A High-power Propellant-free Electromagnetic Propulsion Interacting on Local Space-time Curvature

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Abstract – In this paper a high-power and propellant-free electromagnetic propulsion is proposed based on the General Relativity and nuclear fusion technology. We find that Riemann curvature vanish and geodesic motion is free from gravitational field locally in a special space-time, which demonstrates the feasibility of propellant-free electromagnetic propulsion. To achieve high-power propulsion in Schwarzschild background, we choose current loop as axisymmetric field source and obtain exact solution of Einstein-Maxwell field equation using Killing symmetry and Ernst generation technique. An implementation with superconductor shield is given according to the Meissner effect, calculation implies that the device can be sufficiently free from gravitational field with the aid of existing nuclear fusion engineering.
Keywords –Electromagnetic propulsion; High power; Propellant free; General Relativity; Nuclear fusion

1 Introduction. – Traditional aircrafts consuming propellants is limited by the maximum capacity they can carry. In recent years, solar propulsion, antimatter propulsion, plasma propulsion and other propellant-free propulsion principles have been proposed [1, 2]. However most propulsion mechanics are not clear and have academic controversy [3], also complex engineering implementation.

In 1994, Alcubierre [4] proposed a new type of spaceship drive engine named Alcubierre Drive which is clearly unrealistic as its realization requires Casimir dark energy to curve spacetime. Li Ning et al. [5,6] found that the weight of objects floating above a rotating superconducting disk was reduced by 0.5% to 2%, but they did not propose a rational theoretical explanation.

Above studies provide a idea for new type of propulsion engines. According to general relativity, both matter and energy can curve space-time [7–9]. Although it is impossible to use electromagnetic field curve spacetime to eliminate preexisting matter gravitational field everywhere. But here is a case that, when a free particle is at the point of extreme gravitational potential energy, then its 3-acceleration is zero and the free particle maintains the anti-gravity equilibrium.

This paper will firstly discuss the anti-gravity phenomenon of spacecrafts geodesic motion and local vanishment of the Riemann curvature in special spacetime, which demonstrates the feasibility artificial anti-gravity. Secondly, in the Schwarzschild background, an artificial manifold in the form of static axisymmetric field is given, and the required strength of the electromagnetic field source is solved. Finally, a efficiency field interacting method is proposed.

2 Natural anti-gravity under specific spacetime. –

2.1 Reissner-Nordstrom background spacetime. Assuming that a spacecraft is located near the outer surface of the spherically symmetric and uniformly charged planet, with a distance from the center $r = r_0$. The spaceship is in the standard Reissner-Nordstrom [10] background space-time, and its line element form:

$$ds^2 = -(1 - \frac{2m}{r} + \frac{q^2}{r^2})dt^2 + (1 - \frac{2m}{r} + \frac{q^2}{r^2})^{-1}dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2$$

where $m = \frac{\mathrm{GM}}{\mathrm{c}^2}, q^2 = \frac{\mathrm{G}Q^2}{4\pi\epsilon_0 \mathrm{c}^4}$.

Firstly, approximation of the Newtonian gravitational potential energy with limitation of weak-field and low-velocity [11]:

$$\phi = \frac{q^2}{r^2} - \frac{\mathrm{m}}{r}$$

The distribution of the gravitational potential with r corresponding to different charges scalar Q is shown in Figure 1; when $Q = 0$, it degenerates into the Schwarzschild

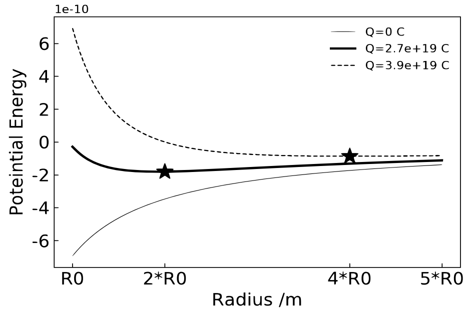

Fig. 1: Gravitational potential distributions for different Q

spacetime. When $Q \neq 0$, the gravitational potential has a non-monotonic distribution, and the asterisk in fig.1 is marked as the extreme point of the gravitational potential where $r = q^2/m$. Assuming that the low-speed spacecraft is just at the extreme point with zero initial speed, according to the principle of minimum potential energy, 3-space position of the spacecraft should stay the same.

Secondly, assuming that the spacecraft has no charge and its mass is negligible, then its geodesic motion equation is [11]:

$$\begin{cases} \frac{d^2t}{d\tau^2} = & \frac{2(q^2-mr)}{r(r^2-2mr+q^2)}\frac{dr}{d\tau}\frac{dt}{d\tau} \\ \frac{d^2r}{d\tau^2} = & \frac{(r^2-2mr+q^2)(q^2-mr)}{r^5}\left(\frac{cdt}{d\tau}\right)^2 + \\ & \frac{(mr-q^2)}{r(r^2-2mr+q^2)}\left(\frac{dr}{d\tau}\right)^2 \end{cases}$$

where symbol c represents the vacuum light speed. And the spacecraft starts from the point where $r = r_0$ with a initial velocity of $v_0 = dr/dt = 0$. When the charge scalar $q^2 = mr_0$, it is easy to know that its geodesic is a time-like curve and always has:

$$\frac{dr}{d\tau} = 0$$

which implies the spacecraft will keep balance at the point $r = r_0$, and the gravitational effect of the planet matter will be eliminated by the static electric field.

If given the disturbance to initial velocity, then we can get the evolution law of the spacecrafts position over coordinate time by numerical simulation in fig.2. With different initial velocities, the spacecraft vibrates up and down around the balance point $r = r_0$. The amplitude increases as the initial velocity increases.

Whats more, for a given initial velocity, if we assume that the total charge on the planet surface can be controlled artificially according to the position of the spacecraft. Referring to Birkhoff's theorem and its corollary [12], the solution is still in form of R-N solution. When the planet's environment is similar to the Earth, the following control law exists:

$$Q = Q_0\left(\frac{r_0}{r}\right)^n$$

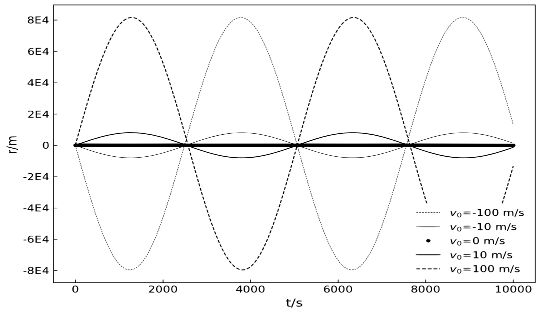

Fig. 2: Evolution trend of spacecraft position at different initial velocities

where $Q_0 = \sqrt{mr_0}, n \in \mathbb{Z}$.

And the larger the n, the stronger the penalty when the spacecraft deviates from the balance point. With a fix initial velocity $v_0 = -100m/s$, the spacecraft position varies over coordinate time under control law is given by numerical calculation, as shown in fig.3. It implies that the motion amplitude decreases significantly as the parameter n increase. In general, the initial velocity is much less than 100m/s, the spacecraft motion under given control law can maintains a good balance at the specified point.

2.2 Local curvature flattening in special space-time. A general static spherically symmetric Weyl solution has a line element:

$$ds^2 = -Adt^2 + 1/Adr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

where A is the function of r with axisymmetric coordinate $\{t, r, \theta, \phi\}$. And all the nonzero components of Riemann curvature tensor are as follows:

$$\begin{aligned} &R^t_{rtr} = -R^t_{rrt} = -\frac{A'}{2A}, R^r_{ttr} = -R^r_{trt} = -\frac{AA''}{2} \\ &R^t_{\theta t\theta} = -R^t_{\theta\theta t} = R^r_{\theta r\theta} = -R^r_{\theta\theta r} = -\frac{rA'}{2} \\ &R^t_{\phi t\phi} = -R^t_{\phi\phi t} = R^r_{\phi r\phi} = -R^r_{\phi\phi r} = -\frac{A'r\sin^2\theta}{2} \\ &R^\theta_{tt\theta} = -R^\theta_{t\theta t} = R^\phi_{tt\phi} = -R^\phi_{t\phi t} = -\frac{AA'}{2r} \\ &R^\theta_{rr\theta} = -R^\theta_{r\theta r} = R^\phi_{rr\phi} = -R^\phi_{r\phi r} = \frac{A'}{2rA} \\ &R^\theta_{\phi\theta\phi} = -R^\theta_{\phi\phi\theta} = (1-A)\sin^2\theta \\ &R^\phi_{\theta\theta\phi} = -R^\phi_{\theta\phi\theta} = A - 1 \end{aligned}$$

It is clear that if there exists r^* satisfying following equations:

$$\begin{cases} A(r)|_{r=r^*} & = 1 \\ A'(r)|_{r=r^*} & = 0 \\ A''(r)|_{r=r^*} & = 0 \end{cases}$$

then all components of Riemann curvature vanish, which means that the spacetime curvature disappears at this

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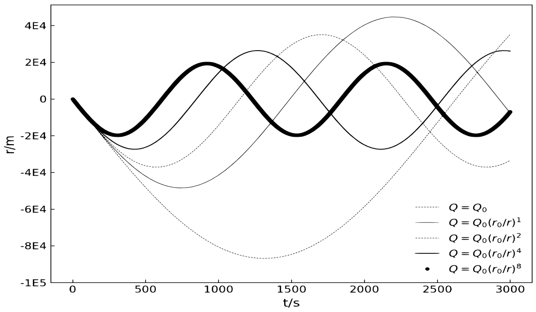

Fig. 3: Evolution trend of spacecraft position under different charge control law

point, and the local space-time becomes Minkowski spacetime. In the line element forms given by papers [13–15] based on the Yukawa effect, $A(r)$ has more than three constant parameters which makes it possible to find a non-singular r^*. At that point, Riemann curvature tensor vanish and the local spacetime has no gravitational field anymore.

In summary, there exists a specific space-time point in the special background spacetime, where free particles get rid of the gravitational effect or spacetime degenerates into Minkowski type. Therefore, it is possible to use electromagnetic field to curve spacetime and eliminate local pre-existing gravitational field.

3 Artificial anti-gravity in Schwarzschild spacetime. – An ordinary planet like the Earth, usually has spherically symmetric body without electric and magnetic charge. Its outer spacetime can be approximately described by Schwarzschild spacetime. In order to realize the anti-gravity effect in this background, it is necessary to generate an energy field which has a more concentrated distribution form than the spherically symmetric R-N field especially in some specific direction.

3.1 Field generated by current loop. Neutron stars in the universe have extremely strong magnetic moments, the value of which is about $p \sim 10^{30} \mathrm{G} \cdot \mathrm{cm}^3$ [16]. Its magnetic field is axisymmetric, and the electromagnetic energy is more concentrated near the polar axis than the spherically symmetric electrostatic field.

Because magnetic field distribution of current loop is similar to that of Neutron star, so we select current loop or solenoid as the artificial gravitation source equipping the spacecraft. Then we will seek to obtain exact metric solution of 4-dimension spacetime manifold induced by current loop, some basic assumptions are as follows:

1. The initial current is near to zero.
2. The initial background is Minkowski spacetime.

Select the cylindrical coordinate$\{t, z, \rho, \phi\}$ According to Maxwell equations, components of electromagnetic field are as follows:$E_z = E_\rho = E_\phi = 0, B_\phi = 0$. Nonzero 4-potential vector component: $A_3 = A_\phi$.

Now we turn up the current slowly and smoothly, and assume that the current is increased by infinitesimal for each step. The manifold evolves from the Minkowski space-time to axisymmetric one. We define the 4-manifold $\mathcal{F}(\mathcal{M}, g_{ab})$ and metric tensor g_{ab} induced by the current loop where the metric satisfies the gauge condition $\nabla_c g_{ab} = 0$. When the current stabilized at a certain level, the curved space-time formed by it is axisymmetric (there is a space-like Killing vector) and static [11]. And the line element can be written in Weyl form [17]:

$$ds^2 = -e^{2\alpha} dt^2 + e^{2(\beta-\alpha)} dz^2 + e^{2(\beta-\alpha)} d\rho^2 + \gamma^2 e^{-2\alpha} d\varphi^2$$

where $\alpha(\rho, z), \beta(\rho, z), \gamma(\rho, z)$ are functions of ρ, z. And the Weyl canonical coordinates are noted as $\{t, z, \rho, \varphi\}$. We use in this paper a system of units in which G=c=1, metric signature$(-, +, +, +)$, a spherical coordinate system $\{t, r, \theta, \varphi\}$. We also employ an abstract index notation discussed by Penrose, and Latin indices like $\{a, b, c, d\}$ represent part of notation for tensor itself. Any equation employing Greek indices like $\{\alpha, \beta, \gamma, \mu, \nu\}$is a relation between tensor component and specially chosen basis. Weyl canonical coordinates $\{t, \varphi\}$ are generated by time-like Killing vectorξ^a, space-like Killing vector ϕ^a:

$$\xi^a = (\partial/\partial t)^a$$
$$\phi^a = (\partial/\partial\varphi)^a$$

Let $\mathcal{K} = \{\mathcal{F}_i |\ i \ \in R\}$ be the homeomorphic set generated during manifold evolution, meanwhile $\mathcal{S} = \{\Omega_i | i \in \mathbf{R}\}$ be the space-like hypersurface set induced by orthonormal time-like Killing vector, studies [18, 19] imply that initial constraints is valid to the system throughout the whole evolution, the same is to tensor fields. So we still have $A = A_3(\rho, z)$.

3.2 Solution of Maxwell Equations. Source-less Maxwell equations in covariant form:

$$\nabla^a F_{ab} = 0 \tag{1}$$
$$\nabla_{[a} F_{bc]} = 0 \tag{2}$$

According to study [11] equation (2) is satisfied already with Weyls metric form. Eq.1 can be simplified as:

$$g^{\mu\nu} \nabla_\nu \left(A_{3,\mu} - \Gamma^3_{3\mu} A_3 \right) = g^{33} R_{33} A_3$$

Simplified:$\left(A_{3,1} e^{2\alpha}/\gamma \right)_{,1} + \left(A_{3,2} e^{2\alpha}/\gamma \right)_{,2} = 0$

3.3 Solutions of Einstein-Maxwell Equations. Corresponding components of field equation:

$$e^{2(2\alpha-\beta)}\left(\alpha,_{zz}+\alpha,_{\rho\rho}+\gamma^{-1}\left(\alpha,_{z}\gamma,_{z}+\alpha,_{\rho}\gamma,_{\rho}\right)\right) = \frac{e^{6\alpha-2\beta}}{\gamma^2}\left(F_{13}^2+F_{23}^2\right) \tag{3}$$

$$\alpha,_{zz}+\alpha,_{\rho\rho}-\beta,_{zz}-\beta,_{\rho\rho}-2\alpha,_{z}^2-\gamma^{-1}\gamma,_{zz}+ \gamma^{-1}\left(\alpha,_{z}\gamma,_{z}+\alpha,_{\rho}\gamma,_{\rho}\right)+\gamma^{-1}\left(\beta,_{z}\gamma,_{z}-\beta,_{\rho}\gamma,_{\rho}\right) = \frac{e^{2\alpha}}{\gamma^2}\left(F_{13}^2-F_{23}^2\right) \tag{4}$$

$$-2\alpha,_{z}\alpha,_{\rho}+\gamma^{-1}\left(\beta,_{z}\gamma,_{\rho}+\beta,_{\rho}\gamma,_{z}\right)-\gamma^{-1}\gamma,_{z\rho} = \frac{2e^{2\alpha}}{\gamma^2}F_{13}F_{23} \tag{5}$$

$$\alpha,_{zz}+\alpha,_{\rho\rho}-\beta,_{zz}-\beta,_{\rho\rho}-2\alpha,_{\rho}^2-\gamma^{-1}\gamma,_{\rho\rho}+ \gamma^{-1}\left(\alpha,_{z}\gamma,_{z}+\alpha,_{\rho}\gamma,_{\rho}\right)-\gamma^{-1}\left(\beta,_{z}\gamma,_{z}-\beta,_{\rho}\gamma,_{\rho}\right) = \frac{e^{2\alpha}}{\gamma^2}\left(F_{23}^2-F_{13}^2\right) \tag{6}$$

$$\gamma^2e^{-2\beta}\left(\alpha,_{zz}+\alpha,_{\rho\rho}+\gamma^{-1}\left(\alpha,_{z}\gamma,_{z}+\alpha,_{\rho}\gamma,_{\rho}-\gamma,_{zz}-\gamma,_{\rho\rho}\right)\right) = e^{2\alpha-2\beta}\left(F_{13}^2+F_{23}^2\right) \tag{7}$$

(3) / (7)

$$\gamma,_{zz}+\gamma,_{\rho\rho}=0 \tag{8}$$

γ is harmonic function of (ρ, z), so its reasonable to assume that $\gamma=\rho$ [19]. (4) + (6) and substitute γ with ρ:

$$\alpha,_{zz}+\alpha,_{\rho\rho}-\beta,_{zz}-\beta,_{\rho\rho}-\alpha,_{z}^2-\alpha,_{\rho}^2+\rho^{-1}\alpha,_{\rho}=0 \tag{9}$$

Use eq.9 to simplify (3)-(5):

$$\alpha,_{zz}+\alpha,_{\rho\rho}+\rho^{-1}\alpha,_{\rho}=\frac{e^{2\alpha}}{\rho^2}\left(F_{13}^2+F_{23}^2\right) \tag{10}$$

$$\alpha,_{\rho}^2-\alpha,_{z}^2-\rho^{-1}\beta,_{\rho}=\frac{e^{2\alpha}}{\rho^2}\left(F_{13}^2-F_{23}^2\right) \tag{11}$$

$$-2\alpha,_{z}\alpha,_{\rho}+\rho^{-1}\beta,_{z}=\frac{2e^{2\alpha}}{\rho^2}F_{13}F_{23} \tag{12}$$

According to equations (11) (12), take substitutions:

$$\begin{cases} F_{13}=-\dfrac{\rho}{e^{\alpha}}v,_{\rho} \\ F_{23}=\dfrac{\rho}{e^{\alpha}}v,_{z} \end{cases}$$

into Maxwell Equations and use eq.9 to eliminate variable β in equations (10)-(12) (To be convenient commas are omitted in following subscripts):

$$\alpha_z v_\rho = v_z\alpha_\rho \tag{13}$$

$$\alpha_{\rho\rho}+\alpha_{zz}+\rho^{-1}\alpha_\rho=v_\rho^2+v_z^2 \tag{14}$$

$$\rho\left(\alpha_\rho\left(\alpha_{\rho\rho}+\alpha_{zz}\right)-v_\rho\left(v_{\rho\rho}+v_{zz}\right)\right)=v_\rho^2-\alpha_\rho^2 \tag{15}$$

Use equations (13)-(14) to simplify eq.15:

$$\alpha_z v_\rho = v_z\alpha_\rho$$
$$\alpha_{\rho\rho}+\alpha_{zz}+\rho^{-1}\alpha_\rho=v_\rho^2+v_z^2$$
$$v_{\rho\rho}+v_{zz}+\rho^{-1}v_\rho=v_\rho\alpha_\rho+v_z\alpha_z$$

Two class of solutions can be obtained easily:

i. $v=0$

$$\nabla^2\alpha=0$$

We take the Laplaces solution in prolate spheroidal coordinate system [21] and have result as follows:

$$g_{00}=e^{2\alpha}=\frac{x-1}{x+1}, A_3=0$$

It is the Schwarzschild exterior solution.

ii. $\alpha=v$

$$\begin{cases} \nabla^2 e^{-\alpha}=0 \\ \beta=\text{const} \end{cases}$$

With aid of Ernst generation technique [22], make substitutions as follows:

$$\begin{cases} v'_\rho=e^{\alpha}v_\rho, v'_z=e^{\alpha}v_z \\ \epsilon_1=e^{\alpha}+v', \epsilon_2=e^{\alpha}-v' \end{cases}$$

Then field equations have the same form as Ernst equations:

$$\left(\epsilon_1+\epsilon_2\right)\nabla^2\left(\epsilon_1\right)=2\left(\nabla\epsilon_1\right)^2$$
$$\left(\epsilon_1+\epsilon_2\right)\nabla^2\left(\epsilon_2\right)=2\left(\nabla\epsilon_2\right)^2$$
$$\left(\epsilon_1+\epsilon_2\right)^2\beta_\rho=4\rho\left(\epsilon_{1,\rho}\epsilon_{2,\rho}-\epsilon_{1,z}\epsilon_{2,z}\right)$$
$$\left(\epsilon_1+\epsilon_2\right)^2\beta_z=4\rho\left(\epsilon_{1,\rho}\epsilon_{2,z}+\epsilon_{1,z}\epsilon_{2,\rho}\right)$$

Adopting Ehlers transformation [23] and get solution set:

$$\epsilon_1=\frac{a+b\epsilon_0^1}{c+d\epsilon_0^1}, \epsilon_2=\frac{-a+b\epsilon_0^2}{c-d\epsilon_0^2}$$

where a,b,c,d are arbitrary constants. Taking solution (2) as a seed, that is:

$$\epsilon_0^1=\frac{2}{\phi}, \epsilon_0^2=0, \nabla^2\phi=0$$

New solutions derived as follows:

$$\begin{cases} g_{00}=-\dfrac{(bc-ad)^2}{\left(\phi c^2-2cd\right)^2} \\ v'=\dfrac{\phi ac+(bc+ad)}{\phi c^2+2cd} \\ \beta=\text{const} \end{cases}$$

To get the solution matching current loop boundary conditions, we take into account the magnetic flux density observed by the ZAMO:

$$B^2 = \frac{1}{2} F_{ab} F^{ab}$$

Spatial 3-magnetic components described by the orthonormal tetrad [20]:

$$\begin{aligned} \hat{B}^\rho &= \frac{1}{2}\epsilon_{2\mu\nu}\sqrt{g^{\mu\mu}g^{\nu\nu}}F_{\mu\nu} \\ &= \sqrt{g^{11}g^{33}}F_{13} \\ &= e^{-\beta}v'_\rho \\ \hat{B}^z &= e^{-\beta}v'_z \end{aligned}$$

Boundary conditions derived from current loop field in Minkowski spacetime:

$$\hat{B}^\rho(z) = -\hat{B}^\rho(-z), \hat{B}^z(z) = \hat{B}^z(-z)$$

Let $\beta = 0$:

$$\phi_\rho(z) = -\phi_\rho(-z), \phi_z(z) = \phi_z(-z)$$

Magnetic flux densitys module:

$$|B| = \frac{(bc - ad)}{(\phi\ c + 2d)^2}||\nabla\phi||$$

It is convenient to transform coordinates to Weyl spherical coordinate system:

$$\begin{cases} r = \sqrt{\rho^2 + z^2} \\ \theta = \arctan\left(\frac{\rho}{z}\right) \end{cases}$$

Corresponding line element:

$$ds^2 = -e^{2\alpha}dt^2 + e^{2(\beta-\alpha)}\left(dr^2 + r^2 d\theta^2 + r^2\sin^2\theta d\varphi^2\right)$$

Laplace solution in spherical coordinates:

$$\phi = \sum_{m=0}^{\infty}\left(A_m r^m + B_m r^{-m-1}\right)P_m(\cos\theta)$$

where P_m represents Legendre's polynomials, m is a positive integer. Components in different coordinate systems have transformation as follows(left side represents axisymmetric one, while right side represents spherical one):

$$\begin{cases} \phi_\rho = \phi_r\sin\theta + \phi_\theta\frac{\cos\theta}{r} \\ \phi_z = \phi_r\cos\theta - \phi_\theta\frac{\sin\theta}{r} \end{cases}$$

Together with boundary conditions in axisymmetric coordinates, solution has following general form:

$$\phi = A_0 + \sum_{m=2k-1}^{\infty} B_m r^{-m-1}P_m(\cos\theta), k \in \mathbf{Z}^+$$

To solve constant parameters, lets look back to Minkowski spacetime and magnetic field near to current loop polar axis has flux density as following form [24]:

$$|B| = \frac{\mu_0 I R^2}{2\left(R^2 + r^2\right)^{3/2}} \sim \mathrm{O}\left(r^{-3}\right)$$

By appending following approximation as conditions:

$$\begin{cases} g_{00} \approx 1 \\ v' \approx \mathrm{O}\left(r^{-2}\right) \\ |B| \approx \mathrm{O}\left(r^{-3}\right) \end{cases}$$

we can obtain ϕ and further g_{00} as follows:

$$\begin{cases} a = -b = -c,\ d = 0,\ e = 1 \\ \phi = 1 + \frac{\hat{\mu}_0 I R^2}{4r^2}\cos\theta, \hat{\mu}_0 = \frac{\sqrt{\mu_0 G}}{c^2} \\ g_{00} = -1 + \frac{2\hat{\mu}_0 I R^2\cos\theta}{4r^2 + \hat{\mu}_0 I R^2\cos\theta} \\ \qquad - \frac{\hat{\mu}_0^2 I^2 R^4\cos^2\theta}{\left(4r^2 + \hat{\mu}_0 I R^2\cos\theta\right)^2} \end{cases}$$

The spacetime degenerates to Minkowski form as radius goes to infinity.

3.4 Current loop in Schwarzschild background. Given that the current loop is placed around the planet exterior surface, where spacetime can be described in form of Schwarzschild metric in spherical coordinates $\{t, r', \theta, \phi\}$:

$$ds^2 = -\left(1 - \frac{2M}{r'}\right)dt^2 + \left(1 - \frac{2M}{r'}\right)^{-1}dr'^2 + r'^2 d\theta^2 + r'^2\sin^2\theta d\phi^2$$

where r' describes the radius and its origin point is located in the center of planet. The corresponding metric components:

$$g'_{00} = -1/g'_{11} = -\left(1 - \frac{2M}{r'}\right)$$

Assuming that the current loop is placed in where $r' = R_0$ and its polar axis coincides with coordinate axis $\theta = 0$:

$$\begin{cases} r' = r + R_0 \\ \sin\theta = 0 \\ \mathrm{d}\theta = 0 \end{cases}$$

The metric of current loop background spacetime can be degenerated locally as:

$$ds^2 = -\left(1 - \frac{2M}{r + R_0}\right)dt^2 + \left(1 - \frac{2M}{r + R_0}\right)^{-1}dr^2$$

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It is clear that symmetry constraints in previous section are not valid anymore affected by background field except for $g_{00} \approx g'_{00}$ at infinity. Omitting high order terms, the and the Ernst complex potential solution can be rewritten as:

$$\phi = e + \frac{h}{r} + \frac{f\cos\theta}{r^2}$$

$$g_{00} = -\frac{(bc-ad)^2}{(ec^2 - 2cd + \frac{hc^2}{r} + \frac{fc^2\cos\theta}{r^2})^2}$$

$$\therefore bc - ad = ec^2 - 2cd$$

Let $h' = \frac{hc^2}{bc-ad}$, $f' = \frac{fc^2}{bc-ad}$:

$$g_{00} = -\frac{r^4}{(r^2 + h'\, r + f'\cos\theta)^2}$$

Referring to studies in [25–27], the transformation from Weyl axisymmetric coordinate system through prolate spheroidal one to spherical one can be written as:

$$\begin{cases} r = \sqrt{r'^2 - 2mr' + m^2\cos^2\theta'} \\ \cos\theta = \dfrac{(r'-m)\cos\theta'}{\sqrt{r'^2 - 2mr' + m^2\cos^2\theta'}} \end{cases}$$

It is clear that $\theta = 0$ around current loop also means $\theta' = 0$:

$$-g_{00} = 1 - \frac{2h'}{r' - m + h' + O\ (r'^{-1})} + \frac{h'^2 - 2f'}{(r' - m + h' + O\ (r'^{-1}))^2} - \frac{f'^2}{((r'-m)^2 + h'(r-m) + f')^2}$$

With high order terms omitted, constant parameters are solved as:

$$\begin{cases} h' = M \\ m = M - R_0 \end{cases}$$

By locally applying following coordinates approximation:

$$\begin{cases} \theta \approx \theta' = 0 \\ r \approx r' - M + R_0 \end{cases}$$

We finally obtain the line element describing the composite gravitational field:

$$ds^2 = -g_{00}dt^2 + 1/g_{00}dr'^2 + r'^2 d\theta'^2 + r'^2\sin^2\theta' d\phi^2$$

where:

$$-g_{00} = 1 - \frac{2M}{r' + R_0 + \frac{f'}{r' - M + R_0}} + \frac{(M^2 - 2f')(r' - M + R_0)^2 - f'^2}{((r' - M + R_0)^2 + M(r' - M + R_0) + f')^2}$$

To solve f', let $M = 0$ and compare magnetic components in different coordinates with notice to the fact that $r' - M + R_0 \gg r'$. So f' is approximate to:

$$f' \approx ||R_0||^2 f = \frac{\hat{\mu}_0 I R^2 ||R_0||^2}{4}$$

When $M \neq 0$ and $f' \to 0$, curved spacetime degenerates to Schwarzschild type.

3.5 Required Source Strength for Anti-gravity. Free particles geodesic around current loop:

$$\frac{d^2x^\mu}{d\tau^2} + \Gamma^\mu_{\nu\sigma}\frac{dx^\nu}{d\tau}\frac{dx^\sigma}{d\tau} = 0$$

For motion much slower than the speed of light, the proper time may be approximated by the coordinate time as:

$$\frac{d^2x^\mu}{dt^2} = -\Gamma^\mu_{00}$$

where $\Gamma^1_{00} \approx -\frac{1}{2}g_{00,1}, \Gamma^i_{00} = 0, i \in (2,3)$. Let $\frac{d^2x^\mu}{dt^2} = 0$, we obtain source strength I where free particle is located at radius $r = \delta$:

$$I = \frac{2\mathrm{GM_e}\left(\delta + \mathrm{R_e} - \frac{\mathrm{GM_e}}{c^2}\right)}{\sqrt{\mu_0}\mathrm{G}R^2\,||\mathrm{R_e}||^2}$$

where $\mu_0 = 1.257\times10^{-6}\mathrm{N\cdot A^{-2}}, \mathrm{G} = 6.67\times10^{-11}\mathrm{N\cdot m^2/kg^2}, \mathrm{c} = 3\times10^8\mathrm{m}/s, \mathrm{M_e} = 5.965\times10^{24}\mathrm{kg}, \mathrm{R_e} = 6.371\times10^6\mathrm{m}$. Let loop radius R and particle position radius δ be unit length, then the result of source strength is:

$$I = 1.364\times10^{16}\mathrm{A}$$

However its still unfeasible to implement such a strong current even if a superconductor limited to certain critical current density is used to carry that. So following work will propose a feasible implementation based on superconductor.

4 Implementation of Artificial Anti-gravity. –

4.1 Static magnetic field modulation. Lets reconsider Laplace solution form in section 3.3:

$$\phi = 1 + \frac{M}{r} + f\cdot g(r^n)$$

The difference is the third term which represents limitations to some specific boundary conditions. And if we substitute it into geodesic function, the corresponding current can be solved as:

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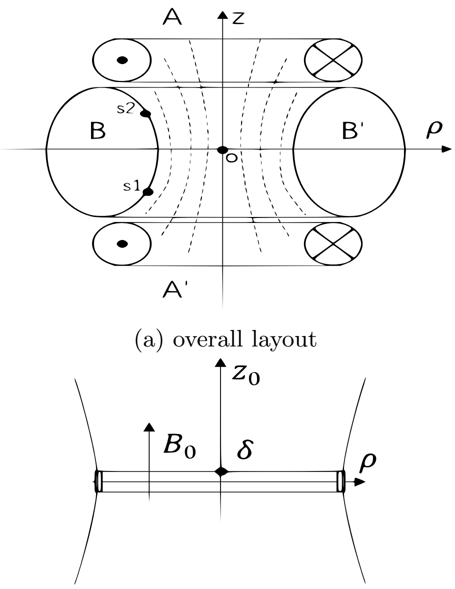
$$I_c = \frac{2\mathrm{GM_e}}{\sqrt{\mu_0}\mathrm{G}R^2 \left\|\mathrm{R_e}\right\|^2 g'(r^n)} \frac{1}{\left(\delta + \mathrm{R_e} - \frac{\mathrm{GM_e}}{\mathrm{c}^2}\right)^{n+1}}$$

When $n = -2, I_c = I_0 = 1.364 \times 10^{16}$A. When $n = -1, I_c = 2.141 \times 10^9$A. Corresponding complex potential vector is:

$$v' = \frac{1-\phi}{\phi} = -\frac{\frac{M}{r} + f \cdot g\left(r^{-1}\right)}{1 + \frac{M}{r} + f \cdot g\left(r^{-1}\right)}$$
$$\approx -\left(\frac{M}{r} + f \cdot g\left(r^{-1}\right)\right) = \mathrm{O}\left(r^{-1}\right)$$

Therefore, it can be concluded that if the magnetic potential vector field can be modulated to vary more slowly with radius, the required current strength can be significantly reduced.

Taking into account the Meissner effect of superconductors [28], an implement is shown in fig.5. A-A' represents Helmholtz coil consist of current loops. Origin point of $\rho - z$ coordinate system coincides with the center of Helmholtz coil, where magnetic field is homogeneous approximately. Magnetic field near to origin point is surrounded by superconductor torus B-B' which takes ellipse as shape of poloidal section.



(a) overall layout

(b) local slice of superconductor

Fig. 4: Helmholtz coil under shield of superconductor

Qualitatively,according to the Meissner effect, total magnetic flux in the central region surrounded by A-A' and B-B' does not change over the z-coordinate:

$$\frac{d\Phi_0}{dz} = 0$$

Assuming that the curve shape of s1-s2 is described by: $\rho = f(z)$ The average magnetic flux density over z is:

$$B(z) = \frac{\Phi_0}{\pi f^2(z)}$$

It is clear that $v' \sim \mathrm{O}\left(r^{-1}\right)$ as $f(z) \sim \mathrm{O}\left(z^{-1}\right)$. Quantitatively, considering the magnetic flux density of the Helmholtz coil in the $\rho - z$ coordinate system:

$$B_0 = \frac{0.8^{1.5}\mu_0 n I}{R}$$

Take a slice of B-B at position $z = 0$ to study as shown in fig.5-b. Inside the body of superconductor B-B, according to London equations and Maxwell equations we can obtain the compound magnetic flux density at where $\rho = 0,\ z = \delta \to 0$ assumed that boundary field maintains continuity [29]:

$$B_{z=\delta} = B_0 - \frac{2\pi B_0 f^3(z)}{\left(f^2(z) + z^2\right)^{3/2}} \left(1 - \frac{3\lambda}{f(z)} \coth\frac{f(z)}{\lambda} + \frac{3\lambda^2}{f^2(z)}\right)$$

Because $f(z) \gg \lambda$:

$$B_{z=\delta} \approx B_0 \left(1 - \frac{2\pi f^3(z)}{\left(f^2(z) + z^2\right)^{3/2}}\right)$$

If given $f(z) = z^{2/3}$, then $B_{z=\delta} = \mathrm{O}\left(z^{-1}\right)$. So the magnetic field can be modulated by adjusting the shape of the curve. Therefore the current intensity required to counteract the background gravity is significantly reduced. In fact, if the central region of Helmholtz coil is filled with dielectric but magnetically conductive material and its relative permeability is about$10^4 \sim 10^5$, then current can be reduced to $2.141 \times (10^6 \sim 10^7)$A. According to study in [30], the single Toroidal Field Coil of CFETR(nuclear fusion tokamak device) can bear about 1.47×10^7A. Therefore the proposed method is feasible using superconductor.

5 Conclusion. – This paper discussed the shortage of normal electromagnetic propulsion and proposed a high-power propellant-free propulsion which interacts on background space-time curvature locally. To achieve an effective propulsion, a prototype consist of superconductor Helmholtz coil and superconductor shield is proposed. Computation demonstrated the feasibility of effective interaction on the gravitational field. Detail control techniques need to be processed in further work.

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