs for instances with 45 and 50 clients

The results in Table 6 compare the solutions obtained by both algorithms. Columns **Best-gap** and **Median-gap** present the percentage deviation between the upper bound of the EM and the best and the median solution found by the IMM. A negative value in either of these columns means that IMM obtained a better solution than the EM. The results clearly show that the exact method performs better in test instances with up to 25 clients. However, the IMM finds solutions close to the ones found by the EM for this group of instances. For test instances with 30, the results show that the IMM outperforms the EM in two of the five instances. On the other hand, the IMM performs better for test instances with 35 or more clients. Overall, the average **Best-gap** for all test instances is negative, while for the **Median-gap** this value is 0.00. These statistics indicate that in average, the IMM will find similar or better solutions than the EM. This is especially evident for the test instances with 35 or more clients.
The results show that the EM performs better then the IMM for instances with up to 30 vertices. This behavior can be explained by the fact that, for small- to medium-scale instances, the MILP-based exact method is able to explore a large portion of the feasible solution space and exploit strong linear relaxations, often reaching optimal or near-optimal solutions within reasonable computational time. In these instances, the heuristic decomposition strategy adopted by the IMM and the heuristic guidance

do not provide a significant advantage, since the search space is still manageable for the EM. Moreover, the separation of inventory and routing decisions in the IMM may introduce minor suboptimality when the full integrated structure can still be handled exactly.

Table 6. Comparison of best solutions between algorithms.

Instances	UB-EM	Best-IMM	Best-gap	Median-gap
InsC5P0T3-1.dat	1,775.49	1,783.41	0.45	0.53
InsC5P0T3-2.dat	1,899.02	1,964.35	3.44	3.44
InsC5P0T3-3.dat	4,110.76	4,194.88	2.05	2.23
InsC5P0T3-4.dat	2,381.27	2,422.07	1.71	3.97
InsC5P0T3-5.dat	2,237.60	2,249.84	0.55	0.55
InsC10P0T3-1.dat	4,828.95	4,860.97	0.66	1.37
InsC10P0T3-2.dat	4,698.02	4,724.14	0.56	0.56
InsC10P0T3-3.dat	3,744.11	3,770.04	0.69	0.95
InsC10P0T3-4.dat	3,870.99	3,905.18	0.88	6.54
InsC10P0T3-5.dat	4,040.98	4,109.95	1.71	1.79
InsC15P0T3-1.dat	5,195.63	5,305.97	2.12	4.21
InsC15P0T3-2.dat	5,007.31	5,018.67	0.23	1.49
InsC15P0T3-3.dat	5,864.45	6,001.59	2.34	5.62
InsC15P0T3-4.dat	4,827.26	4,844.76	0.36	0.36
InsC15P0T3-5.dat	4,712.60	4,725.30	0.27	1.17
InsC20P0T3-1.dat	7,297.67	7,441.19	1.97	1.97
InsC20P0T3-2.dat	6,251.49	6,306.84	0.89	2.53
InsC20P0T3-3.dat	6,880.18	7,116.66	3.44	4.83
InsC20P0T3-4.dat	7,676.71	7,782.68	1.38	1.49
InsC20P0T3-5.dat	8,278.08	8,285.93	0.09	5.93
InsC25P0T3-1.dat	7,974.01	8,070.71	1.21	2.32
InsC25P0T3-2.dat	9,660.05	9,702.18	0.13	0.35
InsC25P0T3-3.dat	10,611.84	10,422.07	-1.79	1.00
InsC25P0T3-4.dat	7,809.74	7,863.91	0.69	1.81
InsC25P0T3-5.dat	12,142.33	12,322.28	1.48	6.19
InsC30P0T3-1.dat	14,504.28	13,848.55	-4.52	-4.13
InsC30P0T3-2.dat	10,672.61	10,659.49	-0.12	3.83
InsC30P0T3-3.dat	10,618.15	10,932.93	2.96	5.01
InsC30P0T3-4.dat	9,223.39	9,429.23	2.23	3.51
InsC30P0T3-5.dat	9,561.75	9,414.22	-1.54	-0.59
InsC35P0T3-1.dat	12,207.68	10,942.18	-10.37	-5.22
InsC35P0T3-2.dat	10,235.58	10,165.66	-0.68	0.01
InsC35P0T3-3.dat	15,418.24	14,194.89	-7.93	-6.61
InsC35P0T3-4.dat	10,988.42	10,609.81	-3.45	-0.67
InsC35P0T3-5.dat	10,834.69	10,466.88	-3.39	1.71
InsC40P0T3-1.dat	15,922.32	14,678.15	-7.81	-5.22
InsC40P0T3-2.dat	11,119.67	10,781.71	-3.04	1.09
InsC40P0T3-3.dat	11,779.37	10,783.07	-8.46	-4.57
InsC40P0T3-4.dat	10,225.01	10,048.39	-1.73	2.17
InsC40P0T3-5.dat	12,195.05	11,878.29	-2.60	0.04
InsC45P0T3-1.dat	11,854.50	13,547.85	-3.95	-2.15
InsC45P0T3-2.dat	15,313.19	14,114.27	-8.97	-7.85
InsC45P0T3-3.dat	13,686.17	13,359.05	-2.39	-1.88
InsC45P0T3-4.dat	14,926.33	14,319.63	-6.93	-4.86
InsC45P0T3-5.dat	11,893.88	11,860.39	-0.28	1.25
InsC50P0T3-1.dat	16,875.41	15,442.80	-9.82	-9.10
InsC50P0T3-2.dat	18,069.74	16,461.38	-11.15	-9.34
InsC50P0T3-3.dat	16,597.47	15,320.10	-9.20	-7.98
InsC50P0T3-4.dat	18,575.31	17,772.00	-5.64	-4.61
InsC50P0T3-5.dat	16,972.31	15,736.43	-7.28	-7.24
Average			-1.77	0.00

On the other hand the IMM performs better for instances with 35 or more clients due to the problem's $NP-hard$ nature. As in any $NP-hard$ problem in the DCIRP, the number of variables, constraints, and possible solutions grows exponentially, as does the time to find the optimal solution. It is important to note that the EM uses a branch-and-bound algorithm, which generates a binary tree. In every leaf (node) of the binary tree, the linear relaxation of the problem at the leaf is solved. As the size of the instances increases, the time required to solve the linear relaxation at each leaf increases, reducing the speed at which the branch-and-bound algorithm processes the nodes in the binary tree. For example, for the instance "InsC20P0T3-1.dat", the branch-and-bound algorithm, after 300 seconds, generated 863,473 nodes in the binary tree, of which 644,817 remained active. On the other hand, for instance "InsC50P0T3-1.dat", the branch-and-bound algorithm, after 300 seconds, had generated 65,178 nodes in the binary tree, of which 52,294 remain active. In other words, as the size of the instances increases, the complexity to find integer solutions increases exponentially. Additionally, the IMM's better performance is explained by its ability to reduce the search space and thus the number of possible solutions by focusing only on potentially good ones. In small instances, this can lead to failure to find the optimal solution, but in large instances, it helps find high-quality solutions with lower computational effort.

6. Conclusions and Future Research

In this research paper, the Distance Constrained Inventory Routing Problem (DCIRP) was introduced and studied. The DCIRP is an extension of the classical Inventory Routing Problem, motivated by the need to design distribution routes for a specialty gas producer subject to strict distance constraints. This combinatorial optimization problem is NP-hard, meaning its complexity grows exponentially as the instance size increases. One of the main contributions of this work is the explicit modeling of the correlation between distance constraints and inventory–routing decisions, which is fully captured within the proposed DCIRP formulation.

A valid MILP formulation was presented, and an exact solution method (EM) based on this formulation was developed and used to solve a set of benchmark instances. Computational results show that EM can obtain optimal or near-optimal solutions for small- and medium-scale instances, demonstrating its effectiveness when the problem size remains manageable. To address larger instances, an Iterative Matheuristic Method (IMM) was designed and implemented. The IMM uses an iterative heuristic decomposition strategy that separates the problem into two coordinated subproblems: inventory management and vehicle routing. From a methodological perspective, the main innovation of the IMM lies in its ability to explicitly account for distance constraints while maintaining coordination between inventory and routing decisions. This is achieved by incorporating minimum-cost and minimum-distance parameters into the inventory subproblem and complementing the approach with a routing-based local search procedure. These elements enable the IMM to efficiently guide the search toward high-quality feasible solutions in larger instances where exact methods become computationally prohibitive.

The computational experiments were conducted on a benchmark set of 50 instances derived from the literature. The results indicate that EM yields the best solutions for instances with up to 30 clients, while IMM becomes increasingly competitive as problem size increases. For instances with 35 or more clients, the IMM obtained a majority of the best solutions, and the overall average gap across all instances favors the IMM. These findings confirm that the proposed matheuristic approach provides an effective trade-off between solution quality and computational effort for large-scale DCIRP instances.

Future research directions include developing more advanced solution approaches to further enhance the performance of the IMM. In particular, metaheuristic techniques such as Particle Swarm Optimization (PSO), Biased Random-Key Genetic Algorithms (BRKGA), Simulated Annealing (SA), Ant Colony Optimization (ACO), or Gravitational Search Algorithms (GSA) could be integrated to solve the routing subproblem (Sub2VRP) more efficiently or to strengthen the coordination between inventory and routing decisions. Additionally, future extensions of the DCIRP could consider stochastic demand, heterogeneous fleets, multiple depots, or time-dependent distance constraints, thereby increasing the practical applicability and realism of the proposed model.

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