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The Theory of Plafales: P vs NP Problem Solution †

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† In memory of Tolik Bendeliani and Victor Kovalyov.

Abstract

In this paper, we consider the properties of the following objects: plafal and geo-space (a general overview). As an application of the created theory, the proof of the equality of complexity classes P and NP will be given*. The geo-plafal is a kernel (computational template) of the proof; constructive theory of serendipity approximations, Stepanets' school and the Bogolyubov principle of the decay of correlations for an infinite systems (dim = 3) is a shell.

Keywords: plafal; plafale; the theory of plafales; P vs NP problem

MSC: Primary: 00-02; 00A05; 68Q15; Secondary: 00A71; 08-00; 18A05; 18A10; 37-00; 54-00; 65D99; 65Y20; 68Q25

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1. Introduction

The first edition of [1] is published in March 2011. The publication's goal is the creation of a new theory in mathematics, where the central object is a plafal[1]. The second edition of [2] is published in February 2013. After the report at the 42nd Polish Conference on Mathematics Applications [3], it is necessary to create applications based on the theory of plafales. Here we will provide the R·D process.

Finite element method. There are created mathematical models of serendipity finite elements: a new approach to construction basis and field functions. A quadruple role of the basis functions of serendipity finite elements is shown [4,5].

IT (finite element method as an algorithmic support). Due to solving the non-standard Dirichlet boundary value problem [4], using the components of the theory of plafales (to obtain the surface of the temperature field in three-dimensional space), there is developed a software for testing non-stationary temperature fields [6].

Cryptography. In September 2014 there is created a symmetric-key algorithm "ECLECTIC-DT-1" [7]. Algorithm's characteristics: block length is 128 bits, key length is 256 bits, 14 rounds. Let us show the algorithm's indicators. Upper bounds of practical security: $EDP \leq 2^{-714}$ (against differential cryptanalysis [8]), $ELP \leq 2^{-714}$ (against linear cryptanalysis [9]). Upper bounds of provable security (for the first four rounds): $d^{\Im} \leq 2^{-96}$ and $l^{\Im} \leq 2^{-96}$. In December 2018 there is created a symmetric-key algorithm "STEEL" [10]. Algorithm's characteristics: block length is 128 bits, key length is 256 bits, 14 rounds. Algorithm's indicators: $EDP \leq 2^{-595}, ELP \leq 2^{-595}, d^{\Im} \leq 2^{-80}$ and $l^{\Im} \leq 2^{-80}$.

Note. Editions [1] and [2] are the bases of the theory of plafales. The paper presents the "linear passage": all objects are introduced to prove P = NP as soon as possible. The theory has to be taken into consideration starting from this paper.

*Using the deterministic machine, P = NP is performed with pre-prepared tape.

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[1] The singular form is a plafal or a plafale. Pl: plafales.

2. Concept of the Plafal

We will use a framework "Standard By Default" [11]. A mathematical entity is described as $\mathcal{U}$-standard according to the following rules: a set, monoid, topological space, poset, family, graph, diagram, cardinal, ordinal etc. is standard when it is small; a category, groupoid, multicategory, locally ordered category etc. is standard when it is light; for $2 \leq n < \infty$, an n-category is standard when it is n-moderate. A mathematical entity is described as $\mathcal{U}^*$-standard according to the following rules: an ∞-groupoid is standard when it is small; an $(\infty, 1)$-category is standard when it is light; for $2 \leq n < \infty$, an (∞, n)-category is standard when it is n-moderate. A function, relation, subset, functor, natural transformation etc. is always standard.

Definition 2.1. A plafal is a tuple $\langle G, \diamond PF, \mathcal{P} \rangle$ and is denoted by PF or PF^i_j, where $G = G(V, E)$ is a simple graph[2], $i = |V| > 0, j = |E| > 0, \diamond PF = \langle S(\widetilde{\mathcal{V}}), S(\widetilde{\mathcal{E}}) \rangle$ is a collection of arbitrary $\mathcal{U}$-standard, $\mathcal{P} = \langle S(\mathbf{v}), S(\mathbf{e}) \rangle$ is a collection of correspondences such that $\forall v \in V$ we have $v \xrightarrow{\mathbf{v} \in S(\mathbf{v})} \widetilde{\mathcal{V}} = \mathcal{U}$-standard, $\widetilde{\mathcal{V}} \in S(\widetilde{\mathcal{V}})$, here $S(\widetilde{\mathcal{V}})$ is a collection of arbitrary $\mathcal{U}$-standard that correspond to the vertices; $\forall e \in E$ we have $e \xrightarrow{\mathbf{e} \in S(\mathbf{e})} \widetilde{\mathcal{E}} = \mathcal{U}$-standard, $\widetilde{\mathcal{E}} \in S(\widetilde{\mathcal{E}})$, here $S(\widetilde{\mathcal{E}})$ is a collection of arbitrary $\mathcal{U}$-standard that correspond to the edges. Generally, $\langle v, \widetilde{\mathcal{V}}, \mathbf{v} \rangle$ is called a vertex of the plafal and is denoted by $\mathcal{V}$; $\langle e, \widetilde{\mathcal{E}}, \mathbf{e} \rangle$ is called an edge of the plafal and is denoted by $\mathcal{E}$. By $V(PF)$ we denote a collection of vertices, by $E(PF)$ we denote a collection of edges. Between $\mathcal{E}$ and $\mathcal{V}_1, \mathcal{V}_2$, which are connected by $\mathcal{E}$, there is not necessarily a logical relation. If $\forall \mathcal{V} \in V(PF) : \mathcal{V} = \langle v, v, \mathbf{v} \rangle$ and $\forall \mathcal{E} \in E(PF) : \mathcal{E} = \langle e, e, \mathbf{e} \rangle$, then we say that $PF = G$ is a trivial plafal and write $_*PF$. So, $PF = \langle V(PF), E(PF) \rangle$.

For example, the representation of $G(V, E)$ on a vector space is a type of PF [12].

Remark 1. $|S(\widetilde{\mathcal{V}})| \leq i$ and $|S(\widetilde{\mathcal{E}})| \leq j$. We can operate with $S(\widetilde{\mathcal{V}})$ and $S(\widetilde{\mathcal{E}})$ that contain the same entities. **Agreement.** We claim that $G(V, E) = G(PF)$, $\langle G, \diamond PF, \mathcal{P} \rangle = \langle G, \diamond PF \rangle$, $\langle v, \widetilde{\mathcal{V}}, \mathbf{v} \rangle = \langle v, \widetilde{\mathcal{V}} \rangle$, $\langle e, \widetilde{\mathcal{E}}, \mathbf{e} \rangle = \langle e, \widetilde{\mathcal{E}} \rangle$. Let us introduce the following notation. $S(PF)$ is a collection of plafales. $S(G(PF))$ is a collection of simple graphs as supports of $S(PF)$.

Claim 2.2. *$\mathcal{V}, \mathcal{E}$ are categories.*

Proof. Let us show for $\mathcal{V}$. $id_v \circ id_v = id_v$, $id_{\widetilde{\mathcal{V}}} \circ id_{\widetilde{\mathcal{V}}} = id_{\widetilde{\mathcal{V}}}$, $\mathbf{v} \circ id_v = \mathbf{v}$, $id_{\widetilde{\mathcal{V}}} \circ \mathbf{v} = \mathbf{v}$. □

Let us provide a definition of the plafal within the framework of category theory.

Definition 2.3. A plafal is a tuple $\langle ASC, Disc, \mathcal{F} \rangle$, where ASC is a 1-dimensional abstract simplicial complex (considering as a category), here $Disc$ is a discrete category of arbitrary $\mathcal{U}$-standard, $\mathcal{F}$ is functor between ASC and $Disc$ such that the above correspondences (see definition 2.1) are performed and morphisms of simplices of ASC and identity morphisms of enteties of $Disc$ are realized.

Definition 2.4. Labeled plafal is a type of the plafal, where all or some of vertices and/or edges are enumerated. We have $V^\bullet(PF) \xrightarrow{\pi_1} \{\overline{1, i}\}$, here $V^\bullet(PF) \subseteq V(PF)$ and $|V^\bullet(PF)| = k$; $E^\bullet(PF) \xrightarrow{\pi_2} \{\overline{1, j}\}$, here $E^\bullet(PF) \subseteq E(PF)$ and $|E^\bullet(PF)| = l$; π_1, π_2 are substitutions[3]. By $PF^{i_k}_{j_l}$ we denote a labeled plafal, k is a quantity of enumerated vertices; l is a quantity of enumerated edges. In the case of $k = 0$, the plafal does not contain enumerated vertices. In the case of $l = 0$, the plafal does not contain enumerated edges. In the case of $k = l = 0$, we claim that $PF^{i_0}_{j_0} = PF^i_j$. By $^\tau\mathcal{V}$ we denote a vertex with assigned number ($1 \leq \tau \leq i$), by $^\varsigma\mathcal{E}$ we denote an edge with assigned number ($1 \leq \varsigma \leq j$).

Notice that a graph labeling is a type of PF such that $\widetilde{\mathcal{V}}, \widetilde{\mathcal{E}} \in \{v, e, \{\text{labeling}\}\}$. We consider a labeled trivial plafal as a graph labeling. $^\tau\mathcal{V}$ and $^\varsigma\mathcal{E}$ do not necessarily coincide with a number of a

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[2] G is a 1-dimensional abstract simplicial complex that does not contain an isolated vertex. Generally, v is a vertex of $G(V, E)$, e is an edge of $G(V, E)$. V is a collection of vertices, E is a collection of edges. For G we have the following: a vertex is a singleton; an edge is an unordered pair.

[3] π_1 is a bijection iff $k = i$; π_1 is an injection iff $k < i$. π_2 is a bijection iff $l = j$; π_2 is an injection iff $l < j$.

graph labeling. Evidently, we can use the following configurations: $\widetilde{\mathcal{V}} = \langle\{\text{labeling}\}, \mathcal{U}\text{-standard}\rangle$ and $\widetilde{\mathcal{E}} = \langle\{\text{labeling}\}, \mathcal{U}\text{-standard}\rangle$.

The illustrations of plafales and graphs are given for the reader's perception.

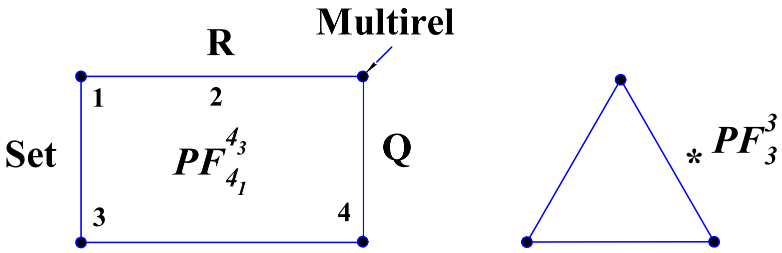

Figure 1. $PF^{4_3}_{4_1}$. Vertices: $\mathcal{V} = \langle v, v\rangle$ (for three vertices), the category of small sets and small multirelations. Edges: the category of sets, the set of real numbers, the set of rational numbers, $\mathcal{E} = \langle e, e\rangle$. Three vertices are enumerated, one edge is enumerated. $_*PF^3_3$ is a trivial plafal.

Let us give an example in cryptography [7], [10]. For a byte $\overline{b_7 \dots b_0}$ we have the following configurations: ${}^{\varsigma}\mathcal{E} = \langle e, \{b_{8-\varsigma}\}\rangle$ and $\mathcal{V} = \langle v, v\rangle$.

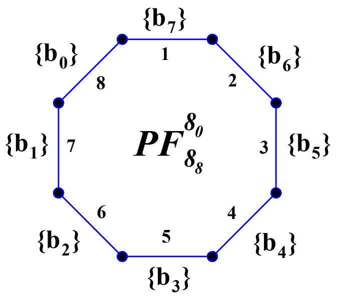

Figure 2. $PF^{8_0}_{8_8}$. Eight edges are enumerated. The plafal does not contain enumerated vertices.

Definition of an $*$-plafal can be formulated by analogy with the plafal, taking into account $\forall v \in V$ we have $v \xrightarrow{\mathbf{v} \in S(\mathbf{v})} \widetilde{\mathcal{V}} = \mathcal{U}^*$-standard and $\forall e \in E$ we have $e \xrightarrow{\mathbf{e} \in S(\mathbf{e})} \widetilde{\mathcal{E}} = \mathcal{U}^*$-standard.

Remark 2. $*$-plafales and their properties will be the object of another paper.

3. The Category of Plafales and Operations

Definition 3.1. A plafal morphism $PF \xrightarrow{\Lambda = \langle f, \Theta\rangle} PF'$ is a tuple of maps's collections such that
(1) $G(PF) \xrightarrow{f} G(PF')$, f is a graph morphism (an abstract simplicial map);
(2) $\diamond PF \xrightarrow{\Theta = \langle\Theta^*, \Theta^{**}\rangle} \diamond PF'$: $S(\widetilde{\mathcal{V}}) \xrightarrow{\Theta^*} S(\widetilde{\mathcal{V}}')$ and $S(\widetilde{\mathcal{E}}) \xrightarrow{\Theta^{**}} S(\widetilde{\mathcal{E}}')$.
Note that (2) is performed in accordance with (1).

The example is given in figure 3.

Claim 3.2. *Plafales and plafal morphisms form the category Plafales, together with the componentwise compositions* $\langle f, \Theta\rangle \circ \langle f', \Theta'\rangle = \langle f \circ f', \Theta \circ \Theta'\rangle$ *and identities* $id_{PF} = \langle id_{G(PF)}, id_{\diamond PF} = \langle id_{S(\widetilde{\mathcal{V}})}, id_{S(\widetilde{\mathcal{E}})}\rangle\rangle$. *The proof is left to the reader.*

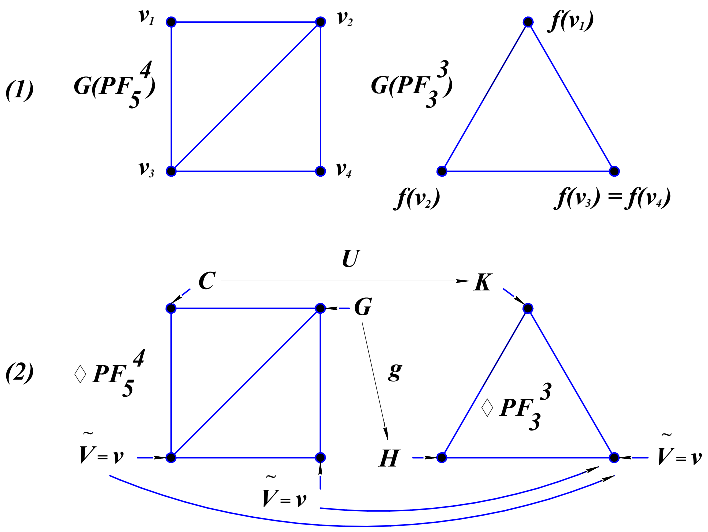

Figure 3. Condition 1. The image of $G(PF_5^4)$ under the strict homomorphism is $G(PF_3^3)$. Condition 2. U is a functor; g is a homomorphism of groups.

Definition 3.3. $\langle \widehat{0}_{G(PF_j^i)} = \langle \varnothing, \varnothing \rangle, \diamond PF = \langle \varnothing, \varnothing \rangle \rangle$ is called an empty plafal and is denoted by PF^{emp}, here $\widehat{0}_{G(PF_j^i)}$ is the empty graph[4] (the initial object in the categories of simple graphs (**SimpGph**)).

Claim 3.4. *In **Plafales**, there're no initial and terminal objects.*

Proof. **Initial.** Omitted. **Terminal.** In **SimpGph**, there's no terminal object. ☐

Definition 3.5. A product of two plafales $PF^{prod} = PF \times PF'$ is defined by the graph product $\prod = G(PF) \times G(PF')$ and the following condition holds:
(i) $\forall \widetilde{\mathcal{V}}^{prod} \in S(\widetilde{\mathcal{V}}^{prod}) : \widetilde{\mathcal{V}}^{prod} = \widetilde{\mathcal{V}} \times \widetilde{\mathcal{V}}'$ and $\forall \widetilde{\mathcal{E}}^{prod} \in S(\widetilde{\mathcal{E}}^{prod}) : \widetilde{\mathcal{E}}^{prod} = e$.
Notice that (i) is realized in accordance with $\prod$.

Definition 3.6. A coproduct of two plafales $PF^{coprod} = PF + PF'$ is defined by the graph coproduct $\coprod = G(PF) + G(PF')$ and the following condition holds:
(i) in the general case, $\diamond PF + \diamond PF'$ is a coproduct of categories.
Let us remark that (i) is performed in accordance with $\coprod$.

Claim 3.7. *In **Plafales**, the (co)-product of any two plafales does not always exist.*

Proof. It is sufficient to consider $S(\widetilde{\mathcal{V}})$ and $S(\widetilde{\mathcal{V}}')$ as collections of objects of the category of fields. ☐

Corollary 3.8. ***Plafales** is not a (quasi)-topos. The proof is streightforward.*

3.1. Basic Operations

Elementary operations

1. Vertex deletion $PF_{jl}^{ik} - \mathcal{V}$. 2. Edge deletion $PF_{jl}^{ik} - \mathcal{E}$. 3. Edge addition $PF_{jl}^{ik} + \mathcal{E}$.

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[4] $i = 0, j = 0$ (see definition 2.1).

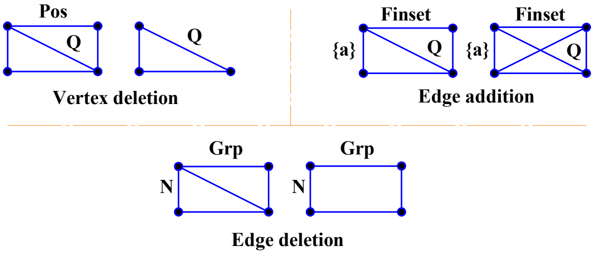

Figure 4. Elementary operations.

Advanced operations

Let us introduce the following notation. PF_ρ is a plafal with assigned number. In the general case, $\widetilde{\mathcal{V}}_\alpha$ and $\widetilde{\mathcal{E}}_\beta$ are the components of PF_α and PF_β respectively. Let us remark that $\widetilde{\mathcal{V}}_\alpha$ and $\widetilde{\mathcal{E}}_\beta$ are not the entities with specified numbers.

In definitions 3.9 – 3.14, we will use $I \subseteq \{\overline{1,n}\}$. In definitions 3.9 – 3.11, conditions (i) are performed in accordance with the union of graphs $H^{un} = \bigcup_{i\in I} G(PF_i)$. In definitions 3.12 – 3.14, conditions (i) are realized in accordance with the intersection of graphs $H^{in} = \bigcap_{i\in I} G(PF_i)$.

Definition 3.9. A union of plafales PF^{un} is defined by H^{un} and the following condition holds:
(i) $\widetilde{\mathcal{V}}_{\text{common}} = \bigcup_{\mu\in I} \widetilde{\mathcal{V}}_\mu$ and $\widetilde{\mathcal{E}}_{\text{common}} = \bigcup_{\nu\in I} \widetilde{\mathcal{E}}_\nu$.

Definition 3.10. A docking of plafales PF^{doc} is defined by H^{un} and the following condition holds:
(i) $\widetilde{\mathcal{V}}_{\text{common}} = \langle \widetilde{\mathcal{V}}_\eta, \ldots, \widetilde{\mathcal{V}}_\mu \rangle, \eta < \ldots < \mu$ and $\widetilde{\mathcal{E}}_{\text{common}} = \langle \widetilde{\mathcal{E}}_\varrho, \ldots, \widetilde{\mathcal{E}}_\chi \rangle, \varrho < \ldots < \chi$.

Definition 3.11. A docking• of plafales $PF^{doc\bullet}$ is defined by H^{un} and the following condition holds:
(i) Condition (i) of definition 3.9 is performed for $1 \le r_1 \le r-1$ common vertices and for $1 \le t_1 \le t-1$ common edges; condition (i) of definition 3.10 is realized for $r_2 = r - r_1$ common vertices[5] and for $t_2 = t - t_1$ common edges[6].

Definition 3.12. An intersection of plafales PF^{in} is defined by H^{in} and the following condition holds:
(i) $\widetilde{\mathcal{V}}_{\text{common}} = \bigcap_{\mu\in I} \widetilde{\mathcal{V}}_\mu$ and $\widetilde{\mathcal{E}}_{\text{common}} = \bigcap_{\nu\in I} \widetilde{\mathcal{E}}_\nu$.

Definition 3.13. A merger of plafales PF^m is defined by H^{in} and the following condition holds:
(i) $\widetilde{\mathcal{V}}_{\text{common}} = \langle \widetilde{\mathcal{V}}_\eta, \ldots, \widetilde{\mathcal{V}}_\mu \rangle, \eta < \ldots < \mu$ and $\widetilde{\mathcal{E}}_{\text{common}} = \langle \widetilde{\mathcal{E}}_\varrho, \ldots, \widetilde{\mathcal{E}}_\chi \rangle, \varrho < \ldots < \chi$.

Definition 3.14. A merger• of plafales $PF^{m\bullet}$ is defined by H^{in} and the following condition holds:
(i) Condition (i) of definition 3.12 is performed for $1 \le r_1 \le r-1$ common vertices and for $1 \le t_1 \le t-1$ common edges; condition (i) of definition 3.13 is realized for $r_2 = r - r_1$ common vertices and for $t_2 = t - t_1$ common edges.

Agreement[7]**.** In definitions 3.10, 3.13, in the case of the chain of operations: $\widetilde{\mathcal{V}}_{\text{common}} = \langle \underbrace{\langle \widetilde{\mathcal{V}}_\alpha, \ldots, \widetilde{\mathcal{V}}_\mu \rangle, \ldots, \langle \widetilde{\mathcal{V}}_\beta, \ldots, \widetilde{\mathcal{V}}_\zeta \rangle}_{k\ge 1}, \underbrace{\widetilde{\mathcal{V}}_\varsigma, \ldots, \widetilde{\mathcal{V}}_\xi}_{m\ge 0} \rangle$, $k + m = |I|$, and taking into account $|I|!$ permutations for $\widetilde{\mathcal{V}}_{\text{common}}$, we get $\widetilde{\mathcal{V}}_{\text{common}} = \langle \widetilde{\mathcal{V}}_\alpha, \ldots, \widetilde{\mathcal{V}}_\varsigma \rangle$, where $\alpha < \beta < \ldots < \varsigma$. For the same reason, this is performed for $\widetilde{\mathcal{E}}_{\text{common}}$.

On the other hand, definitions 3.9 – 3.14 can be formulated in the following form: in general, $S^k(G(PF)) \xrightarrow{\star = \{\cup,\cap\}} S(G(PF)) \xrightarrow{\mathcal{P}} \diamond PF$.

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[5] r is a quantity of common vertices.

[6] t is a quantity of common edges.

[7] We use this agreement only as a recommendation.

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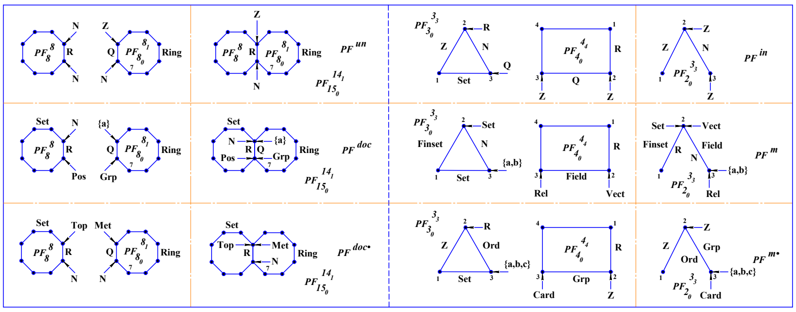

Figure 5. Advanced operations.

In the case of labeled plafales, $PF^{un}, PF^{doc}, PF^{doc\bullet}, PF^{in}, PF^{m}$ and $PF^{m\bullet}$ are not labeled plafales, but they can be transformed into the specified types.

Definition 3.15. A product• of two plafales $PF^{prod\bullet} = PF \times PF'$ is defined by the graph product $\prod = G(PF) \times G(PF')$ and the following condition holds:
(i) $\forall \widetilde{\mathcal{V}}^{prod\bullet} \in S(\widetilde{\mathcal{V}}^{prod\bullet}) : \widetilde{\mathcal{V}}^{prod\bullet} = \widetilde{\mathcal{V}} \times \widetilde{\mathcal{V}}', \forall \widetilde{\mathcal{E}}^{prod} \in S(\widetilde{\mathcal{E}}^{prod}) : \widetilde{\mathcal{E}}^{prod} = \mathcal{U}$-standard.
(i) is realized in accordance with $\prod$. The mentioned $\widetilde{\mathcal{E}}^{prod}$ can be different.

We stress that PF^{prod} is a particular case of $PF^{prod\bullet}$.

Definition 3.16. A decomposition of plafales is a collection of maps such that $S(G(PF)) \xrightarrow{H^d} S(G(PF)) \xrightarrow{S(\mathcal{P})} S(\diamond PF)$ and the following conditions hold:
(i) for k-vertices, which were located at the common vertice, of k-decomposited graphs we have: $\widetilde{\mathcal{V}}_{\text{common}}$ corresponds to one of the above vertices (k-variations) and there is a collection of arbitrary $\mathcal{U}$-standard that correspond to remaining $(k-1)$-vertices. By the same argument, this is performed for l-edges;
(ii) $\widetilde{\mathcal{V}}_{\text{no common}}$ and $\widetilde{\mathcal{E}}_{\text{no common}}$ remain unchanged;
where H^d is an operation of the decomposition of graphs; here $S(\mathcal{P})$ is a collection of $\mathcal{P}$ and $S(\diamond PF)$ is a collection of $\diamond PF$ for $S(G(PF))$ respectively.

Remark 3. The other types of operations will be discussed in a further paper.

4. Geo-Space

Definition 4.1. A geo-space is a tuple $\langle \mathbb{R}^3, U \rangle$ and is denoted by $|PF|^U$, where U is a universe of $\mathcal{U}$-standard such that $\mathbb{R}^3 \cap U = \varnothing$.

The objects of the following type $\langle \text{Object} \subset \mathbb{R}^3, \diamond\text{Object} \subset U \rangle$, which will be represented in this section, are implemented in $|PF|^U$. $\mathbb{R}^3 \cap U = \varnothing$ means that U is located outside of $\mathbb{R}^3$. Let us provide a visual perception by analogy in CS. $\mathbb{R}^3$ is a directory and U is a collection of external files.

4.1. Geo-Plafal

Let us give a definition of a geometric representation of $PF = \langle G, \diamond PF \rangle$ in $|PF|^U$.

Definition 4.2. $\langle |G|, \diamond PF \rangle$ is a geo-plafal and is denoted by $|PF|$, $|G|$ is the geometric realization of G in $\mathbb{R}^3$ such that $|G| \xrightarrow{\mathcal{P}'} \diamond PF$ in accordance with $G \xrightarrow{\mathcal{P}} \diamond PF$.

Let us give a comment to definition 4.2. We use the metric or coherent topologies.

On the other hand, $|PF|$ can be formulated by analogy with definition 2.1.

Definition 4.3. $\langle |\mathcal{K}|, \diamond|\mathcal{K}|, \mathcal{T} \rangle$ is a geo-plafal and is denoted by $|PF|$ or $|PF|^i_j$, where $|\mathcal{K}|$ is a 1-dimensional simplicial (topological) complex[8], $i = |V(|\mathcal{K}|)| > 0$, $j = |E(|\mathcal{K}|)| > 0$, $\diamond|\mathcal{K}| =$

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[8] $|\mathcal{K}|$ is a polyhedron that does not contain an isolated vertex. Generally, $|v|$ is a vertex of $|\mathcal{K}|$, $|e|$ is an edge of $|\mathcal{K}|$. $V(|\mathcal{K}|)$ is a collection of vertices, $E(|\mathcal{K}|)$ is a collection of edges.

$\langle S(|\widetilde{\mathcal{V}}|), S(|\widetilde{\mathcal{E}}|)\rangle$ is a collection of arbitrary $\mathcal{U}$-standard, $\mathcal{T} = \langle S(|\mathbf{v}|), S(|\mathbf{e}|)\rangle$ is a collection of correspondences such that $\forall |v| \in V(|\mathcal{K}|)$ we have $|v| \xrightarrow{|\mathbf{v}| \in S(|\mathbf{v}|)} |\widetilde{\mathcal{V}}| = \mathcal{U}$-standard, $|\widetilde{\mathcal{V}}| \in S(|\widetilde{\mathcal{V}}|)$, here $S(|\widetilde{\mathcal{V}}|)$ is a collection of arbitrary $\mathcal{U}$-standard that correspond to the vertices; $\forall |e| \in E(|\mathcal{K}|)$ we have $|e| \xrightarrow{|\mathbf{e}| \in S(|\mathbf{e}|)} |\widetilde{\mathcal{E}}| = \mathcal{U}$-standard, $|\widetilde{\mathcal{E}}| \in S(|\widetilde{\mathcal{E}}|)$, here $S(|\widetilde{\mathcal{E}}|)$ s a collection of arbitrary $\mathcal{U}$-standard that correspond to the edges. Generally, $\langle |v|, |\widetilde{\mathcal{V}}|, |\mathbf{v}|\rangle$ is called a vertex of the geo-plafal and is denoted by $|\mathcal{V}|$; $\langle |e|, |\widetilde{\mathcal{E}}|, |\mathbf{e}|\rangle$ is called an edge of the geo-plafal and is denoted by $|\mathcal{E}|$. By $V(|PF|)$ we denote a collection of vertices, by $E(|PF|)$ we denote a collection of edges. Between $|\mathcal{E}|$ and $|\mathcal{V}|_1, |\mathcal{V}|_2$, which are connected by $|\mathcal{E}|$, there is not necessarily a logical relation. If $\forall |\mathcal{V}| \in V(|PF|) : |\mathcal{V}| = \langle |v|, |v|, |\mathbf{v}|\rangle$ and $\forall |\mathcal{E}| \in E(|PF|) : |\mathcal{E}| = \langle |e|, |e|, |\mathbf{e}|\rangle$, then we say that $|PF| = |\mathcal{K}|$ is a trivial geo-plafal and write ${}_*|PF|$. So, $|PF| = \langle V(|PF|), E(|PF|)\rangle$.

Remark 4. $|S(|\widetilde{\mathcal{V}}|)| \leq i$ and $|S(|\widetilde{\mathcal{E}}|)| \leq j$. We can operate with $S(|\widetilde{\mathcal{V}}|)$ and $S(|\widetilde{\mathcal{E}}|)$ that contain the same entities. Claim 2.2, definition 2.3 and definition 2.4 are performed for $|PF|$. **Agreement.** We claim that $\langle |\mathcal{K}|, \diamond|\mathcal{K}|, \mathcal{T}\rangle = \langle |\mathcal{K}|, \diamond|\mathcal{K}|\rangle$, $\langle |v|, |\widetilde{\mathcal{V}}|, |\mathbf{v}|\rangle = \langle |v|, |\widetilde{\mathcal{V}}|\rangle$, $\langle |e|, |\widetilde{\mathcal{E}}|, |\mathbf{e}|\rangle = \langle |e|, |\widetilde{\mathcal{E}}|\rangle$. By $S(|PF|)$ we denote a collection of geo-plafales. By $S(|\mathcal{K}|)$ we denote a collection of $|\mathcal{K}|$ as supports of geo-plafales.

Definition 4.4. A geo-plafal morphism $|PF| \xrightarrow{\Lambda^* = \langle \vartheta, o\rangle} |PF|'$ is a tuple of maps's collections such that
(1) $|\mathcal{K}| \xrightarrow{\vartheta} |\mathcal{K}|'$, ϑ is a morphism between polyhedra;
(2) $\diamond|\mathcal{K}| \xrightarrow{o = \langle o^*, o^{**}\rangle} \diamond|\mathcal{K}|'$: $S(|\widetilde{\mathcal{V}}|) \xrightarrow{o^*} S(|\widetilde{\mathcal{V}}|')$ and $S(|\widetilde{\mathcal{E}}|) \xrightarrow{o^{**}} S(|\widetilde{\mathcal{E}}|')$.
Note that (2) is realized in accordance with (1).

Claim 4.5. *Geo-plafales and geo-plafal morphisms form the category Geo-Plafales, together with the component-wise compositions $\langle \vartheta, o\rangle \circ \langle \vartheta', o'\rangle = \langle \vartheta \circ \vartheta', o \circ o'\rangle$ and identities $id_{|PF|} = \langle id_{|\mathcal{K}|}, id_{\diamond|\mathcal{K}|}\rangle$. The proof is left to the reader.*

Definition 4.6. A product of two geo-plafales $|PF|^{prod} = |PF| \times |PF|'$ is defined by the product of polyhedra $\prod = |\mathcal{K}| \times |\mathcal{K}|'$ and the following condition holds:
(i) $\forall |\widetilde{\mathcal{V}}|^{prod} \in S(|\widetilde{\mathcal{V}}|^{prod}) : |\widetilde{\mathcal{V}}|^{prod} = |\widetilde{\mathcal{V}}| \times |\widetilde{\mathcal{V}}|', \forall |\widetilde{\mathcal{E}}|^{prod} \in S(|\widetilde{\mathcal{E}}|^{prod}) : |\widetilde{\mathcal{E}}|^{prod} = |e|$.
Notice that (i) is realized in accordance with $\prod$.

Definition 4.7. A coproduct of two geo-plafales $|PF|^{coprod} = |PF| + |PF|'$ is defined by the disjoint union of polyhedra $\coprod = |\mathcal{K}| + |\mathcal{K}|'$ and the following condition holds:
(i) in the general case, $\diamond|\mathcal{K}| + \diamond|\mathcal{K}|'$ is a coproduct of categories.
Let us remark that (i) is performed in accordance with $\coprod$.

Claim 4.8. *In **Geo-plafales**, there're no initial and terminal objects. The (co)-product of any two geo-plafales does not always exist. **Geo-plafales** is not a (quasi)-topos. The proof is similar to the proofs in the section 3.*

It is easily proved that **Plafales** $\xrightarrow{\mathcal{G} = \langle \mathcal{L}, \diamond \mathbf{F}\rangle}$ **Geo-Plafales** such that **SCpx** $\xrightarrow{\mathcal{L}}$ **Top** and $\diamond\mathbf{F}$ is a collection of identity functors, here **SCpx** is the category of abstract simplicial complexes, **Top** is the category of topological spaces. Note that advanced operations are performed for geo-plafales. The reader will have no difficulty in showing that $|PF| = \langle CW, \diamond PF\rangle$, where CW is a 1-dimensional cellular complex.

4.2. Point

Definition 4.9. $\langle pl, \diamond pl = \mathcal{U}\text{-standard}, \mathcal{X}\rangle$ is a limit point and is denoted by $|PF|^{pl}$, where pl is a limit point in $\mathbb{R}^3$, here $\mathcal{X}$ is a correspondence such that $pl \xrightarrow{\mathcal{X}} \diamond pl = \mathcal{U}$-standard. If $\diamond pl = pl$ (a singleton in **Top**), then we say that $|PF|^{pl} = pl$ is a trivial point and write ${}_*|PF|^{pl}$.

Agreement. We claim that $\langle pl, \diamond pl = \mathcal{U}\text{-standard}, \mathcal{X}\rangle = \langle pl, \diamond pl = \mathcal{U}\text{-standard}\rangle$. By $S(|PF|^{pl})$ we denote a collection of limit points in $|PF|^{U}$. By $S(pl)$ we denote a collection of limit points in $\mathbb{R}^3$ as supports of $S(|PF|^{pl})$.

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Definition 4.10. A morphism $|PF|^{pl}_i \xrightarrow{t_{pl}=\langle\varsigma,\mathcal{R}\rangle} |PF|^{pl}_j$ is a tuple of maps's collections such that
(1) $(pl)_i \xrightarrow{\varsigma} (pl)_j$;
(2) $\diamond(pl)_i \xrightarrow{\mathcal{R}} \diamond(pl)_j$.
Note that (2) is performed in accordance with (1).

Claim 4.11. *$S(|PF|^{pl})$ and morphisms t_{pl} form the category PFpl.*

Proof. $id_{|PF|^{pl}} = \langle id_{pl}, id_{\diamond pl}\rangle$. We get the following commutative diagrams:

$$\begin{tikzcd}
{|PF|^{pl}_1} \arrow[r, "t_{pl\_1-2}"] \arrow[d, shift right=2, "t_{pl\_3-4}\circ(t_{pl\_2-3}\circ t_{pl\_1-2})"'] \arrow[d, shift left=2, "(t_{pl\_3-4}\circ t_{pl\_2-3})\circ t_{pl\_1-2}"] \arrow[dr, "t_{pl\_2-3}\circ t_{pl\_1-2}" near start] & {|PF|^{pl}_2} \arrow[d, "t_{pl\_2-3}"] \arrow[dl, "t_{pl\_3-4}\circ t_{pl\_2-3}" near end] \\
{|PF|^{pl}_4} & {|PF|^{pl}_3} \arrow[l, "t_{pl\_3-4}"]
\end{tikzcd}$$

$$\begin{tikzcd}
{|PF|^{pl}_1} \arrow[r, "t_{pl\_1-2}"] \arrow[dr, "t_{pl\_1-2}"'] & {|PF|^{pl}_2} \arrow[d, "1_{|PF|^{pl}_2}"'] \arrow[dr, "t_{pl\_2-3}"] & \\
& {|PF|^{pl}_2} \arrow[r, "t_{pl\_2-3}"'] & {|PF|^{pl}_3}
\end{tikzcd}$$

□

Remark 5. The concept of $\mathbb{R}^3$ with isolated points and the concept of a flickering point, as a superposition of limit and isolated points, will be discussed elsewhere.

4.3. Manifold

Definition 4.12. $\langle M, \diamond M = \mathcal{U}\text{-standard}, \mathcal{I}\rangle$ is a manifold and is denoted by $|PF|^M$, where M is a topological manifold in $\mathbb{R}^3$, here $\mathcal{I}$ is a correspondence such that M(as a whole object) $\xrightarrow{\mathcal{I}} \diamond M = \mathcal{U}$-standard. If $\diamond M = M$ (an object of **TopMfd**[9]), then we say that $|PF|^M = M$ is a trivial manifold and write $_*|PF|^M$.

Agreement. We note that $\langle M, \diamond M = \mathcal{U}\text{-standard}, \mathcal{I}\rangle = \langle M, \diamond M = \mathcal{U}\text{-standard}\rangle$. By $S(|PF|^M)$ we denote a collection of manifolds in $|PF|^{\mathcal{U}}$. By $S(M)$ we denote a collection of manifolds in $\mathbb{R}^3$ as supports of $S(|PF|^M)$.

Remark 6. The concept of (open) cover $\mathcal{J}$ of M (including the relationship with (pre)-sheaf) and a collection of arbitrary $\mathcal{U}$-standard, which corresponds to $\mathcal{J}$, will be the object of another paper.

Definition 4.13. A morphism $|PF|^M_i \xrightarrow{t_M=\langle\varrho,\mathcal{D}\rangle} |PF|^M_j$ is a tuple of maps's collections such that
(1) $M_i \xrightarrow{\varrho} M_j$, ϱ is a morphism between topological manifolds;
(2) $\diamond M_i \xrightarrow{\mathcal{D}} \diamond M_j$.
Notice that (2) is realized in accordance with (1).

Claim 4.14. *$S(|PF|^M)$ and morphisms t_M form the category PFM. Omitted.*

Definitions 4.6 and 4.7 (in terms of the product manifold and the disjoint union of topological manifolds) and results of claim 4.8 are performed for **PFM**.

4.4. Dynamical System

Let us summarize and use the results of [13], [14].

Definition 4.15. $\langle\langle\mathcal{S}, G\rangle, \langle\mathcal{U}, \Omega\rangle\rangle$ is a dynamical system and is denoted by $_d|PF|^{\mathcal{U}}$, where $\mathcal{S} = \langle\mathbb{R}^3, \tau\rangle$ is a locally compact space satisfies the second axiom of countability and undergoes homeomorphic

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[9] **TopMfd** is the category of topological manifolds.

transformations, here G is a topological group; Ω is a collection of identity functors and results of advanced operations.

Definition 4.16. A homomorphism ${}_d|PF|_i^U \xrightarrow{t_{DS}=\langle\Phi,\Delta\rangle} |PF|_j^U$ is a tuple of maps's collections such that
(1) $\langle\mathcal{S}_i, G_i\rangle \xrightarrow{\Phi=\langle\mu,\mu^*\rangle} \langle\mathcal{S}_j, G_j\rangle$, Φ is a homomorphism between dynamical systems: $\mathcal{S}_i \overset{\mu}{\to} \mathcal{S}_j$, μ is a continuous mapping and $G_i \overset{\mu^*}{\longrightarrow} G_j$, μ^* is a homomorphic mapping;
(2) Δ is a collection of maps between $\langle U_i, \Omega_i\rangle$ and $\langle U_j, \Omega_j\rangle$.
Let us remark that (2) is realized in accordance with (1).

The category of dynamical systems is not considered in this paper.

Perspective

Plafal ($*$-plafal) complex and plafal ($*$-plafal) cell complex are generalizations of the plafal ($*$-plafal), and this will be the object of another paper. We will give a short description. For any abstract simplicial complex (n-dimensional cell complex) let there be a correspondence between each face (k-skeleton) and $\mathcal{U}$-standard ($\mathcal{U}^*$-standard respectively).

5. P = Np (with Pre-Prepared Tape)

The proof of the equality of complexity classes P and NP will be given in a constructive way using the apparatus of the constructive theory of serendipity approximations (CTSA)[10] [4], [15 – 24]. The total volume of CTSA is about 1000 papers (they can be traced evolutionarily by the indicated references). In the general case, by S we denote an instance of an NP-complete problem. Using the deterministic machine, C is a type of S with answer **"Yes"** after the checking, otherwise is $\overline{C}$. The implementation model will be described after theorem 5.1. We will operate with a plane in $|PF|^U$. By $|PF|_{pl}^U$ we denote this topological subspace.

A universal algorithm will be proposed, where the specialization of an NP-complete problem is irrelevant (including SAT). Therefore, if we need to solve a specific NP-complete problem in polynomial time, we apply the specified algorithm.

Let us describe the pre-prepared tape. All instances of an NP-complete, which are printed on the tape, are sorted by unique parameters (2nd stage of theorem 5.1), but a priori we do not know their types (C or $\overline{C}$).

Theorem 5.1. *Based on the theory of plafales, there exists an algorithm that allows to check all instances of an NP-complete problem in polynomial time.*

Proof. 1st stage. Computational template (CT). Let us consider the following labeled geo-plafal: $|PF|_{N_0}^{N_N} = \langle|\mathcal{K}|$ is a boundary of SFE$, \diamond|\mathcal{K}|\rangle$, where SFE is a serendipity finite element (square $D = \{|x| \le 1, |y| \le 1\}$) on N vertices (nodes)[11]. Vertices $|v|_i$ are evenly spaced on the boundary of SFE ($|x| = 1, |y| = 1$) by the rule

$$\mathcal{G} = \begin{cases} i \in \{4\cdot l + 1\}; -1 \le x < 1, y = -1, \\ i \in \{4\cdot l + 2\}; x = 1, -1 \le y < 1, \\ i \in \{4\cdot l + 3\}; -1 < x \le 1, y = 1, \\ i \in \{4\cdot (l + 1)\}; x = -1, -1 < y \le 1, \\ l \in \mathbb{Z}_+ \cup \{0\}. \end{cases}$$

For $\diamond|\mathcal{K}|$ we have the following: ${}^i|\mathcal{V}| = \langle |v|_i, |\widetilde{\mathcal{V}}|_i = \langle S_i, \mathcal{H}_i = \zeta_i(t)\cdot C_i + \theta_i(t)\cdot \overline{C}_i\rangle\rangle$, here $\mathcal{H}_i$ is a linear combination such that $\zeta_i(t) + \theta_i(t) = 1$, $\zeta_i(t), \theta_i(t) \in \mathbb{R}$, where t is time; and $\forall|\mathcal{E}| \in |PF|_{N_0}^{N_N} : |\mathcal{E}| = \langle|e|, |e|\rangle$.

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[10] A. N. Khomchenko is the founder of the CTSA scientific school (since 1982).

[11] Serendipity finite element is a finite element that does not contain interior nodes.

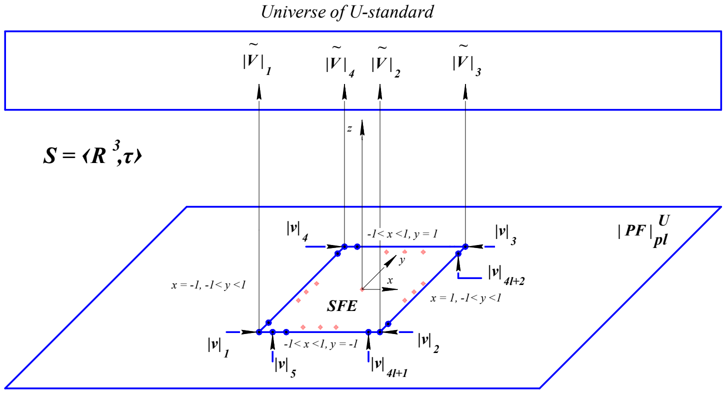

Figure 6. Computational template (in general).

2nd stage. Relationship between S_i and basis function (BF)[12] **$L_i(x,y)$.** Without loss of generality, let us consider the word S_i over a given alphabet into a representation over the alphabet $\{\overline{0,9}\}$. Then to each S_i corresponds a unique "weight" $\chi_i = \frac{S_i}{\sum_{i=1}^N S_i}$; however $\chi_i = \frac{1}{G}\iint_D L_i(x,y)dxdy \geq 0$, here G is the area of D, where N is a quantity of instances of an NP-complete problem. Hence to each χ_i corresponds a unique collection of the basis functions (UCBF) in the CTSA. A relationship between the word S_i and UCBF is structurally shown[13].

3rd stage. The analysis of the field function (FF)[14]**.** Let us configure the CT. Without loss of generality, $\chi_1 < \chi_2 < \ldots < \chi_l < \ldots$. So, ${}^i|\mathcal{V}| = \langle |v|_i(\chi_i), |\widetilde{\mathcal{V}}|_i \rangle$.

We will operate with the non-standard Dirichlet boundary value problem [4]. $f(x,y,t) = \sum_{i=1}^N \lambda_i(t) \cdot L_i(x,y)$ such that $(\star) = \begin{cases} g_i = \frac{1}{S_i}, g_1 > g_2 > \ldots > g_l > \ldots, \\ \lambda_i(t) = \zeta_i(t) \cdot (g_i - g_{i+1}) + g_{i+1}, \\ \lambda_i(t) \in (g_{i+1}, g_i),\ t < T^*, \\ \lambda_i(t) = \left[\begin{array}{l} g_i,\ t \geq T^*, \\ g_{i+1},\ t \geq T^*. \end{array} \right. \end{cases}$ Let us comment. For C we get $\lambda_i(t) = g_i$, otherwise is $\lambda_i(t) = g_{i+1}$ (see figure 7).

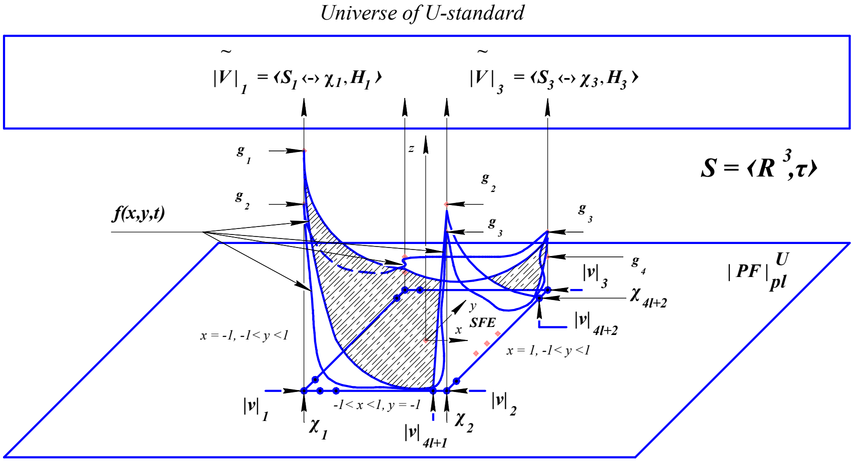

Figure 7. The field function.

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[12] Let us show the key properties of the basis function. $L_i(x_k, y_k) = \delta_{ik}$ and $\sum_{i=1}^N L_i(x,y) = 1$, where δ_{ik} is the Kronecker delta, here i is a number of function and k is a number of node.

[13] In fact, this relationship acts as a "lock" (hard model [25]) such that $S_i \leftrightarrow$ UCBF. This enables us to determine S_i based on λ_i, here λ_i is a value of the FF at a given node $|v|_i$.

[14] The field function (interpolant) $f(x,y,t) = \sum_{i=1}^N \lambda_i(t) \cdot L_i(x,y)$ makes an interpolation on the boundary of SFE and an approximation inside of SFE.

Taking into account the field function $f(x,y,t) = \sum_{i=1}^{N} \lambda_i(t) \cdot L_i(x,y)$ and $\mathcal{G}$, let us provide the claim, which is a generalization of the results of Ph. D. thesis [19].

Claim 5.2. *For SFE on N vertices the condition of invariant of the field function (stability of the field function with respect to the basis function) is the following*

$$\begin{cases} \frac{\sum_{i=1}^{4} \lambda_i(t)}{4} = \frac{\sum_{i=5}^{N} \lambda_i(t)}{N-4}, N = 4 \cdot k, \\ \frac{\sum_{i=1}^{4} \lambda_i(t)}{4} = \frac{\sum_{i \in \{5,9,\dots,4 \cdot l+1\}} \lambda_i(t)}{N-1} + \frac{\sum_{i \in \{4 \cdot l+2, 4 \cdot l+3, 4 \cdot (l+1), 1 \le l < N\}} \lambda_i(t)}{N-5}, N = 4 \cdot k + 1, \\ \frac{\sum_{i=1}^{4} \lambda_i(t)}{4} = \frac{\sum_{i \in \{4 \cdot l+1, 4 \cdot l+2, 1 \le l \le N\}} \lambda_i(t)}{N-2} + \frac{\sum_{i \in \{4 \cdot l+3, 4 \cdot (l+1), 1 \le l < N\}} \lambda_i(t)}{N-6}, N = 4 \cdot k + 2, \\ \frac{\sum_{i=1}^{4} \lambda_i(t)}{4} = \frac{\sum_{i \in \{4 \cdot l+1, 4 \cdot l+2, 4 \cdot l+3, 1 \le l \le N\}} \lambda_i(t)}{N-3} + \frac{\sum_{i \in \{8,12,\dots,4 \cdot (l+1)\}} \lambda_i(t)}{N-7}, N = 4 \cdot k + 3. \end{cases}$$

Proof. For SFE on 8 nodes the above condition is $\frac{\sum_{i=1}^{4} \lambda_i(t)}{4} = \frac{\sum_{i=5}^{8} \lambda_i(t)}{4}$ [4], [19]. Let us consider the BF of SFE on N nodes in the following representation: $L_i(x,y) =$
$= \sum_{k,l=0}^{r} \sigma_{kl} \cdot x^k \cdot y^l = \sum_{k,l=0}^{2} \mu_{kl} \cdot x^k \cdot y^l \cdot Q(x,y)$ such that $\min r = \begin{cases} \frac{N}{4}\ if\ 4 \mid N, \\ [\frac{N}{4}] + 1\ if\ 4 \nmid N. \end{cases}$ Let us show the rule of arrangement of the number of nodes. $N \equiv b \pmod 4$ (see figure 8). In the case of $b \in \{1,2,3\}$, b nodes are spaced on b sides of SFE.

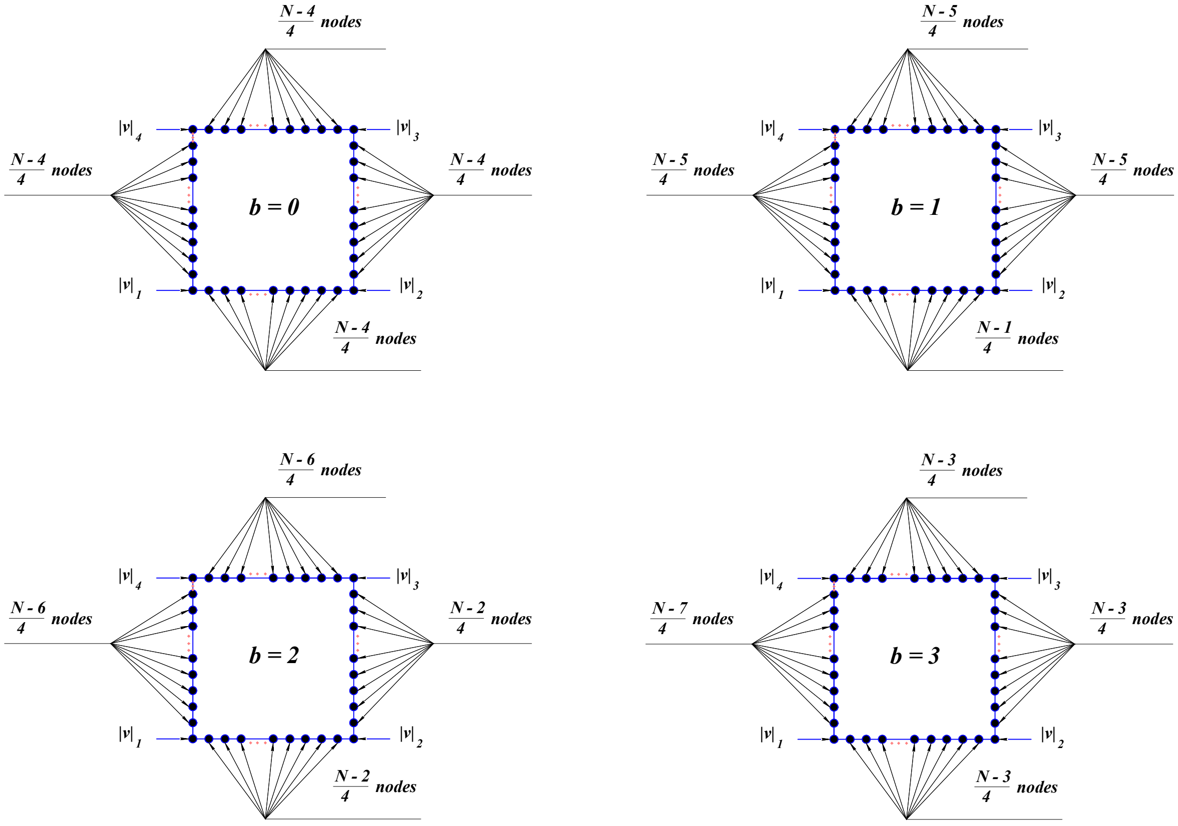

Figure 8. Configuration of nodes.

$K_1(x,y), \dots, K_8(x,y)$ is a collection of the basis functions of SFE on 8 nodes [4], [17], [22]. Therefore, $K_1(-1,-1) = 1$, $K_2(1,-1) = 1$, $K_3(1,1) = 1$, $K_4(-1,1) = 1$, $K_5(0,-1) = 1$, $K_6(1,0) = 1$, $K_7(0,1) = 1$, $K_8(-1,0) = 1$. Let us show one of the collections of the basis functions for SFE·8.

$$K_1(x,y) = \frac{1}{4} \cdot (1-x) \cdot (1-y) \cdot xy, \quad K_5(x,y) = \frac{1}{4} \cdot (1-x^2) \cdot (1-y)^2,$$

$$K_2(x,y) = -\frac{1}{4} \cdot (1+x) \cdot (1-y) \cdot xy, \qquad K_6(x,y) = \frac{1}{4} \cdot (1-y^2) \cdot (1+x)^2,$$

$$K_3(x,y) = \frac{1}{4} \cdot (1+x) \cdot (1+y) \cdot xy, \quad K_7(x,y) = \frac{1}{4} \cdot (1-x^2) \cdot (1+y)^2,$$

$$K_4(x,y) = -\frac{1}{4} \cdot (1-x) \cdot (1+y) \cdot xy, \quad K_8(x,y) = \frac{1}{4} \cdot (1-y^2) \cdot (1-x)^2.$$

Let us introduce the following system.

$$\mathcal{Y} = \begin{cases} \mathcal{W}_1 = \underbrace{\lambda_5(t) \cdot L_5(x,y) + \ldots + \lambda_{4 \cdot l+1}(t) \cdot L_{4 \cdot l+1}(x,y)}_{q_1 = (N + \frac{2 \cdot b^3}{3} - 4 \cdot b^2 + \frac{19 \cdot b}{3} - 4)/4} = K_5(x,y) \cdot g_5 = \\ = K_5(x,y) \cdot (\underbrace{\lambda_5(t) \cdot Q_5(x,y) + \ldots + \lambda_{4 \cdot l+1}(t) \cdot Q_{4 \cdot l+1}(x,y)}_{q_1 = (N + \frac{2 \cdot b^3}{3} - 4 \cdot b^2 + \frac{19 \cdot b}{3} - 4)/4}), \\ \mathcal{W}_2 = \underbrace{\lambda_6(t) \cdot L_6(x,y) + \ldots + \lambda_{4 \cdot l+2}(t) \cdot L_{4 \cdot l+2}(x,y)}_{q_2 = (N - \frac{4 \cdot b^3}{3} + 6 \cdot b^2 - \frac{17 \cdot b}{3} - 4)/4} = K_6(x,y) \cdot g_6 = \\ = K_6(x,y) \cdot (\underbrace{\lambda_6(t) \cdot Q_6(x,y) + \ldots + \lambda_{4 \cdot l+2}(t) \cdot Q_{4 \cdot l+2}(x,y)}_{q_2 = (N - \frac{4 \cdot b^3}{3} + 6 \cdot b^2 - \frac{17 \cdot b}{3} - 4)/4}), \\ \mathcal{W}_3 = \underbrace{\lambda_7(t) \cdot L_7(x,y) + \ldots + \lambda_{4 \cdot l+3}(t) \cdot L_{4 \cdot l+3}(x,y)}_{q_3 = (N + \frac{2 \cdot b^3}{3} - 2 \cdot b^2 + \frac{b}{3} - 4)/4} = K_7(x,y) \cdot g_7 = \\ = K_7(x,y) \cdot (\underbrace{\lambda_7(t) \cdot Q_7(x,y) + \ldots + \lambda_{4 \cdot l+3}(t) \cdot Q_{4 \cdot l+3}(x,y)}_{q_3 = (N + \frac{2 \cdot b^3}{3} - 2 \cdot b^2 + \frac{b}{3} - 4)/4}), \\ \mathcal{W}_4 = \underbrace{\lambda_8(t) \cdot L_8(x,y) + \ldots + \lambda_{4 \cdot (l+1)}(t) \cdot L_{4 \cdot (l+1)}(x,y)}_{q_4 = (N - b - 4)/4} = K_8(x,y) \cdot g_8 = \\ = K_8(x,y) \cdot (\underbrace{\lambda_8(t) \cdot Q_8(x,y) + \ldots + \lambda_{4 \cdot (l+1)}(t) \cdot Q_{4 \cdot (l+1)}(x,y))}_{q_4 = (N - b - 4)/4)}. \end{cases}$$

Using $\mathcal{Y}$, we have $\sum_{i=1}^{N} \lambda_i(t) \cdot L_i(x,y) = \sum_{1 \le i \le 4} \lambda_i(t) \cdot L_i(x,y) + \sum_{1 \le i \le 4} \mathcal{W}_i =$ $= \sum_{1 \le i \le 4} \lambda_i(t) \cdot K_i(x,y) \cdot Q_i(x,y) + \sum_{1 \le i \le 4} \mathcal{W}_i$. Let us remark that $L_1(-1,-1) =$ $= K_1(-1,-1)$, $L_2(1,-1) = K_2(1,-1)$, $L_3(1,1) = K_3(1,1)$, $L_4(-1,1) = K_4(-1,1)$.

Taking into account the above condition $\frac{\sum_{i=1}^{4} \lambda_i(t)}{4} = \frac{\sum_{i=5}^{8} \lambda_i(t)}{4}$ for SFE·8 and the properties of the

BF: $\begin{cases} L_1(x,y) + L_2(x,y) + L_5(x,y) + \ldots + L_{4 \cdot l+1}(x,y) = 1, \\ L_2(x,y) + L_3(x,y) + L_6(x,y) + \ldots + L_{4 \cdot l+2}(x,y) = 1, \\ L_3(x,y) + L_4(x,y) + L_7(x,y) + \ldots + L_{4 \cdot l+3}(x,y) = 1, \\ L_1(x,y) + L_4(x,y) + L_8(x,y) + \ldots + L_{4 \cdot (l+1)}(x,y) = 1, \end{cases}$ we get $\mathcal{I} = \begin{cases} Q_1(0,-1) + Q_2(0,-1) + Q_5(0,-1) + \ldots + Q_{4 \cdot l+1}(0,-1) = 1, \\ Q_2(1,0) + Q_3(1,0) + Q_6(1,0) + \ldots + Q_{4 \cdot l+2}(1,0) = 1, \\ Q_3(0,1) + Q_4(0,1) + Q_7(0,1) + \ldots + Q_{4 \cdot l+3}(0,1) = 1, \\ Q_1(-1,0) + Q_4(-1,0) + Q_8(-1,0) + \ldots + Q_{4 \cdot (l+1)}(-1,0) = 1, \end{cases}$

and in accordance with coordinates of Q_j in $\mathcal{I}$ we have $\frac{\sum_{i=1}^{4} \lambda_i(t)}{4} = \frac{\sum_{i=5}^{8} g_i}{4}$. Our goal is that the choice of the basis functions is insignificant. So, the expected value of difference of two field functions with different collections of the basis functions is equal to zero. To do that, let us show for the first equation of $\mathcal{I}$. $Q_1(0,-1) + + Q_2(0,-1) = 0$ and $Q_5(0,-1) + \ldots + Q_{4 \cdot l+1}(0,-1) = 1$, then $Q_{4 \cdot j+1}(0,-1) = \frac{1}{q_1}$, where q_i is a quantity of nodes on the side of SFE (see $\mathcal{Y}$).

Let us show for $b = 0$. $\forall j \in \mathbb{N} \setminus \overline{\{1,4\}} : Q_j(x,y) = \frac{4}{N-4}$, here (x,y) are coordinates in $\mathcal{I}$. Thus we obtain $\frac{\sum_{i=1}^{4} \lambda_i(t)}{4} = \frac{\sum_{i=5}^{N} \lambda_i(t)}{N-4}$. $\square$

Remark 7. There is an additional condition of symmetry of boundary values in some nodes for SFE·12 with n-parametric interpolation ($n > 13$) and SFE·16 [19].

Further, we will use the results of Stepanets' school[15]. Let us provide the results of [26]. Let $\mathcal{M}$ be a set of all sequences $m = \{m_k\}_{k=1}^{\infty}$ of nonnegative numbers such that $|m| := \sum_{k=1}^{\infty} m_k \leq 1$. Let r be a positive number and $\mathcal{A}_r$ is a set of all nonincreasing sequences $\alpha = \{\alpha_k\}_{k=1}^{\infty}$ of positive numbers such that $\lim_{k\to\infty} \alpha_k = 0$. In the case of $r \in (0,1)$, we have the following condition: $\sum_{k=1}^{\infty} \alpha_k^{1/(1-r)} < \infty$. Denote by $\gamma_n = \{k_1, k_2, \dots, k_n\}$ a collection of n different integers $n = 1, 2, \dots$, and for any $m \in \mathcal{M}$ and $\alpha \in \mathcal{A}_r$, set

$$\mathcal{E}_n(m) = \mathcal{E}_n(\alpha, r, m) := \sum_{k=1}^{\infty} \alpha_k \cdot m_k^r - \sup_{\gamma_n} \sum_{k\in\gamma_n} \alpha_k \cdot m_k^r,$$

and

$$\mathcal{E}_n = \mathcal{E}_n(\alpha, r) := \sup_{m\in\mathcal{M}} \mathcal{E}_n(m) = \sup_{m\in\mathcal{M}} \mathcal{E}_n(\alpha, r, m).$$

(i) Let $\alpha \in \mathcal{A}_r$ and $r \geq 1$. Then $\forall n \in \mathbb{N}\ \exists s^* > n$ such that

$$\mathcal{E}_n(\alpha, r) = (s^* - n) \cdot \left(\sum_{k=1}^{s^*} \alpha_k^{-1/r}\right)^{-r}.$$

s^* is determined by

$$\sup_{s>n} (s-n) \cdot \left(\sum_{k=1}^{s} \alpha_k^{-1/r}\right)^{-r} = (s^* - n) \cdot \left(\sum_{k=1}^{s^*} \alpha_k^{-1/r}\right)^{-r}.$$

The exact upper bound is realized by the sequence $m^* = \{m_k^*\}_{k=1}^{\infty}$ from $\mathcal{M}$ such that

$$m_k^* = \begin{cases} (\alpha_k^{1/r} \cdot \sum_{i=1}^{s^*} \alpha_i^{-1/r})^{-1}, \ k \in [1, s^*], \\ 0, \ k > s^*. \end{cases}$$

(ii) Let $\alpha \in \mathcal{A}_r$ and $r \in (0,1)$. Then $\forall n \in \mathbb{N}$ is performed the following equality

$$\mathcal{E}_n = \mathcal{E}_n(\alpha, r) = \left((s^* - n)^{1/(1-r)} \cdot \left(\sum_{k=1}^{s^*} \alpha_k^{-1/r}\right)^{r/(r-1)} + \sum_{k=s^*+1}^{\infty} \alpha_k^{1/(1-r)} \right)^{1-r},$$

here $s^* > n$ is the biggest natural number such that

$$s - n \leq \alpha_s^{1/r} \sum_{k=1}^{s} \alpha_k^{-1/r}, \text{ for all } s \in (n, s^*].$$

The exact upper bound is realized by the sequence $m^* = \{m_k^*\}_{k=1}^{\infty}$ from $\mathcal{M}$ such that

$$m_k^* = \begin{cases} (\alpha_k^{-1/r} \cdot (s^* - n)^{1/(1-r)} \cdot \left(\sum_{i=1}^{s^*} \alpha_i^{-1/r}\right)^{1/(r-1)} \cdot \mathcal{E}_n^{1/(r-1)}, \ k \in [1, s^*], \\ \alpha_k^{1/(1-r)} \cdot \mathcal{E}_n^{1/(r-1)}, \ k > s^*. \end{cases}$$

Note. Since solutions of these extremal problems may be of an independent interest, in the paper [26], the authors propose a new method of finding these solutions, which leads to the required result by essentially shorter and more transparent way.

Let us combine all of the above into the following provisions. It is easily shown that the conditions of [26] can be implemented for the field function $f(x, y, t)$. Indeed, $\lambda_i(t) > 0, \lambda_i(t) \downarrow, \lim_{i\to\infty} \lambda_i(t) \to$

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[15] In the series of works, O. I. Stepanets' and his successors study approximation properties of the spaces S_φ^p introduced by Stepanets'. Problems of finding exact values of n-term approximations of q-ellipsoids in the spaces considered are reduced to some extremal problems for series with terms that are determined as a product of elements of two nonnegative sequences one of which is fixed and another varies on certain set.

$0, L_i(x,y) = m_i^r$ and it is clear that $\sum_{i=1}^{\infty} \lambda_i^{1/(1-r)}(t) < \infty$ in the case of $r \in (0,1)$. This means that the results of [26] provide an effective tool for the analysis of the field function[16]. Notice that the above results give an opportunity to make the estimate for $\mathcal{N} \to \infty$ (word length).

Ideology of P = NP. Due to checking the first M instances (see 1st stage: configuration of nodes) with total computational complexity of $O(n^k)$, we can identify $(N-M)$ instances without checking them[17], here M is polynomially bounded. **Agreement.** Without loss of generality it can be assumed that a functional type of $\lambda_i(t) = f_i(t)$ is not principled; all required $\{g_i, g_{i+1}\}$ are defined at time $t = T^*$. Note that this configuration is an example of the soft mathematical modeling [25].

Let us implement the results of claim 5.2. We can assume without loss of generality that $\frac{\sum_{i=1}^{4}\lambda_i(t)}{4} \cdot \varpi(N,t) = \frac{\sum_{i=5}^{N}\lambda_i(t)}{N-4}$, where $\varpi(N,t)$ is a stabilization function.

Claim 5.3. *The value of the field function at the barycentre*[18] $\frac{\sum_{i=1}^{N}\lambda_i(t)}{N}$ *equals* $\kappa \cdot \varpi(N,t) \cdot \mathcal{E}_n(\lambda(t),1)$, *here* $\kappa = const$, *see* ***(i)***.

Proof. It is easily shown that $\frac{\sum_{i=1}^{s^*}\lambda_i(T^*)}{s^*} = \kappa \cdot \varpi(s^*,T^*) \cdot \mathcal{E}_n(\lambda(T^*),1)$, where $s^* = M$. Taking into account $\frac{\sum_{i=1}^{4}\lambda_i(T^*)}{4} \cdot \varpi(N,T^*) = \frac{\sum_{i=5}^{N}\lambda_i(T^*)}{N-4}$, by induction, we obtain the statement. ☐

Corollary 5.4. *We get* $\varpi(N,T^*) = \frac{4\cdot\sum_{i=1}^{4}\lambda_i(T^*)}{4\cdot N\cdot\kappa\cdot\mathcal{E}_n(\lambda(T^*),1)-(N-4)\cdot\sum_{i=1}^{4}\lambda_i(T^*)}$, $\frac{\sum_{i=1}^{4}\lambda_i^r(t)}{4} \times \times (\varpi(N,t))^r = \frac{\sum_{i=5}^{N}\lambda_i^r(t)}{N-4}$ *and* $\frac{\sum_{i=1}^{N}\lambda_i^r(t)}{N}$ *equals* $\upsilon \cdot (\varpi(N,t))^r \cdot \mathcal{E}_n(\lambda(t), 0<r<1)$, *here* $\upsilon = const$, *see* ***(ii)***. *The proof is omitted.*

Remark 8. Claim 5.3 links two scientific schools: CTSA and Stepanets' school.

Using claim 5.3, corollary 5.4 and the results of the Bogolyubov principle of the decay of correlations for an infinite three-dimensional systems [27] for the FF (see figure 7), we have the following system $\mathcal{U}$ for determination of $\lambda_i(t)$, here $i > s^*$.

$$\mathcal{U} = \begin{cases} \frac{\sum_{i=1}^{N}\lambda_i(t)}{N} = \kappa\cdot\varpi(N,t)\cdot\mathcal{E}_n(\lambda(t),1),\ (1) \\ \frac{\sum_{i=1}^{4}\lambda_i^r(t)}{4}\cdot(\varpi(N,t))^r \ = \ \frac{\sum_{i=5}^{N}\lambda_i^r(t)}{N-4},\ (2) \\ \frac{\sum_{i=1}^{N}\lambda_i^r(t)}{N} = \upsilon\cdot(\varpi(N,t))^r\cdot\mathcal{E}_n(\lambda(t),0<r<1),\ (3) \\ \frac{d}{dt}F(t) = -\mathcal{L}F(t) + \mathbf{a}\mathcal{L}^{\mathrm{int}}F(t), F(t)|_{t=0} = F(0).\ (4) \end{cases}$$

Let us comment. (4) is the initial-value problem of the BBGKY hierarchy[19]. Taking into account $s^* = M$, for operators $\mathcal{L}$ (which is defined by the Poisson bracket of free particles with boundary conditions on ∂W_{s^*}) and $\alpha\mathcal{L}^{\mathrm{int}}$ (in the case of $t > 0$) we get

$$(\mathcal{L}F(t))_{s^*}(x_1,\dots,x_{s^*}) = \mathcal{L}_{s^*}F_{s^*}(t) = \sum_{i=1}^{s^*}\left\langle p_i, \frac{\partial}{\partial q_i}\right\rangle_{\partial W_{s^*}} F_{s^*}(t,x_1,\dots,x_{s^*}). \tag{5.1}$$

Each particle is characterized by the phase coordinates $(q_i,p_i) \equiv x_i \in \mathbb{R}^3\times\mathbb{R}^3, i \geq 1$.

$$(\mathbf{a}\mathcal{L}^{\mathrm{int}}F(t))_{s^*}(x_1,\dots,x_{s^*}) = \tag{5.2}$$

$$= \sigma^2\sum_{i=1}^{s^*}\int_{\mathbb{R}^3\times\mathbb{S}_+^2} dp_{s^*+1}d\eta\langle\eta,(p_i-p_{s^*+1})\rangle \times (F_{s^*+1}(t,x_1,\dots,q_i,p_i^*,\dots,x_{s^*},$$
$$,q_i-\sigma\eta,p_{s^*+1}^*) - F_{s^*+1}(t,x_1,\dots,x_{s^*},q_i+\sigma\eta,p_{s^*+1})),$$

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[16] We are able to evaluate all "fluctuations" of the field function (according to all possible shifts).

[17] We are able to level the specified surplus of instances.

[18] Taking into account the results of Khomchenko's school and Mӧbius's problem (1827), we are able to manage the integral characteristics of the basis function at the barycentre.

[19] The Bogolyubov (Bogoliubov) - Born - Green - Kirkwood - Yvon hierarchy.

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$\langle \eta, (p_i - p_{s^*+1}) \rangle = \sum_{\alpha=1}^{3} \eta^\alpha (p_i^\alpha - p_{s^*+1}^\alpha), \mathbb{S}_+^2 = \{\eta \in \mathbb{R}^3 | |\eta| = 1, \langle \eta, (p_i - p_{s^*+1}) \rangle > 0\}$, for impulses $p_i^*, p_{s^*+1}^*$: $p_i^* = p_i - \eta \langle \eta, (p_i - p_{s^*+1}) \rangle, p_{s^*+1}^* = p_{s^*+1} + \eta \langle \eta, (p_i - p_{s^*+1}) \rangle$. Also for (4) we take into account s-particles correlation functions $G_{s^*}(t)$ [27], so

$$F_{|Y|}(t, Y) = \sum_{\mathrm{P}:\ Y = \bigcup_i X_i} \prod_{X_i \subset \mathrm{P}} G_{|X_i|}(t, X_i), Y \equiv (x_1, \ldots, x_{s^*}). \tag{5.3}$$

The chaos condition for s-particles correlation functions is as follows [27]:

$$G_{s^*}(t, x_1, \ldots, x_{s^*}) = \tag{5.4}$$

$$= \sum_{n=0}^{\infty} \frac{1}{n!} \int_{\mathbb{R}^3 \times \mathbb{R}^3} dx_{s^*+1} \ldots dx_{s^*+n} \mathbf{U}_{s^*+n}(t, 1, \ldots, s^* + n) \prod_{i=1}^{s^*+n} G_1(0, x_i) \chi_{\Gamma_{s^*+n}}.$$

Note that (5.4) can be obtained on the basis of the solutions of Liouville's equations.

Taking into account $\mathcal{U}$ and (5.1) – (5.4), $f(x, y, t)$ is a phase space: each z-axis at the node is a particle of unit mass and diameter $\rho > 0$ such that $|q_i - q_j| \geq \rho$ and $\lambda_{1 \leq i}^{0 < r < 1}(t) \in (g_{i+1}, g_i)$. This implies that, $\lim_{t \to T^* - 0} \lambda_i(t) \to \{g_i, g_{i+1}\}$ such that $r = \omega(t)$ (taking into account $t = T^*$) and the number of r is polynomially bounded. □

The algorithm is able to indicate a position number of C in $(N - M)$ instances.

Remark 9. For a metrizable space $\mathcal{S} = \langle \mathbb{R}^3, \tau \rangle$ the following condition holds: existence of the space of sequences of integrable translation-invariant functions [27].

The implementation model. 1. *Deterministic Turing machine (DTM).* Using the DTM with polynomial memory, we operate with pre-prepared tape $\chi_1 < \chi_2 < \ldots < \chi_l < \ldots$ (see 2nd stage). Total computational complexity of the analysis of the field function (see 3rd stage) is checking the first M instances with total computational complexity of $O(n^k)$ (see ideology of P = NP), checking $\lambda_{1 \leq i \leq s^*}^{0 < r < 1}(t)$ with total computational complexity of $O(n^l)$ and analysis of $\mathcal{U}$ with total computational complexity of $O(n^m)$. Thus we have leveled the surplus of instances $(N - M)$ and P = NP is performed with pre-prepared tape. 2. *Quantum Turing machine (QTM) and DTM.* Using the QTM, we perform the above tape [28].

Proposition 5.5. *Using the QTM, theorem 5.1 is performed for BQP = NP.*

Remark 10. For proposition 5.5: $\mathcal{H}_i$ is a vector in Hilbert space (see 1st stage).

Historical background. Dual role of the basis functions of serendipity finite elements is proved by O. Zienkiewicz (1968). His group constructed a biquadratic basis (SFE·8), but Zienkiewicz's paradox is formed: in some nodes exist negative loads (unnatural spectrum). In 1982 Khomchenko constructed a 13-parametric bicubic basis (SFE·12) with positive loads [20], [21], thus Zienkiewicz's paradox was solved. Methods of constructing alternative (physically correct) bases are presented in [16].

Note. The proof was given within the framework of Bernstein's school of constructive function theory. S. N. Bernstein presented in a constructive way (1912) the Weierstrass theorem (1885). CTSA is a branch of the aforementioned school.

Appendix A

Let us give a strengthening of the results [26] in the case of $|m_k| \leq 1, \alpha_k \geq 0$.

Let $\mathcal{M}$ be a set of all sequences $m = \{m_k\}_{k=1}^{\infty}$ of real numbers such that

$$|m_k| \leq 1, \ k = 1, 2, \ldots, (k \in \mathbb{N}), \tag{A1}$$

and $|m| := \sum_{k=1}^{\infty} m_k \leq 1$. Let also N be a fixed positive integer and $\mathcal{A}_N$ be a set of all nonincreasing sequences $\alpha = \{\alpha_k\}_{k=1}^{\infty}$ of nonnegative numbers such that $\alpha_k = 0, k > s$.

Denote by $\gamma_n = \{k_1, k_2, \ldots, k_n\}$ a collection of n different integers $n = 0, 1, 2, \ldots$, and for any $m \in \mathcal{M}$ and $\alpha \in \mathcal{A}_N$, set

$$\mathcal{E}_n(m) = \mathcal{E}_n(\alpha, m) := \sum_{k=1}^{\infty} \alpha_k \cdot m_k - \sup_{\gamma_n} \sum_{k\in\gamma_n} \alpha_k \cdot m_k = \sum_{k=1}^{N} \alpha_k \cdot m_k - \sup_{\gamma_n} \sum_{k\in\gamma_n} \alpha_k \cdot m_k$$

and

$$\mathcal{E}_n = \mathcal{E}_n(\alpha) := \sup_{m\in\mathcal{M}} \mathcal{E}_n(m) = \sup_{m\in\mathcal{M}} \mathcal{E}_n(\alpha, m). \tag{A2}$$

Lemma A.1. *Let $\alpha \in \mathcal{A}_N, N \in \mathbb{N}$. Then for any $n = 0, 1, \ldots, n < N$*

$$\mathcal{E}_n(\alpha) = \sum_{k=n+1}^{N} \alpha_k.$$

In this case, the exact upper bound on the right-hand side of the relation (5.6) is realized by the sequence $m^* = \{m_k^*\}_{k=1}^{\infty}$ such that

$$m_k^* = \begin{cases} 1,\ k \in [1, N], \\ -1,\ k \in [N+1, 2\cdot N - 1], \\ 0,\ k > 2\cdot N - 1. \end{cases} \tag{A3}$$

Proof. Let $\mathcal{M}'$ be a set of all sequences $m \in \mathcal{M}$ such that $\alpha_1 \cdot m_1 \ge \ldots \ge \alpha_N \cdot m_N$. Then

$$\mathcal{E}_n = \sup_{m\in\mathcal{M}'} \mathcal{E}_n(m).$$

Indeed, let $m \in \mathcal{M}$. We construct a sequence $m' \in \mathcal{M}'$ such that $\mathcal{E}_n(m) \le \mathcal{E}_n(m')$.

Step 1. If $\max_{k\in[1,N]} \alpha_k \cdot m_k = \alpha_{k_1} \cdot m_{k_1}, k_1 > 1$, then represent the number m_{k_1} in the form $m_{k_1} = \overline{m}_{k_1} + \widetilde{m}_{k_1}$, where $\alpha_1 \cdot (m_1 + \overline{m}_{k_1}) = \alpha_{k_1} \cdot m_{k_1}$. Consider the sequence $m^{(1)} = \{m_k^{(1)}\}_{k=1}^{\infty}$ such that

$$m_k^{(1)} = \begin{cases} m_1 + \overline{m}_{k_1},\ k = 1, \\ \widetilde{m}_{k_1},\ k = k_1, \\ m_k,\ k \ne 1, k_1. \end{cases}$$

Since $\alpha_1 \ge \alpha_{k_1}$, we have $m_1^{(1)} = m_1 + \overline{m}_{k_1} \le m_{k_1}$, $|m^{(1)}| = |m|$, $\mathcal{E}_n(m^{(1)}) \ge \mathcal{E}_n(m)$ and $\max_{k\in[1,N]} \alpha_k \cdot m_k^{(1)} = \alpha_1 \cdot m_1^{(1)}$. If $\max_{k\in[1,N]} \alpha_k \cdot m_k = \alpha_1 \cdot m_1$, then we set $m^{(1)} := m$.

Step 2. If $\max_{k\in[2,N]} \alpha_k \cdot m_k^{(1)} = \alpha_{k_1} \cdot m_{k_2}^{(1)}, k_2 > 2$, we represent the number m_{k_2} in the form $m_{k_2} = \overline{m}_{k_2} + \widetilde{m}_{k_2}$, where $\alpha_2 \cdot (m_2 + \overline{m}_{k_2}) = \alpha_{k_2} \cdot m_{k_2}$. Consider the sequence $m^{(2)} = \{m_k^{(2)}\}_{k=1}^{\infty}$ such that

$$m_k^{(2)} = \begin{cases} m_2^{(1)} + \overline{m}_{k_2},\ k = 2, \\ \widetilde{m}_{k_2}^{(1)},\ k = k_2, \\ m_k^{(1)},\ k \ne 2, k_2. \end{cases}$$

Similarly, we have $m_2^{(1)} = m_2 + \overline{m}_{k_2} \le m_{k_2}$, $|m^{(2)}| = |m^{(1)}| = |m|$, $\mathcal{E}_n(m^{(2)}) \ge \mathcal{E}_n(m^{(1)}) \ge \mathcal{E}_n(m)$ and $\alpha_1 \cdot m_1^{(2)} \ge \max_{k\in[2,N]} \alpha_k \cdot m_k^{(2)} = \alpha_2 \cdot m_2^{(2)}$. If $\max_{k\in[2,N]} \alpha_k \cdot m_k^{(2)} = \alpha_2 \cdot m_2^{(1)}$, then we set $m^{(2)} := m^{(1)}$.

Step N. Continuing this procedure, at Nth step we finally construct the sequence $m^{(N)} = \{m_k^{(N)}\}_{k=1}^{\infty} \in \mathcal{M}$ such that $|m^{(N)}| = |m|$, $\alpha_1 \cdot m_1^{(N)} \ge \alpha_2 \cdot m_2^{(N)} \ge \ldots \ge \alpha_N \cdot m_N^{(N)}$ and $\mathcal{E}_n(m^{(N)}) \ge \mathcal{E}_n(m^{(N-1)}) \ge \ldots \ge \mathcal{E}_n(m^{(1)}) \ge \mathcal{E}_n(m)$. Setting $m' := m^{(N)}$ we see that it satisfies all the necessary relations.

By virtue of (5.5), for any sequence $m \in \mathcal{M}'$ we have

$$\mathcal{E}_n(m) = \sum_{k=n+1}^{N} \alpha_k \cdot m_k \leq \sum_{k=n+1}^{N} \alpha_k.$$

To finish the proof it is sufficient to make sure that the sequence $m^* = \{m_k^*\}_{k=1}^{\infty}$ of the form (5.7) belongs to the set $\mathcal{M}$ and

$$\mathcal{E}_n(m^*) = \sum_{k=n+1}^{N} \alpha_k.$$

$\square$

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List of key terms

PF, PF_j^i – plafal; ${}_*PF$ – trivial plafal; PF_i – plafal with assigned number
$\mathcal{V}$ – vertex of the plafal; $\mathcal{E}$ – edge of the plafal; $\diamond PF$ – collection of $\mathcal{U}$-standard
$S(\diamond PF)$ – collection of $\diamond PF$; $S(PF)$ – collection of plafales
${}^{\tau}\mathcal{V}$ – vertex with assigned number; ${}^{\varsigma}\mathcal{E}$ – edge with assigned number
$PF_{j_l}^{i_k}$ – labeled plafal; PF^{emp} – empty plafal
PF^{prod} – product of plafales; PF^{coprod} – coproduct of plafales
PF^{un} – union of plafales; PF^{doc} – docking of plafales
PF^{in} – intersection of plafales; PF^{m} – merger of plafales
$|PF|^{U}$ – geo-space; $|PF|, |PF|_j^i$ – geo-plafal; ${}_*|PF|$ – trivial geo-plafal
$|PF|^{prod}$ – product of geo-plafales; $|PF|^{coprod}$ – coproduct of geo-plafales
$|PF|^{pl}$ – limit point in $|PF|^{U}$; ${}_*|PF|^{pl}$ – trivial point in $|PF|^{U}$
$|PF|^{M}$ – manifold in $|PF|^{U}$; ${}_*|PF|^{M}$ – trivial manifold in $|PF|^{U}$
${}_d|PF|^{U}$ – dynamical system; $|PF|_{pl}^{U}$ – plane in $|PF|^{U}$

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