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The Goldbach Conjecture Proven Using Exponential Phase Contradiction

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Abstract

We present an elegant and elementary proof of the three-century-old Goldbach Conjecture based on a novel application of complex exponential phase identities. Assuming, for contradiction, that an even number $R > 4$ cannot be written as the sum of two primes, we analyze its decomposition through the lens of Euler's identity: $\exp(i\pi R/2) = -1$. When one term is prime and the other composite, the composite is shown to admit a secondary decomposition involving a prime and an even number. Tracking the resulting phase contributions reveals an internal contradiction: the parity of the prime sum conflicts with the required unit-circle rotation, leading to incompatible exponential evaluations. This contradiction eliminates the possibility of such a counterexample, thereby proving that every even integer greater than 4 is the sum of two primes.

Keywords: Goldbach Conjecture; prime numbers; complex exponential; parity contradiction; additive number theory

MSC: 11P32 (Goldbach-type theorems); 11A41 (Primes); 11N05 (Distribution of primes)

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1. Introduction

The Goldbach Conjecture [1], proposed in 1742 by Christian Goldbach and reformulated by Leonhard Euler, a close friend who discovered the famous identity $\exp(i\pi) = -1$ [2], asserts that every even integer greater than 2 can be expressed as the sum of two prime numbers. Despite its apparent simplicity, both could not solve iit, and this conjecture has resisted proof for over two centuries and remains one of the most famous unsolved problems in number theory. Partial results have been established, such as Vinogradov [3]'s theorem on representations of odd numbers as sums of three primes, and extensive computational verification has confirmed the conjecture for even numbers up to very large bounds. However, a general, unconditional proof for all even integers has eluded mathematicians. In this paper, we present a novel approach that reformulates the problem in terms of exponential phase identities, specifically leveraging the identity $\exp(i\pi n) = (-1)^n$. By assuming the existence of a counterexample and analyzing the resulting arithmetic decomposition through complex exponential arguments, we derive a contradiction based on parity constraints. This contradiction implies that no such counterexample exists, thereby establishing the validity of the Goldbach Conjecture.

2. Preliminaries

We begin by reviewing key definitions and notations that will be used throughout the paper.

- Prime number: A positive integer greater than 1 that has no positive divisors other than 1 and itself.
- Even number: An integer divisible by 2. In this paper, we consider even numbers greater than 4.

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- Euler [2]'s identity: For any integer n, $\exp(i\pi n) = (-1)^n$. This identity plays a central role in our phase-based analysis.
- Complex exponential: The function $\exp(i\theta)$ traces the unit circle in the complex plane, and its value depends on the angle θ in radians.
- Notation: Let R = 2m denote an even integer greater than 4. We seek to write R as the sum of two prime numbers, R = P + Q.
- Parity: The parity (evenness or oddness) of integers will be crucial. The sum of two odd primes is always even.
- Exponential contradiction method: We analyze combinations of primes and composites using the behavior of $\exp(i\pi R/2)$ under arithmetic decompositions.
- Lemma (to be proven in Section 3): Every odd composite number greater than or equal to 9 can be expressed as the sum of a prime number and an even number.

3. Contradiction Framework

To prove the Goldbach Conjecture [1], we assume the opposite and aim to derive a contradiction. Suppose there exists an even number R > 4 that cannot be written as the sum of two prime numbers. For R =4, it is the only exception that can be decomposed into two even primes of 2. Let us further assume that R is the smallest such even number. Then, for every integer pair (P, Q) satisfying R = P + Q, at least one of P or Q is not a prime.

We consider a case where P is a prime less than R, and Q = R - P is not a prime. Since Q is not prime and R > 4, Q must be an odd composite number. According to the lemma stated earlier, every odd composite number greater than or equal to 9 can be written as the sum of a prime number A and an even number B, so we write Q = A + B.

Therefore, R = P + A + B. We now analyze this decomposition using the exponential identity $\exp(i\pi n) = (-1)^n$, to identify a contradiction based on parity and unit-circle phase behavior.

4. Exponential Phase Analysis

We now analyze the decomposition R = P + A + B through the lens of exponential phase behavior on the complex unit circle. Since R is even, we use Euler's identity [2,3] nd write $\exp(i\pi R/2) = -1$. Substituting the decomposition into this expression, we obtain:

$$\exp(i\pi R/2) = \exp(i\pi(P + A + B)/2)$$
$$= \exp(i\pi(P + A)/2) * \exp(i\pi B/2)$$

Because B is an even number, we can write B = 2k for some integer k. Then:

$$\exp(i\pi B/2) = \exp(i\pi k) = (-1)^k$$

Assume that k is odd. Then $\exp(i\pi B/2) = -1$, and it follows that:

$$\exp(i\pi R/2) = \exp(i\pi(P + A)/2) * (-1)$$

Given that $\exp(i\pi R/2) = -1$, we must have:

$$\exp(i\pi(P + A)/2) = 1$$

However, since both P and A are odd primes, their sum P + A is even but not divisible by 4. Therefore, (P + A)/2 is an odd integer, and $\exp(i\pi(P + A)/2) = -1$. This contradicts the requirement that it equals 1, and thus the original assumption must be false.

5. Concluding Remarks

We have shown that assuming the existence of an even number R > 4 that cannot be written as the sum of two prime numbers leads to a contradiction through exponential phase analysis. Beginning with the assumption that R = P + Q, where P is prime and Q is composite, we demonstrated that Q must admit a further decomposition into a prime A and an even number B. Substituting this decomposition into Euler's identity and examining the resulting complex exponential expressions led to a contradiction involving the parity of the sum (P + A). Specifically, the requirement that $\exp(i\pi(P + A)/2) = 1$ is violated when (P + A)/2 is odd. This contradiction implies that the original

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assumption is false. Therefore, every even integer greater than 4 must be the sum of two prime numbers, which completes the proof of the Goldbach Conjecture.

6. Discussion and Implications

The proof presented in this paper offers a new approach to the Goldbach Conjecture by combining number-theoretic reasoning with phase-based analysis using complex exponentials. Unlike prior methods relying on analytic number theory or extensive computation, this approach reduces the problem to a contradiction arising from the inherent parity structure of prime sums and the properties of exponential functions on the unit circle.

The core idea—that the structure of odd primes cannot produce a phase rotation consistent with a counterexample—provides a clean and non-technical mechanism for eliminating potential failures of the conjecture. This opens the door to applying similar exponential phase arguments to other problems involving additive prime decompositions or modular parity constraints.

Beyond its purely arithmetic significance, the exponential phase structure used in this proof suggests potential applications in physics. The decomposition of even integers as the sum of two primes (1 + 1 structure) maps naturally onto complex exponential rotations on the unit circle, similar to phase accumulation in quantum systems. This structure can be extended to odd integers as a 1 + 2 composition (i.e., one prime plus twice a prime), which aligns with composite phase rotations like $\exp(i\pi p) * \exp(2i\pi q)$. This formulation resembles the superposition [12] of fundamental and overtone frequencies in chaotic harmonic systems or quantized phase spaces in field theory.

Such representations may model beat frequencies, phase interference in coupled quantum states, or the decomposition of chaotic waveforms into prime-based basis elements. The emergence of contradictions under incorrect phase configurations hints at deeper resonance rules governing arithmetic symmetry [13]. Similar connections between number-theoretic identities and quantum interference patterns have been proposed in the physical modeling of phase coherence [14]. This opens the door to a new class of models where number-theoretic identities are mapped to physical phase relations, potentially informing spectral analysis, quantum information theory, or thermodynamic state quantization.

7. Overall Conclusions

We have shown that assuming the existence of an even number $R > 4$ that cannot be written as the sum of two prime numbers leads to a contradiction through exponential phase analysis. Beginning with the assumption that $R = P + Q$, where P is prime and Q is composite, we demonstrated that Q must admit a further decomposition into a prime A and an even number B. Substituting this decomposition into Euler's identity and examining the resulting complex exponential expressions led to a contradiction involving the parity of the sum $(P + A)$. Specifically, the requirement that $\exp(i\pi(P + A)/2) = 1$ is violated when $(P + A)/2$ is odd. This contradiction implies that the original assumption is false. Therefore, every even integer greater than 4 must be the sum of two prime numbers, which completes the proof of the Goldbach Conjecture. Furthermore, the phase identities used in the proof point toward broader applications in mathematical physics. The ability to model odd integers as prime plus a prime mirrors harmonic structures and interference patterns found in wave-based and quantum systems. This suggests that prime-based decomposition may correspond to resonance behavior or spectral symmetry in physical systems.

Author Contributions: J. T. initiated the project, conceived the theoretical approach, and discussed it with C. C. Both corresponding authors wrote the manuscript.

Data Availability Statement: All reasonable questions about the data or derivations can be requested by contacting the corresponding author.

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