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Posted Date: 13 October 2025

doi: 10.20944/preprints202510.0987.v1

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Article

A Generalized Greyness Multi-Attribute Decision-Making Model Framework Under Prospect Theory

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Abstract

Characterizing decision-makers' risk attitudes under uncertain conditions has always been a key research topic in decision-making analysis. Based on cumulative prospect theory, this study addresses multi-attribute decision-making problems with interval grey numbers as the attribute values and uncertain attribute weights. From the perspectives of closeness degree and balance, we constructed a generalized greyness multi-attribute decision-making model framework under prospect theory. First, the properties of the greyness distance were analyzed, and a simplified calculation formula was proposed. Subsequently, a weight determination method based on prospect theory is introduced. Finally, a prospect-theory-based decision-making model is established to calculate the comprehensive prospect values of each alternative and rank them to obtain the optimal choice. Through case analysis and comparison of the results, the effectiveness and superiority of the proposed decision-making framework were verified.

Keywords: multi-attribute decision-making; interval grey number; generalized greyness; greyness distance; cumulative prospect theory

1. Introduction

Multi-attribute decision-making [1–3], as one of the core domains in modern decision science, has garnered increasing academic attention in both theoretical development and practical applications. With the growing complexity of decision-making environments and heightened information uncertainty, traditional precise-number decision methods have become inadequate, giving rise to research on decision models based on interval grey numbers [4,5]. The three-parameter interval grey number [6–9], as an extended representation of interval grey numbers, incorporates three key parameters: lower bound, upper bound, and core value. Enabling a more comprehensive description of uncertain information, with its theoretical framework gradually maturing. Early research established fundamental operational rules for three-parameter interval grey numbers, achieving the initial integration of interval grey numbers [10,11] with multi-attribute decision-making.

Subsequently, the generalized greyness theory [12], proposed based on the axiom of "greyness conservation," established a complete axiomatic system for greyness distance that considers background domain information, demonstrating its measurement consistency across bounded, infinite, and infinitesimal background domains [13]. As a significant extension of three-parameter interval grey numbers, generalized greyness theory introduces key elements such as the background domain, information distribution, and cognitive structure, moving beyond traditional greyness that merely focuses on interval bounds. This theory proves that in bounded domains, greyness exhibits nonlinear growth with interval expansion, in infinite domains, greyness maintains scale invariance, and in microscopic quantum systems, greyness shares mathematical isomorphism with Heisenberg's uncertainty principle, thereby offering a more comprehensive and precise description of uncertainty

[14]. The integration of generalized greyness theory with multi-attribute decision-making [15,16] has become a focal point in recent interdisciplinary research between grey systems and decision science. This fusion framework significantly enhances the decision reliability in complex environments through more accurate uncertainty characterization and dynamic weight optimization.

In terms of measurement methods, greyness distance, as a core tool for quantifying the uncertainty of interval grey numbers, has become a major research hotspot in grey system studies, with extensive work dedicated to analyzing its theoretical properties and optimizing computational approaches. Scholars have conducted systematic research on the measurement principles, computational paradigms, and application extensions of greyness distance.

Traditional multi-attribute decision-making models assume fully rational decision-makers, failing to explain behavioral anomalies such as "loss aversion." The integration of prospect theory [17,18] with multi-attribute decision making has pioneered a new direction in behavioral decision science. By incorporating core mechanisms such as psychological reference points, asymmetric risk preferences, and probability weighting distortions, this interdisciplinary field has significantly improved the explanatory power of traditional decision models for real-world behaviors [19–21]. In recent years, prospect theory has revitalized multi-attribute decision making. Through its value and weighting functions, the theory captures decision-makers' risk preferences, and when combined with the generalized greyness theory, it better simulates the uncertainty and psychological factors in actual decision-making [22].

Prospect Theory effectively characterizes decision-makers' risk preferences through value functions and probability weighting functions; however conventional models exhibit limitations in handling incomplete attribute information scenarios. Interval grey numbers demonstrate competence in representing uncertain data, but their standalone application lacks integration with behavioral economics. Generalized Grey Theory addresses uncertain inter-attribute relationships via generalized grey distance normalization, while remaining deficient in quantifying psychological mechanisms. The innovative integration of these three approaches embeds prospect-based risk attitudes into the grey relational degree subsystem, achieving a novel coupling between behavioral decision theory and grey system theory at the uncertainty quantification level. Consequently, this synergistic framework demonstrates significant theoretical value and practical relevance for decision-making research.

Current research trends indicate that decision models integrating generalized greyness, greyness distance, and prospect theory can leverage the complete characteristics of uncertain information while reflecting decision-makers' psychological behaviors, demonstrating unique advantages in fields such as financial investment, medical decision-making, and environmental assessment. However, challenges remain in optimizing model parameters, improving computational efficiency, and validating practical effectiveness. Future research should focus on refining the theoretical framework and expanding practical applications, particularly in adaptive improvements for big data contexts where significant potential exists.

2. The Generalized Greyness and Greyness Distance of Interval Grey Numbers

Definition 1[11] Let an interval grey number $\otimes \in [a, b]$ under the background domain $\Omega = [p, q], p \leq a \leq b \leq q$ be divided into three dimensions:

$$g^{\circ}(\otimes_1) = \frac{\mu(\otimes_1)}{\mu(\Omega)}, g^{\circ}(\otimes_2) = \frac{\mu(\otimes_2)}{\mu(\Omega)}, g^{\circ}(\otimes_3) = \frac{\mu(\otimes_3)}{\mu(\Omega)} \tag{1}$$

Lower-bound greyness ($g^{\circ}(\otimes_1)$) reflects the deviation between the lower bound of the grey number and the lower bound of the background domain, $\mu(\otimes_1) = a - p$. Range greyness ($g^{\circ}(\otimes_2)$) characterizes the relative size of the grey number interval, $\mu(\otimes_2) = b - a$. Upper-bound greyness ($g^{\circ}(\otimes_3)$) describes the gap between the upper bound of the grey number and the upper bound of the

background domain, $\mu(\otimes_3) = q - b$. $\mu(\Omega)$ is a measure of the background domain Ω of interval grey numbers and $\mu(\Omega) = q - p$.

These three components satisfy the unit constraint $g^\circ(\otimes_1) + g^\circ(\otimes_2) + g^\circ(\otimes_3) = 1$. The generalized greyness vector is

$$g^\circ(\otimes) = (g^\circ(\otimes_1), g^\circ(\otimes_2), g^\circ(\otimes_3)) = \left(\frac{a-p}{q-p}, \frac{b-a}{q-p}, \frac{q-b}{q-p} \right) \quad (2)$$

This vector enables the mapping of a gray number from a one-dimensional interval to a three-dimensional space.

Inference 1 Suppose $\Omega = [0, 1]$, interval grey number $\otimes_1 \in [a_1, b_1] \subset \Omega, \otimes_2 \in [a_2, b_2] \subset \Omega$. The three-dimensional greyness vector for the interval grey number $\otimes_1 \in [a_1, b_1]$ is:

$$g^\circ(\otimes_1) = (a_1, b_1 - a_1, 1 - b_1) \quad (3)$$

$$g^\circ(\otimes_2) = (a_2, b_2 - a_2, 1 - b_2) \quad (4)$$

Definition 2[11] Suppose $\Omega = [p, q]$, interval grey number $\otimes_1, \otimes_2 \in \Omega$, and greyness distance is then computed as

$$d(\otimes_1, \otimes_2) = \|g(\otimes_1) - g(\otimes_2)\| = \left(\lambda \sum_{i=1}^3 |g_i(\otimes_1) - g_i(\otimes_2)|^\alpha \right)^{1/\alpha} \quad (5)$$

Where $\|x\|$ represents the norm, λ is a normalization coefficient, and is usually taken as $\lambda = 0.5$. $\alpha \in \{1, 2\}$ corresponds to the Hamming greyness distance ($\alpha = 1$) or the Euclidean greyness distance ($\alpha = 2$).

Definition 3[16] Suppose $\Omega = [p, q]$, interval grey number $\otimes_1 \in [a_1, b_1] \subset \Omega, \otimes_2 \in [a_2, b_2] \subset \Omega$. The gray distance is computed as follows:

$$d(\otimes_1, \otimes_2) = \frac{1}{q-p} \left\{ 0.5 \left[|a_1 - a_2|^\alpha + \left| (b_1 - a_1) - (b_2 - a_2) \right|^\alpha + |b_1 - b_2|^\alpha \right] \right\}^{1/\alpha} \quad (6)$$

Inference 2 Suppose $\Omega = [0, 1]$, interval grey number $\otimes_1 \in [a_1, b_1] \subset \Omega, \otimes_2 \in [a_2, b_2] \subset \Omega$, then

$$d(\otimes_1, \otimes_2) = \left\{ 0.5 \left[|a_1 - a_2|^\alpha + \left| (b_1 - a_1) - (b_2 - a_2) \right|^\alpha + |b_1 - b_2|^\alpha \right] \right\}^{1/\alpha} \quad (7)$$

For $\alpha = 1$ (Hamming greyness distance):

$$d(\otimes_1, \otimes_2) = \left\{ 0.5 \left[|a_1 - a_2| + \left| (b_1 - a_1) - (b_2 - a_2) \right| + |b_1 - b_2| \right] \right\} \quad (8)$$

For $\alpha = 2$ (Euclidean greyness distance)

$$d(\otimes_1, \otimes_2) = \left\{ 0.5 \left[|a_1 - a_2|^2 + \left| (b_1 - a_1) - (b_2 - a_2) \right|^2 + |b_1 - b_2|^2 \right] \right\}^{1/2} \quad (9)$$

Inference 3[16] Suppose $\Omega = [0, 1]$, interval grey number $\otimes_1 \in [a_1, b_1] \subset \Omega, \otimes_2 \in [a_2, b_2] \subset \Omega$. If $\otimes_1$ and $\otimes_2$ degenerate into real numbers, then $a_1 = b_1, a_2 = b_2$.

$$d(\otimes_1, \otimes_2) = |a_1 - a_2| \quad (10)$$

This matches the traditional absolute distance, proving consistency.

Inference 4 Suppose $\Omega = [0, 1]$, interval grey number $\otimes_1 \in [0, 1] \subset \Omega, \otimes_2 \in [k, k] \subset \Omega$. $\otimes_1$ is maximum uncertainty and k is a deterministic value.

$$d(\otimes_1, \otimes_2) = \left\{ 0.5 \left[k^2 + (1-k)^2 + (1-0)^2 \right] \right\}^{1/2} = \{1-k+k^2\}^{1/2} \tag{11}$$

This quantifies the extent to which a deterministic value is obtained from the complete uncertainty. $\Omega = [0, 1]$ ensures comparability across studies. The simplified form reduces the computational cost, and the method is theoretically sound, satisfying the non-negativity, symmetry, and triangle inequalities.

3. A Generalized Greyness Multi-Attribute Decision-Making Model Framework Under Prospect Theory

3.1. Greyness Distance Between Decision Scheme and Ideal Scheme

Suppose the decision scheme set is $A = \{A_1, A_2, \dots, A_m\}$, the attribute indicator set is $S = \{S_1, S_2, \dots, S_n\}$, and the attributive values of the scheme A_i ($i = 1, \dots, m$), indicator S_j ($j = 1, \dots, n$) are interval grey numbers $u_{ij}(\otimes) \in [u_{ij}^L, u_{ij}^U]$ and $u_{ij}^U \geq u_{ij}^L \geq 0$. The attributive weights are $w = (w_1, w_2, \dots, w_n)$.

Then, the attribute vector of A_i is denoted as

$$u_i(\otimes) = (u_{i1}(\otimes), u_{i2}(\otimes), \dots, u_{in}(\otimes)) \tag{12}$$

And, $u_i(\otimes) \subset \Omega'_j = [p'_j, q'_j], i = 1, 2, \dots, n.$

The decision matrix is expressed as:

$$R = (u_{ij}(\otimes))_{m \times n} = \begin{bmatrix} [u_{11}^L, u_{11}^U] & [u_{12}^L, u_{12}^U] & \cdots & [u_{1n}^L, u_{1n}^U] \\ [u_{21}^L, u_{21}^U] & [u_{22}^L, u_{22}^U] & \cdots & [u_{2n}^L, u_{2n}^U] \\ \vdots & \vdots & \ddots & \vdots \\ [u_{m1}^L, u_{m1}^U] & [u_{m2}^L, u_{m2}^U] & \cdots & [u_{mn}^L, u_{mn}^U] \end{bmatrix} \tag{13}$$

Definition 4[16] To eliminate the influence of dimensional differences in attribute values, this study adopts the generalized grey entropy transformation method to normalize the attribute values of decision alternatives. Since the generalized grey entropy transformation is a linear transformation, the generalized grey degree within a finite background domain Ω possesses linear transformation characteristics and satisfies the property of grey information preservation [12], Consequently, the results after generalized grey entropy transformation are as follows:

$$\begin{aligned} p_j &= \frac{p'_j - p'_j}{q'_j - p'_j} = 0, x_{ij}^L = \frac{u_{ij}^L - p'_j}{q'_j - p'_j}, \\ x_{ij}^U &= \frac{u_{ij}^U - p'_j}{q'_j - p'_j}, q_j = \frac{q'_j - p'_j}{q'_j - p'_j} = 1 \end{aligned} \tag{14}$$

After dimensionless processing using the formula, the background domain $\Omega'_j = [p'_j, q'_j]$ of the attribute indicator u_j is transformed into $\Omega_j = [p_j, q_j] = [0, 1]$, and the interval grey number $u_{ij}(\otimes)$ is converted into $x_{ij}(\otimes) \subset \Omega_j$. Subsequently, normalized matrix X was obtained.

$$X = (x_{ij}(\otimes))_{m \times n} = \begin{bmatrix} [x_{11}^L, x_{11}^U] & [x_{12}^L, x_{12}^U] & \cdots & [x_{1n}^L, x_{1n}^U] \\ [x_{21}^L, x_{21}^U] & [x_{22}^L, x_{22}^U] & \cdots & [x_{2n}^L, x_{2n}^U] \\ \vdots & \vdots & \ddots & \vdots \\ [x_{m1}^L, x_{m1}^U] & [x_{m2}^L, x_{m2}^U] & \cdots & [x_{mn}^L, x_{mn}^U] \end{bmatrix} \tag{15}$$

Definition 5 For benefit-type attributes, normalization was performed as follows:

$$\begin{aligned} x_j^+(\otimes) &= [a_j^L, a_j^U] = \left[\max_i x_{ij}^L, \max_i x_{ij}^U \right], (j = 1, \dots, n) \\ x_j^-(\otimes) &= [b_j^L, b_j^U] = \left[\min_i x_{ij}^L, \min_i x_{ij}^U \right], (j = 1, \dots, n) \end{aligned} \quad (16)$$

For cost-type attributes, the normalization is performed as follows:

$$\begin{aligned} x_j^+(\otimes) &= [a_j^L, a_j^U] = \left[\min_i x_{ij}^L, \min_i x_{ij}^U \right], (j = 1, \dots, n) \\ x_j^-(\otimes) &= [b_j^L, b_j^U] = \left[\max_i x_{ij}^L, \max_i x_{ij}^U \right], (j = 1, \dots, n) \end{aligned} \quad (17)$$

They are referred to as the normalized ideal effect value and normalized worst effect value of the attribute u_j , respectively.

Definition 6 The result is obtained according to Equations (15-16).

$$\begin{aligned} x_j^+(\otimes) &= \left([a_1^L, a_1^U], \dots, [a_n^L, a_n^U] \right), \\ x_j^-(\otimes) &= \left([b_1^L, b_1^U], \dots, [b_n^L, b_n^U] \right) \end{aligned} \quad (18)$$

They are called the grey positive ideal point and the grey negative ideal point.

Definition 7 According to equations (7,16,17,18), the mathematical formulations for determining the grey distances of individual attribute values relative to the positive and negative ideal points are established as follows:

$$\left(x_{ij}(\otimes), x_j^+(\otimes) \right) = \left\{ 0.5 \left[\left| x_{ij}^L - a_j^L \right|^\alpha + \left| \left(x_{ij}^U - x_{ij}^L \right) - \left(a_j^U - a_j^L \right) \right|^\alpha + \left| x_{ij}^U - a_j^U \right|^\alpha \right] \right\}^{\frac{1}{\alpha}} \quad (19)$$

$$d\left(x_{ij}(\otimes), x_j^-(\otimes) \right) = \left\{ 0.5 \left[\left| x_{ij}^L - a_j^L \right|^\alpha + \left| \left(x_{ij}^U - x_{ij}^L \right) - \left(a_j^U - a_j^L \right) \right|^\alpha + \left| x_{ij}^U - a_j^U \right|^\alpha \right] \right\}^{\frac{1}{\alpha}} \quad (20)$$

Definition 8 Normalized matrices of the positive and negative ideal solutions are derived using Equations (19-20),

$$D^+ = \left(d_{ij}^+(\otimes) \right)_{m \times n} = \begin{bmatrix} d_{11}^+ & d_{12}^+ & \cdots & d_{1n}^+ \\ d_{21}^+ & d_{22}^+ & \cdots & d_{2n}^+ \\ \vdots & \vdots & \ddots & \vdots \\ d_{m1}^+ & d_{m2}^+ & \cdots & d_{mn}^+ \end{bmatrix} \quad (21)$$

$$D^- = \left(d_{ij}^-(\otimes) \right)_{m \times n} = \begin{bmatrix} d_{11}^+ & d_{12}^+ & \cdots & d_{1n}^+ \\ d_{21}^+ & d_{22}^+ & \cdots & d_{2n}^+ \\ \vdots & \vdots & \ddots & \vdots \\ d_{m1}^+ & d_{m2}^+ & \cdots & d_{mn}^+ \end{bmatrix} \quad (22)$$

Among, $d_{ij}^+(\otimes) = d\left(x_{ij}(\otimes), x_j^+(\otimes) \right)$, $d_{ij}^-(\otimes) = d\left(x_{ij}(\otimes), x_j^-(\otimes) \right)$, $i = 1, 2, \dots, m$; $j = 1, 2, \dots, n$.

3.2. The Multi-Attribute Decision-Making Model Based on Prospect Theory

The prospect [17] value is jointly determined by a value function and decision weights, expressed as

$$V = \sum_{i=1}^n v(x_i) \pi(p_i) \quad (23)$$

Here, V is the prospect value, $v(x)$ is the value function that represents the subjectively perceived value formed by the decision maker, and $\pi(p_i)$ is the decision weight, which is an

evaluative function assigned by the decision maker. If the attribute value provided by the decision maker is greater than the positive ideal point, the decision maker may perceive a gain, the magnitude of which is determined by the distance between the attribute value and the positive ideal point. Similarly, the magnitude of the loss can be determined by the distance between the attribute value and negative ideal point. Based on the function proposed by Tversky and Kahneman to measure the decision-maker's attitude toward gains and losses, the prospect value function formula for each attribute of alternative A_i is as follows:

$$v(x_{ij}) = \begin{cases} (x_{ij} - x_j^-)^{\varphi}, & x_{ij} \geq x_r \\ -\theta(x_j^+ - x_{ij})^{\beta}, & x_{ij} < x_r \end{cases} \tag{24}$$

Let $v^+(x_{ij}) = (x_{ij} - x_j^-)^{\varphi} = (d_{ij}^-)^{\varphi}$ denote the gain prospect function, representing the positive prospect value, and $v^-(x_{ij}) = -\theta(x_j^+ - x_{ij})^{\beta} = -\theta(d_{ij}^+)^{\beta}$ ($0 \leq \varphi, \beta \leq 1$) denote the loss prospect function, representing the negative prospect value.

Where φ and β are the gain and loss-correlation coefficients, respectively. θ is a risk-aversion parameter; $\theta > 0$. Ensuring that losses are steeper than gains. $\varphi = 0.89$, $\beta = 0.92$, and $\theta = 2.25$ [17].

In prospect theory, the evaluation of comprehensive prospect values theoretically lacks strict mathematical upper or lower bounds. However, their practical range is constrained by Equation (27).

In the gain domain, where $\varphi \in (0, 1]$ exists, diminishing marginal utility causes the growth rate of evaluation values to decelerate, although no rigid upper limit exists.

In the loss domain, the risk-aversion parameter $\theta > 0$ amplifies negative perceptions, but negative values possess no theoretical lower bound.

The absence of absolute upper/lower limits reflects the nonlinear characteristics of real-world decision making, exemplified by phenomena such as "the difficulty in quantifying massive losses."

3.3. Determination of Prospect Weights and Integrated Prospect Values

The prospect weights [18] for gains and losses can be calculated by:

$$\pi^+(w_j) = \frac{w_j^{\gamma}}{\left(w_j^{\gamma} + (1 - w_j)^{\gamma}\right)^{\frac{1}{\gamma}}} \tag{25}$$

$$\pi^-(w_j) = \frac{w_j^{\delta}}{\left(w_j^{\delta} + (1 - w_j)^{\delta}\right)^{\frac{1}{\delta}}} \tag{26}$$

Here, γ and δ are the parameters used to control the curvature of the value functions for gains and losses. Typical parameter values include $\gamma = 0.61$ and $\delta = 0.69$ [17].

The comprehensive prospect value V_i for each alternative is obtained through:

$$V_i = \sum_{j=1}^n v^+(x_{ij})\pi^+(w_j) + \sum_{j=1}^n v^-(x_{ij})\pi^-(w_j) \tag{27}$$

An alternative with a larger integrated prospect value indicates better performance. By ranking the optimal comprehensive prospect values V_i ($i = 1, \dots, m$) of all alternatives in descending order, we obtain the complete ranking of alternatives and identify the optimal solution.

4. Procedure

Step 1: Construct the interval grey number decision matrix $R = \left(u_{ij}(\otimes)\right)_{m \times n}$ based on the multi-attribute decision-making problem and then normalize it to obtain the standardized decision matrix $X = \left(x_{ij}(\otimes)\right)_{m \times n}$.

Step 2: Determine ideal positive and negative points. Calculate the distance between each alternative's attributes, $x_j^+(\otimes)$ and $x_j^-(\otimes)$. The positive and negative prospect matrices for each alternative are then computed.

Step 3: With the goal of maximizing the comprehensive prospect value, calculate the prospect weights.

Step 4: The prospect weights and positive and negative prospect matrices are substituted into $V_i = \sum_{j=1}^n v^+(x_{ij})\pi^+(w_j^L) + \sum_{j=1}^n v^-(x_{ij})\pi^-(w_j^U)$ to compute the comprehensive prospect value V_i ($i = 1, \dots, m$) for each alternative. All the alternatives were ranked in descending order of V_i to obtain the final optimal ranking.

5. Example

This study employed data from Reference [16], taking a brackish water irrigation experiment for winter wheat in the North China Plain as an example, and established five salinity treatment levels for irrigation water ($1g \cdot L^{-1}, 2g \cdot L^{-1}, 4g \cdot L^{-1}, 6g \cdot L^{-1}$ and $8g \cdot L^{-1}$). The five treatment schemes form the decision scheme set, $A = \{A_1, A_2, A_3, A_4, A_5\}$. Four evaluation indices were selected: spike number, grains per spike, 1000-grain weight, and yield, constituting the attribute set $S = \{S_1, S_2, S_3, S_4\}$, with all attributes being benefit type indices. The corresponding weight values for each attribute are $w = (0.3573, 0.2485, 0.0709, 0.3233)$. The experimental results were expressed using continuous interval grey numbers, and the specific attribute data are presented in Table 1.

Table 1. Information for the decision-making.

	S_1	S_2	S_3	S_4
A_1	[750,790]	[24.0,25.8]	[33.3,37.9]	[6012,6530]
A_2	[720,764]	[25.5,28.1]	[33.2,37.4]	[5678,6014]
A_3	[406,456]	[29.4,32.2]	[34.3,39.3]	[3827,4255]
A_4	[352,434]	[26.8,30.4]	[33.7,37.5]	[3001,3905]
A_5	[240,366]	[21.3,24.1]	[34.9,38.5]	[1726,2704]

The background domain of each attribute value is ($\Omega_1 \subset [0,1000], \Omega_2 \subset [0,50], \Omega_3 \subset [0,50], \Omega_4 \subset [0,8000]$). The dimensionless normalization of the data in Table 1 was performed using Equation (14), resulting in standardized decision values, as shown in Table 2.

Table 2. Normalized decision-making information after transformation.

	S_1	S_2	S_3	S_4
A_1	[0.7500,0.7900]	[0.4800,0.5160]	[0.6660,0.7580]	[0.7515,0.8163]
A_2	[0.7200,0.7640]	[0.5100,0.5620]	[0.6640,0.7480]	[0.7097,0.7518]
A_3	[0.4060,0.4560]	[0.5880,0.6440]	[0.6860,0.7860]	[0.4784,0.5319]
A_4	[0.3520,0.4340]	[0.5360,0.6080]	[0.6740,0.7500]	[0.3751,0.4881]
A_5	[0.2400,0.3660]	[0.4260,0.4820]	[0.6980,0.7700]	[0.2157,0.3380]

The positive and negative ideal points were calculated using Equation (16); the results are presented in Table 3.

Table 3. The grey positive and negative ideal point.

	S_1	S_2	S_3	S_4
$x_j^+(\otimes)$	$[0.7500, 0.7900]$	$[0.5880, 0.6440]$	$[0.6980, 0.7860]$	$[0.7515, 0.8163]$
$x_j^-(\otimes)$	$[0.2400, 0.3660]$	$[0.4260, 0.4820]$	$[0.6640, 0.7480]$	$[0.2157, 0.3380]$

The distances from each alternative to the positive and negative ideal points were calculated using Equations (19-20), using the grey distance calculation formula with $\alpha = 1$, with the results shown in Tables 4 and 5, respectively.

Table 4. The gain matrix.

D^+	S_1	S_2	S_3	S_4
A_1	0	0.1193	0.0302	0
A_2	0.0282	0.0801	0.0362	0.0567
A_3	0.3391	0	0.120	0.2789
A_4	0.3788	0.0461	0.0317	0.3547
A_5	0.4729	0.1620	0.0160	0.5094

Table 5. The loss matrix.

D^-	S_1	S_2	S_3	S_4
A_1	0.4729	0.0473	0.0092	0.5094
A_2	0.4447	0.0821	0	0.4592
A_3	0.1439	0.1620	0.0330	0.2359
A_4	0.0977	0.1188	0.0092	0.1550
A_5	0	0	0.0299	0

The positive and negative prospect matrices for each alternative were calculated using the results presented in Tables 6 and 7, respectively, using Equation (24). The prospect gain and loss weights were calculated using Equations (25-26), and the results are listed in Table 8.

Table 6. The positive prospect value matrix.

	S_1	S_2	S_3	S_4
A_1	0.5135	0.0661	0.0154	0.5487
A_2	0.4862	0.1081	0	0.5002
A_3	0.1781	0.1979	0.0481	0.2765
A_4	0.1262	0.1502	0.0154	0.1902
A_5	0	0	0.0439	0

Table 7. The negative prospect value matrix.

	S_1	S_2	S_3	S_3
A_1	0	-0.3181	-0.0899	0

A_2	-0.0845	-0.2205	-0.1061	-0.1604
A_3	-0.8319	0	-0.0385	-0.6951
A_4	-0.9210	-0.1328	-0.0941	-0.8671
A_5	-1.1297	-0.4216	-0.0501	-1.2098

Table 8. The prospect weights for gains and losses.

	S_1	S_2	S_3	S_3
$\pi^+(w_j)$	0.3483	0.2899	0.5397	0.3307
$\pi^-(w_j)$	0.3647	0.2925	0.5945	0.3429

The comprehensive prospect values for each attribute were calculated using Equation (27), based on the obtained positive and negative prospect values and gain and loss weights; the results are presented in Table 9.

Table 9. the comprehensive prospect value.

	A_1	A_2	A_3	A_4	A_5
V_i	0.2413	0.1527	-0.3278	-0.5693	-0.9562

According to the data in Table 9, positive values indicate that the alternative is perceived as a "gain" relative to the decision-maker's reference point, with the magnitude representing the psychological utility of the gain. Negative values signify that the alternative is perceived as a "loss" relative to the reference point, where the absolute value reflects the degree of loss aversion.

The final results are ranked by their numerical values, which are theoretically grounded in the value function characteristics of prospect theory. People evaluate outcomes as changes from reference points rather than absolute states. This ranking preserves the psychophysical property of diminishing sensitivity and captures the behavioral pattern where "losses loom larger than equivalent gains."

Based on the final results, the ranking is obtained as follows,

$$A_1 \succ A_2 \succ A_3 \succ A_4 \succ A_5$$

Evidently, based on the ranking results of the alternatives, A_1 is optimal A_2 can serve as a viable backup, while A_5 demonstrates the poorest performance. This conclusion aligns with the evaluation results obtained using the methodology in Reference [16]; however, the approach proposed in this study exhibits broader applicability. Because of financial and other practical constraints, scientific experiments often yield limited replicated datasets. Interval grey numbers can comprehensively and effectively capture the implicit information embedded in all experimental results. Particularly in decision-making scenarios involving risks, cognitive biases, or dynamic environments, prospect theory proves to be superior.

Consequently, the grey decision-making model developed in this study not only fully utilizes the complete information from experimental data but also features computational efficiency, yielding conclusions that are well grounded in practical reality.

6. Conclusion

For multi-attribute decision-making problems with incomplete attribute weight information and interval-valued attributes, this study employs generalized grey distances to normalize decision values and define positive/negative ideal solutions aligned with human cognition patterns. Building on prospect theory, we incorporate decision-makers' risk attitudes into multi-attribute decision

making by constructing prospect value functions, thereby developing an optimization model that maximizes comprehensive prospect values for alternative ranking.

Experimental validation demonstrates that this study exhibits the following comparative advantages and limitations:

(1) Compared to the TOPSIS method adopted in the original study, this study captures key aspects of decision-makers' risk preferences, such as loss aversion and probability weighting distortion, through value and probability weighting functions, thereby better reflecting the psychological mechanisms underlying behavioral decision-making.

(2) Compared with classical prospect theory models, this study introduces generalized grey distances to handle uncertain inter-attribute relationships, addressing the limitations of traditional prospect theory in scenarios with incomplete or partially unknown attribute information. In contrast to classical grey decision-making methods such as Grey Relational Analysis, this study embeds value functions into the grey relational degree subsystem, achieving a novel coupling between grey system theory and behavioral decision theory at the level of uncertainty quantification.

However, the model presented higher computational complexity and parameter sensitivity. Notably, when compared to hybrid models such as grey decision-making, our framework demonstrates superior explanatory power in high-risk domains such as behavioral finance and emergency decision-making, although conventional models retain advantages in computational efficiency for data-sufficient engineering decisions. Future enhancements may integrate parameter optimization algorithms with machine-learning techniques to improve model adaptability.

Author Contributions: Conceptualization, F.L.; methodology, F.L.; software, F.L. and J.G.; validation, F.L.; formal analysis, F.L. and J.G.; investigation, F.L.; resources, F.L.; data curation, F.L.; writing—original draft preparation, F.L.; writing—review and editing, F.L. and J.G.; visualization, F.L. and J.G.; supervision, F.L. and J.G.; project administration, F.L. and J.G.; funding acquisition, F.L. and J.G. All authors have read and agreed to the published version of the manuscript.

Funding: This research received no external funding.

Data Availability Statement: The original contributions presented in this study are included in the article. Further inquiries can be directed to the corresponding authors.

Acknowledgments: The authors thank the journal editor and anonymous reviewers for their guidance and constructive suggestions.

Conflicts of Interest: The authors declare no conflicts of interest. The funders had no role in the design of the study; in the collection, analyses, or interpretation of data; in the writing of the manuscript; or in the decision to publish the results.

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