
Control System Numerical Simulation and Modelling Under Typical Manoeuvres of Autonomous Unmanned Underwater Vehicle for Planar Motion

[Stanislav Gurylenko](#)* and Nina Ilchenko

Posted Date: 3 April 2026

doi: 10.20944/preprints202604.0160.v1

Keywords: autonomous underwater vehicle (AUV); autonomous unmanned underwater vehicle (AUUV); control system; planar motion; PID controller; numerical simulation; modelling; AUUV maneuverers; AUUV planar motion; AUUV circular motion; AUUV zig-zag motion



Preprints.org is a free multidisciplinary platform providing preprint service that is dedicated to making early versions of research outputs permanently available and citable. Preprints posted at Preprints.org appear in Web of Science, Crossref, Google Scholar, Scilit, Europe PMC.

Copyright: This open access article is published under a [Creative Commons CC BY 4.0 license](#), which permit the free download, distribution, and reuse, provided that the author and preprint are cited in any reuse.

Disclaimer/Publisher's Note: The statements, opinions, and data contained in all publications are solely those of the individual author(s) and contributor(s) and not of MDPI and/or the editor(s). MDPI and/or the editor(s) disclaim responsibility for any injury to people or property resulting from any ideas, methods, instructions, or products referred to in the content.

Article

Control System Numerical Simulation and Modelling Under Typical Manoeuvres of Autonomous Unmanned Underwater Vehicle for Planar Motion

Stanislav Gurynenko ^{1,*} and Nina Ilchenko ²

¹ Department of Computer-Integrated Optical and Navigation Systems, National Technical University of Ukraine "Igor Sikorskyi Kyiv Polytechnic Institute", Kyiv, Ukraine

² Independent researcher, Scientific and research institute, Kyiv, Ukraine

* Correspondence: stas_gurynenko@ukr.net

Abstract

This article is representing numerical research and modeling of autonomous unmanned underwater vehicle's (AUUV) control system for planar motion. A mathematical model of the control system is designed for an underwater vehicle, the structure of which consists of a main thruster and control surfaces. Based on the dynamic AUUV's mathematical model two types of planar motion equations have developed: simple planar motion equations and extended planar motion equations. Both equations types include AUUV's geometrical characteristics and hydrodynamics coefficients which have determined from computer aided design and CFD simulation. The difference between simplified and extended equations of planar motion consists in the inclusion of an additional planar coordinate and extra hydrodynamic coefficients. For each type of motion equations as input signal typical maneuverers, such as constant value, sin value and Kempf maneuverer (zig-zag maneuverer) have implemented. The system's output signals from external actions in the form of typical maneuverers the estimated of necessity of regulator. As regulator were chosen and used the PID-regulator for velocity and yaw control. The results of this study also demonstrate what the engine thrust required to achieve the desired speed, and identify which equations of motion are appropriate for specific maneuvers performed by the apparatus during its operation and mission execution.

Keywords: autonomous underwater vehicle (AUV); autonomous unmanned underwater vehicle (AUUV); control system; planar motion; PID controller; numerical simulation; modelling; AUUV maneuverers; AUUV planar motion; AUUV circular motion; AUUV zig-zag motion

1. Introduction

The autonomous unmanned underwater vehicle's (AUUV) planar motion is a basic motion which can executed as on the water surface and underwater. The stability and precision of motion depend on AUUV's actuating mechanism system. Exist several types how actuating mechanism (control) system could be realized in AUUV. It could be main thruster and pair of servo thrusters (side and vertical locations) or it could be main thruster and fins and rudders. In most cases widely used control system which contain main thruster with fins and rudders (Bouraou et al., 2017). The crucial feature of the AUUV with fins and rudders is the ability of unmanned underwater vehicle's rudders to provide stable and relatively precise control of the vehicle.

The stable and precise AUUV's control depends of vehicle's design and the response quickness of the vehicle to the commanded rudder movement.

In order to develop control system mainly development process should be presented, which describes vehicles valuable parameters and characteristics which should be achieved. The beginning

of the development and research is the determination of the type of structure of the apparatus. Taking into account the design features, the calculation of hydrodynamic coefficients is carried out. The obtained hydrodynamic coefficients are used in modelling the equations of the dynamics of the vehicle's motion in the aquatic environment (Bouraou & Gurylenko, 2023).

The hydrodynamic and vehicle's design researches by the authors (Bouraou et al., 2021; Gurylenko 2023; Gurylenko et al., 2023) show how hull and apparatus in general behave in water under typical movements and maneuverers. The result of these studies is an understanding of the layout of the AUUV and the numerical value of the hydrodynamic coefficients for motion equations.

The design of control system in general is solution of system of six differential motion equations. This system of motion equations should contain as all as possible parameters which describe apparatus construction and also should include forces and momentums which generated by thrusters or controlling planes. The final system in general is non-linear system. To get system's solution mostly computer and numerical simulation are used. However, to obtain a complete solution of the system requires significant computational time. To reduce computational time, equations that are responsible for a specific simple motion are isolated from the complete system of equations. Commonly, for AUUV simple movements are planar movement and movement of diving/surfacing.

In order to design and develop of any vehicle, especially of AUUV, it is necessary to have any procedures and regulation documents which governing the vehicle in its environment.

However, in regulation documents of autonomous unmanned underwater vehicle (AUUV) inputs are not often designed for dynamic modeling but for evaluating the performance of such a vehicle (ITTC Recommended Procedures and Guidelines, Rev. 02, 2017). The typical maneuverers such as turning circle, meander and "zigzag" are designed to evaluate the performance of AUUVs (Issac et al. 2008). Despite that these maneuverers could be used for AUUVs dynamic model parameters estimation.

The document (ITTC Recommended Procedures and Guidelines, Rev. 02, 2017) also applies to methodologies for conducting full-scale tests. Using the provisions of this document, it is advisable to test the apparatus, compare the obtained results with theoretical calculations, and, if necessary, make adjustments to the model or the apparatus itself.

In research (Perrault et al., 2003) by authors the response sensitivity of C-SCOUT underwater vehicle under turning circles and horizontal and vertical zigzag manoeuvres are examined. Authors used a computer model of an axi-symmetric underwater vehicle. Using numerical simulation, the influence of hydrodynamic parameters such as added mass, the lift and drag forces on constituent components, and the point of application of the lift and drag forces on apparatus was estimated.

In the pipeline (Sutulo & Soares, 2005) based on simplified nonlinear mathematical models authors are studied dynamic properties of directionally unstable surface displacement ships in manoeuvring motion. By means of numerical simulation based on the First-order and second-order Nomoto equations, which have written in normalized form to minimize the number of the defining parameters zigzag motion focusing on the case of directionally unstable ships were studied. For the open-loop system under sinusoidal excitation and for the closed-loop system imitating the ship in zigzag manoeuvres authors discovered anomalous responses were discovered when the rudder deflection amplitude is close to the loop's half-width.

Applied scientific researches presented in (Issac et al., 2007; Issac et al., 2008) provide us with a series of manoeuvring tests using the MUN Explorer AUV. Researchers are described the methods and procedures used in accomplishing the series of manoeuvres tasks, such as: 18 turning circles, 12 zigzags, 2 straight-line tests and a helix. Accomplishment of all manoeuvres were done in a slightly different manner compared with the classic maneuvers in order to make use of the special vehicle mission planning software (Issac et al., 2007; Issac et al., 2008). From the obtained data and their further analysis authors found out the zigzag manoeuvres, the vehicle has a good path-follow qualities. In the case of turning circles, the radius of turn in most cases turned out to be somewhat more than the demanded radius. Also authors note obtained data records shown vehicle's ability perform extreme manoeuvres with the accuracy which apparatus can follow a pre-defined path.

Based on the results of field tests and studies conducted in (Issac et al., 2007; Issac et al., 2008), the Nomoto indices (Azarsina & Williams, 2013) for the studied apparatus can be estimated. Using the obtained estimates of the Nomoto indices, a first-order model of the Nomoto equations is solved using computer modeling to predict the apparatus' rate of turn during horizontal zigzag maneuvers in response to a rectangular input signal for the rudder deflection angle.

The paper (Liu et al., 2015) examines a combined modeling approach and virtual prototype creation using ADAMS and Matlab/Simulink. The paper describes the design of the vehicle and its components. Detailed equations of motion and attitude control are provided. The design of the navigation system is briefly described. The resulting virtual prototype, consisting of the AUV control system and control algorithm, can be used for AUV simulation analysis and functional verification through the interaction of intelligent and dynamic control.

A research (Villa et al., 2020) describes the development and theoretical study of the Girona500 AUV with five thrusters. The authors describe the vehicle's operating algorithms and provide the motion equations taking into account the Girona500 AUV parameters. The authors consider two guidance, navigation, and control (GNC) system algorithms (simple and extended). The simple GNC algorithm considers three different types of PID controllers (speed, speed-position, and position). The extended GNC algorithm ensures trajectory following, as well as the collection and processing of data from an underwater sensor. The researchers consider the implementation problem in two control scenarios to validate the developed GNC architecture and verify the position PID controller model. The developed guidance, navigation, and control (GNC) system architecture was implemented as block diagrams and verified using simulation in MATLAB/SIMULINK.

The reviewed articles above are very useful in practical case. However, most of considered researches examine already made apparatus and describe approaches how to improve it according to the some regulations requirements. From the cited works only minor part describe development process from begging to the end which follow common information model of research and development of AUUV (Gurylenko & Bouraou, 2023).

The proposed research come a next step after preliminary design and numerical study of the hull of AUUV. This article discusses a model of the planar motion of an AUUV, the design of which was studied in the works (Bouraou et al., 2021; Gurylenko 2023; Gurylenko et al., 2023). In turn, planar movement is conditionally divided into simple movement and complex movement. Simple planar motion of AUUV includes surge motion and change in direction. Complex planar motion of AUUV additionally includes sway motion.

The results presented in this article are as follows:

- Two systems of equations of motion are shown: simple plane motion and complex plane motion;
- For each system of equations, as control system with open loop contour, responses to typical maneuverers are shown;
- Responses to typical maneuverers are shown for each system of equations, as control system with negative feedback (closed loop contour);
- Responses to typical maneuverers are shown for each system of equations with PID-regulator.

2. Methodology

Autonomous unmanned underwater vehicle planar motion is one of the simplest and, at the same time, one of the most fundamental motion. Analyzing the planar motion of an AUUV allows to evaluate the maneuverability of the AUUV, the minimum required thrust, and the response time of the corresponding control surfaces. Planar motion's mathematical model could be obtained from 6DoF dynamic equations in way of some assumptions around motion and restriction of some degrees of freedom. The implementation of some regulator block in planar motion equations convert this system into AUUV control system model. Varying of the regulator's coefficients allows us to select the most appropriate values of these coefficients, at which the output value of the motion parameters will correspond to the specified (desired) ones, and the settling time will be minimal.

3. Mathematical Model of Motion

The mathematical model of an AUUV can be follows the classic approach of movement dynamics. The space movement's equations are developed based on the forces and moments acting on the AUUV. In general, the obtained equations system is nonlinear. Also, for further simplicity of the description of input and output values, as well as for the implementation of different subsystems, the resulting system convert to the state-space form.

In present research, to describe the AUUV motion (Earth-fixed Inertial frame ($E-\xi\eta\zeta$) and body-fixed frame ($O-XYZ$) are introduced, as shown in Figure 1.

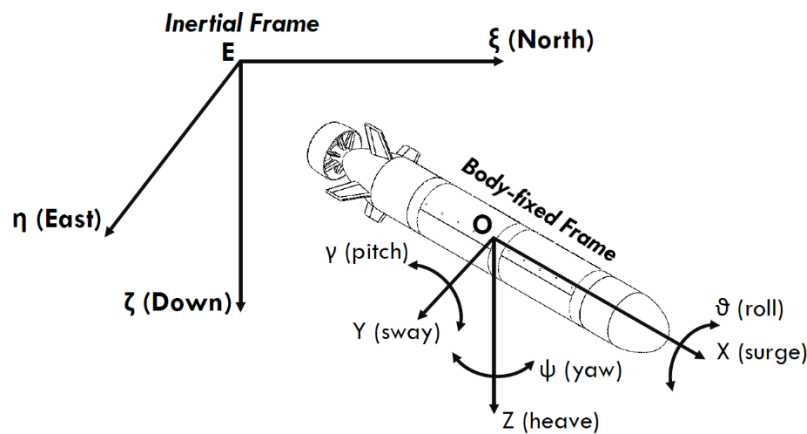


Figure 1. AUUV coordinate frame.

The origin of the body-fixed frame located in the center of mass of the AUUV. Apparatuses center of mass and the center of buoyancy which are coincides with the center of gravity of it. The AUUV OX axes is the forward-cruising direction (Lukomskii, & Chugunov, 1988). According to the second law of dynamic the AUUV motion equations in the body-fixed frame can be expressed as follows:

$$\left\{ \begin{array}{l}
 \text{Surge: } (M + \lambda_{11}) \frac{dV_X}{dt} + \lambda_{12} \frac{dV_Z}{dt} + (Mx_c + \lambda_{46}) \frac{d\omega_Y}{dt} = T - \frac{1}{2} C_X \rho S V_X^2 - \\
 P \sin \gamma - C_X^{\omega_Z} \rho S L V_X \omega_Y - (M + \lambda_{33}) V_Z \omega_Y + (M + \lambda_{22}) V_Y \omega_Z; \\
 \text{Sway: } (M + \lambda_{33}) \frac{dV_Y}{dt} - (Mx_c - \lambda_{35}) \frac{d\omega_Z}{dt} - (Mh - \lambda_{34}) \frac{d\omega_X}{dt} = -C_Y^\beta \rho S V_X V_Y - \\
 C_Y^{\delta_e} \rho S V_X^2 \delta_e - (M - C_Y^{\omega_Z} \rho S L) V_X \omega_Z - C_Y^{\omega_X} \rho S L V_X \omega_X + P \cos \gamma \sin \vartheta; \\
 \text{Heave: } (M + \lambda_{22}) \frac{dV_Z}{dt} + \lambda_{12} \frac{dV_X}{dt} + (Mx_c + \lambda_{46}) \frac{d\omega_Y}{dt} = (C_Z^0 + C_Z^{\delta_e} \delta_e) \rho S V_X^2 - \\
 C_Z^\alpha \rho S V_X V_Z - (M - C_Z^{\omega_Y} \rho S L) V_X \omega_Y - P \cos \gamma \cos \vartheta; \\
 \text{Roll: } (I_X + \lambda_{44}) \frac{d\omega_X}{dt} - (Mh - \lambda_{34}) \frac{dV_Y}{dt} - \lambda_{45} \frac{d\omega_Z}{dt} = m_X^\beta \rho S L V_X V_Y + m_X^{\alpha\beta} \rho S L V_Y V_Z - \\
 (m_X^{\delta_e} \delta_e + m_X^{\delta_e} \delta_e) \rho S L V_{AX}^2 - [m_X^{\omega_X} \omega_X - m_X^{\omega_Z} \frac{Mh}{\rho S L^2} \omega_Z] \rho S L^2 V_X - Gh \cos \gamma \sin \vartheta; \\
 \text{Pitch: } (I_Y + \lambda_{66}) \frac{d\omega_Y}{dt} + (Mx_c + \lambda_{25}) \frac{dV_Z}{dt} + (Mh - \lambda_{46}) \frac{dV_X}{dt} = m_Y^\alpha \rho S L V_Z V_X - \\
 [\frac{Mx_c}{\rho S L^2} + m_Y^{\omega_Y}] \rho S L^2 V_X \omega_Y - G(x_c \cos \gamma \cos \vartheta + h \sin \gamma); \\
 \text{Yaw: } (I_Z + \lambda_{55}) \frac{d\omega_Z}{dt} - (Mx_c - \lambda_{35}) \frac{dV_Y}{dt} + \lambda_{45} \frac{d\omega_X}{dt} = m_Z^\beta \rho S L V_X V_Y - m_Z^{\delta_e} \rho S L V_X^2 \delta_e - \\
 [(\frac{Mx_c}{\rho S L^2} + m_Z^{\omega_Z}) \omega_Z + (\frac{Mh}{\rho S L^2} m_Z^{\omega_X}) \omega_X] \rho S L^2 V_X - Gx_c \cos \gamma \sin \vartheta.
 \end{array} \right. \quad (1)$$

As shown in Equation (1), motion equations are nonlinear and interrelated. These equations contain many hydrodynamic coefficients, which can be defined by series of numerical simulation or series of nature experiments of stand-in AUUV model to confirm results getting by numerical studies. Also, as may be inferred from Equations (1) hydrodynamic coefficients is exert influence on AUUV controllability.

3.1. Simple Planar Motion Model

Due to the complexity of the Equations (1) the model is simplified in engineering research. Hence, to get model of planar motion equations some assumptions are made:

- Added mass are remove (equal to zero) or can be esteem as part of apparatuses mass;
- Roll and Pitch angular movements are equal to zero or such small that can be neglected;
- Apparatus does not undergo Sway and Heave motions.

Taking into account the above assumptions, Equations (1) take the following form:

$$\begin{cases} \text{Surge: } M \frac{dV_X}{dt} + \frac{1}{2} C_X \rho S V_X^2 = T; \\ \text{Yaw: } I_Z \frac{d\omega_Z}{dt} + m_Z^{\omega_Z} \omega_Z \rho S L^2 V_X = -m_Z^{\delta_6} \rho S L V_X^2 \delta_6. \end{cases} \quad (2)$$

AUUV movement model in inertial frame can be expressed as:

$$\begin{cases} \frac{d\psi}{dt} = \omega_Z; \\ V_\xi = V_X \cos(\psi); \\ V_\eta = V_X \sin(\psi). \end{cases} \quad (3)$$

3.2. Extended Planar Motion Model

The Equations (2) describe ideal AUUV motion. However, in real life cross-axes movement influence take place. Also, current stream should been taken into account. Regarding to these notice, Equations (1) take the following form:

$$\begin{cases} \text{Surge: } M \frac{dV_X}{dt} + \frac{1}{2} C_X \rho S V_X^2 = T - M V_Y \omega_Z; \\ \text{Sway: } M \frac{dV_Y}{dt} - (M x_c - \lambda_{35}) \frac{d\omega_Z}{dt} = -C_Y^\beta \rho S V_X V_Y - C_Y^{\delta_6} \rho S V_X^2 \delta_6 - (M - C_Y^{\omega_Z} \rho S L) V_X \omega_Z; \\ \text{Yaw: } I_Z \frac{d\omega_Z}{dt} - (M x_c - \lambda_{35}) \frac{dV_Y}{dt} = m_Z^\beta \rho S L V_X V_Y - m_Z^{\delta_6} \rho S L V_X^2 \delta_6 - \left(\frac{M x_c}{\rho S L^2} + m_Z^{\omega_Z} \right) \omega_Z \rho S L^2 V_X. \end{cases} \quad (4)$$

In this study, to simplify calculations, the Equations (4) do not contain current stream value. AUUV movement in inertial frame the same as shown in Equations (3).

4. Numerical Simulation

The AUUV control system dynamic model is created in Matlab/Simulink software (Лазарев, & Бондар, 2011). General model view is shown on Figure 2.

Control system basic elements are main block, AUUV parameters + environment parameters block (light-blue), control point adjuster block (cyan), two regulators: one for velocity value (green) and second for direction (yaw) value (magenta). Control system additional elements provide switch-on/switch-off of feedback and regulators. Control system main block can include or Equations (2) or Equations (4) and Equations (3). Control point adjuster block can generate typical maneuverers such as: forward motion with specified direction angle, turning circle, sine and “zigzag”. Initializing program allows to predefined maneuverers motion parameters such as: desired velocity, desired turn radius, frequency of steering rudder re-laying.

In this section the results of simple planar motion for direct solution, for solution with feedback and for solution with feedback and regulator are presented.

4.2. Extended Planar Motion Simulation

In this section the results of extended planar motion for direct solution, for solution with feedback and for solution with feedback and regulator are presented.

5. Results Discussion

Figures 3–6 depict direct solution of equations of simple planar motion. As can be seen from plots, has a certain stability in gaining thrust and achieving desired velocity of movement, however AUUV output direction angle and movement trajectory do not correspond to the desired.

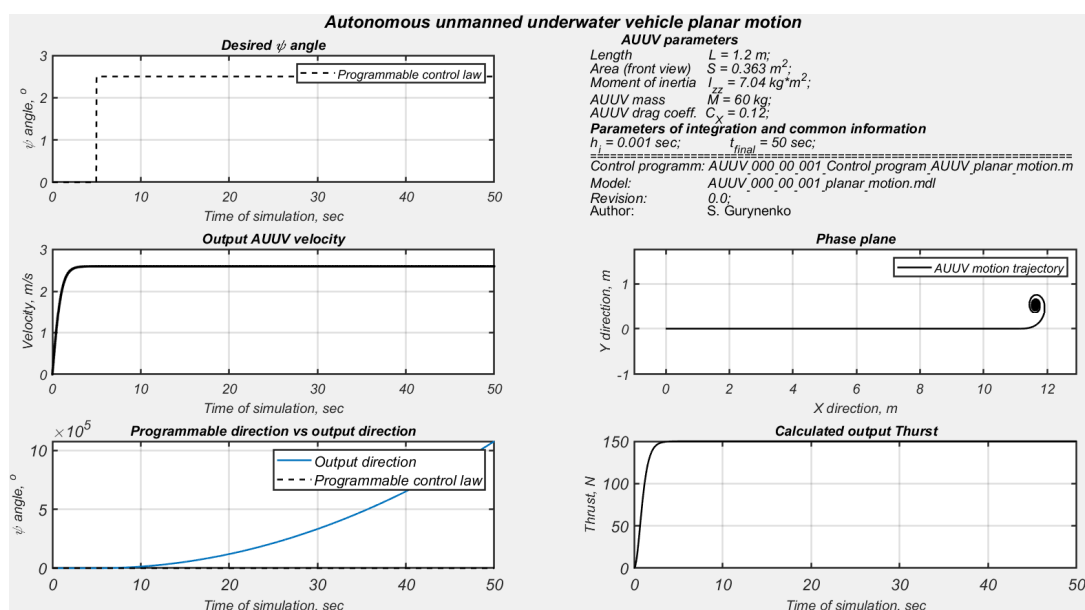


Figure 3. AUUV motion under direct solution (control input—constant direction 2.5°).

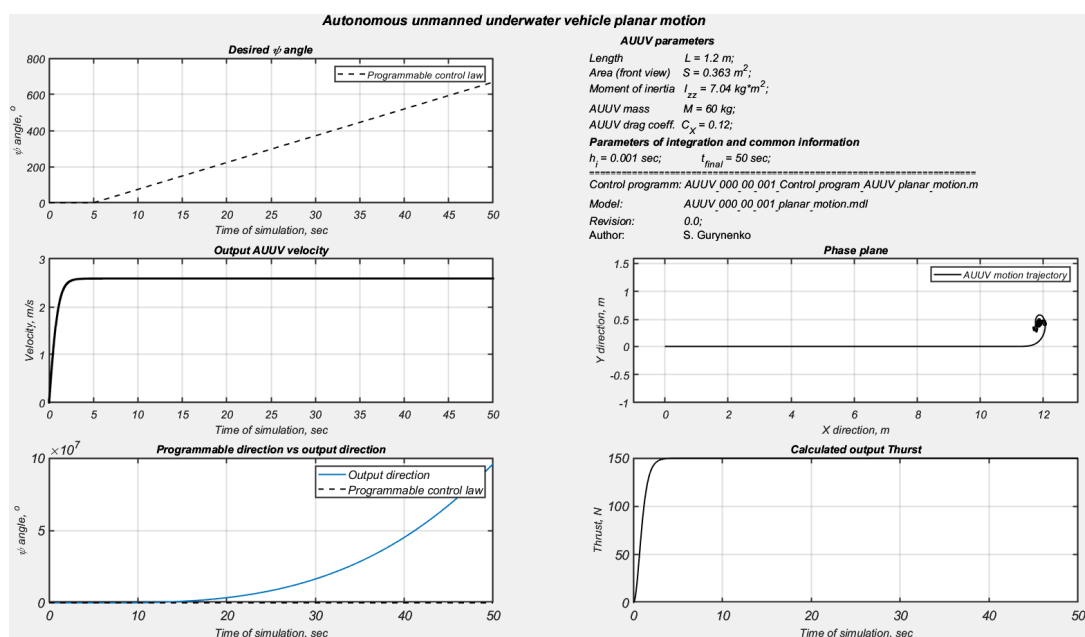


Figure 4. AUUV motion under direct solution (control input—ramp direction with desired turn radius 10 m).

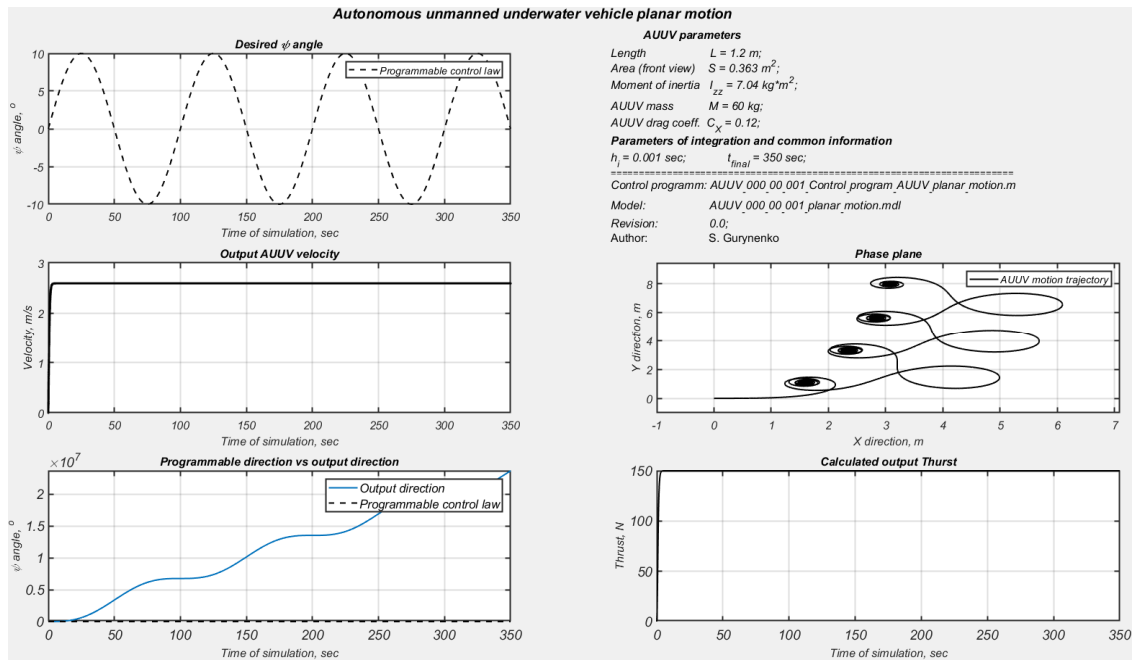


Figure 5. AUUV motion under direct solution (control input—sine direction with 0,01 Hz control-surface reversal frequency).

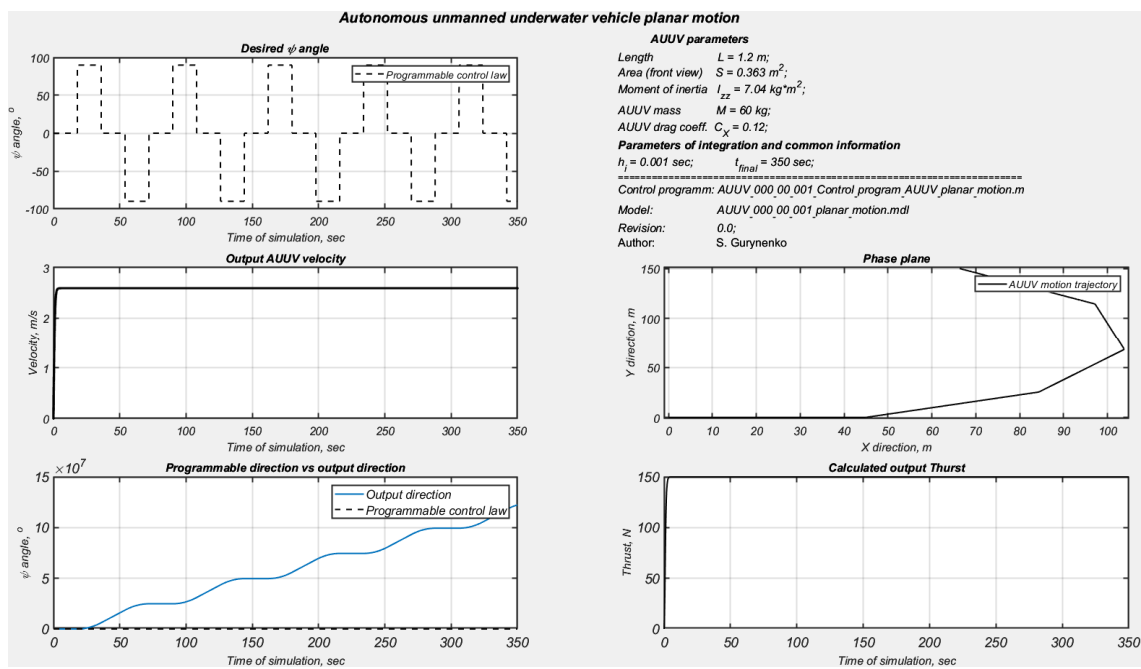


Figure 6. AUUV motion under direct solution (control input—relay control-surface reversal (zig-zag motion)).

Figures 7–10 represent AUUV simple planar motion under solution with feedback. For constant input of desired direction angle expected motion trajectory of apparatus is observed—straight trajectory with constant inclination, however, some oscillation of output apparatus's direction angle is presented. For inputs such as ramp direction and sine direction apparatus output trajectories are circle and sine-shape trajectory respectively. For relay control input AUUV motion under solution with feedback output trajectory does not correspond to predictable, moreover, some oscillation of output direction also presented.

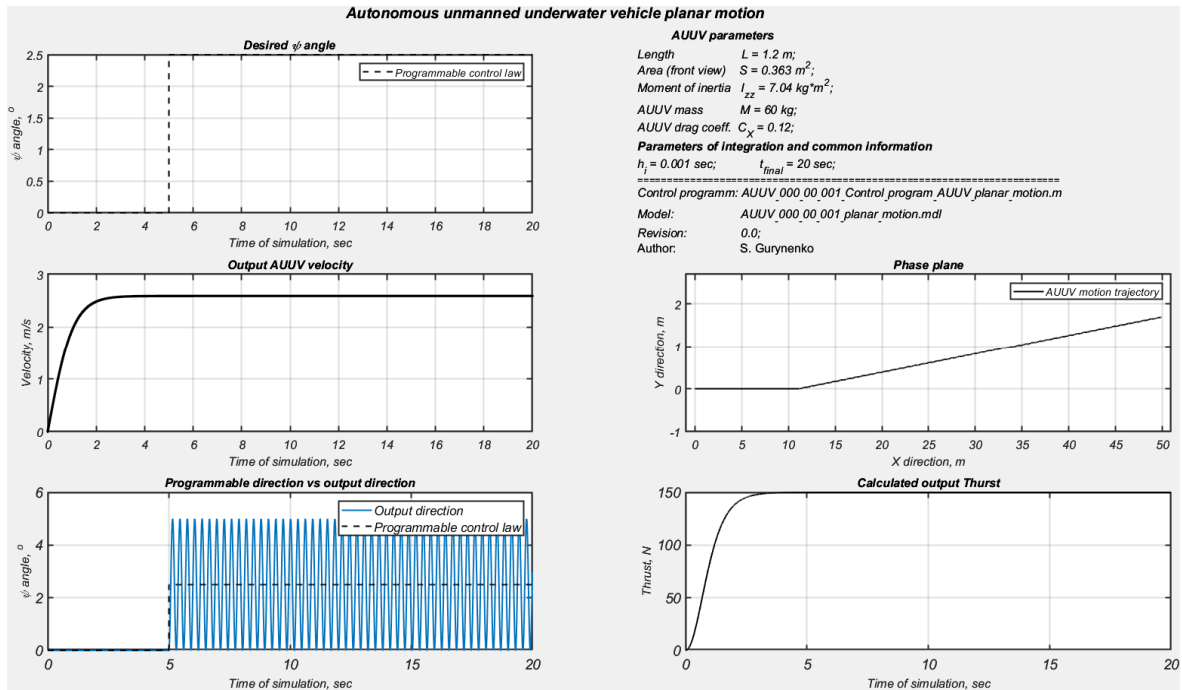


Figure 7. AUVV motion under solution with feedback (control input—constant direction 2.5°).

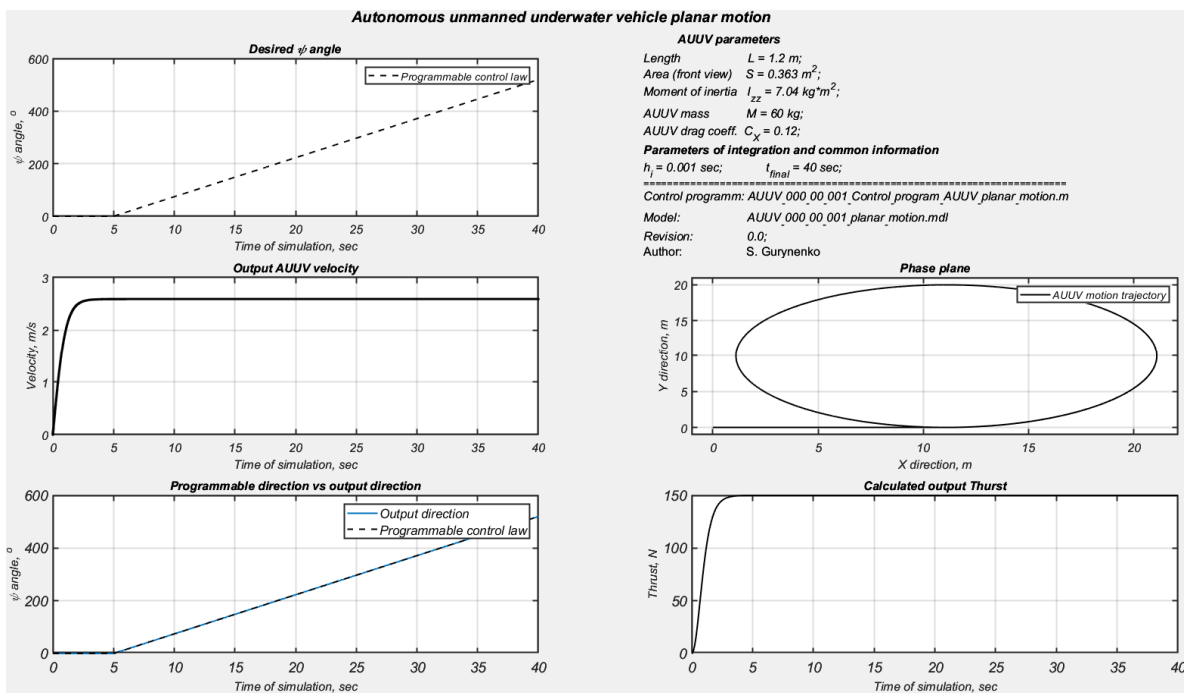


Figure 8. AUVV motion under solution with feedback (control input—ramp direction with desired turn radius 10 m).

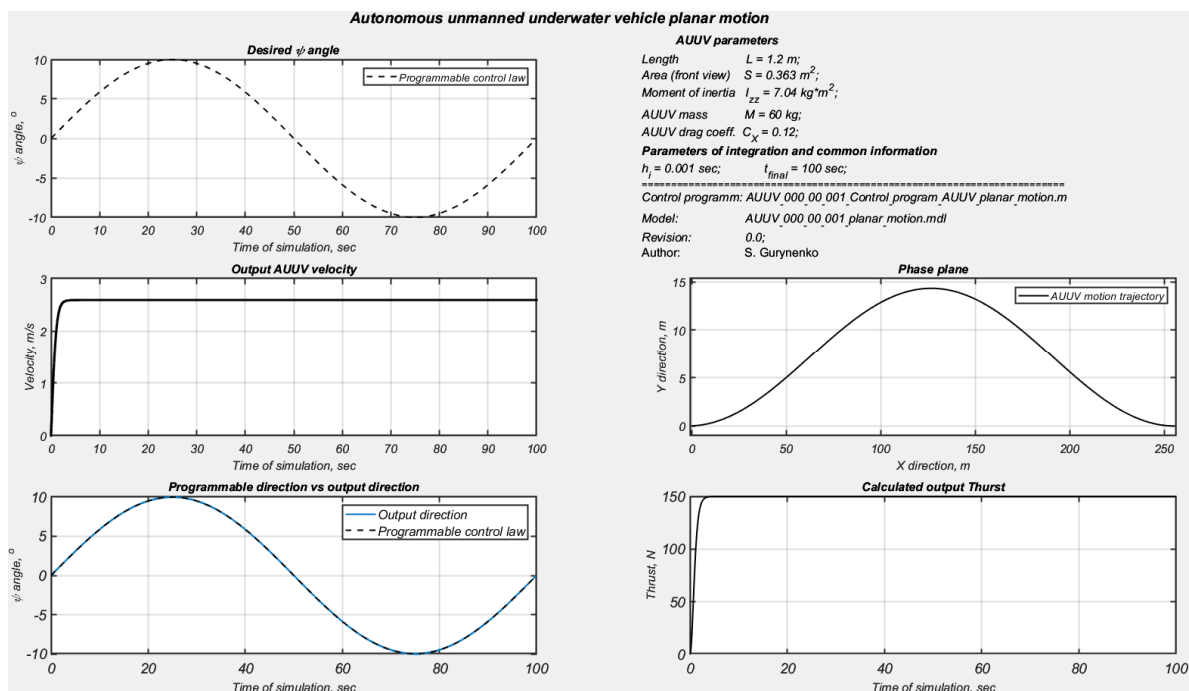


Figure 9. AUUV motion under solution with feedback (control input—sine direction with 0,01 Hz control-surface reversal frequency).

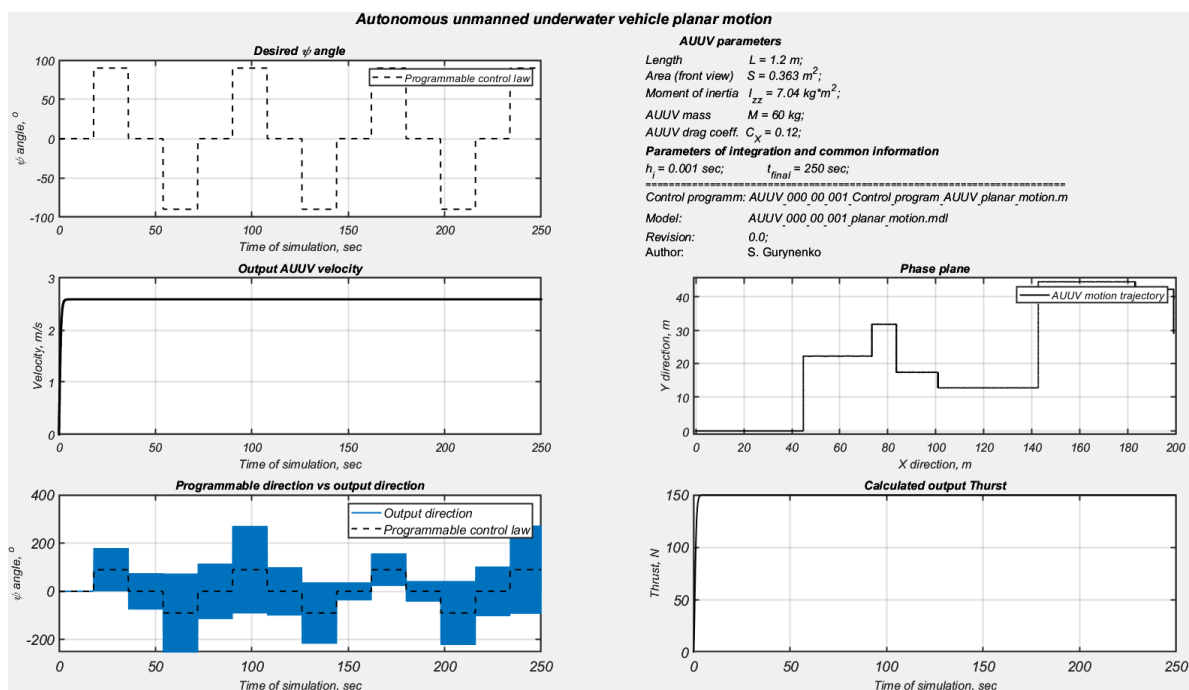


Figure 10. AUUV motion under solution with feedback (control input—relay control-surface reversal (zig-zag motion)).

Figures 11–14 show AUUV simple planar motion under solution with feedback and regulator. A PID-controller was used as the regulator. The properties, application, and configuration methods of this controller are described in (Wang, 2020). The obtained results analysis demonstrates that the AUUV clearly repeat desired direction changes. Also, as a consequence of direction change AUUV gained properly and desired movement trajectory. This result is achieved by varying the controller coefficients, however in practice it is often necessary to select the controller parameters using the method of sequential approximations.

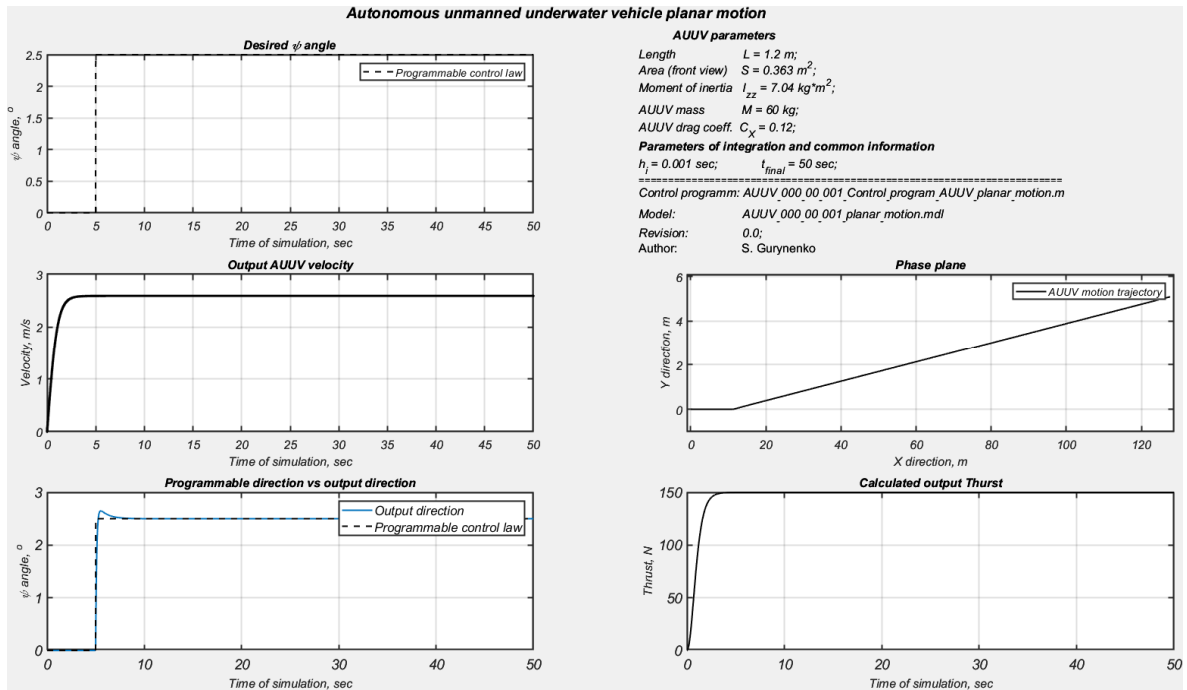


Figure 11. AUUV motion under solution with feedback and regulator (control input—constant direction 2.5°).

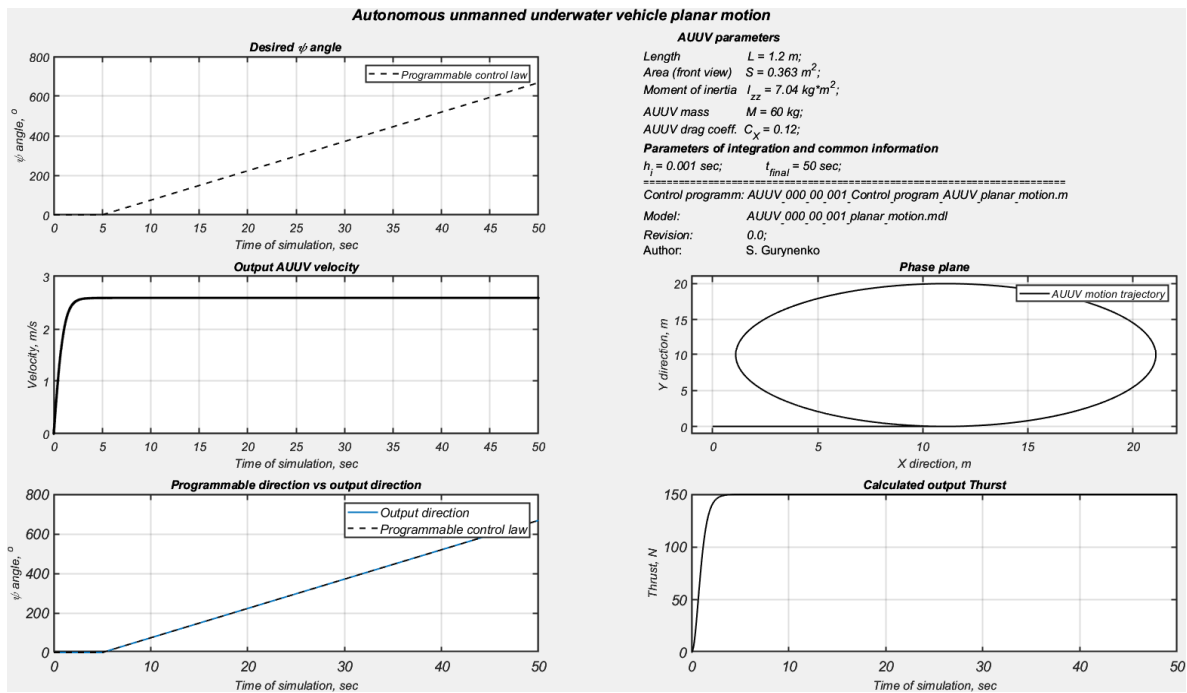


Figure 12. AUUV motion under solution with feedback and regulator (control input—ramp direction with desired turn radius 10 m).

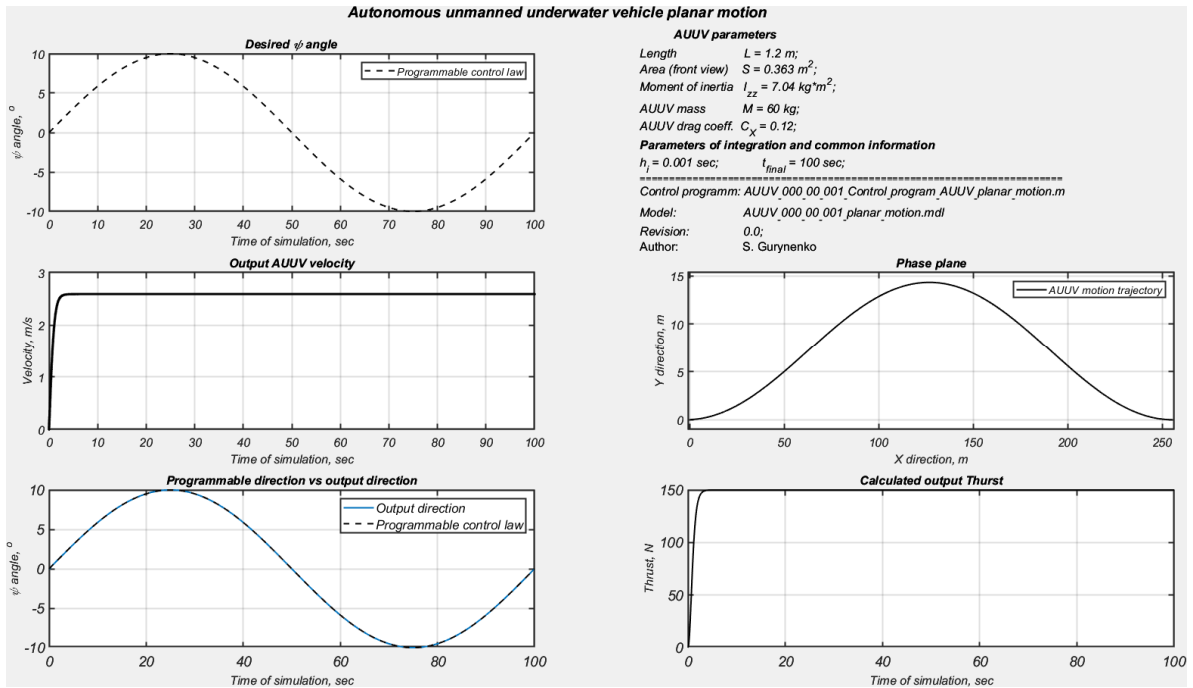


Figure 13. AUUV motion under solution with feedback and regulator (control input—sine direction with 0,01 Hz control-surface reversal frequency).

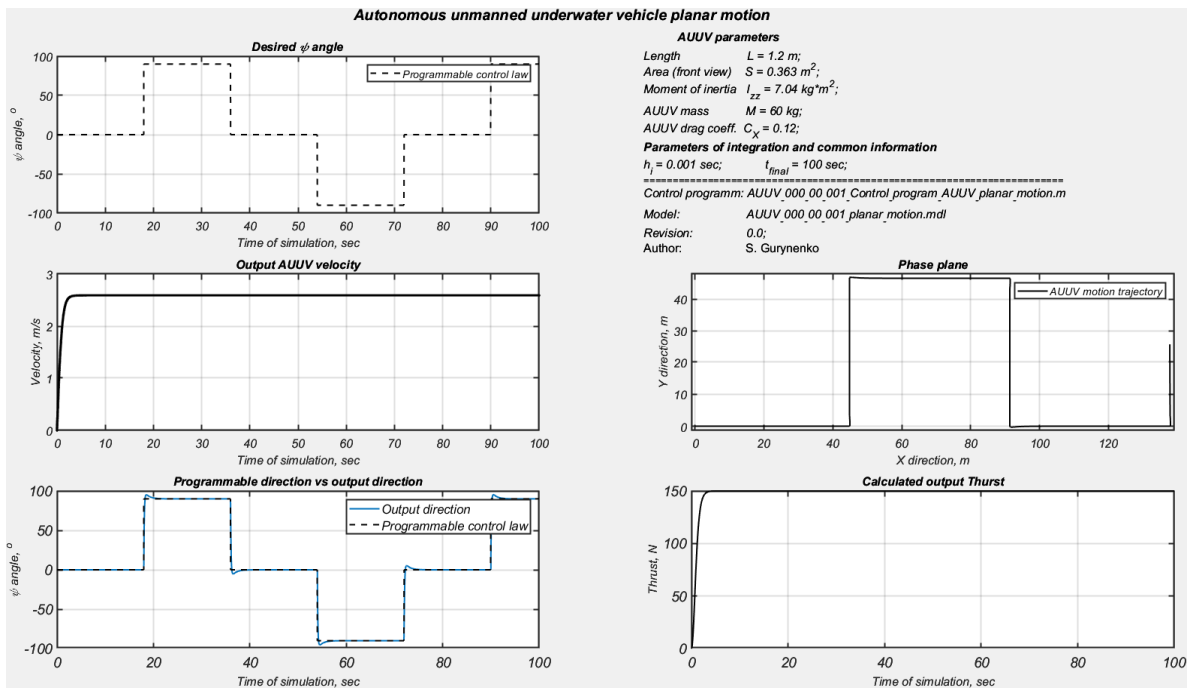


Figure 14. AUUV motion under solution with feedback and regulator (control input—relay control-surface reversal (zig-zag motion)).

Figures 15–18 show AUUV extended motion under direct solution. From obtained results unstable motion of AUUV is shown. As can be seen from the graphs, the apparatus achieve desired velocity as is expected, however after a certain point in time, the trajectory of the vehicle's movement becomes uncontrollable and many times increases. Such unstable motion and uncontrollable trajectory increases are caused by presents of additional hydrodynamics coefficients in motion equations.

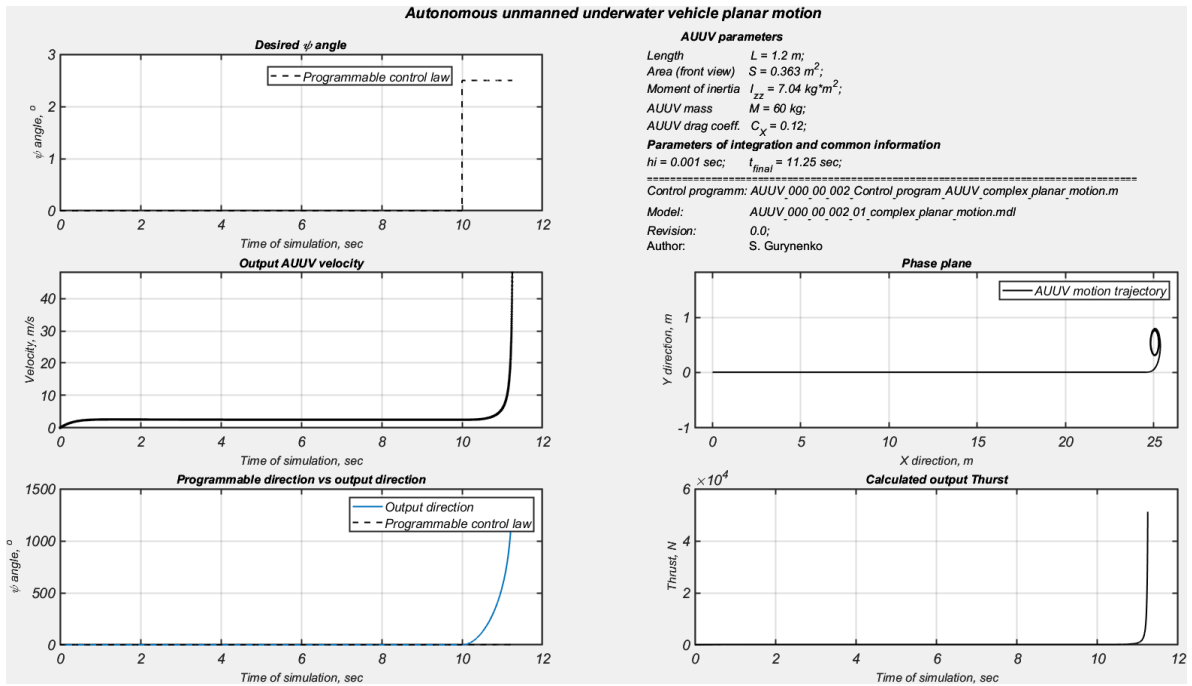


Figure 15. AUUV extended motion under direct solution (control input—constant direction 2.5°).

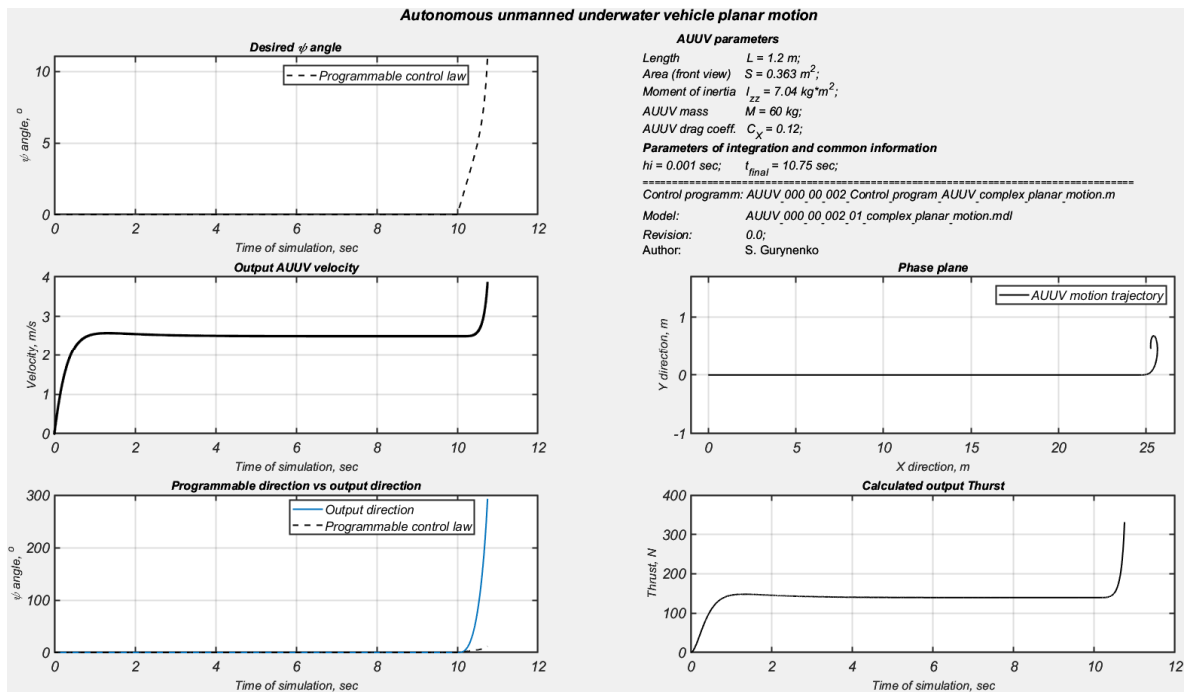


Figure 16. AUUV extended motion under direct solution (control input—ramp direction with desired turn radius 10 m).

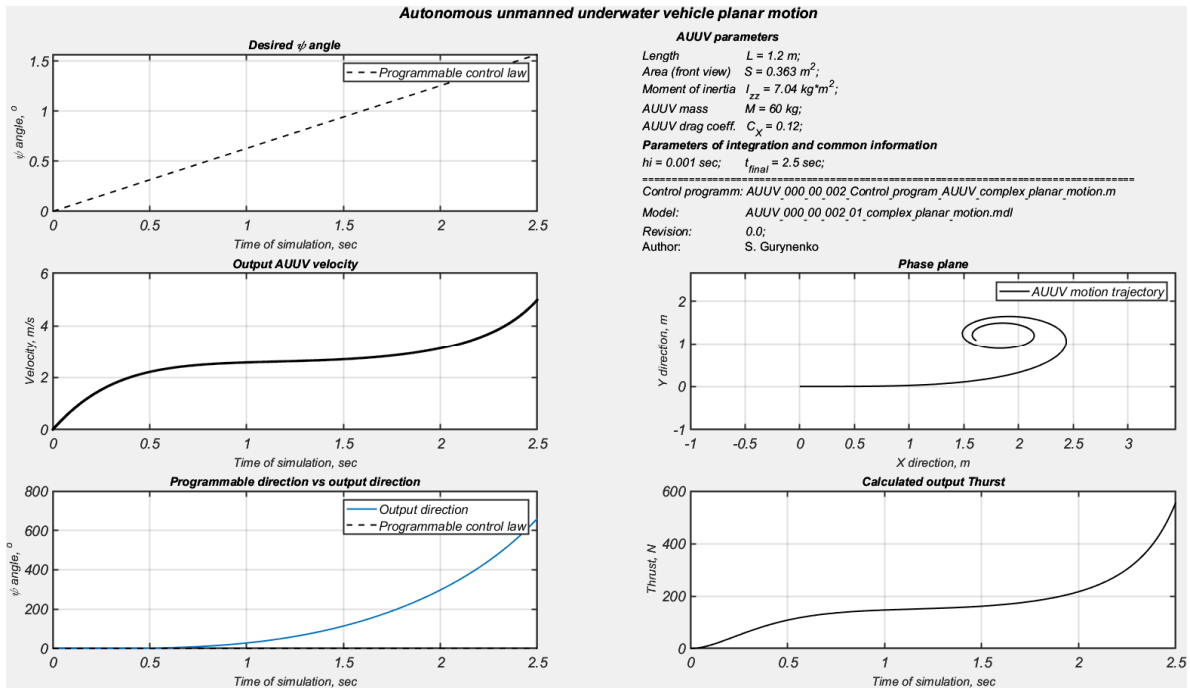


Figure 17. AUUV extended motion under direct solution (control input—sine direction with 0,01 Hz control-surface reversal frequency).

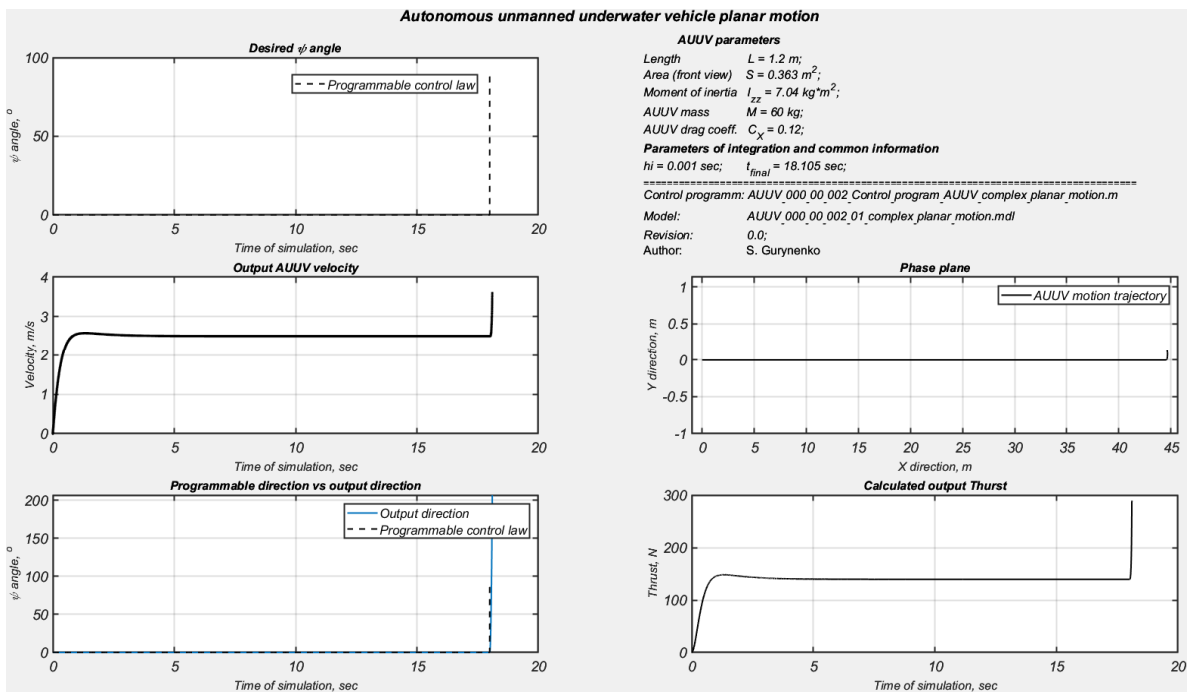


Figure 18. AUUV extended motion under direct solution (control input—relay control-surface reversal (zig-zag motion)).

Figures 19–22 present the solution of the closed system of extended motion equations for the AUUV. Figure 19 illustrates the system response under a constant input of angular influence. The output value of the vehicle heading exhibits a damped oscillatory behavior, while the output value of the trajectory represents a straight line with a constant slope. For input angular influence in the ramp form and sinusoidal functions, the vehicle trajectory becomes a circle and a sinusoid, respectively. In the case of input of relay-type control influence, the vehicle trajectory takes on a zig-zag form, however, the output heading of the AUUV demonstrates damped oscillatory behavior

(Figure 22). Also, when the vehicle changes direction, an oscillatory overcontrol in velocity is observed.

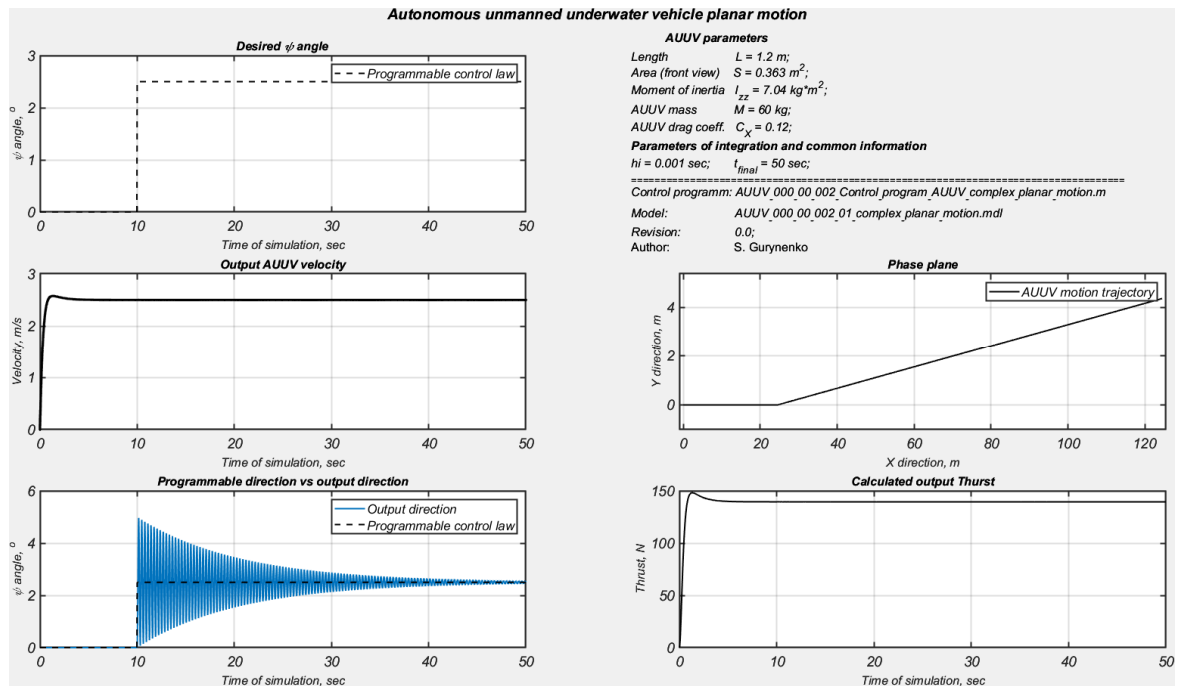


Figure 19. AUUV extended motion under solution with feedback (control input—constant direction 2.5°).

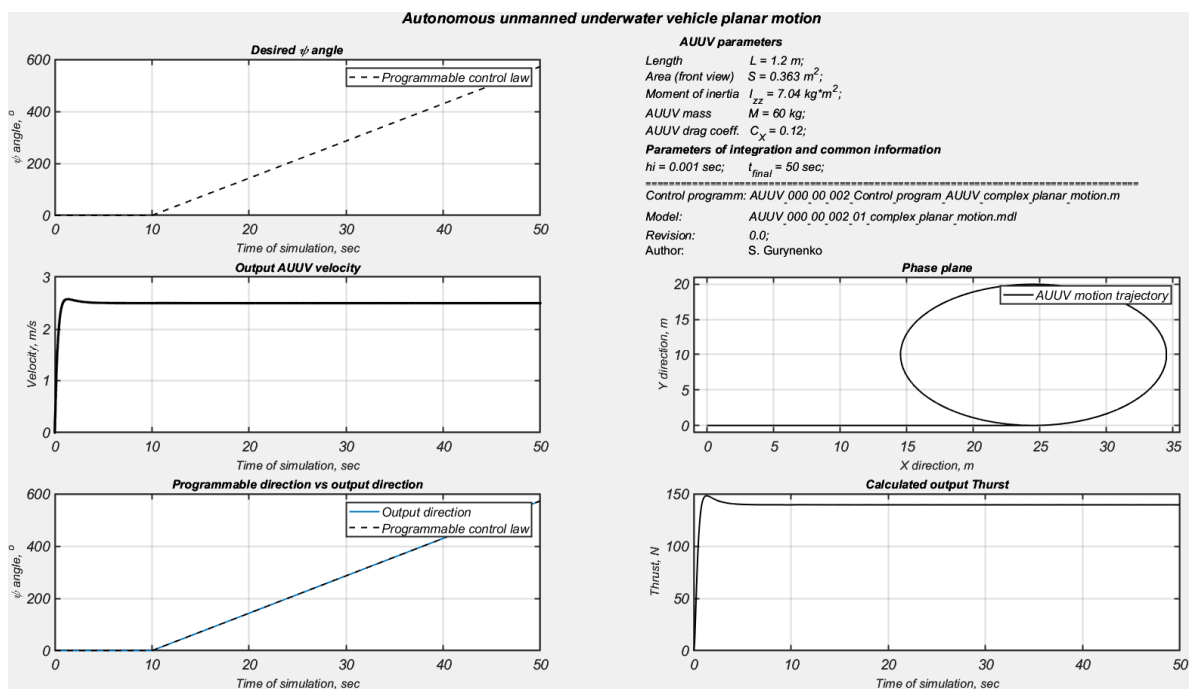


Figure 20. AUUV extended motion under solution with feedback (control input—ramp direction with desired turn radius 10 m).

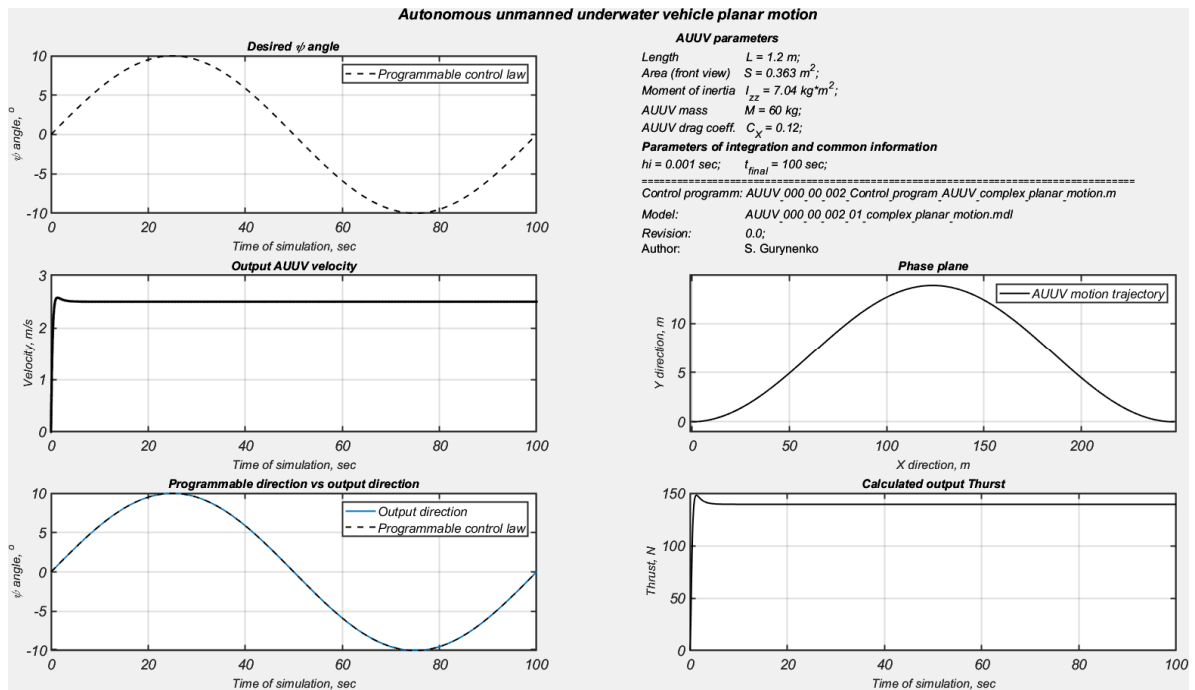


Figure 21. AUUV extended motion under solution with feedback (control input—sine direction with 0,01 Hz control-surface reversal frequency).

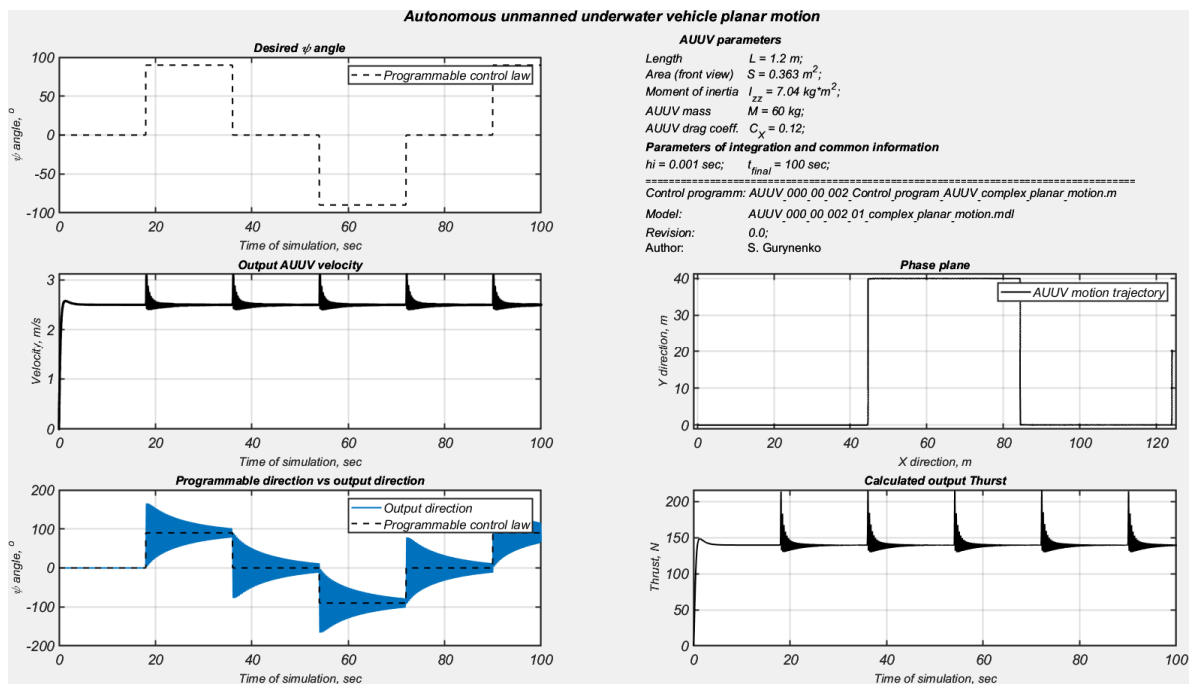


Figure 22. AUUV extended motion under solution with feedback (control input—relay control-surface reversal (zig-zag motion)).

Figures 23–26 illustrate the extended planar motion of the AUUV obtained from the solution with feedback and regulator. A PID-controller was used as the regulator. Analysis of the obtained results shows that the vehicle accurately tracks the input influence and follows the expected trajectory. In the case of input of relay-type control influence, an overshoot in velocity is observed during trajectory switching (small spikes visible in Figure 26).

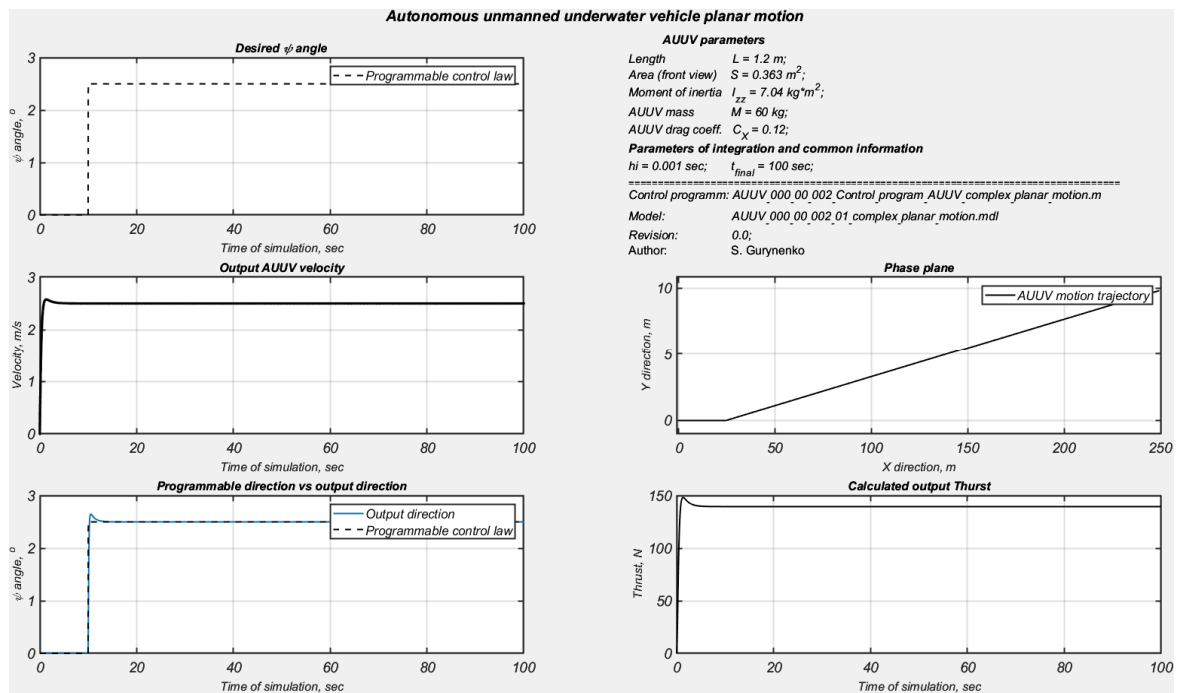


Figure 23. AUUV extended motion under solution with feedback and regulator (control input—constant direction 2.5°).

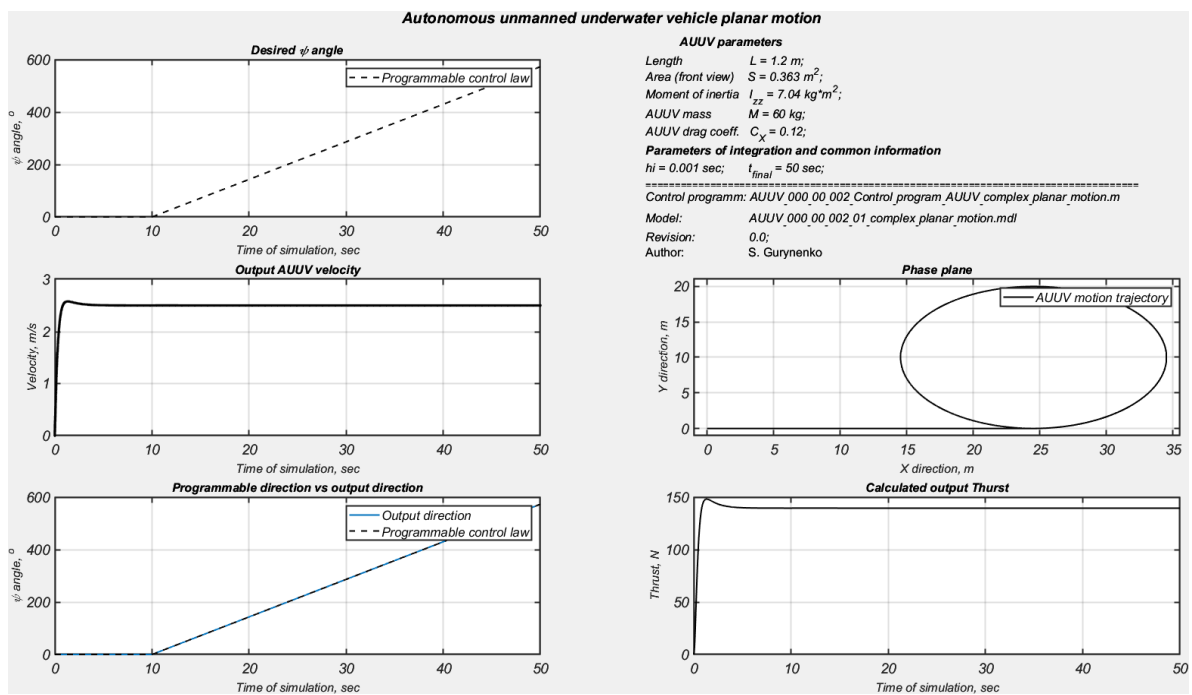


Figure 24. AUUV extended motion under solution with feedback and regulator (control input—ramp direction with desired turn radius 10 m).

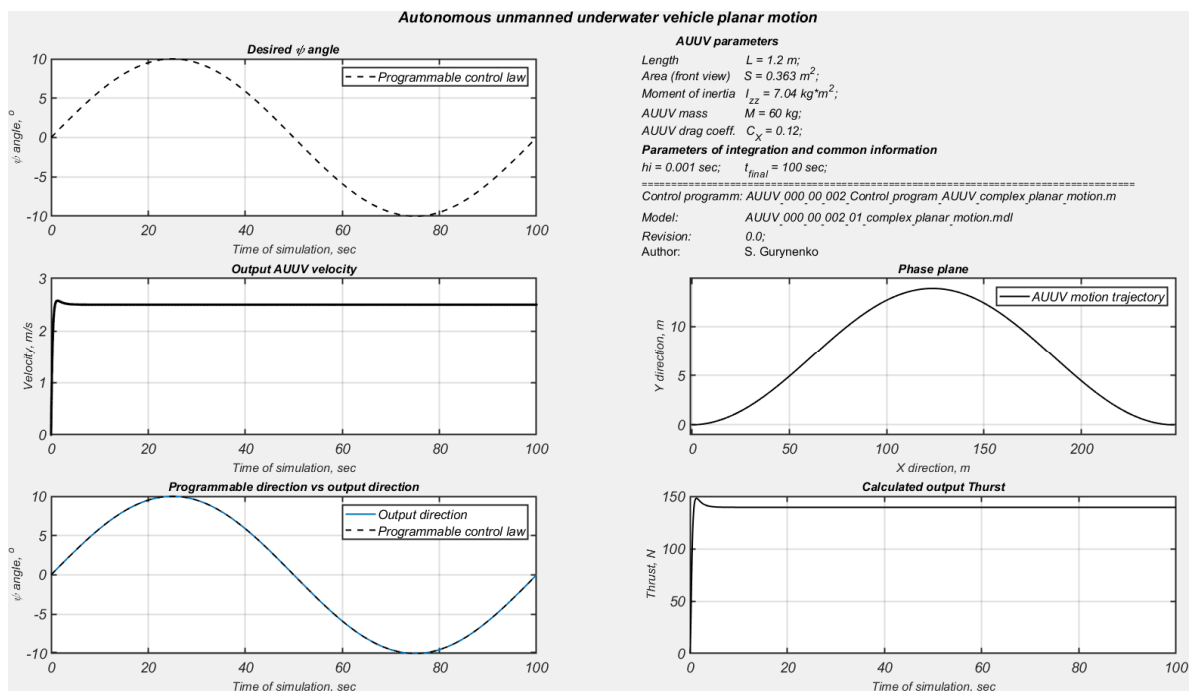


Figure 25. AUUV extended motion under solution with feedback and regulator (control input—sine direction with 0,01 Hz control-surface reversal frequency).

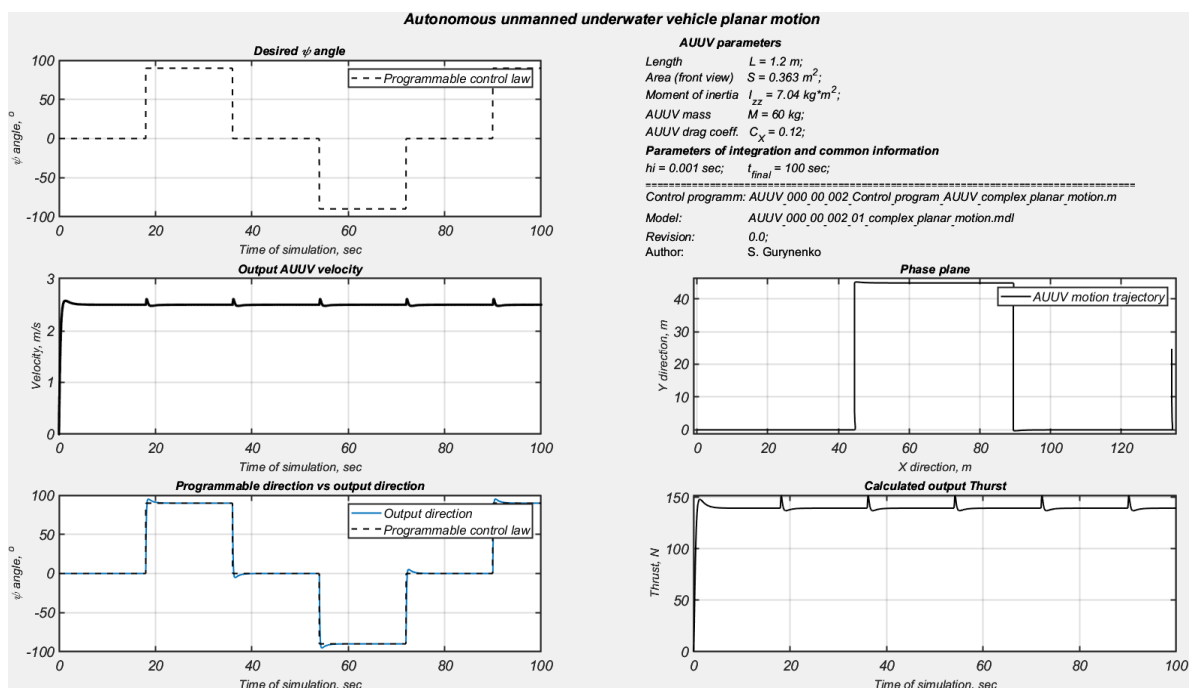


Figure 26. AUUV extended motion under solution with feedback and regulator (control input—relay control-surface reversal (zig-zag motion)).

6. Conclusions

This paper considers the numerical simulation of a control system for an autonomous unmanned underwater vehicle performing planar motion. The control system is developed using both simplified equations of motion and extended equations of motion. For each type of motion equation, a simulation model was developed with the capability to enable feedback control and to incorporate a PID-controller.

The simulation results show that when simplified planar motion equations are used with direct solution, the trajectory of the vehicle converges to a single point, that is, the vehicle tends toward a final position while rotating in place. The introduction of negative feedback improves the controllability of the vehicle. However, under relay control the trajectory of the output signal does not correspond to the expected one, and certain oscillations of the heading angle are observed. The implementation of a PID-controller in the system improves the controllability of the vehicle. With the controller, the AUUV responds adequately to input commands and follows trajectories corresponding to the specified input actions.

The extended equations of planar motion differ from the simplified ones by the presence of an additional equation for one planar coordinate and additional hydrodynamic coefficients. Simulation results demonstrate that when the system with extended equations of motion is solved directly, the vehicle becomes unstable, that is, its angular motion increases without bound and, as a consequence, the trajectory also diverges. The introduction of negative feedback significantly improves the controllability of the vehicle. However, for a step constant input, a damped oscillatory process of the output angular motion is observed. In addition, under relay control, when the direction of motion changes, a damped oscillatory process is observed for both velocity and output angular motion. When a PID-controller is used, the oscillatory processes are reduced. During changes in the motion trajectory, velocity overshoot is clearly observed (small peaks on the corresponding plots). This behavior is caused by the presence of the additional coordinate equation and the additional hydrodynamic coefficients. The extended equations make it possible to evaluate the influence of each coefficient on the behavior of the vehicle and allow a more detailed investigation of the system under various input conditions. For correct calculations these additional hydrodynamic coefficients must be determined at the stage of numerical CAE and CFD modeling or obtained through full-scale hydrodynamic testing of the vehicle hull design.

Further research will focus on the analysis of equations of planar motion under side velocity (undercurrent) and under the influence of sea waves. Also, described approach will be implemented for analysis of equations of describing diving and surfacing motions.

Acknowledgments: Not applicable.

Funding: Not applicable.

Disclosure Statement: The Authors have no conflicts of interest to declare that are relevant to the content of this article. The Authors did not receive support from any organization for the submitted work.

Notations

In this section variables, functions, abbreviations, etc. are listed with explanations.

Variables and functions

M	AUUV mass
G	AUUV weight
L	AUUV length
S	AUUV surface in front section view
T	Main thrust
P	Resultant force of weight and Archimedes' force
h	Depth
ρ	Density (fresh water, sea water)
$\lambda_{11}, \lambda_{12}, \lambda_{16}, \lambda_{22}, \lambda_{25}, \lambda_{33}, \lambda_{34}, \lambda_{35}, \lambda_{44}, \lambda_{45}, \lambda_{55}, \lambda_{66}$	Added mass
$\frac{dV_X}{dt}, \frac{dV_Y}{dt}, \frac{dV_Z}{dt}$	AUUV linear acceleration in each direction
V_X, V_Y, V_Z	AUUV linear velocity in each direction
$\frac{d\omega_X}{dt}, \frac{d\omega_Y}{dt}, \frac{d\omega_Z}{dt}$	AUUV angular acceleration around each axis

$\omega_X, \omega_Y, \omega_Z$	AUUV angular velocity around each axis
$\frac{d\psi}{dt}$	Direction (Yaw) angular velocity
ψ, ϑ, γ	Orientation angles (direction (Yaw), Roll, Pitch)
V_ξ, V_η	North and East velocity in Earth frame
I_X, I_Y, I_Z	AUUV momentum of inertia of each axis
x_c, y_c, z_c	AUUV center of mass coordinates displacement
δ_z	Horizontal rudder angle control input
δ_θ	Vertical rudder angle control input
C_X	AUUV drag coefficient
$C_X^{\omega_Z}, C_Y^{\omega_X}, C_Z^{\omega_Y}$	AUUV drag coefficient of the rotational derivative
$C_Y^\beta, C_Z^0, C_Z^\alpha$	Proportionality coefficient
$C_Y^{\delta_\theta}, C_Z^{\delta_z}$	Proportionality coefficient that characterize the effectiveness of the relevant control surface for linear motion
$m_X^\beta, m_X^{\alpha\beta}, m_Y^\alpha, m_Z^\beta$	AUUV hydrodynamic proportionality coefficient of the rotational derivative
$m_X^{\omega_X}, m_X^{\omega_Z}, m_Y^{\omega_Y}, m_Z^{\omega_X}, m_Z^{\omega_Z}$	Dimensionless coefficient of the rotational derivative
$m_X^{\delta_\theta}, m_Z^{\delta_z}$	Proportionality coefficient that characterize the effectiveness of the relevant control surface for angular motion

References

- Azarsina, F., & Williams, C. D. (2013). Nomoto Indices for Constant-Depth Zigzag Manoeuvres of an Autonomous Underwater Vehicle. *International Scholarly Research Notices*, 2013(1), 219545. <https://doi.org/10.5402/2013/219545>.
- Banazadeh, A., Seif, M. S., Khodaei, M. J., & Rezaie, M. (2017). Identification of the equivalent linear dynamics and controller design for an unmanned underwater vehicle. *Ocean Engineering*, 139, 152-168.
- Bishop, Richard C., & Dorf Robert H. (2011). *Modern control systems*.
- Bouraou, N., & Gurylenko, S. (2023). Analysis of automatic control systems of a multipurpose autonomous unmanned underwater vehicle with complex motion dynamics. *Bulletin of Kyiv Polytechnic Institute. Series Instrument Making*, (65(1), 5-12. [https://doi.org/10.20535/1970.65\(1\).2023.283195](https://doi.org/10.20535/1970.65(1).2023.283195).
- Bouraou, N., Iatsko, L., Rasulov, M., & Bobryk, V. (2017). Overview of the state of modern autonomous unmanned underwater vehicles. *Bulletin of the Engineering Academy of Ukraine*, (4), 12-17.
- Bouraou, N., Velychko, S., & Gurylenko, S. (2021). Dynamics simulation of autonomous unmanned underwater vehicle in simple motion. *KPI Science News*, (3), 64-73. <https://doi.org/10.20535/kpissn.2021.3.243586>.
- Goodwin, G. C., Graebe, S. F., & Salgado, M. E. (2001). *Control system design* (Vol. 240). Upper Saddle River: Prentice Hall.
- Griffiths, G. (Ed.). (2002). *Technology and applications of autonomous underwater vehicles* (Vol. 2). CRC Press.
- Gurylenko, S. (2023). Simulation, CFD calculation and estimation of hydrodynamics coefficients of an autonomous unmanned underwater vehicle. *International Scientific Technical Journal "Problems of Control and Informatics"*, 67(6), 5-13. <https://doi.org/10.34229/1028-0979-2022-6-1>.
- Gurylenko S., & Bouraou N., (2023). A Rview of mathematical and algorithmic methods of control systems a multi-purpose autonomous unmanned underwater vehicle with complex motion dynamic. *MEASURING AND COMPUTING DEVICES IN TECHNOLOGICAL PROCESSES*, (2), 42-48. <https://doi.org/10.31891/2219-9365-2023-74-6>.
- Gurylenko, S., Bouraou, N., & Surgok, V. (2023). Simulation and Analysis of the Complex Movement of an Autonomous Unmanned Underwater Vehicle. *Elektronnoe Modelirovanie*, 45(3). <https://doi.org/10.15407/emodel.45.03.081>

- Hinostrroza, M. A., & Xu, H. (2017). Experimental and numerical simulations of zig-zag manoeuvres of a self-running ship model. Guedes Soares, C., Teixeira, AP (Eds.), *Maritime Transportation and Harvesting of Sea Resources*.
- Issac, M. T., Adams, S., He, M., Bose, N., Williams, C. D., Bachmayer, R., & Crees, T. (2007). Manoeuvring Experiments Using the MUN Explorer AUV. 2007 Symposium on Underwater Technology and Workshop on Scientific Use of Submarine Cables and Related Technologies, 256–262. <https://doi.org/10.1109/ut.2007.370791>.
- Issac, M. T., Adams, S., Bose, N., Williams, C. D., Bachmayer, R., & Crees, T. (2008). Analysis of horizontal zigzag manoeuvring trials from the MUN Explorer AUV. *OCEANS 2008*, 1–7. <https://doi.org/10.1109/oceans.2008.5151952>.
- ITTC Recommended Procedures and Guidelines, Rev. 02 (2017), Full Scale Manoeuvring Trials, Section 7.5-04-02-01.
- Jimoh, I. A., & Yue, H. (2024). Path following model predictive control of a coupled autonomous underwater vehicle. *IFAC-PapersOnLine*, 58(20), 183-188. <https://doi.org/10.1016/j.ifacol.2024.10.052>.
- Liu, G., Chen, G., Jiao, J., & Jiang, R. (2015). Dynamics Modeling and Control Simulation of an Autonomous Underwater Vehicle. *Journal of Coastal Research*, 73, 741–746. <https://doi.org/10.2112/si73-127.1>.
- Lukomskii Yu.A., & Chugunov V.S. (1988). *Sistemi upravleniya morskimi podvzhnimi obektami*. L.: Sudostroenie, 272.
- Nicholson, J. W., & Healey, A. J. (2008). The present state of autonomous underwater vehicle (AUV) applications and technologies. *Marine Technology Society Journal*, 42(1), 44-51. <https://doi.org/10.4031/002533208786861272>.
- Nouri, N. M., Valadi, M., & Asgharian, J. (2018). Optimal input design for hydrodynamic derivatives estimation of nonlinear dynamic model of AUV. *Nonlinear Dynamics*, 92(2), 139-151.
- Perrault, D., Bose, N., O'Young, S., & Williams, C. D. (2003). Sensitivity of AUV response to variations in hydrodynamic parameters. *Ocean Engineering*, 30(6), 779-811. [https://doi.org/10.1016/S0029-8018\(02\)00043-4](https://doi.org/10.1016/S0029-8018(02)00043-4).
- Roberts, G. N., & Sutton, R. (Eds.). (2006). *Advances in unmanned marine vehicles* (Vol. 69). Iet. <https://doi.org/10.1049/PBCE077E>.
- Sutulo, S., & Soares, C. G. (2005). Numerical study of some properties of generic mathematical models of directionally unstable ships. *Ocean engineering*, 32(3-4), 485-497. <https://doi.org/10.1016/j.oceaneng.2004.05.008>.
- Valeriano-Medina, Y., Martínez, A., Hernández, L., Sahli, H., Rodríguez, Y., & Cañizares, J. R. (2012). Dynamic model for an autonomous underwater vehicle based on experimental data. *Mathematical and Computer Modelling of Dynamical Systems*, 19(2), 175–200. <https://doi.org/10.1080/13873954.2012.717226>.
- Villa, J., Vallicrosa, G., Aaltonen, J., Ridaou, P., & Koskinen, K. T. (2020). Model-based Guidance, Navigation and Control architecture for an Autonomous Underwater Vehicle. *Global Oceans 2020: Singapore—U.S. Gulf Coast*, 1–6. <https://doi.org/10.1109/ieeconf38699.2020.9389247>.
- Vukić, Z., & Mišković, N. (2016). State and perspectives of underwater robotics-role of laboratory for underwater systems and technologies. *Pomorski zbornik*, (1), 15-27. <https://doi.org/10.18048/2016.00.15>.
- Wadoo, S. (2017). *Autonomous underwater vehicles: modeling, control design and simulation*. CRC press.
- Wang, L. (2020). *PID control system design and automatic tuning using MATLAB/Simulink*. John Wiley & Sons.
- Лазарев, Ю. Ф., & Бондар, П. М. (2011). *Основи теорії чутливих елементів систем орієнтації*. К.: НТУУ «КПІ».

Disclaimer/Publisher's Note: The statements, opinions and data contained in all publications are solely those of the individual author(s) and contributor(s) and not of MDPI and/or the editor(s). MDPI and/or the editor(s) disclaim responsibility for any injury to people or property resulting from any ideas, methods, instructions or products referred to in the content.