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Sadi Carnot: Misunderstood for Two Hundred Years

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Abstract: The overall theme is that for two hundred years Sadi Carnot's 1824 essay has been misunderstood. Carnot's reasoning has been misrepresented by the dominant mechanistic formulation of the Clausius, Boltzmann, Gibbs tradition. To understand the origin and nature of the misunderstanding it is necessary to recognize that there are the two histories and two current formulations of thermodynamics: the mechanical and the engineering. Both these formulations turn out to have a long history. In examining three historic periods, one just before Sadi wrote. A second period 1650-1750 centered on Huygens and Leibniz. The third period reaches back to the ancient Greek geometers, symbolized in Archimedes' method. Atkins pointed out that there were two histories of thermodynamic, Boltzmann's and Carnot's. These have much longer histories in the Analytic and Synthetic Traditions. Two subthemes run throughout my inquiry helping to distinguish and clarify the two research programs. First are their different ontologies: particles versus metabolic engines. Second is their different perspectives on inquiry itself. The mechanical adopts a detached spectator perspective while the engineering has the perspective of a participant inside reality. Adrian Bejan's recent recovery of engineering thermodynamics provides both clues and substance pointing to the proper understanding of Carnot's essay.

Keywords: Sadi and Lazare Carnot; mechanical vs engineering thermodynamics; particle vs metabolic engine ontology; Huygens; Leibniz; Newton; Archimedes and Eudoxus; Euler-Lagrange; Analytic and Synthetic

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Introduction

Sadi Carnot's thermodynamics has been regularly, almost exclusively, misunderstood by the reformulations of Clapeyron and Clausius. [1] The conceptual framework of Clausius, Boltzmann, Gibbs thermodynamics is classical mechanics. I refer to their formulations and their tradition as Mechanical Thermodynamics. On the other hand, Sadi Carnot's original formulation is thermodynamics of engines. I refer to Sadi's formulation and its tradition, including its pre-history and subsequent history, as Engineering Thermodynamics.

There are two characteristic differences between these two formulations of thermodynamics. The first has to do with their conceptually distinct ontologies. Mechanical thermodynamics has a particle ontology while engineering thermodynamics has an engine ontology. A second distinguishing feature concerns their perspectives. Mechanical thermodynamics attempts to understand thermodynamic systems from outside, typical of the mechanical tradition. In the spectator perspective of classical mechanics, inquiry is conceived as detached, seeking the time-space invariant laws governing objective reality – out there. In engineering thermodynamics, the inquiry and the inquirer are inside the system. The inquirer is an active participant, an embodied agent, participating in the evolution of reality.

The two ontologies are apparent in Peter Atkins' claim that thermodynamics has two competing histories. In his book, *The Second Law,* [2] Atkins maintained that: "The aims adopted, and the attitudes struck by Carnot and Boltzmann epitomize thermodynamics. Carnot traveled toward thermodynamics from the direction of the engine, then the symbol of industrial society: his aim was to improve its efficiency. Boltzmann traveled to thermodynamics from the atom, the symbol of emerging scientific fundamentalism: his aim was to increase our comprehension of the world at the

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deepest levels then conceived." Atkins then surprised me by claiming that "Thermodynamics still has both aspects, and reflects complementary aims, attitudes, and applications."

It took me, literally, three years to convince myself that Atkins was correct, not only that there were two distinct histories of thermodynamics but also that there are two corresponding current formulations of thermodynamics. The crucial difference between the ontologies of the mechanical and engineering thermodynamics is apparent already in Atkins' characterization of the two formulations. Mechanical thermodynamics has a particle ontology, while in engineering thermodynamics reality is composed of engines.

In researching Atkins' claim, I discovered Donald Cardwell (Manchester) [3] who like Atkins had suspected that the dominant mechanical formulation of thermodynamics was perhaps misunderstanding.

"Almost traditionally, it seems, accounts of the development of the concepts of work and energy have tended to describe them within the classical framework of Newtonian mechanics. … I would like to suggest that this may be to take too narrow a view of the case. It is to project backwards our present specialist arrangement of scientific knowledge, our present divisions between the sciences, and to assume that past development was strictly guided by these divisions. And this is to make questionable historical and sociological assumptions."[1]

Cardwell went on to highlight that thermodynamics did not arise from *within* the mechanics research program, or from any concern with the inadequacies of that program. Thermodynamics arose quite independently from the work of engineers. Cardwell proposed that the real history of thermodynamics should be seated in the history of the development and study of machines and engines.

A further encouragement, supporting Atkins and Cardwell, came from my colleague, Robert Ulanowicz (Maryland). With only a brief mention of the issue, he enthusiastically offered the following anecdote. 'When I was finishing my PhD in chemical engineering at Johns Hopkins, in my orals, there was of course the obligatory thermodynamics question. If I had said anything about particles bouncing around, the next day I would have been looking for a job selling real estate.' [4] The Hopkins program was clearly not supportive of Boltzmann's particle-based formulation of thermodynamics.

Sadi Carnot, in the first pages of his famous essay, provides us with a picture of a world of engines, of coupled cyclic processes. "Everyone knows that heat can produce motion. That it possesses vast motive-power no one can doubt, in these days when the steam-engine is everywhere so well known. To heat also are due the vast movements which take place on the earth. It causes the agitations of the atmosphere, the ascension of clouds, the fall of rain and of meteors, the currents of water which channel the surface of the globe, and of which man has thus far employed but a small portion. Even earthquakes and volcanic eruptions are the result of heat. From this immense reservoir we may draw the moving force necessary for our purposes."

Crucial for my theme, he also places humanity *inside* the overall system. What I like to refer to as Carnot's Epiphany is that we are metabolic heat engines, embedded and embodied, in a world of heat engines. As I will argue, we are participant engineers in a world of engineering. We are active agents in an evolving world of agency.

American pragmatist John Dewey [5] recognized that the mechanical and the engineering research programs had two quite different representations of inquiry. He characterized these as that of the spectator and that of the participant. In the classical mechanical research program, the inquirer is a spectator, a detached observer seeking to understand the time-space invariant laws governing 'objective reality' – out there. The spectator does not interact with reality since to do so would introduce a disturbance in the natural processes, making it impossible for the inquirer to sort out

[1] Cardwell, D. S. L., (1967) "Some Factors in the Early Development of the Concepts of Power, Work and Energy", The British Journal for the History of Science, Vol. 3, No. 3 (Jun. 1967), pp. 209-224

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what changes were due to his actions and what were the natural behaviors of 'objective reality' – out there. In Dewey's representation of participant inquiry, the participant is a natural embodied component of reality. Reality is composed of active participants, and, per hypothesis, as engineers they are asking different types of questions. Engineers are problem solvers, seeking knowledge that enables them to move from a current state of affairs to a future more desirable state.

Following Dewey's reasoning, if the scientific spectator and the engineering participant are asking different types of questions, then their knowledge of reality will differ and what they can say about reality will differ. The spectator gives us, passively, a qualitatively fixed description of reality. The participant offers knowledge useful in the participant active role in a qualitatively evolving reality. The question of the proper representation and understanding of thermodynamics presses on us is the further difficult question of the relation of scientific knowledge and engineering knowledge. This is equivalent to the question of the relation between the mechanical formulation and the engineering formulation of thermodynamics. It is hoped that the results reported here will serve to stimulate new and renewed research into these fundamental issues.

Strategy and Structure

To understand the origin and nature of the misunderstanding of Carnot's essay, it is essential to fully reveal both the two histories and the two conceptually distinct formulations thermodynamics: the mechanical and the engineering.

Ontology and Perspective

Both these formulations turn out to have a long history. I examine three historic periods; each reveals different characteristics relevant to understanding Sadi Carnot's essay. The first period is just before Sadi wrote and includes the influence of the work of his father, Lazare Carnot and others around that time at Ecole Polytechnic. The second period is between 1650-1750 and centers on the contributions of Christiaan Huygens and colleagues including Gottfried Leibniz. Lazare Carnot recognized the historical path from Leibniz's project to his own work. I was led to the third period, by the fact that all those contributors in the first two periods claimed an intellectual ancestry to Archimedes. The third period reveals a foundational connection to the ancient Greek geometry.

Stimulated by both clues and the substance of Adrian Bejan's recent recovery of engineering thermodynamics I found that the two long histories can be organized into two recognized traditions: the Analytic for the mechanical thermodynamics and the Synthetic for the engineering thermodynamics.

As mentioned in the Introduction, two subthemes run throughout and serve to distinguish and clarify the two research programs. First are their different ontologies: particles or engines. Second is their distinctive perspectives on their inquiries themselves.

From Carnot to Clausius and Back

In Clausius's novel reinterpretation of Sadi's essay the separation of the conceptual frameworks of mechanical thermodynamics and engineering thermodynamics is already apparent. Clausius, initially, seems to recognize the importance of Sadi's contribution. Clausius states: "Carnot proves when work is produced by heat, heat passes from a warm body to a cold one. Carnot regards the change of heat to correspond to the work produced."

Clausius continues: "[However] Carnot says expressly that no heat is lost in the process, that the quantity remains unchanged." Clausius adds that Carnot emphasizes, "This is a fact which has never been disputed. To deny it, would be to reject the entire theory of heat, of which it forms <u>the principal foundation</u>."

In Clausius's response to Carnot's foundational claim he clearly expresses the incompatibility of their conceptual frameworks. Clausius: "I am not, however, sure that the assertion, that in the production of work a loss of heat never occurs, is sufficiently established by experiment. Perhaps the contrary might be asserted with greater justice."[2]

It is not difficult to imagine at this point that Clausius and Carnot are reasoning with two different concepts of 'heat'. For Clausius 'heat' is the external 'driving force' that produces work. This is analogous to Newton's First Law where an external force is required to alter the state of motion of a body, to produce work. For Clausius 'heat', the driving force, is progressively lost as work is performed. As 'heat' is lost, the capacity to perform work declines. In Clausius's Second Law as 'heat', which embodies the capacity to perform work, decreases, entropy increases.

On the other hand, for Sadi, as work is performed, the quantity of 'heat' remains unchanged. Sadi is emphatic: "This is a fact which has never been disputed; it is first assumed without investigation and then confirmed by various calorimetric experiments. To deny it, would be to reject the entire theory of heat, of which it forms the principal foundation."

In Carnot's engine thermodynamics, since there is no loss of 'heat' there is no Second Law and no concept of entropy. Sadi's insistence that 'heat' is conserved, not lost, in the production of work, should be understood as his First Law of Engineering Thermodynamics.

But what does Sadi mean by 'heat'?

Sadi's Caloric

One of the sources of confusion in understanding Sadi's essay arises from his tendency to represent heat in terms of the concept of 'caloric', a concept that had been recently introduced in the new science of chemistry and popularized by Lavoisier. Worth emphasizing that what Sadi and those he cites mean by 'temperature' is empirical and not the Kelvin scale derived from mechanical presuppositions.[3]

For the purposes of this essay, rather than trying to sort out the various issues surrounding the concept of caloric, I will just focus on what Sadi referred to as the First Principle of his theory – conservation of 'something'. And, as to the something, the historical context will come to the rescue.

Lazare Carnot and all the preceding and subsequent contributors to engineering thermodynamics accepted that what was conserved in the universe was what was referred to as *vis viva* or living force. The colloquial origin of the expression 'living force' is straight forward. In the history of humanity, it was people or their domesticated animals that perform useful work.

Linking humans, animals and the steam engine, James Watt [3] coined the expression 'horsepower' as the standard measure of the potential strength or power of a steam engine. James Watt had attached a pencil and paper to his steam engine such that the pencil would trace a cyclic curve as the engine operated. What was drawn came to be called 'the indicator diagram'. What Watt wanted to measure was the motive power as he modified the designs of his steam engines. The relevant measure for Watt was the area enclosed by the cycle. The greater the area the greater the motive power of the engine. Watt considered the indicator diagram a trade secret, using it to significantly enhance steam engine design and performance.

Both Sadi's cycle and Watt's indicator diagram are pressure-volume diagrams. The diagram of Sadi's cycle is not a graph of a normal analytic function in Cartesian analytics. The area inside the

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[2] Clausius, M.R. "On the Moving Force of Heat, and the Laws regarding the Nature of Heat itself which are deducible therefrom." Philosophical Magazine, Vol II, No. VIII, July 1851

[3] Sadi Carnot: "It is considered unnecessary to explain here what is quantity of caloric or quantity of heat (for we employ these two expressions indifferently), or to describe how we measure these quantities by the calorimeter. Nor will we explain what is meant by latent heat, degree of temperature, specific heat, etc. The reader should be familiarized with these terms through the study of the elementary treatises of physics or of chemistry." [1] Footnote page 37

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cycle is the important measure of the motive power or strength of the engine. The relation between the up-down curves of the cycle and the area enclosed, I will argue, is treated differently in the two formulations of thermodynamics.[4]

Calorimetry of Organisms, Metabolic Engines

Sadi's empirical claim that although "it is first assumed without investigation", it is "then confirmed by various calorimetric experiments" does not appear to have been taken seriously. It is more common to hear that Clausius's claim that 'heat is lost' is well established by experiment. And, perhaps, for experiments designed and interpreted *within* the mechanical framework Clausius is supported. But what might be the evidence that Sadi seems to be confidently referencing that shows that 'heat is not lost in the production of work'. Sadi's claim is about the world of cyclic engines not the world of particles behaving mechanically.

Modern calorimetric measurement by Hansen, Battley and others show that for organisms 'energy in and energy out' are always, at least very nearly, equal. Obviously if this were not the case organisms would either become cold or heat up with the performance work. Warm blooded animals are metabolic engines, *active* agents, not *passive* like the particles of the mechanical ontology.[5]

Further empirical support for Sadi's First Principle comes if we consider the Earth, the ecosystem, as an engine. It is well established that the energy that the Earth receives from the Sun is equal to the energy that the Earth radiates into space. Energy-in equals energy-out. If this were not the case the Earth would progressively either heat up or cool down.

However, in Sadi's engineering thermodynamics since there is no external driving force to produce work, an unsettling question is asked by Clausius and promoters of mechanical thermodynamics. If 'heat' or 'living force' is conserved how is work generated?

Clausius, Boltzmann and Gibbs

Although Clausius's initial reinterpretation of Sadi's essay did not refer to particles, it is generally accepted that, in terms of fundamental principles, Clausius's and Boltzmann's formulations of thermodynamics are equivalent. Clausius introduced the concept of entropy while Boltzmann gave an account in terms of particle motion. What is common to both is the need for an external force, a 'driving force', to produce motion. The representation of the kinetic gas law by Boltzmann assumes that the motion of particles is rectilinear. Clausius, Boltzmann, and later Gibbs, in their formulations of thermodynamics, all accept the defining presuppositions of classical mechanics: symmetry, equilibrium and conservation. Consequently, what I have referred to as mechanical thermodynamics is to be understood in the modern context as Clausius, Boltzmann, Gibbs thermodynamics (CBG).[6]

Two examples serve to emphasize that difference. Those promoting the mechanical formulation of thermodynamics frequently cite the example of placing a glass of cold water in a warm room. The

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[4] What is important to my narrative is that only in engineering thermodynamics can it be realized that pressure and volume are conjugate variables, conceptually complementary.

[5] Hansen, Lee D., Richard S. Criddle, and Edwin H. Battley, (2009) "Biological calorimetry and the thermodynamics of the origination and evolution of life", Pure Appl. Chem., Vol. 81, No. 10, pp. 1843–1855. Battley, E.H. (1999). In *Handbook of Thermal Analysis and Calorimetry*, Vol. 4, *From Macromolecules to Man*, R. B. Kemp (Ed.), pp. 219–266, Elsevier, Amsterdam.

[6] Sean Carroll (2010) states that "Clausius's rule that temperatures tend to even themselves out, rather than spontaneously flowing from hot to cold, is precisely equivalent to the statement that the entropy as defined by Boltzmann never decreases in a closed system." *From Eternity to Here: The Quest for the Ultimate Theory of Time* (p. 156). Penguin Publishing Group. Kindle Edition.

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glass of water will soon come to the same temperature as the room. Actually, the room will also become somewhat cooler. Similarly, a glass of warm water in a cold room will soon come to the temperature of the cold room. Actually, the room will also become somewhat warmer. The overall message in that in passive particle ontologies, differences in temperature tend to equilibrate. I suspect bias in concluding that this show that heat always flows in the direction of hot to cold.

However, in Sadi's ontology of active metabolic agents, when you place a human in room much warmer than natural body temperature the human body operates like a refrigerator maintaining a cooler body temperature by exporting, radiating, heat, warming the already warm room. The same human placed in a cold room operates like a furnace internally generating heat to maintain natural body temperature. The human body radiates heat to the cooler environment. At least in these limited images the tendency of 'body plus environment' toward static equilibrium is not well-represented.

It is in the contrast between the glass of water and the human organism that I become more sympathetic to Atkins hypothesis as to the relation between the two current formulations of thermodynamics: (Atkins) "Thermodynamics still has both aspects, and reflects *complementary* aims, attitudes, and applications."

Problems with the Clausius, Boltzmann, Gibbs thermodynamics

Although there are many critiques in the modern literature of the dominant CBG mechanical thermodynamics I don't feel that my theme is much advanced by dwelling on them. But I will at least mention a few. First, Clausius's 'increasing entropy' only applies to closed or isolated systems, and common-sense observation suggests that there aren't any. Closed and isolated systems are idealizations.

Sean Carroll (Cal Tech) argues [6] that Boltzmann's H-Theorem presupposes what it is intended to prove. He takes Loschmidt's argument that you can't derive time-asymmetry from time-symmetry (time-irreversible processes from time-reversible processes) to suggest that there is an equal probability that entropy could decrease or increase. Carroll also notes that there are numerous definitions of entropy.[7]

Marko Popovic provides one of the best reviews of the status (2017) of the entropy concept.[8] In addition, Carroll points out that if we were to just come across a universe with the same composition as ours, its most likely state, indeed, its overwhelmingly most probable state, would be equilibrium. How then do we account for the observed non-equilibrium state of our universe? Carroll goes on to point out that if Boltzmann's account of the laws of mechanical thermodynamics is correct there is no obvious account of the non-equilibrium state of our universe. Put more strongly, there is *no possible account* of our non-equilibrium state. David Albert (Columbia) has posited as the Past Hypothesis that we just happen to find ourselves in a universe that started in an extremely low entropy state. [7] This, of course, is precisely what needs to be explained.

One of my favorite candid reflections comes from Brian Greene (Columbia) after he first learned of Loschmidt Paradox. [8] "When I first encountered this idea many years ago, it was a bit of a shock. Up until that point, I had thought I understood the concept of entropy fairly well, but the fact of the

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[7] Carroll (2010) "Nevertheless, there's no question that the Gibbs formula for entropy is extremely useful in certain applications, and people are going to continue to take advantage of it. And that's not the end of it; there are several other ways of thinking about entropy, and new ones are frequently being proposed in the literature. There's nothing wrong with that; after all, Boltzmann and Gibbs were proposing definitions to supersede Clausius's perfectly good definition of entropy, which is still used today under the rubric of "thermodynamic" entropy." *From Eternity to Here: The Quest for the Ultimate Theory of Time* (p. 171). Penguin Publishing Group. Kindle Edition.

[8] Popovic, Marko, "Researchers in an Entropy Wonderland: A Review of the Entropy Concept" arXiv:1711.07326v1 [physics.chem-ph, doi.org/10.48550/arXiv.1711.07326

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matter was that, following the approach of textbooks I'd studied, I'd only ever considered entropy's implications for the future. And, as we've just seen, while entropy applied toward the future confirms our intuition and experience, entropy applied toward the past just as thoroughly contradicts them. It wasn't quite as bad as suddenly learning that you've been betrayed by a longtime friend, but for me, it was pretty close." (*Fabric of the Cosmos*, page 168)

A simple way to understand the CBG argument that the entropy of the universe tends to maximum is that it tends to an equilibrium state. At equilibrium everything is the same. The trend is from differences to sameness. Albert's Past Hypothesis is simply asking how the differences were created in the first place, if everything is trending to sameness. Maxwell investigated the possibility of what came to be called Maxwell's demons who would be able to increase differences. Neither Maxwell nor any of the many later physicists who worked on the question succeeded in finding a way for the demons to create any net differences.[9]

PART TWO

To understand Sadi's essay and the real engineering thermodynamics, we need to take an historical deep dive. Engineering thermodynamics has a remarkably long history.

Through my research over the years, I became aware of three relevant historical periods. The first encompassed Sadi's father, Lazare Carnot, and the engineers at the Ecole Polytechnique, 1780-1824. Second, earlier, was the period 1650-1750 initiated perhaps by Christiaan Huygens, but included the Johann and Daniel Bernoullis, Jean le Rond d'Alembert, Isaac Newton and Gotfried Leibniz. The most extensive conceptual articulation of this second period came from Leibniz. Lazare Carnot recognized his connection to the project initiated but left incomplete by Leibniz and his collaborators. Finally, the third period reaches back to ancient Greek geometry. I had been struck by the fact that all these historical contributors mentioned so far claimed an intellectual ancestry to ancient Greek geometry, in particular, but perhaps only symbolically, to Archimedes.

I became aware that both of Atkins's two histories of thermodynamics had much longer histories. Cardwell had speculated that the real history of thermodynamics involved the entire history of machines. Gradually I began to suspect a pattern that could be organized around two traditions: the Analytic and the Synthetic. And I came to the hypothesis that mechanical thermodynamics had its history in the Analytic tradition, and engineering thermodynamics was seated in the history of the Synthetic tradition.

Bejan's Clue – Finite-Size and Finite-Time

These different individuals and isolated pieces of the history of thermodynamics came together for me following a recent encounter in 2022, with Adrian Bejan, a professor of mechanical engineering at Duke University. Bejan had been struggling to articulate an engineering version of thermodynamics from early in the 1980s. Although Bejan is well-known and celebrated in the engineering community, he is largely unknown or dismissed in the science community.[10]

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[9] I explored these issues in a 2019 presentation at a conference in St. Petersburg celebrating the 150th Anniversary of Mendeleev's period table. I pointed out that the emergence of very hot stars in a very cold space was evidence that the universe evolved in precisely the opposite direction expected by the entropy increase model of mechanical thermodynamics. What was emerging was new more powerful gradients, dynamic energetic differences. My presentation is available on YouTube: https://youtu.be/fKWLoB0ZUtw?si=hVhshEGw5BEbsXIz

[10] Adrian Bejan, Ph.D., the J.A. Jones Distinguished Professor of Mechanical Engineering at Duke University in North Carolina, has been named the 2024 recipient of The American Society of

What I refer to as 'Bejan's clue' is something he mentions early on, repeats several times over the years and lately emphasizes: the importance of his 'embrace of the finite'. The theme of the clue is central to his 1995 book *Entropy Generation Minimization: The Method of Thermodynamics Optimization of Finite-Size Systems and Finite-Time Processes*.

In 1996 he proposed his 'Constructal Law', to express his core engineering thermodynamic insight. Although the Constructal Law is much celebrated within the thermal engineering community, I found it difficult to understand it succinctly in the broader philosophical context. I recently published my initial attempt to grasp of his core insight.[11]

Over time Bejan has come to appreciate that he has an intellectual 'tiger by the tail'. I want to praise Bejan for his humility in sharing his ongoing efforts to understand and to properly represent engineering thermodynamics.

Two Historical Traditions – the Analytic and the Synthetic

For me, somewhat in hindsight, it has been this commitment to finitude that has opened the path to integrating the diverse contributions to the history of engineering thermodynamics. For me, Bejan's clue, was analogous to a rosette stone.

Bejan soon realized that his fundamental commitment to finitude was not new: "Object and finite size are old concepts in human thought, much older than the mirage of the infinitesimal size. … Most of this work (i.e. constructal law) could have been done two centuries ago, before thermodynamics. It is a mystery that this was not done then."[12]

I had occasion to mention to Bejan in 2022, something that by then he had already become aware, that much of engineering thermodynamics based on finitude, rejecting infinitesimals, was developed two centuries earlier in the work of Lazare and Sadi Carnot. Bejan, like Lazare Carnot, has recognized the historical link to the contributions a century earlier of Leibniz and his cohort of colleagues, Huygens, the Bernoullis, Newton and d'Alembert.

Mechanical Engineers (ASME) Medal. The award, established in 1920, is the highest award that the Society can bestow and recognizes "eminently distinguished engineering achievement." Bejan is honored for unprecedented creativity, breadth, and permanent impact on engineering; for developments in the new science of energy, motion, form, and evolution; and for building bridges to design in biological, geophysical, and sociological systems. An eminent scholar in his field, Bejan is credited with several groundbreaking developments. He unified thermodynamics with heat transfer, fluid dynamics, and the science of form (i.e., flow configuration, image, design), as a counterweight to the doctrine of reductionism; discovered, taught, and applied the Constructal Law of evolution in nature; and brought together biologists, physicists, engineers, sociologists, philosophers, economists, managers, and athletes with creative books for the public, including *Design in Nature* (2012), *The Physics of Life* (2016), *Freedom and Evolution* (2020), and *Time and Beauty* (2022). His influential work and prolific publication record have earned him 18 honorary doctorates from 11 countries. He holds a position among the top 0.01% of most-cited and impactful scientists, is the sixth most impactful scholar in mechanical engineering worldwide, and the 11th across all engineering disciplines, according to the citations impact database in PLOS Biology. Bejan earned his B.S., M.S., and Ph.D. degrees at the Massachusetts Institute of Technology in 1971, 1972, and 1975, respectively.

[11] Bristol, Terry (2024) "What hath Bejan wrought?" International Communications in Heat and Mass Transfer Volume 156, August 2024 doi.org/10.1016/j.icheatmasstransfer.2024.107641

[12] Bejan, Adrian (2000) *Shape and Structure from Engineering to Nature*, page 313.

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The hint to the earliest period came from the fact that all the historical contributors mentioned so far explicitly claimed intellectual ancestry to the ancient Greek geometers, symbolically to Archimedes. The historical path of engineering thermodynamics can be seen to reach back 2500 years.

Bejan's 'clue', the 'embrace of the finite', was at the same time a vehement rejection of infinitesimals.[13] Following what I had learned from tutorial sessions with Antonito Drago (Naples, Pisa), a scholar of the Carnots, I knew that there were two historic traditions that reasoned oppositely about the finite, infinitesimals, and infinities. The Analytic tradition reasons in terms of completed infinitesimals and infinities. The Synthetic tradition reasons about infinitesimals and infinities as inherently incomplete; 'as small, or large, as you like', but always finite. In probing Bejan's clue, I organized my investigation following the Analytic Synthetic themes.

I will argue that mechanical thermodynamics rests fully within the Analytic tradition, and engineering thermodynamics, including Sadi's contribution, is contained in, to be identified with the Synthetic tradition.[14]

Two Approaches to Quadrature

The difference between the Analytic and Synthetic traditions is illustrated in their different methods of reasoning in the ancient problem of determining the area of a circle. The common initial method was to place a polygon, say a square, inside a circle, touching the inner edges of the circle. The area of the square is a very rough first estimate of the area of the circle. Next, one increases the number of sides of the polygon, to a pentagon, hexagon, and so forth. As you increase the number of sides, again with the vertices touching the inside of the circle, the area of the polygon more closely approaches the shape and the area of the circle.

Now, in the Analytic tradition, you imagine that you can complete this process, allowing the polygon to have an infinite number of sides with the length of the outer sides of the final polygon as infinitesimals, then you succeed in expressing the area of a circle in terms of linear constructions. The sum or integral of the inscribed components of the polygon with a completed infinity of number of sides corresponds to the area of the circle. This procedure and reasoning, allows one to express the area of the circle in terms of linear constructions. Generalizing the method of reasoning, one can express the area under any continuous curve filling the area under the curve with an infinite number of infinitesimal linear constructions, then summing or integrating. This method of reasoning is well-known and fundamental to Analytic Calculus.

The Synthetic tradition insists that infinite series, toward the large or small, are always incomplete. A common expression is that an infinite series has no final step, can never be completed. The series of inscribed polygons with an increasing number of sides is always incomplete, getting 'as close as you like' but never reaching equivalence with the circle. The Synthetic tradition rejects the idea that there could be a polygon with an infinite number of infinitesimal sides inscribed in a circle.

The Synthetic approach to quadrature is fundamentally different and is typically identified with Archimedes. [10] His approach requires two opposite processes. He both inscribes polygons with

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[13] Bejan, Adrian, (2019) *Freedom and Evolution*, page 223: "The 'fundamental' is the truth that lies as 'foundation' (means bottom). The secret–the brick–is of finite size not infinitesimal. The difference between finite size and infinitesimal is like the difference between black and white, night and day, dead and alive, and pregnant and not pregnant. The infinitesimal does not have freedom, flow, organization, and evolution. The finite size does." "The fundamental is the building block. The brick is macroscopic, not infinitesimal."

[14] The Analytic and Synthetic traditions were explicitly discussed since the time of Pappus of Alexandria (died 350 BC). Pappus wrote *Mathematical Collections*. In Book VII he provides one of the most extensive discussions of the methods of analysis and synthesis in geometry.

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increasing number of sides and circumscribes polygons with increasing number of sides. The inscribed and circumscribed polygons can 'get as close as you like' to the circle but never reach it. Each process, each series, is always incomplete. What Archimedes method establishes is not a fixed area for a circle but a ratio, the ratio of the linear diameter to the circular circumference of the circle. Archimedes is asking a different type of question. He is asking for the ratio of a line and a circle where the line (the diameter) and the circle (the circumference) are co-defined ideals. These are ideals because for Greek geometry there are no perfect lines and no perfect circles. By using a reductio argument, characteristic of all Synthetic geometric proofs, Archimedes reasons that the ratio cannot be used to express a fixed area. Indeed, Archimedes ratio cannot be a fixed quantity at all.

The Analytic method reasons that the ratio is a definite number, pi. With a definite number it becomes possible to calculate the supposed definite area of the circle. However, consistent with Archimedes' Synthetic reasoning, pi, is not a definite number. We call it an irrational number, even more uniquely, it is a transcendental irrational number. Apparently, the area of the circle must also be irrational.

For the Analytic tradition the area of the circle, or the area under any continuous curve, can be expressed by means of an infinite series of infinitesimal linear constructions. The crucial consequence is that the reasoning adopted by the Analytic tradition *erases the conceptual difference* between lines and circles (curves). As an undergrad I characterized the analytic 'limit' argument as 'jumping over the asymptote'. The idea of an asymptote, or asymptotic limit, is that you can approach it, getting closer and closer but never reach it. For the Synthetic tradition lines and circles are essentially different, conceptually, qualitatively distinct, *by their very nature*. Indeed, the mathematics of lines and the mathematics of circles are conceptually different, so that you can't express one in terms of the other, you can't legitimately reduce one to the other.

Awareness of the difference between the Analytic and Synthetic approaches reaches far back in history. That lines and circles are incommensurable, indeed co-defined opposites can be traced back through Pappus and Archimedes, back to Eudoxus and Apollonius.[15]

Eli Maor emphasizes the Greek 'horror infiniti'.[16] [11] John Stillwell [12] refers to the Greek 'fear of the infinitesimal'. By excluding infinitesimal reasoning, completed infinitesimal processes, the Greeks preserve the conceptual distinction, the conceptual opposition of lines and circles, of translational and rotational motion.

The Synthetic tradition's insistence on the incompleteness of reasoning about infinitesimals has two closely related consequences, essential in understanding Sadi's engine. First, real entities are not infinitesimals, but finite. What is observable is macroscopic. What can be reasoned about is macroscopic. The finiteness of the elements of his ontology is clearly expressed in the subtitle of Bejan's 1995 book [13], where he emphasizes 'finite-size systems and finite-time processes'. Bejan again insists that infinitesimals are 'not observable realities', not real. The supposed ontology of the Analytic tradition, such as Newton's 'point masses', is composed of unobservable infinitesimals. For Bejan's engineering thermodynamics and the Synthetic tradition whatever is observable, whatever is actual, 'real', must be finite, macroscopic.

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[15] Archimedes method of exhaustion originated with Eudoxus. The first theory of ratios and proportions, Books IV and V of Euclid's *Elements*, are due to Eudoxus. Two aspects are worth mentioning: First Eudoxus's 'ratios' are *composites of incomplete inscribed and circumscribed processes*. Second, ratios are between finites, infinitesimals are excluded. Eudoxus's theory led Apollonius to develop a new type of geometry of conics where the curves are composites.

[16] Maor also quotes, Anaxagoras (500-428 B.C.) "There is no smallest among the small and no largest among the large; But always something still smaller and something still larger." In Eli Maor (1991) *To Infinity and Beyond. A Cultural History of the Infinite*. Page 2

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The second powerful consequence of the Synthetic tradition's insistence on the incompleteness of reasoning about infinitesimals is an unexpected bonus: the reality of conceptual opposites. As reasoned, so far, these opposites are lines and circles. What is most interesting and unexpected, for me at least, is that 'embrace of the finite' entails an essential, irreducible ontological duality in the composition of motion and forms. All finite-time processes, motions, trajectories or paths are composite, having two opposite *types* of components. One simple way to express this is to say that all motion (change of state) has both a linear and circular component. All finite-size forms, structures or systems are composed of two opposite *types* of dynamic components. As I will elaborate, forms are couples, involve cyclic coupling. Per hypothesis, these are engines.

You can't linearly add, as in vector calculus, lines and circles. This is why for Lazare, Newton and Leibniz the parallelogram rule fails. Euler proposes to avoid the problem using completed infinitesimal reasoning to express circles in terms of linear constructions. Euler's analytic calculus preserves the parallelogram rule.

For an intuitive appreciation of the difference between the Analytic and Synthetic traditions, the reader might recall that after your first or second course in Euclidean geometry, you were perhaps shocked to hear, to realize, that as a matter of fact, there are no actual straight-lines and no actual circles in reality. The geometry that we are taught in high school is not the original Synthetic geometry found in Euclid's *Elements* but Euler's Analytic reinterpretation.

In Synthetic geometry lines and circles are ideals, differential forms. All real circles have an irreducible linear component, and all real lines have an irreducible circular (angular) component. The Analytic approach, erasing the difference between lines and circles, erases the difference between purely linear, ideal translational and purely circular, ideal rotational motions. In the Synthetic perspective all motions and structures are composite, with irreducible translational and rotational components.

A Newtonian Reflection

To better understand my brief characterization of the Analytic and Synthetic traditions, I began to reflect on Newtonian physics, which initially I took to be classical mechanics and a prime representation of the Analytic tradition. I must confess that what I found was that the 'real' Isaac Newton of his *Principia*, was fully in the Synthetic tradition. Robert Westfall, in his biography of Newton, [14] remarks that commentators on the *Principia* have expressed surprise and are confounded by the fact that it contains no Analytic calculus.[17]

[17] Westfall (*Never at Rest*): "The mathematics of the *Principia* has always presented something of a puzzle. Why did the inventor of the calculus present his masterwork in the dress of classical geometry? Newton himself compounded the puzzle by several self–serving assertions at the time of the calculus controversy."

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Both Newton and Leibniz explicitly rejected two of the core presuppositions defining Analytic calculus: infinitesimals and the parallelogram rule.[18] [19] [20]

I must admit to being rather shocked to realize that the Newtonian physics that I had been taught from early on through much of my university education was Leonard Euler's Analytic re-definition of Newton's Synthetic based mathematical physics. Whenever one encounters such a shocking revelation it is most reassuring to find that at least one other respected scholar, in this case Marius Stan (Boston College) who has, also quite recently come to the same conclusion.[21]

Most of us educated in the sciences and mathematics were told that Newton had invented Analytic calculus, along with Gottfried Leibniz. What comes to mind here is Kuhn's remark that those of us trained in the sciences and mathematics were not so much educated as indoctrinated.

The relevant point that supports the main theme of this essay is that Euler's Analytic rewrite of Newton's Synthetic physics and calculus must have played a major role in the misrepresentation and misunderstanding Sadi's Synthetic engineering thermodynamics. I will address this more directly below.

A Reflection Newton-Euler Mechanics

Nonetheless, a useful way to explore what I have so far developed is to look at Euler's Analytic version of Newtonian physics. What is commonly taught is that the straight-line motion in Newton's First Law is to be understood as a series of infinitesimal linear steps. However, each infinitesimal step does not embody any distance or length; it's like a razor-sharp cut. Each infinitesimal step is also instantaneous and does not embody any duration. Each instant is static in itself. Each infinitesimal step is a sort of spatial and temporal cut embodying 'nothing'. The criticism of this representation is not new. In Zeno's Arrow Paradox [15], the sum, or analytic integral, of these infinitesimals is still without length and duration, it's a spatial-temporal nothing. What I want to emphasize here is that each infinitesimal 'cut' lacks Bejan's finite-size and finite-time.

Like his unobservable point masses, Newton-Euler's straight-line motion is differential, an idealization. What is missing in the straight-line idealization? There are at least two ways to express

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[18] Westfall (*Never at Rest*) notes that "To replace the parallelogram of forces, he [Newton] devised a further law of motion which has come down to us in a different wording as the third law." Westfall location 11227, Kindle edition. And further that "One of the first problems Newton faced as he wrote the expanded work was its mathematical foundation. *De motu* had employed an implicit method based on differential segments of curves. Newton now formalized the approach by drawing on the concept of first and last ratios, which he had developed to eliminate infinitesimals from his fluxional calculus. He embodied it in eleven lemmas which he later designated as Section I of Book I." Westfall location 11293.

[19] Worth noting is that the 'concept of first and last ratios' seems obviously analogous to Archimedes' method of inscribing and circumscribing.

[20] Leibniz states his view that 'infinitesimals are fictions' in Philos Schriften, Vol VII, page 90. In his *Theory of Fluxions, vol 1*, page 39, MacLaurin claims that "Leibniz owns them [infinitesimals] to be no more than fictions." In a Letter to de Volder, 19 January 1706 Leibniz refers to the problem of infinitesimals and the continuum as the "labyrinth of the continuum", Philos Schriften, Vol. II, page 282.

[21] Marius Stan, (2021) "Euler, Newton, and Foundations for Mechanics", in *The Oxford Handbook of Newton*, eds. C. Smeenk and E. Schliesser. Stan points out that "key 'Newtonian' results are really due to Euler."

what is missing, what has been 'idealized out'. First, recalling that for Bejan and the Synthetic tradition, real motion involves an essential composite duality of linear and curvilinear components. Newton-Euler's straight-line idealization lacks the curvilinear, angular or rotational component. To handle curvature Newton had to introduce to an entirely separate, conceptually incommensurable, theory of gravity. Second, since each of Newton-Euler's linear steps is instantaneous, an infinitesimal instant, what is also missing is actual time, duration. Newton-Euler's straight-line motion, by infinitesimal steps, has no time component. The natural state of the Newtonian motion is not only spatially uniform but also temporally uniform. Without intervention of a new external force motion is essentially uniform, by its very nature, there is neither acceleration nor deceleration.[22] [23]

In the Synthetic tradition and for Bejan, real motion is *composed* of the two geometrically opposite, linear and curvilinear components as well as a finite time component.[24] One of the 'real' Newton's commentators was Colin McLaurin (1698-1746) who points out that Newton's fluxional calculus, using ratios of converging series, is analogous to Archimedes' method.[25]

Euler and Lagrange

As suggested, Euler rewrote Newton's Synthetic geometric mathematical physics in the Analytic form using completed infinitesimal reasoning. Again, I personally found it a bit shocking to realize that Newton and Leibniz both rejected infinitesimals and the applicability of the parallelogram rule.

It is fair to say that Euler is the real father of Analytic calculus and laid the foundations for modern Analytic vector calculus. On the other hand, it was Lagrange who shamelessly extended it, inventing variational calculus and giving a mechanical interpretation the principle of least action, placing it in a foundational role. Euler and later Hamilton pointed out that Lagrange's 'action' could be either least or most.

Recognizing that all classical Eulerian laws are expressible as differential equations, we gain an appreciation of why, all classical transitions are necessarily time-reversible (or time-independent).

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[22] Kepler discovered empirically that the velocity of Mars in its orbit was not uniform but accelerated as the planet approached the Sun and decelerated as it moved away. Previously it was assumed that celestial motion was perfectly circular and uniform.

[23] Preference for the linear frame of reference is captured in Galileo's Boat theme where he argues that physical experiments are the same whether on land or in a ship traveling in a straight-line with a constant, uniform velocity. Galileo Galilei (1632) *Dialogue Concerning the Two Chief World Systems*. This is Galileo's equivalence principle. In Emmy Noether's famous theorem, she argues that the uniform passage of time is what entails conservation of energy. See Dwight E. Neuenschwander (2018) *Emmy Noether's Wonderful Theorem*.

[24] In Bejan's engineering thermodynamics, transitions are always both path-dependent and time-dependent. In classical mechanical thermodynamics transitions are path-independent and time-independent. Bejan tells us the story of his moment of his revelation that real, engineering thermodynamic transitions (changes) were path-dependent and time-dependent.

[25] Colin MacLaurin (1698-1746) *A Treatise on Fluxions* (1742) "Sir Isaac Newton accomplished what was wished for, by inventing the method of fluxions, and proposing it in a way that admits of strict demonstration, which requires the supposition of no quantities, but such as are finite, and easily conceived. The computations in this method are the same as in the method of infinitesimals; but it is founded on accurate principles, agreeable to the ancient geometry." location 1248, Kindle edition. *Maclaurin* proposed a rigorous foundation for the method *of fluxions* based on Archimedean classical geometry.

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Bejan repeatedly points out that there is no net change, no net duration, no qualitative evolution, in a worldview built on infinitesimals.[26] [27]

Per hypothesis, the dominance of the Euler-Lagrange mechanical physics and mathematics has been a primary factor in the dominance of the mechanical formulation and the two hundred years of misrepresentation of Sadi's engineering thermodynamics.

In the Preface to his *Méchanique Analytique*, Lagrange famously remarks "The reader will find no figures in this work. The methods which I set forth do not require either constructions or geometrical or mechanical reasonings: but only algebraic operations, subject to a regular and uniform rule of procedure." Lagrange's statement amounts to a complete rejection of the Synthetic geometric tradition. This works for Lagrange because, using completed infinitesimal reasoning, he erases the conceptual difference of lines and circles. He can express any circular component as the sum or integral of infinitesimals.

One of Bejan's most powerful insights is that by eliminating 'figures', Lagrange can give no account of the shapes and structures of reality. Bejan's further epiphany comes with his critique of Clausius's concept of entropy. "To account for coupled thermomechanical behavior he [Clausius] had to formulate a second principle, the second law, in addition to the conservation of energy. With his new principle came the concept of entropy, which was completely foreign to science. Today the new principle is the construction of geometric form, and the new concept is objective, or purpose."[28]

Bejan realized that since there are no finite entities in the infinitesimal worldview, there are no shapes, no structures. Historically, this became embarrassingly apparent with the three-body problem, which entails recognition of a shape. What is important to realize is that the classical Analytic program doesn't provide *any* account, any understanding of the geometric forms, the shapes, structures and functional processes that we commonly observe in reality. This is a consequence of the Euler-Lagrange program. By eliminating entropy and with no need for Clausius's second law, Bejan's engineering thermodynamics looks very much like Sadi Carnot's.

Bejan comes to view his engineering thermodynamics as a 'science of form'. The mature Leibniz tells us his whole philosophy can be understood as a 'science of form.' Bejan, and arguably Leibniz, both realize that "natural macroscopic structure is not only spatial but also temporal."[29]

In trying to understand Bejan's insights and engineering thermodynamics research program, what only lately struck me is that Bejan is asking new types of questions about reality, and offering new types of answers, new accounts of what we observe in reality. He is concerned to understand the shapes and structures of reality, and their evolving designs, in terms of purposeful functions. These questions don't come up, don't make sense in Euler's Analytic representation of reality.

As Lazare and Sadi Carnot had recognized, the ontology of engineering thermodynamics includes processes and forms that are cyclically coupled [metabolic] engines. Lacking the holistic perspective 'rational mechanics' is inherently unable to make sense of the engine.

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[26] Bejan, Adrian, *Freedom and Evolution* (p 223) "The infinitesimal does not have freedom, flow, organization, and evolution. The finite size does."

[27] I exclude quantum theory from classical mathematical physics. Quantum theory is a thermodynamic theory, but, as Bohr emphasized, it is not a classical mechanical type of theory. In quantum theory changes are not time reversible. I have recently argued [17] that quantum theory can only be properly understood in terms of engineering thermodynamics.

[28] Bejan, Adrian. *The Physics of Life: The Evolution of Everything* (p. 240). St. Martin's Publishing Group. Kindle Edition.

[29] Bejan, A. (2000) *Shape and Structure, from Engineering to Nature*, Cambridge University Press. page 3

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At one point Bejan asks what the engineer brings to the already, apparently, full table of modern Euler-Lagrange Analytic physics. He appropriately answers, *time*. In engineering thermodynamics time is irreversible, allowing us to make sense of the evolution of systems. But something else is left out of Euler's differential straight-line: the curvilinear component, I think the engineer brings something more to the table than time. Initially I thought it was just finite distance coming along with finite duration. Then I thought it must be a volume. Now I imagine that what the irreducible combination of a linear and curvilinear components is bringing to the table is form: shapes and structures.[30] [31]

The Most Recent Historical Period Leading to Sadi's Essay

Of the three historical periods I will review to illuminate Sadi's essay, the first is the closest in time. Gillispie and Pisano make a strong case for the common intellectual agenda of Lazare and Sadi Carnot. Earlier Gillispie [19] wrote about the father-son relation: "It appeared to me that Sadi Carnot's analysis may be read as an application of his father's invention of the science of machines to heat engines. In the new book, Gillispie continues: "I have suggested as much in what follows."[20]

In the Preface to his mature 1803 essay, *Fundamental Principles of Equilibrium and Motion*, [21] Lazare begins: "There are two ways of looking at mechanics and its principles." He specifies: "The first is to consider it as the *theory of forces*, that is, of the causes that impart motions. The second is to consider it as the *theory of motions* themselves. In the first case, the reasoning is based on whatever causes, which impress or tend to impart motion to the bodies, to which they are supposed to be applied. In the second case, we look at the movement as already imprinted, acquired and residing in the bodies; and one seeks only the laws according to which these acquired motions propagate, are modified or destroyed in each circumstance."[32]

The distinction should be familiar. It's between classical mechanics in the Analytic tradition and his proposed engineering mechanics in the Synthetic tradition. The latter becoming engineering thermodynamics.

He rejects the first way, classical mechanics, as being based on: "a metaphysical and obscure notion of *forces."* And on the related concept of cause: "The notion of the relation of forces considered as causes is therefore no clearer than the notion of forces themselves."

An additional comment is particularly important: "All the demonstrations where the word *force* is used have an absolutely unavoidable obscure nature: and that is why, in this sense, there cannot be, for example, according to me, any rigorous demonstration of the parallelogram of forces." To reject the parallelogram rule is equivalent to rejecting linearity, the primacy of vectors and vector addition.

Instead of the A causes B (A–>B) of mechanics, Lazare moves to a holistic perspective: "if we take the standpoint of not distinguishing the cause from the effect" such that as A impacts B, B equally impacts A, "then the reasoning becomes intelligible, but then we return precisely to the second way

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[30] Bejan: "Today, in courses and textbooks, I teach the philosophy that the science of form is a discipline. I did not discover the discipline—this took shape naturally in the evolving drawings of many scientists, most notably Gaspard Monge (1746–1818), the founder of descriptive geometry. *Freedom and Evolution: Hierarchy in Nature, Society and Science* (p. 151). Springer International Publishing. Kindle Edition.

[31] Although I am focusing here on Lazare, there was a group at Ecole Polytechnique all inspired by Gaspard Monge, including Navier, Coriolis, Ampere and Poinsot.

[32] All the quotation in this section are Lazare's, from the Preface of his 1803 *Fundamental Principle of Equilibrium and Motion*.

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of considering the question, that is to say, that then mechanics is nothing but the theory of *the laws of the communication of motions*."

Next Lazare distinguishes his approach from that of Lagrange: "My theory could not be based on the principle of virtual velocities, whose importance nowadays is well known by the happy use made of it by Lagrange in its *Mechanics*, but which is not applicable without modification to the impact of bodies. I, therefore, have started from a different principle, but which is very analogous, or, rather which is only this same principle of virtual velocities extended properly; this generalization consists in substituting for the *virtual* velocities, which are infinitely small, *finite velocities* which I have called geometric."[33]

Lazare continues: "The result is a sort of new theory of a class of motions, which is less the domain of mechanics than of geometry."[34]

Geometric Motions

Understanding what Lazare calls 'geometric motion' will help us understand Sadi's cycle. Geometric motion concerns the interplay between any two of his ontological entities. In mechanical causality A impart a force to B (A–>B), accelerating B. In Lazare's holistic perspective what A imparts to B, B equally impart to A (A<–>B). In the first case A transfers and loses energy to B. In geometric motion the interplay is a 'mutual exchange' with the net energy loss being zero. Per hypothesis, what is mutually exchanged between A and B is living force which is conserved.

Lazare offers a generalization of the paired or coupled geometric motion. "The sum of the quantities of motion of any number of bodies, estimated in a given direction, does not change, in spite of the collisions or the mutual action of the particular bodies, until an external agent disturbs them." The historical link to Huygens' 'central reasoning' is even clearer in Item 105: "*In any system of forces in equilibrium around a given point, the sum of the moments of the forces with respect to any axis drawn in the space is equal to zero.*"

Lazare emphasizes that the geometric motion of his ontological elements cannot be understood as mechanical. Lazare's rejection of the parallelogram rule is equivalent to rejecting linearity and vectors. The 'actions' between the ontological elements is composite with an irreducible linear and circular component. Lazare calls the circular component the 'angular' component of the composite. He characterizes the paths of the A<–>B exchange as occurring along complementary angles, acute and obtuse. If the interplay was mechanical with two linearly opposite mechanical forces on a straight line, the result would be a static equilibrium. Instead, the result of the geometric exchange is a dynamic equilibrium, an ongoing cycling.[35]

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[33] Lazare is making the same shift to finitude made by Bejan, Newton and Leibniz, Archimedes and Eudoxus. This underscores the value of studying the history of physics, mathematics and engineering.

[34] The link to Huygens and his 'central reasoning is clear in Item 105 of *Fundamental Principles*: "*In any system of forces in equilibrium around a given point, the sum of the moments of the forces with respect to any axis drawn in the space is equal to zero.*" This is much more general than a simple linear, static equilibrium.

[35] *Fundamental Principles*, COROLLARY V P. 220. "Whenever the forces in equilibrium around a machine are reduced to two, whatever may be the geometrical motion transferred to the system, it is clear that, according to the theorem, one force will be soliciting and the other resisting; that is to say, that one of these will make an acute angle with the direction of its velocity and the other an obtuse angle with the direction of its velocity; in fact, the product of a force by its velocity estimated in the direction of its force is equal to this same force multiplied by the cosine of the angle that it forms with its velocity

What Lazare refers to "*moving forces* [motive forces] are comparable to weights, so far as they are applied to machines to overcome resistances or to produce whatever motions; and as men, animals, water flows, wind, spring, water reduced to vapor by fire, etc., are often employed for this object, it is to these agents, or rather to the effects they immediately produce on the bodies to which they are applied, that we give the name of moving forces."[36]

Lazare adds, "The agent is called *the motor [engine]* whilst the pressure that it exerts, or the motion it produces, is called the *moving force*. What is called the *living force of the body* is the product of its mass multiplied by the square of its velocity." He points out the conceptual novelty of living force as a "new species of quantity". "Hence, it is on these considerations that the notion of living forces is based."

Lazare continues: "Experience proves that men, animals and the other agents of this nature can exert forces comparable to those of weights, either through their own weight or through the spontaneous efforts of which they are capable. This new way of understanding forces [living forces] is therefore at least as natural and as important as the former [mechanical] one."

Sadi's classmate at Ecole Polytechnique, Andre-Marie Ampere, coined the expression *kinematics* to characterize 'communication of motion'. The laws governing communication of motion make no reference to 'external forces causing' the changes.[37]

To gain a modern intuitive sense of kinematics, notice that both special and general relativity are kinematic, eschewing mechanical forces.

Lazare emphasizes: "The result is a sort of new theory of a class of motions, which is less the domain of mechanics than of geometry. These geometrical motions are those which the different parts of a system of bodies can undertake, without interfering with one another, and which consequently do not depend on the action and reaction of the bodies, and can be determined by geometry alone, and independently of the rules of dynamics." (Preface)

Louis Poinsot, another of Sadi's classmates, in his 'geometrical mechanics', went on to characterize Lazare's treatment of the interplay (exchange) of any two elements in his ontology in terms of *couples*.[38] Per hypothesis, couples are the embodiment of duality in forms.

Ampere also focused on the concept of couples and coupling. His characterization appears to be similar to Carnot's geometric motion. "A couple is a pair of forces, equal in magnitude, oppositely directed, whose lines of action do not coincide. Although there is no resultant force on the system, these opposite forces produce a torque about an axis which is normal to the plane of the forces." The 'torque normal to the plane of the forces' is reminiscent of Lazare Carnot's 'motive force'.[39]

My Imaginary Transition from Particle Ontology to the Agent-Engine Ontology

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(26). Hence the sum of these two products cannot be reduced to zero, without one of the cosines being negative; that is to say, without one of the angles being obtuse whilst the other is acute."

[36] *Fundamental Principles,* Item 53.

[37] Ampere, A-M (1834) Essai sur la philosophie des sciences.

[38] Louis Poinsot, *Eléments de statique*, 2nd ed (1811, preface) "One should do well to convince oneself that the consideration of couples is not that of a singular case, but of an essential element that is lacking in mechanics. Cited in *I. Grattan-Guinness / Historia_Mathematica 41(2014) 82–102))*

[39] Assis, AKT and JPMC Chaib (2011) *Ampere's Electrodynamics: Analysis of the Meaning and Evolution of Ampere's Force between Current Elements, together with a Complete Translation of His Masterpiece, Theory of Electrodynamic Phenomena, Uniquely Deduced from Experience*; p 257

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I had for some time searched for a logical or rational path from the particle ontology of the mechanical framework to the agent-engine ontology of engineering framework. I have found that the relation between these two is not simple or straight forward. If Atkins is right that they are complementary, then it may be that there is no way to provide a logical path from one to the other. In Kuhn's terms the two conceptual systems are incommensurable

In my struggles, I did develop some imagery, although not rigorous, it is still, I think, instructive and illuminating.

Lazare highlighted the equality of cause and effect. In classical mechanics the standard image is that causes B; A–>B. There is a transfer of force or energy in contact causality. In the new framework there is a shift to an 'exchange paradigm' such that as A hits B, B equally hits A; A<–>B. In this holistic perspective on interactions, the net change is zero. Lazare argues that for a system of bodies interacting, exchanging, with each other, the net energy exchange is zero. He refers to the motion of a system of bodies so conceived as geometric motion.

I imagined the paradigm shift as follows: a Newtonian particle is acted on by an external force that would normally, classically, accelerate it. However, at the same time there is friction, I imagined to be an equal and opposite resisting force. These forces would balance resulting in a static equilibrium. To make the shift from the particle ontology, I pictured the two opposing forces to be *'infolded'*. The result of this *'infolding'* is the creation of a new ontology of composite entities that embody two opposing forces.

In Lazare's framework the infolded forces are in geometric motion and no longer linear opposites. Each element of the infolded entity acts on the other with a composite motion having both a linear and a circular component. The parallelogram rule is no longer applicable. These new infolded entities contain, embody, a conceptually new 'internal energy'. Lazare saw the interplay as involving an irreducible angularity, with opposite acute and obtuse angles. The overall result, I imagined, is a new ontology of coupled entities, in coupled pairs and collectively in continuous cyclic exchanges. Since what is exchanged is conserved the result is a dynamic equilibrium. The internal energy is continually, spontaneously self-renewing.[40]

There is no longer any need for external forces to initiate or 'cause' motion. In Lazare's framework of the communication of motion, motion is accepted as 'already present', as spontaneous. There are 'no pushes and no brakes'. Another, not so obvious feature is that there are no isolated systems, no isolated couples.[41] All entities, by their very nature, are 'connected' to all other entities in the universe. The easiest and historically most common image is, for instance, the human organism, each component is actively in contact with every other. The systems of the human body, the circulatory, respiratory, nervous, musculature, skeletal, etc. are also actively renewing couples. In this 'exchange paradigm' all 'energy' is internal and conserved. If we think of these entities as individuals in some sense, with their internal energy, they have some freedom, some ability to act, to perform work.

The Underground River

[40] Contrast this with Newtonian reality that requires an external force to initiate motion. Without an external initiative the Newtonian reality is unchanging—static. Newton posited that God was the external agent that set his clockwork reality in motion. Newton also suggested that because of friction, God would periodically need to re-initiate activity. It is worth noting here that 'change' in mechanics cannot be made sense of without the action of an external agent.

[41] One point worth highlighting is that Clausius's claim that 'energy is conserved, and entropy tends to a maximum' only applies to isolated systems. In Carnot's engineering thermodynamics all systems are interconnected, and so in some sense 'open'. Isolated systems are idealizations.

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Nobel economist Kenneth Arrow spoke of the periodic surfacing of intellectual traditions in the history of ideas, 'like an underground river bursting to the surface, attracting startled attention, then disappearing again'.[42]

Arrow's 'underground river' comment was stimulated by a presentation in 1929 by Allyn Young: "Increasing Returns and Economic Progress." "It was a vigorous dissent from the conventional wisdom of the day. He spoke of qualitative change, disequilibrium, increasing complexity, 'cumulative causation' – all code words for processes not yet fully understood. Returning to Harvard he boarded a ship and died of influenza in the epidemic in 1929, at the age of 53."

The conventional economic wisdom was that economics was to be understood within a mechanical framework, static equilibrium economics. Young's idea of increasing returns could not be made sense of in the framework of equilibrium economics, where all economic activity tended to move the system 'downward' toward equilibrium. His brief insight of a progressive, constructive evolutionary economics was strikingly well supported by the evidence, but incomprehensible within static equilibrium economics. Young's ideas were later developed by Paul Romer who focused on the progressive constructive nature of 'increasing returns'. Romer, 2018 Nobel Laureate in Economics [23] developed Young's ideas leading to 'New Growth Economics', a dynamic participatory evolutionary design model of economics.

Duality Rediscovered by Huygens et al.

The most powerful and well-developed resurfacing of the Synthetic tradition, recalling the ancient precursors, was in the period 1675-1750 in Europe. The prominent initial contributors included Galileo, Christiaan Huygens, Isaac Newton, Gottfried Leibniz, Johann and Daniel Bernoulli, and Jean le Rond d'Alembert. In terms of the modern perception of this period, Leibniz has survived as the most prominent 'mouthpiece' of this group. Nonetheless their ideas again went underground for a generation only resurfacing with Maupertuis, then were again underappreciated until Lazare and Sadi Carnot. Around the time of the Carnots, Gaspard Monge rediscovered Greek geometry influencing a generation of students at Ecole Polytechnique. The foundational work of Monge and the developments of the Carnots were subsequently appreciated by Claude-Louis Navier (1785-1836, Simeon Denis Poisson (1781-1840), Gaspard-Gustave de Coriolis (1792-1843), Louis Poinsot (1777-1859), Andre-Marie Ampere (1775-1836), Jean-Victor Poncelet (1788-1867) and eventually in the late 19th and early 20th centuries by Henri Poincare (1854-1912) and Pierre Duhem (1861-1916). The important point is that all these contributors claimed, usually quite explicitly, intellectual ancestry from Archimedes. They were all working in the Synthetic Tradition that developed into engineering thermodynamics.

Christiaan Huygens, a keen student of Greek geometry, was the first to bring the underground river of the Archimedean tradition once again to the surface.

In his H*orologium Oscillatorium*, [24] Huygens reported his experiments with the cyclic oscillations of the pendulum. Correcting Galileo's circular hypothesis, in 1659, he established that the up-down swings were equal only when their paths were cycloidal.[43]

The paths of the cycloidal swings, unlike purely, ideally circular swings, were composite, with two irreducibly opposite components, linear and circular. Huygens had rediscovered duality. Johann Bernoulli, 1696, proposed a problem to the community, to find the path of swiftest descent of an object in a gravitational field. The vertical option was excluded so the object's motion must always have

[42] Cited in Warsh, David, *Knowledge and the Wealth of Nations* [22]

[43] Huygens had discovered the cycloidal tautochrone curve where the time of the down swing and the upswing of a pendulum are equal.

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some horizontal linear component. This has come down to us as the brachistochrone problem. The path was composed of linear and circular (due to gravity) components. And the path was a cycloid.[44]

Johann Bernoulli recognized and was the first to publish an essay identifying the natural brachistochrone curve arguing the need for a paradigm shift from mechanics to the holistic communication of motion; the same shift argued for later by Lazare Carnot.[45]

Later in 1738, Johann Bernoulli's son Daniel Bernoulli published his famous *Hydrodynamica* [25] wherein both the composite character of changes of state and the conservation of living force were central principles.

What I have outlined here is that Huygens, the Bernoullis, and Newton along with d'Alembert and Leibniz discovered, or rediscovered, the composite duality of motion. I have argued above that duality was a fundamental feature of the Synthetic geometry of the Greeks.[46] Because the linear and circular components are conceptual opposites, of 'different species', the parallelogram rule is inapplicable. Euler and Lagrange, of course, bypass this problem by using completed infinitesimal reasoning to express the circular component in terms of linear constructions.[47]

Leibniz's Meta-Paradigm Shift: From Statics to Dynamics – Analytics to Synthetics

Princeton historian Charles Gillispie, who rediscovered Lazare Carnot for the English-speaking community [19], characterizes the 1675-1850 period as undergoing an intellectual paradigm shift from "the science of mechanics to the science of machines [engines]."

Thomas Kuhn famously characterized advances within the history of science as paradigm shifts. [26] Kuhn argued that advances, by their very nature, were revolutionary because they always involved some irreducible conceptual discontinuity. Kuhn emphasized that advances were not logico-mathematical deductions from prior concepts.

Leibniz proposed an advance from Statics to Dynamics. It was Leibniz who introduced the term 'Dynamics' into physics, engineering and philosophy. Leibniz's Dynamics is one of the clearest early precursors of what matured as engineering thermo*dynamics*. Leibniz asserted the meta-shift in his critique of Descartes who had argued that what was conserved in the universe was the product of

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[44] Lagrange: "The so-called principle of the conservation of living forces was first utilized by Huygens (1629-1695), in 1673, when he solved the problem of the center of oscillation of a compound pendulum." "He [Huygens] has also thought that this problem [on the center of oscillation of a rigid body under the action of terrestrial gravity] must be regarded as entirely new and since he could not resolve it by existing methods he invented a new but indirect principle which has become popular since then under the name of the Conservation des Forces Vives", conservation of vis viva, living force. Cited by Lagrange in his history of mechanics, in *Analytique Mechanique.* [18]

[45] Johann Bernoulii (1724), Discours sur les loix de la communication du mouvement.

[46] Newton, the real Newton, as discussed above, was also a member of this group. Newton anonymously published the solution to the brachistochrone challenge. For Newton to have solved this required recognition that there are two opposite components, confirming that the real Newton was in the Synthetic tradition. Bernoulli guessed that the anonymous author was Newton, commenting, "I recognize the lion by his claw."

[47] Looking back further in history, seeing the world in terms of pairs of opposites was foundational to Greeks physics. Opposites are co-defined, conceptual opposites, like local and non-local, so that, by their very nature, one cannot be reduced to the other.

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mass times velocity, mv, momentum, linear kinetic energy, neglecting rotation. Leibniz, following Huygens, argued that it was mass times velocity squared, mv^2, that was conserved, and he referred this as *vis viva* or living force.

What Leibniz proposed cannot be understood as an advance *within* the conceptual framework of the mechanical research program. The shift is from the conceptual system of mechanics to the conceptual system of engineering and is radically discontinuous. What Leibniz proposed should be understood as a meta-paradigm shift. It is worth recalling here the consensus of historians that thermodynamics did not arise within the mechanics research program, or from any awareness of problems within that program. Thermodynamics was discovered, or rediscovered, independently through the work of engineers like Huygens and his colleagues.

What Gillispie characterizes as 'the science of mechanics' rested within the Analytic tradition, and the 'science of machines [engines] was contained in the Synthetic tradition. The 'meta-paradigm shift' was not from *within* the Analytic framework defined by the presupposition of classical mechanics, but *from* classical mechanics to a more advanced conceptually more comprehensive engineering framework. Leibniz's shift is from the framework of the mechanical formulation of thermodynamics to the framework of the engineering formulation of thermodynamics.

The meta-shift from Descartes' linear motion to Huygens' composite cycloidal motion with irreducible linear and circular components, meant a reconceptualization of 'motion' itself. Although the terms 'motion' and 'work' occur in both mechanical thermodynamics and engineering thermodynamics, they are conceptually different and mean different things. To distinguish this composite motion from simple linear mechanical motion, Leibniz coined another new expression. He called the composite motion an 'action'.[48] Statics was characterized in terms of 'contact causality', A–>B. The cause of the change in motion of B comes from A, an 'external' cause. Dynamics on the other hand has a holistic perspective, 'not distinguishing cause form effect', so that when A hits B, B hits A just as hard; A<–>B.[49]

Leibniz realized that his meta-shift from Statics to Dynamics also entailed a new ontology. The classical mechanical ontology is a sort of billiard ball world of linear contact causality. Leibniz's world is composed of finite 'infolded' entities with their own internal energy. Their motion is not being caused by external causal impacts. Motion/change no longer requires an external mechanical cause (force) but is spontaneous, cyclically coupled and eternal. Internal energy, understood in a new dualistic sense, is living force, and is conserved. He imagined an organic world including people with internal energy walking around and doing things. Their actions were not 'caused' by external contact causality. Leibniz came to realize that these entities are engines. Leibniz's composite 'actions' are what agents do, for a reason. The engines are agents. They are participant engineers.

Justin Smith [27] reviews Leibniz's meta-shift in his book: *Divine Machines: Leibniz and the Sciences of Life*. Smith argues that Leibniz's reality is thermodynamic. Leibniz even suggests that 'organisms are hydraulico-pyrotechnic engines' and are best understood in terms of their 'final cause [reason] … each part is coordinated for the intended use'. Smith points out that Leibniz's meta-shift to Dynamics unifies traditional physics and biology in a new way. The best parts-whole image is organic.

Because of the conceptual discontinuity in the meta-shift the concept of an engine cannot be understood in terms of any mechanics. The concept of an engine is in this sense autonomous relative to mechanical concepts. Leibniz expressed this by saying reality is 'engines all the way down'. In other words, the engine is 'the fundamental' and cannot be understood in terms of anything else. The

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[48] It will perhaps not be lost on readers at this stage that Max Planck, when he introduced the expression 'quantum of action' suggested that clarification of 'action' might be gained by reference to Leibniz's concept.

[49] What I am calling 'holistic' here is also traceable to Huygens' 'center reasoning' which is the basis of the 'real' Newton's Third Law and Lazare Carnot's geometric motion.

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conceptual discontinuity makes it difficult, indeed, perhaps impossible to provide any logically coherent account of the relation between the mechanical and engineering formulations of thermodynamics. And this, I believe, has been one of the impediments to an appreciation and proper understanding of Sadi Carnot's essay.

Euler, Lagrange and the Analytic Vector Calculus

Descartes' physics did not disappear following Leibniz's critique. And although, as Gillispie suggested, there was a general cultural shift for a period, there were also new developments in the Analytic tradition. Leonard Euler and his student Joseph-Louis Lagrange argued that the use of completed infinitesimals reasoning provided a more useful method of calculation. Euler-Lagrange reaffirmed and continued to develop Analytic mathematics and physics. [28] They could erase the duality using completed infinitesimal reasoning, expressing the circular aspect in terms of lines and linear constructions. What they developed became the analytic calculus and the vector calculus, the latter picked up by Hamilton with his introduction of quaternions.

Lagrange was a contemporary of the Carnots and taught for a period at the Ecole Polytechnique. In his 1797 publication, *Treatise on Metaphysical Principles of the Infinitesimal Analysis*, [29] Lazare specifically addresses Lagrange and his methods, acknowledging that they can be useful in approximating certain calculations.

Euler is the author of the Analytic version of Newton's Synthetic mechanics that has become the standard formulation taught in most mathematical physics curriculums.

Despite the meta-shift, the advocates of the Analytic, the 'science of mechanics', didn't just go away. Indeed, the Analytic tradition associated with Euler, Lagrange and Hamilton was foundational for the subsequent advances of classical science, modern mathematical physics (viz. prior to quantum theory) and the mechanical formulation of thermodynamics.

One mark of the difference between the Analytic and Synthetic tradition during the 18th and 19th centuries can be seen in the acceptance in the Euler-Lagrange-Hamilton Analytic program of the parallelogram rule in the composition of forces. For the Synthetic tradition the composition of forces, or motions, rejects the applicability of the parallelogram rule. The translational (linear) and rotational (circular) components are incommensurable duals, complementary opposite types that cannot be added together as if they were linear vectors. The rejection of the parallelogram rule entails, or is equivalent to, rejection of vector calculus.

Lazare Carnot's – Participatory Engineering Worldview

In 1803, Lazare Carnot published *Fundamental Principles of Equilibrium and Motion*, [21] the matured sequel to his two earlier essays, *On Machines in General,* in 1776 and 1783 [30]. In this publication Lazare provides us with insights about what I have referred to as the participant framework of engineering thermodynamics. Lazare rejects the 'force-cause approach to mechanics' making a meta-paradigm shift to the 'communication of motion' approach. Lazare is not just rejecting the 'rational mechanics' worldview, he is moving to articulate a new participant engineering worldview.

He begins with the claim that there is no account of engineering practice, in 'rational mechanics'.[50] What Lazare is endeavoring to present is a participant worldview in which both engineers and engineering practice make sense. He gives substance to his argument by citing a 'well-known principle of engineering practice', "that *one always loses in time or in velocity what one gains in strength [power]."* The idea is quite simple. If I use a pulley system to raise a weight, it is slower, takes longer, but requires less strength, less power per unit time. Indeed, this principle 'that everyone

[50] All quotes of Lazare in this section are from the Preface to *Fundamental Principles of Equilibrium and Motion.*

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repeats' applies to the use of machines in general, machines animated by engines (viz. animals, steam engines, etc.). [51]

Lazare then argues that none of the theories of 'rational mechanics' can explain this principle. He ridicules the 'absurd' attempts by the proponents of rational mechanics, where they "always suspect that there is something magical about the machines." Rational mechanics has no account of engineering practice and engineers. Even more interesting is that rational mechanics has no account of simple machines.[52]

One powerful entailment that Lazare identifies in this principle of participant engineering practice is that engineers have *options* in how they might accomplish a task. For instance, using a pulley system the engineer might choose to raise a weight faster or slower, taking less time or more time. Engineering practice has a sort of 'constrained freedom'. In climbing a steep mountain, the engineer is likely to choose a zig-zag path of switch backs. The resulting path has both linear and curvilinear components. The zig-zag path is also in the alternating path structure of the pulley.

Rational mechanics simply doesn't consider alternative paths because the idea that there are options doesn't make sense in deterministic mechanics. Because there are always options, albeit with constraints, the engineer must *choose*. Engineering *actions* are necessarily the result of choice, and the choice is for a reason.[53]

In the participant engineering worldview, engineering practice takes place in a possibility space. For the engineer the future is always partially indeterminate, to some irreducible extent. In the engineering worldview each current state has a potential that is partially indeterminate. The future to be actualized, is constrained, but is never 'fully constrained' by the present. An engineering professor colleague tells me that he teaches his students that they are 'actualizing opportunity'. This contrasts with scientific rational mechanics, where there is no freedom, where each state *uniquely determines* the subsequent state.

To underscore the representation of scientific inquiry as that of a spectator, there aren't even any scientists in the scientific worldview, no scientific inquirers, no scientific knowledge. The whole idea of useful knowledge doesn't make sense in a fully deterministic worldview. One way of expressing this is to say that the scientific worldview is not 'self-referentially coherent', it is unable to account for itself as contained in the supposed scientific reality. It is unable to make sense of how the scientific worldview itself was learned. Paul Feyerabend, in this book, *Against Method* [31], drove home this last argument as part of his devastating critique of the dominant 20th century attempts at a mechanistic, logico-mathematical philosophy of science. Leading philosopher of engineering Henry Petroski [32] took the next step arguing that 'everything you have thought of as science is actually

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[51] This idea is expressed by Hero of Alexandrea, in his treatise on *Mechanics*: the smaller the force we have available to lift a given weight, the longer it will take us. Hero expresses the general principle as follows: "In this machine (wheel and axle) and all the machines like it which are productive of great force [power] there is retardation, because in the proportion that the moving force which moves the large weight is weak, so in this proportion it must be extended in time. For just as is the proportion of one force to another, so is that of time to time [inversely]. (Hero, *Mechanics*, Book II, Chap. 22, translation from the Arabic text of L. Nix.)" Clagett, Marshall. *Greek Science in Antiquity* (pp. 117-122). Hauraki Publishing. Kindle Edition.

[52] All simple machines have irreducible shapes and structure.

[53] Florman captures the engineer's situation by analogy with European existentialism. The engineer must choose, necessarily, but what he chooses is always partially, irreducibly, indeterminate. Florman, Sam (1976) *The Existential Pleasures of Engineering*.

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engineering'. In other words, what has been represented to us as arising within the Analytic tradition is an abstraction from what could only have arisen through engineering practice, through participant action properly represented in the Synthetic tradition.

The engineer's actions *necessarily* require a choice, and choices are for reasons. The 'reasons' presuppose a purpose, a value, an intended objective. Leibniz uses the expression 'conatus' to represent action as a *seeking*. Engineering practice is not deductive and not 'merely' applied science. Finding an engineering solution requires exploration and experimentation. The participant engineer's chosen action is not a deduction, it's a seeking. And since actions are composite, actions are not mechanically reversible. The 'choice' is also not a selection from given existing or pre-existing set of alternatives, from possible futures that could be completely pre-specified. Engineering paths are not mechanically derivable. Engineering paths are 'contingent', involving an irreducible qualitative novelty. Since actions are not deductive, it will at least appear that there is always an element of chance in the action. Because each choice is partially indeterminate each choice is also a questioning.

Engineering practice is problem solving, always involving the search for a solution. And in engineering problem solving, the precise nature of the problem is never known completely in advance. The engineering enterprise is exploratory, and discovery of a solution, being non-deductive, requires a narrative account.[54]

Bejan, seeking to distinguish and clarify engineering thermodynamics offers a crucial critique: "To account for coupled thermomechanical behavior Clausius had to formulate a second principle, the second law, in addition to the conservation of energy. With his new principle came the concept of entropy, which was completely foreign to science. … "Today the new principle is the construction of geometric form, and the new concept is objective, or purpose."

(in The Physics of Life: The Evolution of Everything (p. 240). St. Martin's Press).

Bejan's engineering thermodynamic universe, arguably the universe of Lazare and Sadi Carnot, has a constructively evolving design. As composite engineering work is performed the organizational design of the universe evolves, qualitatively. This qualitative characteristic cannot, by its very nature, be made sense of within a non-dualistic mechanical thermodynamic framework. Composite, dualistic engineering work, by its very nature, is irreversible and consequently cumulative, part of an irreducibly historical design evolution narrative. As engineering work is performed there is an advance in the opportunity to perform work. The possibility space evolves, the capacity to perform work improves. Unlike the CBG representation of reality as 'running down', the universe of the Carnots' and Bejan is 'running up'.

Summary and Conclusions

The misunderstanding of Sadi's essay arose from and has continued due to the dominance of the Euler-Lagrange Analytic tradition. Sadi's engineering thermodynamics has been misrepresented in terms of the mechanical thermodynamics of Clausius, Boltzmann and Gibbs.

To clarify both the differences and the natures of the two formulations, I counterposed their ontologies, the particle ontology of mechanical thermodynamics and the metabolic engine ontology of engineering thermodynamics. Following Dewey's spectator-participant distinction I argued for a fundamental difference in their epistemological perspectives, in their understandings of inquiry itself. They see a different reality and so ask different types of questions of reality.

Bejan offered a crucial clue: the fundamental importance of finiteness in engineering thermodynamics. This led to contrasting two traditions, the Analytic and the Synthetic, that reason

[54] Nobel Laureate Hoffman emphasizes that real inquiry is dynamic, a developing search that requires a narrative description. Hoffman, Roald (2014) "Perspective: The Tensions of Scientific Storytelling", American Scientist, July–August, Vol. 102, No. 4 pp. 250-253

oppositely about finiteness, infinitesimals and infinities. It became apparent that commitment to finiteness entailed a conceptual duality, initially expressed in terms of linear and circular components in all paths and in all forms. Unexpectedly Bejan's clue also revealed a deep connection to Archimedes' method and to ancient Greek geometry.

To better understand Sadi's essay I explored three historical periods. Initially, the recent period established the strong influence of his father, Lazare Carnot. Lazare had recognized the influence of the earlier period 1650-1750 where Huygens, the Bernoullis, d'Alembert, Newton and Leibniz rediscovered the composite duality of motion and work, again traced back to Archimedes. My review of the contributions of these three periods indicated that they were all concerned with engineering thermodynamics, and all part of the Synthetic tradition.

Beginning in the middle period with Euler, there was the reaffirmation of the Analytic tradition. What began as a method to ease calculations was later extended by Lagrange and taken to offer an objective representation of reality. Using the completed infinitesimal reasoning of the analytic calculus, Lagrange offer a description of reality in terms of mechanical, scientific differential equations.

A wholly unexpected finding was that the Analytic Newtonian physics almost universally taught to science students still today was not the physics of the 'real' Newton presented in his *Principia*. Newton and Leibniz both rejected infinitesimals and the parallelogram of forces, the latter being the basis of Euler-Lagrange vector calculus.

The single most important result of my investigation has been to clarify and establish that Sadi Carnot's essay can only be properly understood in the engineering thermodynamics. To understand engineering thermodynamics the history appears to be essential to grasp that the conceptual foundations are those of the Synthetic tradition.

With the proper understanding of the conceptual foundations of engineering thermodynamics foundation new research opportunities opens. To begin with there are a few obvious obscurities and confusions about what Sadi is arguing in his essay arising from the mechanical misrepresentations. To list a few:

a) Sadi's 'cold sink' is does not seem essential in Clausius' account.
b) Sadi's 'limit of the efficiency of an engine' is certainty not due to friction, as is commonly remarked. And such a limit is not a natural consequence in the analytic mechanical representation.
c) Sadi's 'limit of the efficiency' is not about entropy or a second law. Entropy and a second law cannot be made sense of consistent with Sadi's First Principle, conservation of heat (living force).
d) Sadi's argument for the impossibility of perpetual motion uses a reductio argument, characteristic of the Synthetic tradition. Lazare, Leibniz and Huygens, all within the Synthetic tradition, make a similar argument, also using a reductio.
e) That most of modern classical physics was not Newton's but Euler's Analytic re-presentation is, I think, seriously underappreciated. Equally shocking is that Analytic geometry still taught through high school at least is not the geometry of Euclid's *Elements.* The 'real' Euclid's *Elements,* clearly expressed in the contributions of Eudoxus and Apollonius, is Synthetic geometry. Arguably this is crucial for understanding Sadi's essay in that engineering thermodynamics calls for a New Geometry. Aspects of the New Geometry come out in quantum theory with its noncommutative geometry as argued in my cited essay.
f) Bejan's clue of finiteness led to a fresh appreciation of duality. Duality manifests in Sadi and Lazare's engineering worldview in terms of cyclically coupled processes and elements. Duality provides a key insight into the nature of metabolic engines and helps us see why mechanics and

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mechanical thermodynamics, having erased the conceptual duality, cannot 'see', let alone account for, the metabolic engines fundamental to engineering thermodynamics.

g) The appreciation of the duality of engineering thermodynamics will be quite helpful in biology, in understanding 'life's [metabolic] engines'.

h) The duality uncovered in my research seems to pertain to conjugates and complementarity quite generally. As I argued in my cited essay quantum theory can perhaps only be understood within an engineering thermodynamic framework.

i) The engineering worldview of Sadi and Lazare is not static but dynamic, progressively, qualitatively evolving. Bejan's modern re-emergence of engineering thermodynamics has begun to give empirical substance to the constructive concept of reality.

j) Lazare's 'well-known' 'principle of engineering practice' was stated by Hero and Philo in ancient Alexandria in their effort to understand machines and engines. This suggests research to support Cardwell's contention that the 'real' history of thermodynamics is inseparable from the engineering history of machines and engines.

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