Preprints.org

Article

Not peer-reviewed version

Saturated Thermal Conduction in Magnetized Advection-Dominated Gas NON-rotating α-Discs onto Non-Rotating Blackholes and Shock: The Role of the Gravitational Pressure

Orchidea Maria Lecian *

Posted Date: 23 March 2026

doi: 10.20944/preprints202603.1706.v1

Keywords: saturated thermal conduction; Schwarzschild spacetimes; gaseous accretion object; α-discs

Preprints.org is a free multidisciplinary platform providing preprint service that is dedicated to making early versions of research outputs permanently available and citable. Preprints posted at Preprints.org appear in Web of Science, Crossref, Google Scholar, Scilit, Europe PMC.

Copyright: This open access article is published under a Creative Commons CC BY 4.0 license, which permit the free download, distribution, and reuse, provided that the author and preprint are cited in any reuse.

Preprints.org (www.preprints.org) | NOT PEER-REVIEWED | Posted: 23 March 2026 doi:10.20944/preprints202603.1706.v1

Disclaimer/Publisher's Note: The statements, opinions, and data contained in all publications are solely those of the individual author(s) and contributor(s) and not of MDPI and/or the editor(s). MDPI and/or the editor(s) disclaim responsibility for any injury to people or property resulting from any ideas, methods, instructions, or products referred to in the content.

Article

Saturated Thermal Conduction in Magnetized Advection-Dominated Gas NON-rotating α-Discs onto Non-Rotating Blackholes and Shock: The Role of the Gravitational Pressure

Orchidea Maria Lecian

Sapienza University of Rome, Rome, Italy; orchideamaria.lecian@uniroma1.it

Abstract

Accretion hot gaseous thick discs of the α model with advection and with saturated thermal conduction onto a non-rotating blackhole with an external magnetic field are newly analytically studied in the General-Relativistic formalism. The configurations of the external magnetic field apt to magnetize the disc are newly analytically found; the allowed configurations are selected after the initial conditions, which are dictated after the derivative of the total pressure, and are therefore such that no new force arises due to the external magnetic field. The heat function per unit volume is newly analytically written. The allowed thermodynamical process of the gaseous material are isoentropic adiabatic isothermal processes. The shock is analyzed. The characterizations apply also to the thin-disc model. The toymodel is here considered of a nonrotating central blackhole on which the accretion is accomplished, nevertheless, the possible result of accretion in modifying the radius of the fictitious singularity is disregarded; furthermore, the turbulence is neglected with the aim of isolating the heat-transport phenomena.

Keywords: saturated thermal conduction; Schwarzschild spacetimes; gaseous accretion object; α-discs

---

1. Introduction

In the present paper, the instance of Schwarzschild blackhole spacetime with gaseous magnetized advection-dominated accretion thick discs is taken. The conservation of mass flux equation, the radial momentum equation, the azimuthal momentum equation, the induction equation, and entropy generation equation are here written in the Schwarzschild case of the non-gravitating gaseous accretion object onto a Schwarzschild blackhole $U \equiv 0$ are written and newly analytically solved: the allowed configurations are selected. As a new result, not all the magnetic fields are allowed, bit only those determined after the analytical solution of the radial momentum equation. Furthermore, not all gaseous discs thermodynamical types are allowed, but only the isoentorpic adiabatic isothermal ones are proven to be of General-Relativistic Astrophysical interest. Moreover, the General-Relativistic forces are proven to be extended to the complete thick disc region, i.e., no regions of the thick disc can be treated in the non-Relativistic manner.

In the work of Oda et al. [1], 3D-MHD simulations are run for accretion discs which are steady, magnetized, optically-thin and single-temperature. The analysis of the action of the magnetic field in the azimuthal direction is motivated ibidem after the numerical simulations. A 'magnetic-pressure-dominated' transition is described.

In the work of Oda et al. [2], accretion discs onto blackholes are studied, where the accretion objects are hypothesized to be tow-temperatures and optically-thin.

Ibidem, the comparison with the experimental evidence of X-ray spectra is implemented.

© 2026 by the author(s). Distributed under a Creative Commons CC BY license.

Preprints.org (www.preprints.org) | NOT PEER-REVIEWED | Posted: 23 March 2026 doi:10.20944/preprints202603.1706.v1

In the work of Sarkar et al. [3], discontinuous shock transitions are found for rapidly-rotating blackhole objects. The forces descending form the radial variation of the toroidal magnetic field flux advection rate are examined. Dissipative shocks are described.

Ibidem, a maximum energy is determined for the post-shock corona,whose interest is explicated in the experimental verification.
The role of the magnetic field in the observation evidence of radio jets is evidentiated.

The magnetized accretion flow is studied to connect the properties of the shock with those of the magnetic flux advection rate for Kerr blackholes.

The advection rate of the toroidal magnetic flux is found to be dictated after the accretion in the radial direction.The phenomena re compared with the work of Oda et al. [1] and with the work of Oda et al. [2].

The further action of the magnetic field is found to make the temperature of the disc diminish. The 'cooling efficiency' is found to be greater in the post-shock corona.

The pressure calculated at the post-shock condition is found to be diminishing.

In the work of Sarkar et al. [4], the forces descending form the effect of the thermal-saturated conduction on magnetized advection-dominated accretion objects onto rotating blackholes is investigated. As a result, steady, global transonic shock accretion is described.

Ibidem, the shock locations are evidentiated; they are described to be traces further away from the blackhole horizon as the saturation shock parameter φ_s increases.

Ibidem, the initial conditions are taken at the outer edge of the disc; the dynamo effects are commented about.

As discussed also in Section 2, most of the present items of investigation do not take into account General-Relativistic forces; in the present paper, differently, all the Relativistic forces are taken into account. This aim is at first accomplished after considering a simple toy-model of non-rotating α disc: the possible result of accretion as changing the radius of the fictitious singularity is neglected; furthermore, the turbulence is neglected for the sake of outlining the phenomenon of heat transport. The refinements of the present model will be presented in further work.

In the analyzed investigations, the GR approaches were set as follows.

In the work of Singh et al. [24]the case of rotating blackhole is considered for the understanding of the role of thermal conduction.

In the work of Fagei [31], the consequences of thermal conduction on the hydrodynamics of quasi-spherical matter flow configurations is analyzed. Te techniques are specified for the perfect fluid and for the gaseous material; in the latter case, the findings are applied to comparison with the experimental data from the accretion object of Sgr A*.

In the present paper, as new results for comparison, the density is newly analytically written from the request of vanishing saturated conduction flux, the heat function per unit volume is newly analytically written, the ϕ component of the magnetic field allowed for the magnetization of the so-obtained thick disc is selected after the radial momentum equation. The magnetic field is found to be Lorentz-free. More in detail, no 'magnetic-pressure-dominated' transitions are allowed in the General-Relativistic treatment.

From the conservation of mass flux equation, the Schwarzschild blackhole is found to be a not accreting one.

The speed of sound is calculated, and the sonic behaviour of the thick disc is analytically studied. The shock is analytically written; the specific energy is newly analytically found to be continuous at the shock front, and the shock 'jump' is found not to be discontinuous.The specific energy is found to depend on the initial conditions of the gas.

The equations governing the presented model are formulated not to comprehend mass outflow.

The notation of [5] is followed for the comparison with [4].

The characterizations here newly achieved apply also to the thin-disc model.

The paper is organized as follows.

© 2026 by the author(s). Distributed under a Creative Commons CC BY license.

Preprints.org (www.preprints.org) | NOT PEER-REVIEWED | Posted: 23 March 2026 doi:10.20944/preprints202603.1706.v1

In Section 2, some of the investigations which motivate the present work are discussed.

In Section 4, the configurations of the external magnetic field are newly indicated.

In Section 5, the new analytical characterization of the external magnetic field is written.

In Section 6, the shock is newly analytically studied.

In Section 7, the main new results of the present paper are discussed. The comparison is indicated with the recent work [4].

2. Introductory Material

In the work of Khesali et al. [6], advection-dominated accretion flow onto a central object is investigated in the presence of a toroidal magnetic field. The α model is considered for a quasi-spherical accretion flow. The forces arising from the external magnetic field are examined. The Keplerian regime is taken for comparison. The calculations are in the non-Relativistic framework. A transonic point into the accretion object is found for every value of the magnetic pressure. The paradigms of self-similarity are implemented.

In the work of Dyda et al. [7], the methods of non-Relativistic MHD are implemented for a disc chosen as one with an 'inner region' of ideal gas and an 'outer region' of ideal plasma; two different densities are assigned for the disc and for the corona. The work is motivated after the prospective investigation of the advection on the matter an those of the magnetic field on a turbulent regime. The time dependence of the evolution of the system is researched. The Paczy'nski-Wiita potential is taken into account. Non-isoentropic processes are considered.

In the work of Ghoreyshi et al. [8], advection-dominated optically-thin discs are described with toroidal magnetic field. The properties of the thermal instabilities are investigated. The role of the radial viscosity is studied. The investigation is performed in the non-Relativistic regime. In the work of Samadi et al [9], after the work of Khesali et al [6], the advection-dominated, differentially-rotating, viscous, resistive, rotating accretion object onto a (rotating) blackhole is considered in the non-General-Relativistic framework with a toroidal magnetic field.

The result is ibidem presented, that the forces arising from the toroidal magnetic field oppose to that of gravity in the vertical direction; the determination of the vertical height of the thick disc is calculated.

In the work of Ghoreyshi et al [10] after the work of Khesali et al [6], resistive accretion flow with a magnetic field which induced anisotropic pressure is considered. The Relativistic forces are neglected; a Newtonian potential is favoured. The investigation is outlined to be apt for comparison with experimental verification of the configuration of the central object in Sgr A*.

In the work of Jafari et al. [11] after the work of Khesali et al. [6], numerical MHD simulation is run about magneto-rotational, turbulent, advective accretion object onto a rotating central object with a poloidal (external) magnetic field. The corresponding dynamo mechanism is discussed. All the GR forces are neglected.

In the work of Dyda et al. [12] after the work of Dyda et al [7], MHD numerical simulation is run about the generation of a magnetic field in proto-stellar discs. The behaviour of the poloidal field is studied. The resulting dynamo is analyzed. The GR forces are neglected. The work is ibidem commented to be of interest for comparison with experimental evidence from T Tauri stars.

In the work of Ghasemnezhad et al. [13] after the work of Ghoreyshi et al. [8], the aspects of a radial viscosity force and those of anisotropic thermal conduction which influence the hot accretion flows are analyzed. Thermal conduction and cooling mechanisms are analyzed.

The model is considered ibidem of interest for comparison with experimental evidence of the accretion flows onto Sgr A*. The results are ibidem commented to be of use for he comparison with GRMHD semi-analytical studies of ADAF's [14]-[19].

© 2026 by the author(s). Distributed under a Creative Commons CC BY license.

Preprints.org (www.preprints.org) | NOT PEER-REVIEWED | Posted: 23 March 2026 doi:10.20944/preprints202603.1706.v1

3. Methodology

In the owork of Singh et al. [24], the case of accretion onto rotating blackhole is described: the mass flux conservation equation, the radial momentum equation, the azimuthal force equationand the energy equation equation are written.

Ibidem, the gravitational potential considered is an effective one and not one directly connected with the metric tensor of the spacetime. In the following, the gravitational potential Φ is taken as

$$\Phi = -\frac{1}{2}g^{00}. \tag{1}$$

Ibidem, furthermore, the radial momentum equation contain the (derivative of the) thermodymaical pressure (with repscet to the radial component of the posion 4-vector). Differently, in the present work, the (derivative of the) total pressure P_{TOT} is considered, i.e.

$$P_{TOT} \equiv P_{td} + P_{grav}, \tag{2}$$

i.e., in Eq. (2), the contributions to the pressure considered are the thermodynamical pressure P_{td} plus the gravitational presssure P_{grav}, with

$$\frac{1}{w}\frac{\partial}{\partial x^r} = P_{grav} \equiv -\frac{1}{2}\frac{\partial}{\partial r}g^{00}. \tag{3}$$

being w the heat function per unit volume.
It is here stressed that the role of the gravitational pressure is to establish the inertial frame in which $v_r \equiv 0$.
As a result, the mass flux conservation equation in written as

$$\dot{M} = 0; \tag{4}$$

the radial momentum equation is written as

$$\frac{1}{H\rho}\frac{d}{dr}P_{TOT} + \frac{d}{dr}Q_{eff} = 0 \tag{5}$$

being Q_{eff} the effective exchanged heat; the azimuthal momentum eqution is reconducted to

$$\frac{1}{\Sigma r}\frac{d}{dr}(r^2 W_{r\phi}) = 0, \tag{6}$$

which is solved as

$$W_{r\phi} = -\alpha P_{TOT} : \tag{7}$$

,the energy equation is reconducted to

$$\frac{1}{r}\frac{d}{dr}(r\mathcal{F}_s) = 0 \tag{8}$$

where the saturated conduction flux is considered.

It s therefore remarked that the advection dominated character of the flow is now newly kept in the solution of the momentum equation (after the position of Eq. (7)) as

$$W_{r\phi} = -\alpha P_{vert\ int} \tag{9}$$

© 2026 by the author(s). Distributed under a Creative Commons CC BY license.

Preprints.org (www.preprints.org) | NOT PEER-REVIEWED | Posted: 23 March 2026 doi:10.20944/preprints202603.1706.v1

being $P_{vert\ int}$ the vertically-integrated total pressure.
Moreover, the energy equation is now newly solved after $\mathcal{F}_s$ the saturated conduction flux whose definition is here newly extended after after [28] as

$$\mathcal{F}_s \equiv 5\phi_s\rho\left(\frac{P_{TOT}}{\rho}\right)^{3/2}. \tag{10}$$

The vertically-integrated thickness H of the radial momentum equation is defined after extending the results of [26] and of [27] and after specifying the results for the non-rotating case as

$$H = \frac{P_{TOT}}{\rho r}. \tag{11}$$

In the work of Fagei [31], the gravitational potential taken for the disc is the Newtonian one. The governing equations are written in spherical cooridnates in order to study the quasi-spherical accretion, and considered on the equatorial plane.
The continuity equation, the radial force equation, the azimuthal force equation and the energy equation are written. The continuity equation is in the present analysis newly reconducted to

$$\frac{d}{dt}\rho = 0; \tag{12}$$

the radial force equation is here newly verified as

$$-\frac{1}{\rho}\frac{\partial}{\partial r}P_{TOT} = 0; \tag{13}$$

the azimuthal force equation is now newly controlled identically vanishing; the energy equation is now newly set as

$$Q_{adv} = Q_{vis} - Q_{rad} + Q_{con} = 0 \tag{14}$$

being Q_{adv} the total heat exchanged because of advection, Q_{vis} the total heat exchnged because of viscosity, Q_{rad} the total heat exchanged because of radiation, and $+Q_{con}$ the total heat exchanged because of conduction.

The energy equation is here commented to according to the purposes of the work of Fagei [31]: the difinition of kinematical viscosity from [32] is not here taken: differently, the dynamical viscosity ν is here considered as

$$\nu = \frac{\mu}{\rho} \tag{15}$$

after which the phenomenon of viscosity is decoupled form the notion of velocities, with $\alpha = const$, $\alpha < 1$.
It is now in order to further define the quantities which qualify the governing equations.
In collisional plasma, the mean free path is requested to be small than the temperature scale length.
This way, ϕ_{cond} the heat flux due to thermal conduction is calculate as

$$\phi_{cond} = -k\nabla T, \tag{16}$$

being T the temperature. In collisional plasma, one has that

$$L_T = \frac{T}{\mid \Delta T \mid} \tag{17}$$

being L_T the temprature scale length.

© 2026 by the author(s). Distributed under a Creative Commons CC BY license.

Preprints.org (www.preprints.org) | NOT PEER-REVIEWED | Posted: 23 March 2026 doi:10.20944/preprints202603.1706.v1

Furthermore, in palsma, the maximum value of the heat flux is expressed after

$$\Delta Q = \frac{3}{2} n_e k T_e v_{char} \tag{18}$$

with v_{char} a characteristic velocity of the order of the electron velocity [35].

A Maxwellain distribution is chosen for the electrons in [31] from [33] and from [34], from which

$$v_{char} = \left(\frac{8}{9\pi}\right)^{1/2} \left(\frac{kT}{m_e}\right)^{1/2} : \tag{19}$$

from [34], the saturated flux ϕ_{sat} is calculated as

$$\phi_{sat} = 0.4 m_e \kappa T \sqrt{\frac{2\kappa T m_e}{\pi m_e}} \tag{20}$$

where, int he above, the pedex $_e$ refers to the electron quantities.
The discussion

It is here newly understood that one further role of the gravitational pressure is to select the thermodynamical configurations available for the General-Relativistic process, e.g. from the comrehensive aspect of Eq. SCRIVI, Eq. (13)

It is further noticed that the analysis of the saturated flux should be upgraded to a Relativistic distribution.

4. The Configurations of the Magnetic Field

The configuration is here considered of a thick non-rotating non-self-gravitating accretion disc onto a non-rotating blackhole of radius of the fictitious singularity r_G, where viscosity and thermal conduction are present; the pertinent stress-energy tensor $T^{\mu\nu}$ of the fluid is taken from [5] Eq. 136.1

$$T^{\mu\nu} = -p g_{\mu\nu} + w u_\mu u_\nu + \tau_{\mu\nu}, \tag{21}$$

where $\tau_{\mu\nu}$ accounts for the two dissipative processes.
Let Σ be the vertically integrated mass density as from [20]

$$\Sigma \equiv 4\rho_o h(r) \tag{22}$$

with the definition of $h(r)$ from [21] and from [22] as

$$h \equiv \sqrt{\frac{r}{\gamma \Phi'(r)}} \tag{23}$$

being ρ_o the initial conditions; with the choice from [5], Φ is define from the $_{tt}$ component of the metric tensor: in the Schwarzschild case, $g_{tt} \equiv 1 - 2\Phi$, from which $h(r)$ is here calculated as

$$h(r) \equiv b\sqrt{\frac{r}{\gamma \frac{d\Phi}{d(r/r/G)}}} \equiv b\sqrt{r^3} 2\gamma r_G^2 \tag{24}$$

being b the speed of the sound.

It is proven in the following indeed that the only allowed configuration of a Keplerian (non-self-gravitating) gaseous accretion object onto a Schwarzschild blackhole is an isoentropic adiabatic process.

The vertically-averaged mass density Σ is therefore

$$\Sigma \equiv 4\rho_0 h(r) \tag{25}$$

© 2026 by the author(s). Distributed under a Creative Commons CC BY license.

Preprints.org (www.preprints.org) | NOT PEER-REVIEWED | Posted: 23 March 2026 doi:10.20944/preprints202603.1706.v1

From [4], the conservation of mass flux equation, the radial momentum equation, the azimuthal momentum equation, the induction equation, and entropy generation equation are here rewritten in the Schwarzschild case of the non-gravitating gaseous accretion object onto a Schwarzschild blackhole $U \equiv 0$ as

$$\dot{M} = 2\pi u \Sigma \frac{r}{r_G} \equiv 0, \tag{26a}$$

$$\frac{1}{\rho}\left[\frac{dP_{TOT}}{d(r/r_G)} + \frac{< B_\phi^2 >}{4\pi\rho r/r_G}\right] = 0, \tag{26b}$$

$$\frac{d}{d(r/r_G)}\left(\frac{r^2}{r_G^2}\mathcal{T}_{\frac{r}{r_G}\phi}\right) = 0, \tag{26c}$$

$$\vec{\nabla} \wedge \vec{j} = 0, \tag{26d}$$

$$UT\Sigma\frac{d}{d(r/r_g)}s = 0 = Q^- - Q^+ - Q_{cond} \tag{26e}$$

being s the entropy and Q_{cond} the heat exchange due to saturated thermal conduction; the notation $<>$ in Eq. (26b) indicates azimuthal-averaged quantities.

From the particle number conservation equation Eq. (26a), one learn that the blackhole mass rate change is vanishing, i.e., in particular, the blackhole mass is not accreting.

From the induction equation Eq. (26d), after the definition of the total pressure as from [5] Eq. 134.16

$$\frac{d}{d(r/r_G)}P_{TOT} \equiv -\frac{1}{2}w\frac{d}{d(r/r_G)}ln(g_{tt}) \tag{27}$$

where w is the heat function per unit volume, one obtains that the magnetic field $\vec{B}$ is Lorentz-free.

From the azimuthal momentum equation Eq. (26c), one directly calculates that the $\frac{r}{r_G}\phi$ component of the shear stress tensor is

$$\mathcal{T}_{\frac{r}{r_G}\phi} \equiv c_2\frac{r_G^2}{r^2}, \tag{28}$$

being c_2 the proper dimensional constant.

The advection rate of the toroidal magnetic $\dot{\mathcal{F}}$ field is vanishing as

$$\dot{\mathcal{F}} \equiv \left(\int B_\phi dz\right)|_{U=0} = 0; \tag{29}$$

this way, the so-called 'disk equatorial-plane mean azimuthal toroidal magnetic field' $B_0(r/r_G)$ is calculated as

$$B_0(r/r_G) \equiv (2^5\pi)^{1/4}\sqrt{\frac{\tilde{R}T\Sigma}{\mu h\beta}} \tag{30}$$

with μ the mean molecular weight of the gaseous material, and β defined as the ratio of the pressure of the gas P_{gas} to the magnetic pressure P_{mag} as

$$\beta \equiv \frac{P_{gas}}{Pmag} \tag{31}$$

and with $\tilde{R}$ the gas constant.

5. Characterization of the Configurations Allowed After the Presence of a Magnetic Field

The temperature T of the isothermal process is therefore calculated as

$$T \equiv \frac{B_0^2()r/r_G}{(2^5\pi)^{1/2}}\mu\beta h\frac{1}{\tilde{R}\Sigma}. \tag{32}$$

© 2026 by the author(s). Distributed under a Creative Commons CC BY license.

doi:10.20944/preprints202603.1706.v1

The vertically-integrated pressure W is therefore calculated as equating the $\frac{r}{r_G}\phi$ component of the shear stress tensor as

$$W \equiv -\frac{1}{\alpha T} c_2 \frac{r_S^2}{r^2} \tag{33}$$

6. Characterization of the Shock

The shock is newly found to be global with

$$W_+ \equiv W_- \tag{34}$$

for the vertically-integrated pressure evaluated before the shock W_- and after the shock W_+.
The specific energy $\mathcal{E}(r)$ is newly found to be conserved during the shock (i.e., 'accross the shock front') as

$$\mathcal{E}\left(\frac{r}{r_G}\right) = \Phi\left(\frac{r}{r_G}\right) + \frac{< B_\phi^2 >}{4\pi\rho} + b^2 = -\frac{1}{2}\left(\frac{r_G}{r}\right) + \frac{< B_\phi^2 >}{4\pi\rho} + b^2. \tag{35}$$

The flow of the specific entropy is conserved (as vanishing) as from [23]

$$s\left(\frac{r}{r_G}\right) \propto u \equiv 0 \tag{36}$$

From Eq. 135.9 form [5], one also has that

$$s_+ - s_- \equiv 0 = \frac{1}{12}\left[\frac{1}{wTv^2}\frac{\partial^2}{\partial p^2}(wV^2)\right](P_+ - P_-)^2. \tag{37}$$

Eq. (37) is explained as follows. As the total pressure is not a constant quantity of the considered spacetimes manifold, one has that

$$p_+ \neq p_-. \tag{38}$$

The heat function per unit volume is therefore written as

$$w = c_6\left(\frac{P_{TOT}(r)}{V^2(r)} - \frac{P_{tot}(r_O)}{V^2(r_O)}\right) \tag{39}$$

where the initial condition $r_O \equiv r_S$ can be taken, r_S being the Schwarzschild radius.

The saturated conduction flux $\mathcal{G}_s$ written as

$$\mathcal{G}_s \equiv 5\varphi_s \rho b^3, \tag{40}$$

where φ_s is the saturated conduction parameter, i.e., a constant such that

$$\varphi_s \leq 1 \tag{41}$$

The saturated thermal conduction heat is newly calculated as vanishing from Eq. (26e) for the isothermal adiabatic process as

$$Q_{cond} \equiv 0 = -\frac{2h(r)}{r/r_G}\frac{d}{d(r/r_g)}\left(\frac{r}{r_G}\mathcal{G}_s 5\varphi_s\rho b^3\right) \tag{42}$$

As

$$b \equiv \sqrt{\frac{2P_{TOT}}{\rho}} \tag{43}$$

© 2026 by the author(s). Distributed under a Creative Commons CC BY license.

Preprints.org (www.preprints.org) | NOT PEER-REVIEWED | Posted: 23 March 2026 doi:10.20944/preprints202603.1706.v1

from Eq. (39) and from Eq. (42) one obtains

$$\rho = \left[\left(\frac{c_6}{V^2(r)} \frac{1}{w + \frac{P_{TOT}(r_O)}{V^2(r_O)}} \right)^{3/2} \rho(r_O) b^3(r_O) \frac{r_O}{r} \right]^{-2}. \tag{44}$$

The initial condition $r_o \equiv r_S$ can be taken.
From Eq. (26b), the azimuthal average of the B_ϕ component of the magnetic field is obtained straightforward, and the B_ϕ component of the magnetic field of the admitted configuration is newly calculated as

$$B_\phi \equiv \mp\sqrt{w} \frac{\sqrt{\pi}}{r\sqrt{r - r_S}}; \tag{45}$$

it is now momentous to comment on the result Eq. (45), i.e., about that only one configuration of the magnetic field is apt to obey the radial momentum equation. As an outcome, the magnetic contain the factor $\sqrt{r - r_S}$ within the denominator. To keep the magnetic field non-diverging,t he appropriate condition on the heat function per unit volume are newly found as a set of the suitable majorizations within the limit procedures.

7. Discussion

The allowed configurations of the magnetized gaseous advection-dominated α disc are calculated int he General-Relativistic framework. All the General-Relativistic forces are newly taken into account.

The configuration of an external magnetic field apt to magnetize the considered gaseous-phase accretion flow is newly determined.

The density of the gas as a function of the heat function per unit volume of the total pressure and of the speed of sound is written. No forces due to the external magnetic field are arising in the definition of the density.

The heat function per unit volume, which determines the stress-energy tensor of the fluid, is newly analytically calculated.

More generally, the role of the derivative of the total pressure is newly found to indicate the constants defining the configuration of the external magnetic field.

Comparison is brought with the work of Sarkar et al. [4] and with the items of bibliography therein. More in detail, the heat flux, the saturated-conductivity flux, the magnetic flux are newly written for comparison with the implementation the Astrophysical mechanisms.

The characterizations here newly found are suitable to be applied also to the thin-disc model.

References

1. H. Oda, M. Machida, K.E. Nakamura et al., Steady Models of Optically Thin, Magnetically Supported Black Hole Accretion Disks, PASJ 59, 457 (2007).
2. H. Oda, M. Machida, K.E. Nakamura et al., Global Structure of Optically Thin, Magnetically Supported, Two-Temperature, Black Hole Accretion Disks, PASJ 64, 15 (2012).
3. Effect of magnetic flux advection on the dynamics of shock in accretion flow around a rotating black hole B. Sarkar, A. Rao, Research in Astronomy and Astrophysics 20(3), id.040 (2020). DOI: 10.1088/1674-4527/20/3/40
4. B. Sarkar , I.K. Dihingia, , R. Misra, Thermal Conduction and Thermal-Driven Winds in Magnetized Viscous Accretion Disk Dynamics, New Astronomy 118, id.102377 (2025). DOI: 10.1016/j.newast.2025.102377
5. L.D. Landau, E.M. Lifshitz, Course of Theoretical Physics Volume 6, Second Edition, Pergamonn Press (1987).
6. A. Khesali, K. Faghei,Time dependence of advection-dominated accretion flow with a toroidal magnetic field Monthly Notices of the Royal Astronomical Society 398(3), 1361-1367 (2009). DOI: 10.1111/j.1365-2966.2009.15086.x
7. S. Dyda, R.V.E. Lovelace, G.V. Ustyugova et al., Advection of matter and B-fields in alpha-discs Monthly Notices of the Royal Astronomical Society, Volume 432, Issue 1, p.127-137 (2013). DOI: 10.1093/mnras/stt429

© 2026 by the author(s). Distributed under a Creative Commons CC BY license.

Preprints.org (www.preprints.org) | NOT PEER-REVIEWED | Posted: 23 March 2026 doi:10.20944/preprints202603.1706.v1

8. S.M. Ghoreyshi, M. Shadmehri, The local stability of the magnetized advection-dominated discs with the radial viscous force , Monthly Notices of the Royal Astronomical Society 476(4), 4830-4839 (2018) . DOI: 10.1093/mnras/sty537
9. M. Samadi, S. Abbassi, M. Khajavi, The effect of toroidal magnetic field on the thickness of a viscous-resistive hot accreting flow Monthly Notices of the Royal Astronomical Society 437(4), 3124-3132 (2014). DOI: 10.1093/mnras/stt2052
10. S.M. Ghoreyshi, A.Khesali, The role of resistivity in hot accretion flows with anisotropic pressure: Comparing magnetic field models Publications of the Astronomical Society of Japan 75(1), 52-70 (2023). DOI: 10.1093/pasj/psac089
11. A. Jafari, E.T. Vishniac, Magnetic Field Transport in Accretion Disks, The Astrophysical Journal 854(1), article id. 2 (2018). DOI: 10.3847/1538-4357/aaa75b
12. S. Dyda, R.V.E. Lovelace, G.V. Ustyugova et al., Magnetic Field Amplification via Protostellar Disc Dynamos, Monthly Notices of the Royal Astronomical Society 477(1), 127-138 (2018). DOI: 10.1093/mnras/sty614
13. M. Ghasemnezhad, M. Khosravi, The role of radial viscosity force and anisotropic thermal conduction in hot accretion flow New Astronomy 113, id.102276 (2024). DOI: 10.1016/j.newast.2024.102276
14. A.E. Broderick, A. Loeb, Astrophys. J. 697, 1164 (2009).
15. Hawley J.F. Astrophys. J. 528, 462 (2000).
16. P.C. Fragile, A. Gillespie, T. Monahan et al., Astrophys. J. Suppl. 201, 9 (2012).
17. P.C. Fragile, A. Olejar, P. Anninos, Astrophys. J. 796, 22 (2014).
18. C.J. White, J.M. Stone, C..: Gammie, Astrophys. J. Suppl. 225, 22 (2016).
19. H. Olivares, O. Porth, J. Davelaar et al. Astron. Astrophys. 629, A61 (2019).
20. R. Matsumoto, S. Kato, J. Fukue, A.T. Okazaki, Viscous transonic flow around the inner edge of geometrically thin accretion disks, Publications of the Astronomical Society of Japan 36, 71-85 (1984).
21. N.I. Shakura, R.A. Sunyaev, A theory of the instability of disk accretion on to black holes and the variability of binary X-ray sources, galactic nuclei and quasars, MNRAS 175, 613 (1976).
22. T.K. Das, Generalized Shock Solutions for Hydrodynamic Black Hole Ac- cretion, ApJ. 577, 880 (2002).
23. W.-M. Gu, J.-F Lu, Standing Rankine Hugoniot Shocks in Black Hole Accretion disks, Chinese Physics Letters 21, 2551-2554 (2004). doi:10.1088/0256- 307X/21/12/064
24. M. Singh, S. Das, Role of thermal conduction in relativistic hot accretion flow around rotating black hole with shock, JCAP02(2025)068.
25. I. K. Dihingia, S. Das, D. Maity, S. Chakrabarti, Phys. Rev. D 98, 083004 (2018), 1806.08481.
26. H. Riffert, H. Herold, ApJ 450, 508 (1995).
27. J. Peitz and S. Appl, MNRAS 286, 681 (1997), astroph/9612205.
28. L. L. Cowie, C. F. McKee, ApJ 211, 135 (1977).
29. G. Rezgui, R. Preiner, The role of saturated thermal conduction in shaping electric current dynamics in jet-launching disks, Journal of High Energy Astrophysics Volume 47, (2025), 100394.
30. K. Menou, Hot accretion with saturated conduction (2005), e-Print: astro-ph/0507189
31. K. Fagei, Dynamics of hot accretion flow with thermal conduction, Monthly Notices of the Royal Astronomical Society 420(1), 118-125, (2012). DOI: https://doi.org/10.1111/j.1365-2966.2011.20006.x
32. Narayan R., Yi I., ApJ 444, 231 (1995).
33. Williams M. M. R., Mathematical Methods in Particle Transport Theory. Wiley Interscience, New York, 1971.
34. Cowie L. L., McKee C. F., ApJ 211, 135 (1977):
35. Parker E. N., Interplanetary Dynamical Processes, Interscience, New York, 1963.
36. Di Matteo T., Quataert E., Allen S. W., Narayan R., Fabian A. C., MNRAS 311, 507 (2000).
37. Sarkar, Biplob and Dihingia, Indu Kalpa and Misra, Ranjeev, Thermal Conduction and Hydrodynamical Winds in Magnetized Viscous Accretion Disk Dynamics, SSRN (2024).: DOI http://dx.doi.org/10.2139/ssrn..
38. Gruzinov, A. 1998, astro-ph/9809265

Disclaimer/Publisher's Note: The statements, opinions and data contained in all publications are solely those of the individual author(s) and contributor(s) and not of MDPI and/or the editor(s). MDPI and/or the editor(s) disclaim responsibility for any injury to people or property resulting from any ideas, methods, instructions or products referred to in the content.

© 2026 by the author(s). Distributed under a Creative Commons CC BY license.