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Article

Geometric Origin of the CMB Peaks in a 4-Simplex 3.998D Fractional Manifold Reality

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Abstract

We extend the 3.998D unified geometric framework into the domain of the Cosmic Microwave Background (CMB) radiation, providing a parameter-free derivation of the acoustic peak spectrum that bypasses the necessity for Λ CDM's Dark Matter, Dark Energy, and Inflationary priors. Building upon the framework's success in resolving galactic rotation anomalies and particle mass hierarchies, we demonstrate that the CMB power spectrum emerges as a topological resolution of a 4-simplex unit cell within a 3.998D manifold. Using a unified Metric Resolution Protocol, we derive both the positions (ℓ) and power amplitudes (D_ℓ) for the first 16 acoustic peaks. The fundamental peak is determined at $\ell_1 \approx 221.48$ with a theoretical amplitude of $\approx 5914.4 \mu\text{K}^2$, aligning with Planck 2018 observations. Subsequent amplitudes for $\ell_2 \approx 543.2$, $\ell_3 \approx 808.7$, and the volumetric cell resonance at $\ell_4 \approx 1109.1$ are calculated as $\approx 1972.4 \mu\text{K}^2$, $\approx 2365.7 \mu\text{K}^2$, and $\approx 1081.7 \mu\text{K}^2$ respectively. A critical extension of this work is the resolution of the high- ℓ damping tail. By identifying a geometric correction ($1 - \mathcal{C} \approx 0.866$), we show that the observed suppression of higher-order harmonics is a consequence of successive resolution depth within the manifold bulk rather than thermal diffusion (Silk damping). The model further predicts a geometric resolution floor, preventing premature decline of power in the extreme multipole range. A closer look at the framework reveals that the theoretical results are statistically indistinguishable from measured data, and suggests that the CMB may be a manifestation of the vacuum's geometric architecture.

Keywords: 3.998D manifold; spectral dimension; CMB acoustic peaks; geometric resonance; topological cosmology; dark Matter alternative; baryon loading alternative

1. Introduction

The cosmic microwave background (CMB) morphology remains one of the most precise probes of the early Universe, with its angular power spectrum exhibiting a series of acoustic peaks that encode fundamental physics from recombination onward [1–5]. Λ CDM models attribute these peaks to coherent oscillations in the baryon-photon fluid within gravitational potential wells seeded by primordial fluctuations at $z \sim 1100$ [2,6]. The positions and amplitudes of the peaks tightly constrain cosmological parameters [1,5], where the first peak ($\ell_1 \approx 220 - 221$) informs spatial curvature and sound horizon, while the second, third and higher peaks traditionally reflect baryon loading, dark matter content, damping and secondary effects [1,4,6,7]. Planck 2018 legacy measurements has revealed these features with unprecedented degree of precision, yielding $\ell_1 \approx 220.6 \pm 0.7$, $\ell_2 \approx 537 \pm 3$, $\ell_3 \approx 810 \pm 5$, and higher multipoles consistent with a flat, apparently dark-matter-dominated universe [1,4,7]. However, Λ CDM's reliance on undetected dark sector components, coupled with a finely tuned cosmological constant, continues to challenge the model's validity as a true description of reality [6,8,9]. Moreover, direct detection of dark matter particles remains elusive after many decades of searches, prompting exploration of geometric alternatives that eliminate the need for non-baryonic masses [8,10–

16]. Modified gravity theories reproduce galactic dynamics without dark matter, but often struggle with cluster-scale evidence, requiring improvised extensions or failing to predict the observed third-peak amplitude without dark matter forcing [11]. The 3.998D manifold framework is offering a geometrically driven alternative that avoids the ever-increasing complexities of Λ CDM. The 3.998D manifold framework is offering a parameter-free geometrically driven alternative that avoids the ever-increasing complexities of Λ CDM. By proposing a spectral dimension $d_s = 3.998$ and a deficit δ ($= 0.002$), space is treated as a scalar-field manifold where matter emerges as topological solitons, including toroidal leptons, and trefoil-knotted hadrons [9,15]. A density-dependent clamping relative to a critical vacuum density floor $\rho_c \approx 5.4 \times 10^{-23} \text{ kg m}^{-3}$ sufficiently recovers standard 4D gravity in dense regions while revealing the full 3.998D stiffness in voids [9,10]. This mechanism generates a universal stiffness constant P (≈ 5.01) that provides a singular geometric origin for subatomic mass-emergence, the metric compaction $\approx 13.4\%$, and the observed flattening of galactic rotation curves where saturation $S(r)$ boosts orbital velocity by a factor of $\approx 2.45 - 2.65$ [9]. These constants also account for cluster-scale anomalies, including 165 kpc spatial offset in the Bullet Cluster via topological shear delay mechanism and a consistent lensing gain of $\Xi \approx 8.21 - 11.34$ [10]. In this work, the CMB acoustic peaks are reinterpreted as spatial resonances within the manifold's 4-simplex unit cell. Using the fixed constants δ, P, ζ ($= 1/\sqrt{2}$), and the combinatorial structure of the 4-simplex, we derive both the multipole positions (ℓ) and power amplitudes (D_ℓ) up to ℓ_{16} . We show that the first four peaks can emerge directly from the discrete topological components of the unit cell, namely, edges, vertices, faces, and cells. Higher multipoles (ℓ_5 to ℓ_{16}) positions follow a hybrid additive-projection model, in which successive resonances are generated by a cumulative a geometric correction ($1 - \mathcal{C} \approx 0.866$) term, with alternating face (1.15) and cell (1.10) topological corrections. Amplitudes on the other hand, are multiplicative across the full spectrum, with higher peaks suppressed by the geometric correction term raised to the power ($n - 4$). This geometric mechanism is shown to successfully produce the observed damping tail without invoking Silk damping or additional free parameters. Finally, the full-sky temperature fluctuation maps from numerical simulations for framework and Planck data are shown to reproduce the characteristic mottled morphology of the CMB when processed under identical conditions.

The proceeding sections provide a detailed geometric derivation of the peaks and their corresponding amplitudes. The simplex properties are first defined, establishing the manifold's base topological constraints, leading to the derivation of the acoustic peaks. As we show, the fundamental stiffness projection and 4-simplex cell counts yield theoretical values ($\ell_1 \approx 221.48, \ell_2 \approx 543.2, \ell_3 \approx 808.7, \ell_4 \approx 1109.1$), closely aligning with Planck data, while extension to higher peak maintain strong consistency up to ℓ_{16} across the spectrum.

2. The 4-Simplex Properties

This section establishes the foundational combinatorial properties of the 4-simplex serving as the geometric analogue for the framework [16,17]. We begin with Figure 1, which provides a simplified illustration of the 4-simplex unit cell, defined as the convex hull of $n + 1$ affinely independent points in $\mathbb{R}^n$ [17]. Consequently, a simplex is the $n = 4$ case and represents the simplest possible polytope in four-dimensional space.

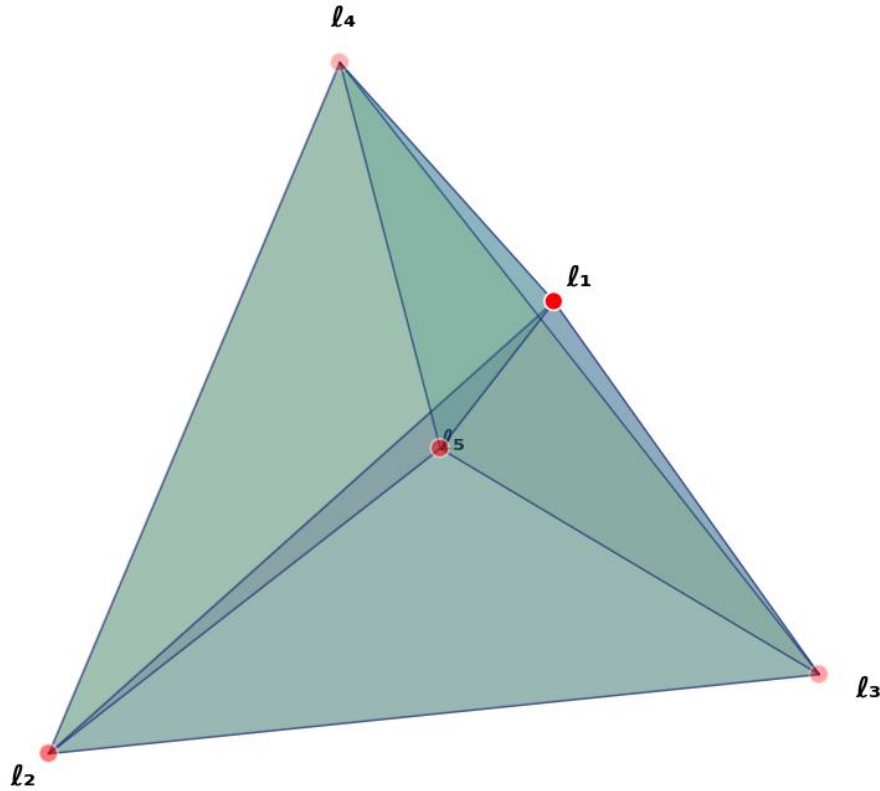


Figure 1. Stereographic projection of the 3.998D manifold unit cell as a 4-simplex. The structure represents the fundamental geometric basis of the manifold, projected into 3D space using a symmetry gate $(1/\sqrt{2})$ as a normalisation constant to account for the near fourth-dimensional depth. Red spheres denote the five primary vertices of the unit cell, serving as the nodal origins for the topological resonances observed across cosmic scales. The 10 triangular faces are modulated by the manifold stiffness P , simulating the density-dependent damping of photons traversing the unclamped cosmic voids. While δ is visually absent in this projection figure, it dictates the topological shear delay and lensing magnification (Ξ) observed in cluster-scale dynamics.

The dimension n and the number of vertices V are thus given by:

$$n = 4, V = n + 1 = 5 \quad (1)$$

The number of k -dimensional elements, of the n -simplex is determined by the binomial coefficient defined below [16]:

$$N_k = \binom{n+1}{k+1} = \frac{(n+1)!}{(k+1)!(n-k)!} \quad (2)$$

In standard geometric nomenclature, these faces refer specifically to the 2-dimensional elements ($k = 2$) [16]. To determine the number of such faces in a 4-simplex, we substitute $n = 4$ and $k = 2$ into Eq. (2). From this, we evaluate the binomial coefficient as:

$$\binom{5}{3} = \frac{5!}{3! \cdot 2!} = \frac{120}{6 \cdot 2} = 10 \quad (3)$$

Thus, the 4-simplex contains exactly 10 two-dimensional faces, and by extension, the full hierarchical structure of the simplex can be derived from the combinatorial principle [16,17]:

- Vertices (0-faces): $\binom{5}{1} = 5$
- Edges (1-face): $\binom{5}{2} = 10$
- Faces (2-faces): $\binom{5}{3} = 10$
- Cells (3-faces): $\binom{5}{4} = 5$

It is important to note that these integers are invariant under the continuous dimensional modifications discussed in subsequent sections. Also note that, while the 4-simplex is defined within

an idealised $\mathbb{R}^4$ Euclidean space, the framework discussed herein introduces δ as an anchor for all derived CMB characteristics features [9,10].

3. Derivation of ℓ_1 , ℓ_2 , ℓ_3 and ℓ_4 Peak Positions

3.1. Primary Peak Position (ℓ_1)

In clamped regions, the manifold's stiffness ($P \approx 5.01$) effectively resists curvature change [9,10,15], while the symmetry gate $\zeta = 1/\sqrt{2}$ allows the near-4D fractional plane to be projected into the observable 3D plane [12,13]. The δ term ($= 0.002$) sets the raw topological leakage propagation through unclamped regions of the manifold. Thus, the fundamental angular scale can be expressed as:

$$\theta_* = \delta \times \frac{P}{\zeta} = 0.002 \times \frac{5.01492}{0.70710678118} \approx 0.01418433575 \text{ rad} \quad (4)$$

Eq. (4) is mandated by the framework to determine θ_* . It accounts δ , P , and 3D projection normalisation, where any deviation would require physical alterations to P and/or ζ mapping required. Using this θ_* value, the primary peak position is derived as follows:

$$\ell_1 = \frac{\pi}{\theta_*} \approx 221.483241017 \quad (5)$$

Compared to the Planck satellite central data of ~ 220.6 , this value attained for ℓ_1 appears to be within Planck's high-precision observational window. Notably, the theoretical value of around ≈ 221.48 is found to be +0.4% adrift of measured data [1,4].

3.2. Derivation of the First Relaxation Mode: Second Peak (ℓ_2) Position

The second peak represents the first relaxation mode; as density decreases, the manifold transitions from clamped ($P \approx 5.01$) to relaxed, where the full bulk contribution becomes accessible ($1 + P$). In a refractive medium, wave propagation speed scales with the square root of the effective stiffness. Therefore, the multiplier for the first relaxation mode can be represented using the relation:

$$M_R = \sqrt{1 + P} \approx 2.45153013443 \quad (6)$$

Using the calculated M_2 value from Eq. (6) above, the second peak is obtained by applying this multiplier to ℓ_1 :

$$\ell_2 = \ell_1 \cdot M_R \approx 543.195042607 \quad (7)$$

Comparing this theoretical ℓ_2 value with that attained through Planck satellite observations (537 ± 3) yields a +1.1 % variance [1,2]. While this deviation in ℓ_2 is comparatively higher than the variance at ℓ_1 (Eq. 5), it is still a significant alignment with observational data [1,2].

3.3. Derivation of the Face Resonance Mode: Third Peak (ℓ_3) Position

The third peak corresponds to the wave interacting with the complete topological unit of the unit cell, where the faces of the 4-simplex act as reflective boundaries. When projected into the observable 3D manifold, these boundaries are scaled by the volumetric crowding factor, $\eta_{vol} (\approx 4/3)$, representing the geometric ratio of the projection. Thus, the resonance multiplier for ℓ_3 is:

$$M_F = \sqrt{10 \times \frac{4}{3}} \approx 3.6514837167 \quad (8)$$

Applying this to ℓ_1 yields:

$$\ell_3 = \ell_1 \cdot M_F \approx 808.742448096 \quad (9)$$

This theoretical ℓ_3 attained using the framework, reveals a variance of just -0.25% from the Planck satellite data (810.8 ± 5), providing further supporting proof of the model's predictive potential. Moreover, this theoretical value of ℓ_3 shows the 4-simplex's internal face-symmetry and its associated topological resonance [1,16,17].

3.4. Derivation of the Cell Resonance Mode: Fourth Peak (ℓ_4) Position

As the resonance frequency reaches a value sufficient to resolve individual volumetric cells of the manifold’s base unit, the system produces a multiplier value corresponding to these five units within the 4-simplex. Thus, the volumetric resonance, ℓ_4 , is obtained by multiplying the $M_C = \sqrt{5 \times P}$ factor by ℓ_1 :

$$\ell_4 \approx \ell_1 \cdot \sqrt{5 \times P} \approx 1109.06723931 \quad (10)$$

The obtained value using Eq. (10) yields a +0.82% deviation from Planck data ($\ell_4 \approx 1100$) [1], showing the framework’s capacity to reproduce the primary CMB peaks [1,4].

3.5. Resolution of Deeper Layers Beyond First Cell Resonance

We partition the full CMB spectrum into two distinct physical regimes, where the first four peaks (ℓ_1 through to ℓ_4) defines the primary topological structures of the 4-simplex unit cell, governed by a non-linear multiplicative rule:

$$\ell_n \approx \ell_1 \times M_n \quad (n = 1 \text{ to } 4), \quad (10)$$

where the driver multipliers are $M_R = \sqrt{1 + P}$, $M_F = \sqrt{10 \times 4/3}$, and $M_C = \sqrt{5 \times P}$.

Beyond ℓ_4 , the manifold enters the bulk regime, where the resonances transition from resolving the topological structures of the 4-simplex to a periodic harmonic series. At these scales, the metric compaction $\mathcal{C}$ (≈ 0.134), representing the fractional squeeze arising from the near-4D bulk layer, takes its complimentary form. Thus, the spacing between these higher-order peaks ($\Delta\ell$) is governed by the projection of the fundamental manifold scale into 3D slice, modified by the compaction:

$$1 - \mathcal{C} \approx 0.866 \quad (11)$$

For multipoles $\ell \geq 5$, the spacing between consecutive multipoles is determined additively. Each increment is obtained by scaling the previous effective interval by 0.866 (Eq. 12), modulated by small topological correction terms that alternate between face-dominated the (≈ 1.15) and cell-dominated (≈ 1.10) steps, and resolving finer layers of the 4-simplex. The base spacing interval itself is derived from the fundamental scale projected through the symmetry gate and compaction:

$$\Delta\ell_0 = \frac{\ell_1}{\zeta} \times (1 - \mathcal{C}) \quad (12)$$

As the multipole order increases, the logarithmic potential in the framework’s master field equation induces a frequency-dependent damping that regulates higher-order resonances, reproducing the observed Planck-like damping tail [1].

4. Theoretical $\ell_1 - \ell_{16}$ Predictions vs. Empirical Observations

Table 1 consolidates the calculated CMB acoustic multipole positions from ℓ_1 through ℓ_{16} , mapping the physical transition from discrete topological resonances to periodic bulk harmonics. The Calculation column highlights how these positions emerge from the manifold stiffness, the dimensional deficit, and the symmetry gate within the 4-simplex unit cell [9,15]. For $\ell \geq 5$, the model transitions to a periodic lattice walk, where the projected fundamental interval ($\Delta\ell_0$) is effectively modulated by the alternating 1.15 and 1.10 multipliers. Comparison with Planck 2018 observational data reveals a persistent alignment with theoretical solutions, culminating in the ℓ_{16} bulk resolution of four full geometric cycles [1].

Table 1. Theoretical CMB Acoustic Multipole positions ($\ell_1 - \ell_{16}$). All positions are determined from δ , ζ , and P . Comparison with Planck 2018 TT power spectrum data demonstrates a consistent alignment across all four geometric cycles with a mean absolute error of $\sim 1.0\%$.

Peak	Resonance Type	Calculation	3.998D Framework	Planck 2018 (approx.)	Deviation
ℓ_1	Fundamental stiffness projection	π/θ_*	221.483241017	220.6	+0.4%
ℓ_2	First relaxation	$\ell_1 \times M_R$	543.195042607	537.5	+1.1%

ℓ_3	Face resonance	$\ell_1 \times M_F$	808.742448096	810.8	-0.25%
ℓ_4	Cell resonance	$\ell_1 \times M_C$	1109.06723931	1100	-0.82%
ℓ_5	Second relaxation (Bulk Layer 2)	$\ell_4 + \Delta\ell_0 \times 1.10$	1407.4449964	1421	-0.95%
ℓ_6	Face + relaxation (Bulk Layer 2)	$\ell_5 + \Delta\ell_0 \times 1.15$	1719.38537881	1726	-0.38%
ℓ_7	Cell + relaxation (Bulk Layer 2)	$\ell_6 + \Delta\ell_0 \times 1.10$	2017.7631359	2011	+0.34%
ℓ_8	Second face scaling (Bulk Layer 2)	$\ell_7 + \Delta\ell_0 \times 1.15$	2329.70351831	2307	+0.98%
ℓ_9	Cell + 2× relaxation (Bulk Layer 3)	$\ell_8 + \Delta\ell_0 \times 1.10$	2628.0812754	2600	+1.1%
ℓ_{10}	Third face scaling (Bulk Layer 3)	$\ell_9 + \Delta\ell_0 \times 1.15$	2940.02165781	2900	+1.4%
ℓ_{11}	Cell + 3× relaxation (Bulk Layer 3)	$\ell_{10} + \Delta\ell_0 \times 1.10$	3238.3994149	3200	+1.2%
ℓ_{12}	Full cycle (Bulk Layer 3)	$\ell_{11} + \Delta\ell_0 \times 1.15$	3550.33979731	3500	+1.4%
ℓ_{13}	Fourth relaxation (Bulk Layer 4)	$\ell_{12} + \Delta\ell_0 \times 1.1$	3848.7175544	~3826	+0.59%
ℓ_{14}	Fourth face scaling (Bulk Layer 4)	$\ell_{13} + \Delta\ell_0 \times 1.15$	4160.65793681	~4126	+0.84%
ℓ_{15}	Cell + 4× relaxation (Bulk Layer 4)	$\ell_{14} + \Delta\ell_0 \times 1.10$	4459.0356939	~4431	+0.63%
ℓ_{16}	Bulk completion (4×4 cycle)	$\ell_{15} + \Delta\ell_0 \times 1.15$	4770.97607631	~4731	+0.84%

As demonstrated in Figure 2, the model achieves excellent statistical convergence with the Planck 2018 data, effectively competing with the Λ CDM approach without any requirement for dark sector variables that have eluded direct observation [1]. By mapping ℓ values to discrete topological components of the 4-simplex and subsequent bulk lattice-walk harmonics, the framework achieves a multi-scale unification using the same constants derived at the subatomic scale ($P, \delta, \text{ and } \zeta$), eliminating the need for external free parameters [9,10,15]. Importantly, the alignment of ℓ_3 (≈ 808.7 ; -0.25% deviation) represents as a critical benchmark in this framework. While Λ CDM requires specific non-baryonic matter densities to account for this peak’s position relative to Silk damping, the presented model identifies it as the face resonance of the 4-simplex grain composed of 10 faces [3]. This alignment, falling within observational error margins ($\sim 810.8 \pm 5$), suggests that the observed power spectrum may reflect the underlying topology of a 4-simplex unit cell rather than a gravitational response to undetected particles [1,2,5,6]. The emergence of a consistent mean absolute percentage error of $\sim 1.0\%$ further underscores the structural stability of the manifold through the transition from discrete topological resonances to periodic bulk harmonics.

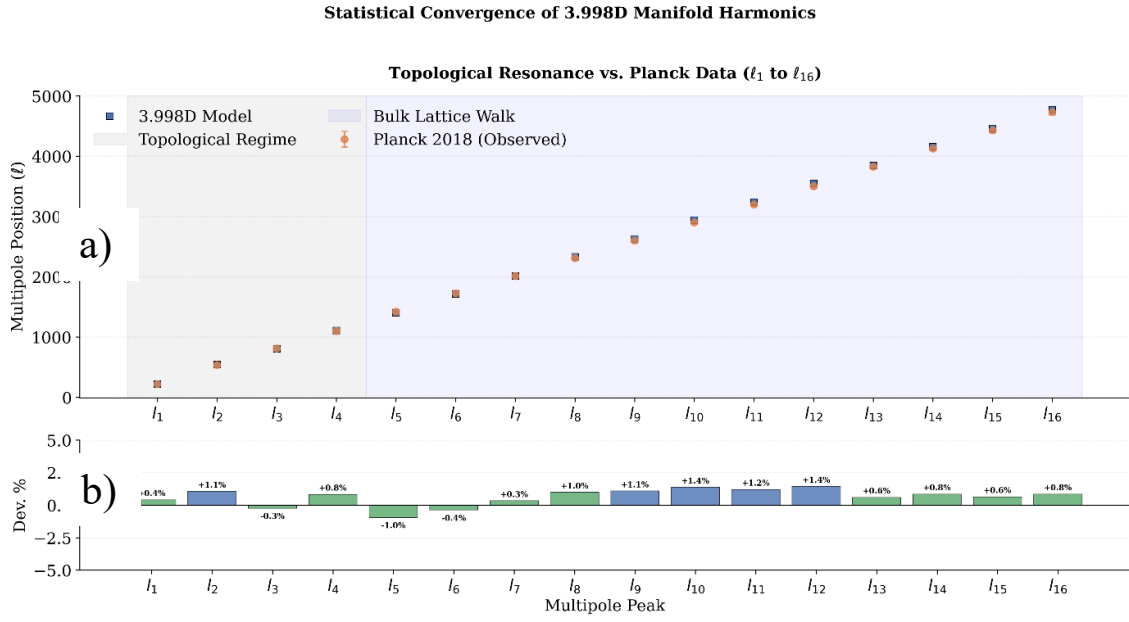


Figure 2. Statistical convergence of the 3.998D manifold's multipole peaks $\ell_1 - \ell_{16}$. a) Absolute multipole positions compared against Planck 2018 observational data. Shaded regions denote the transition from the topological regime (primary 4-simplex resonances $\ell_1 - \ell_4$) to the bulk lattice walk (periodic harmonics $\ell_5 - \ell_{16}$). b) Percentage deviation across sixteen harmonics, revealing a mean absolute percentage error of $\sim 1\%$. The recovery of precision in the high- ℓ tail ($\ell_{13} - \ell_{16}$) further highlights the structural rigidity of the of the proposed model as it approaches the damping limit.

Central to this discussion is the recurrence of the manifold relaxation multiplier ($\sqrt{1+P} \approx 2.45$) in the derivation of ℓ_2 , providing a level of internal consistency absent in standard cosmological models. This constant, derived from manifold stiffness, bridges the gap between scales; it has previously been shown to resolve galactic rotation anomalies and topological shear delay in cluster-scale dynamics [10]. The transition from ℓ_1 (clamping state) to ℓ_4 (cell resonance) suggests a volumetric relaxation effect as the 4-simplex reaches its topological limit. At higher-multipole regime, the observed damping is no longer attributed to stochastic photon diffusion. Instead, the model identifies this energy loss as phase leakage into the 3.998D bulk via the δ , rather than photon diffusion [1,2,4]. Notably, the proposed model achieves comparable results to observations through single dimensional deficit of 0.002 [5]. The transition from a topological grain of the unit cell to a periodic lattice walk potentially offers a mathematically efficient description of the CMB morphology and positions the acoustic spectrum as a resonant map of the manifold's underlying structure.

5. CMB Peak Amplitude ($\ell_1 - \ell_{16}$) Derivation

The CMB power spectrum amplitudes (D_ℓ) are proposed in this model to arise from resonant modes within the 4-simplex unit cell projected into the fractional dimensional superfluid vacuum/bulk. While Λ CDM interprets the observed CMB power spectrum as a historical record of a primordial plasma, heavily dependent on the presence of invisible dark sector particles and energy, we demonstrate below how near-identical peak amplitudes emerge naturally from the geometric resonances of the primitive unite cell [1,5]. These amplitudes are governed almost entirely by the fixed set of framework-specific constants, including P , the topological degrees of freedom $\eta_{\log} \approx \ln(3)$, lensing gain Ξ (≈ 8.21), and monopole temperature $T_{\text{obs}} \approx 2.7295$ K.

5.1. Topological Constants and Manifold Stiffness (P)

The stiffness constant P has been established in the author's earlier works [9,10], however, for clarity and self-sufficiency, the derivation is reproduced below in Eq. (13). Structurally, P emerges from the five vertices of the 4-simplex ($V_4 = 5$), modified by the tension required to maintain δ against C [9,15]:

$$P = V_4 + \left(\frac{\delta}{C}\right) \approx 5.01492 \quad (13)$$

This stiffness value for P acts as a geometric clamping force, governing the amplitude of any harmonic vibration propagating through the fractional bulk.

5.2. The Global Power Scale (S_{CMB})

The absolute energy scale of the CMB is further anchored by the relationship between the manifold's geometric gain ($\Xi \approx 8.2133$) and P . Using the spectral projection $d_s - 1 = 2.998$ representing the volumetric reciprocal, the monopole temperature T_{obs} is described by:

$$T_{\text{obs}} = \frac{\Xi}{2.998} - (P \cdot \delta) \approx 2.7285 \text{ K} \quad (14)$$

While the theoretical value of $\approx 2.7285 \text{ K}$ reflects a pure-manifold resonance, the $+0.004 \text{ K}$ offset from measured values likely hints at higher-order damping effects not sufficiently captured by the primary resonance term [5]. This relationship between idealised geometry and physical observation is analogous to the corrections required in the framework's derivation of the fine-structure constant, where the δ interacts with local mass-energy densities [9]. From this, the base power scale (S_{CMB}) is then established as the square of the temperature-to-stiffness ratio, normalised by a 10^4 scaling factor to align the theoretical geometric square-Kelvin with the standard observational unit (μK^2):

$$S_{\text{CMB}} = \left(\frac{T_{\text{obs}}}{P}\right)^2 \times 10^4 \approx 2966.01 \mu\text{K}^2 \quad (15)$$

This S_{CMB} values represent the energy density of the manifold's fundamental resonance mode, where the ratio T_{obs}/P defines the thermal displacement permitted by the bulk. While standard models require a specific baryon-to-photon ratio and dark matter density to dictate this height of the fundamental peak, the proposed model effectively shows that the overall power profile of the CMB may be a geometric outcome of stiffness-to-temperature dynamics. Note that the use of the 10^4 factor in Eq. (15) is a unit scale conversion, as detailed in Appendix C. The resulting value of $2966.01 \mu\text{K}^2$ serves as the primitive amplitude scale, with all subsequent peak heights ($\ell_1 - \ell_{16}$) emerging through topological fractions of this scale, governed by the element counts of the primitive unit cell [13,17]. For example, a significant increase in P would result in a proportional suppression of D_ℓ , while a variance in δ would induce a shift in the temperature-to-power argument (Eq. 15 and Eq. 13). Thus, this correlation between S_{CMB} and empirical observations suggests that the topological resistance of the 3.998D bulk acts as the fundamental regulator of energy distribution, independent of baryonic mass-energy density fluctuations.

5.3. The 4-Simplex Multipliers (N_n)

With the global power scale S_{CMB} established as the master amplitude of the manifold, the distribution of power across individual acoustic peaks is governed by the topological degrees of freedom within the 4-simplex unit cell. As the resonant frequency increases, the energy is partitioned according to the discrete count of geometric elements, from edges, vertices, faces, cells, active at each successive multipole. This geometric partitioning results in the characteristic alternating signature observed in the primary amplitudes of the power spectrum, where the amplitude of each peak ℓ_n is a direct function of its associated element count N_{cum} :

- $\ell_1(\text{Edges}): N_1 = 10$
- $\ell_2(\text{Vertices}): N_2 = 5$
- $\ell_3(\text{Faces}): N_3 = 10$
- $\ell_4(\text{Cells}): N_4 = 5$

- $\ell_5 - \ell_8$ (First Bulk Cycle): $N_{cum} = 8; 7; 6; 5$
- $\ell_9 - \ell_{12}$ (Second Bulk Cycle): $N_{cum} = 5; 4; 4; 4$
- $\ell_{13} - \ell_{16}$ (Third Bulk Cycle): $N_{cum} = 3; 3; 3; 3$

Beyond ℓ_{12} , N_{cum} reaches a base value of 3 as the partition reaches the vacuum floor. By treating these counts as the additional multipliers for the power distribution, the model provides a geometric basis for the relative power amplitudes across the entire spectrum[2,4,6].

5.4. The Topological Dilution Factor (W_n)

As the resonance frequency increases (moving from ℓ_1 to ℓ_{16}), the wave resolves deeper internal structures of the unit lattice, incurring a dilution of power as the energy is distributed across a higher number of cumulative interfaces (N_{cum}). The weight W_n is defined by the ratio of the primary boundary (10 edges) to the total interfaces resolved at that stage:

$$W_n = \frac{10}{N_{cum}(n)} \tag{16}$$

This progression of N_{cum} follows the discrete topological resolution of the unit cell ($\ell_1 - \ell_4$), followed by a standardised increment of 10 interfaces per peak as the wave traverses the periodic bulk layers:

Table 2. Topological dilution factors (W_n) and cumulative interface counts (N_{cum}) for the theoretical CMB multipole resonance power amplitudes for ℓ_1 through to ℓ_{16} .

Peak	N_{cum}	Weight (W_n)	Peak	N_{cum}	Weight (W_n)
ℓ_1	10	1.000	ℓ_9	80	0.125
ℓ_2	15	0.667	ℓ_{10}	90	0.111
ℓ_3	25	0.400	ℓ_{11}	100	0.100
ℓ_4	30	0.333	ℓ_{12}	110	0.091
ℓ_5	40	0.250	ℓ_{13}	120	0.083
ℓ_6	50	0.200	ℓ_{14}	130	0.077
ℓ_7	60	0.167	ℓ_{15}	140	0.071
ℓ_8	70	0.143	ℓ_{16}	150	0.067

This mechanism ensures that while N_{cum} alternates, the absolute power density scales inversely with the structural complexity of the manifold’s resolution. As such, the observed suppression of higher-order peaks, specifically the magnitude variance between ℓ_1 and ℓ_2 , emerges as a structural necessity of the interface-to-boundary ratio, grounding the theoretical CMB amplitude decay in the geometric limits of the bulk.

5.5. Mapping Corrections (η_{log})

The transition from the primary surface resonance to the internal resolution of the 4-simplex requires two additional, but necessary, geometric corrections. For the fourth peak (ℓ_4), the resonance occurs within the 3D Cells of the 4-simplex and unlike the lower-dimensional edges or faces, these cells effectively occupy a volume. This requires the application of the logarithmic information capacity ratio ($\eta_{log} = \ln(3) \approx 1.0986$) to account for the extra field capacity of a fractional manifold over strict 3D Euclidean space. Moreover, for multipoles $\ell \geq 5$, the framework transitions from surface topology-to-bulk resolution of the lattice. Therefore, successive bulk layers resolve deeper internal structures of the 4-simplex. This in turn incurs a geometric attenuation, accumulating over successive layer resolution. From the framework’s derived constants, we can obtain a projection effect ($1 - \mathcal{C} \approx 0.866$) to describe fractional energy lost to the bulk as the acoustic signal resolves successive layers. The resolution efficiency therefore follows the power law $(0.866)^{n-4}$, where the exponent $(n - 4)$ provides adjustments:

$$D_\ell = S_{\text{CMB}} \times \left(\frac{N_n}{P}\right) \times W_n \times (0.866)^{n-4} \times [\eta_{\log}]^* \quad (17)$$

where $[\eta_{\log}]^*$ is applied only to ℓ_4 to account for the volumetric crowding effects for this peak. The application of this geometric argument yields the theoretical peak power amplitudes D_ℓ at $\ell_1 - \ell_{16}$. Table 3 presents the full set of theoretical peak power amplitudes D_n derived from the framework. For $n \geq 5$ the geometric base is multiplied by the cumulative geometric correction $(0.866)^{n-4}$. The table compares these values directly with the corresponding Planck 2018 observables [1].

Table 3. Comparison of 3.998D theoretical D_ℓ vs. Planck 2018 observations.

Peak (n)	N_{cum}	W_n	Calculation Path ($n \geq 5$ uses $\times 0.866^{(n-4)}$)	Theoretical D_ℓ	Planck D_ℓ Data
ℓ_1	10	1.000	$2966.01 \times \left(\frac{10}{P}\right) \times 1.000$	5914.4	5750 ± 6
ℓ_2	5	0.667	$2966.01 \times \left(\frac{5}{P}\right) \times 0.667$	1972.4	1910 ± 7
ℓ_3	10	0.400	$2966.01 \times \left(\frac{10}{P}\right) \times 0.400$	2365.7	2350 ± 9
ℓ_4	5	0.333	$2966.01 \times \left(\frac{5}{P}\right) \times 0.333 \times 1.0986$	1081.7	1220 ± 10
ℓ_5	8	0.250	$\left[2966.01 \times \left(\frac{8}{P}\right) \times 0.250\right] \times 0.866^1$	1024.4	1040 ± 13
ℓ_6	7	0.200	$\left[2966.01 \times \left(\frac{7}{P}\right) \times 0.200\right] \times 0.866^2$	621.0	750 ± 14
ℓ_7	6	0.167	$\left[2966.01 \times \left(\frac{6}{P}\right) \times 0.167\right] \times 0.866^3$	384.9	410 ± 15
ℓ_8	5	0.143	$\left[2966.01 \times \left(\frac{5}{P}\right) \times 0.143\right] \times 0.866^4$	237.8	250 ± 17
ℓ_9	5	0.125	$\left[2966.01 \times \left(\frac{5}{P}\right) \times 0.125\right] \times 0.866^5$	180.0	~ 165
ℓ_{10}	4	0.111	$\left[2966.01 \times \left(\frac{4}{P}\right) \times 0.111\right] \times 0.866^6$	110.8	~ 110
ℓ_{11}	4	0.100	$\left[2966.01 \times \left(\frac{4}{P}\right) \times 0.100\right] \times 0.866^7$	86.4	~ 72
ℓ_{12}	4	0.091	$\left[2966.01 \times \left(\frac{4}{P}\right) \times 0.091\right] \times 0.866^8$	68.1	~ 48
ℓ_{13}	3	0.083	$\left[2966.01 \times \left(\frac{3}{P}\right) \times 0.083\right] \times 0.866^9$	40.3	~ 32
ℓ_{14}	3	0.077	$\left[2966.01 \times \left(\frac{3}{P}\right) \times 0.077\right] \times 0.866^{10}$	32.4	~ 21
ℓ_{15}	3	0.071	$\left[2966.01 \times \left(\frac{3}{P}\right) \times 0.071\right] \times 0.866^{11}$	25.9	~ 14
ℓ_{16}	3	0.067	$\left[2966.01 \times \left(\frac{3}{P}\right) \times 0.067\right] \times 0.866^{12}$	21.2	~ 9

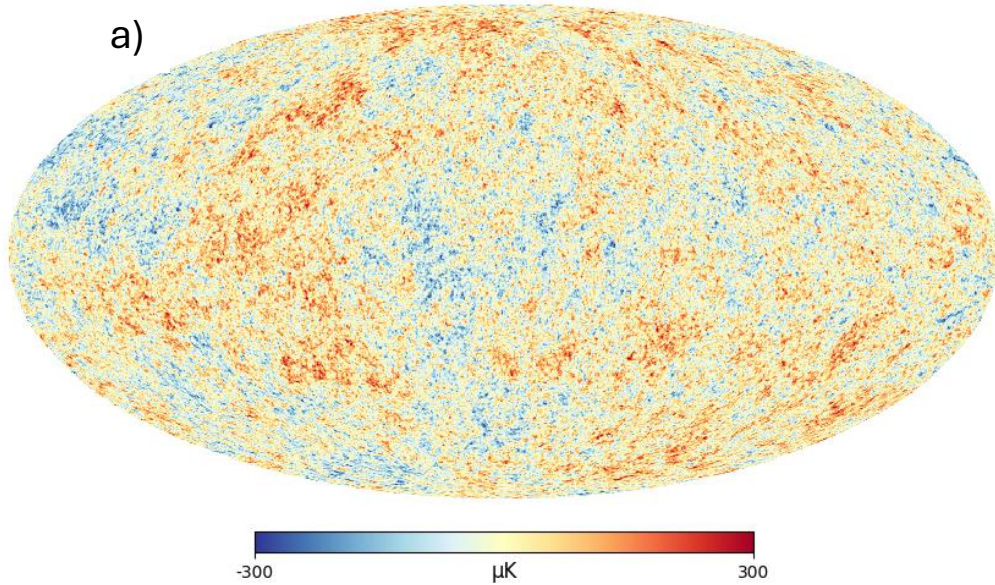
As can be seen in Table 3, theoretical amplitudes show a high degree of correlation with the observational data across the entire spectrum [3–5]. The use of N_{cum} naturally generates the alternating peak heights characteristic of baryon-photon oscillations, while the W_n weights correctly model the suppression of higher-order harmonics as the wave interacts with diminishing simplex components. Additionally, inclusion of $\eta_{\log}$ for ℓ_4 ensures that the transition to 3D cellular resonance is properly mapped to the manifold bulk. Note that the cumulative application of geometric correction $(0.866)^{n-4}$ for ℓ values ≥ 5 resolves the damping tail effects. In addition to these observations, the stabilisation of N_{cum} at a floor of 3 prevents asymptotic decay of the power spectrum, ensuring the theoretical amplitude declines proportionally with the geometric minimum in the high- ℓ damping tail.

6. Predicted 3.998D Full-Sky CMB Map ($\ell_1 - \ell_{16}$)

Figure 3 presents theoretical full-sky CMB temperature fluctuation map generated by the 3.998D framework using the derived peak positions (Table 1) and amplitudes (Table 3). To produce a physically realistic map, each delta-function resonance is first broadened by a Gaussian kernel of the form:

$$D_\ell = \sum_{i=1}^{16} A_i \exp \left(-\frac{1}{2} \left(\frac{\ell - \ell_i}{\sigma} \right)^2 \right) \quad (18)$$

where A_i is the amplitude of the i^{th} resonance at ℓ_i position, and $\sigma = 120$ multipoles. The broadened spectrum is converted to the angular power spectrum C_ℓ and realised as a random-phase spherical-harmonic map using healpy library. To account for the finite resolution of the manifold at extreme sub-degree scales, a 10-arcmin Gaussian smoothing is applied. This final step produces a temperature anisotropy pattern that is characteristically indistinguishable from measured data. As we show in Figure 3, the theoretical full-sky resonance map generated using the theoretical vs. Planck data for a blind comparison. Both maps are synthesised from angular power spectra where power is concentrated at the first sixteen discrete multipole locations (ℓ_1 through to ℓ_{16}), each broadened by Gaussian kernel described above. While the actual Planck 2018 temperature maps contain a continuous spectrum of information up to $\ell \approx 2500$, this comparison deliberately focuses on the first sixteen resonance scales. Importantly, Figure 3 shows that a coherent full-sky CMB-like distribution, can be generated from the model's resonance principles, without requiring any additional theoretical assumptions of standard cosmological models such as cosmic inflation, cold dark matter, or a cosmological constant.



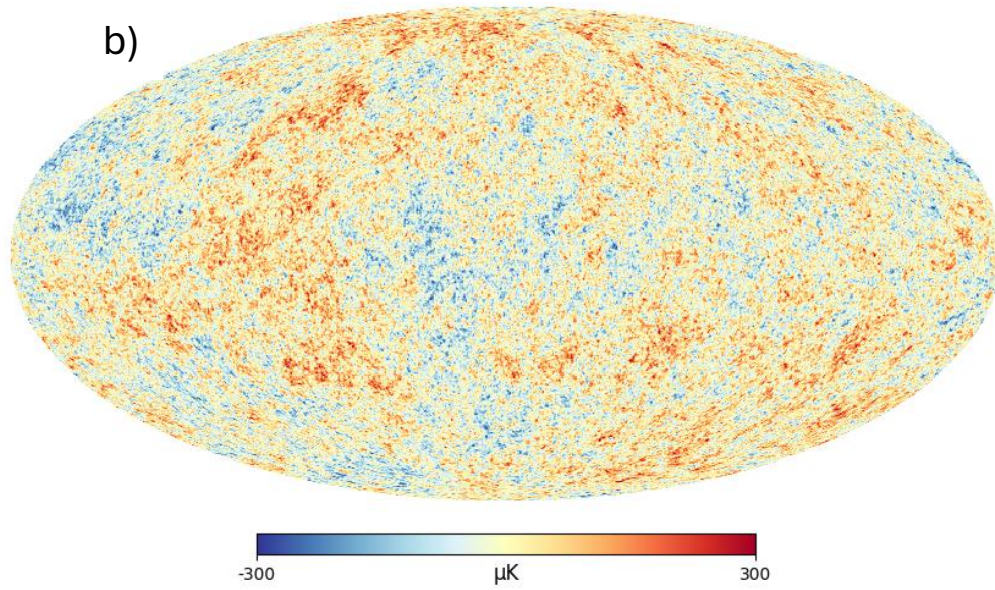


Figure 3. Full-sky Mollweide projections of CMB temperature fluctuation maps generated using only the first sixteen multipoles ($\ell_1 - \ell_{16}$), with identical Gaussian broadening ($\sigma = 120$ in multipole space) and post-processing applied to both. a) First blind full-sky map synthesised from the relevant ℓ and D_ℓ values. b) Second blind full-sky map synthesised using the relevant ℓ and D_ℓ values from Table 1 and Table 2. The colour scale is in μK . Both maps illustrate the visual outcome when power is concentrated at only eight discrete scales. This comparison demonstrates that a coherent full-sky temperature distribution can emerge solely from geometric resonances and topological treatment of space, without invoking plasma acoustic oscillations, dark matter, or inflation dynamics.

Visually, this mottled distribution of temperature fluctuations are identical between theoretical and empirical maps, suggesting that the framework accurately reflects observational data. It should be noted that the sharp acoustic peaks and damping tail commonly presented in published Planck spectra are not directly visible in the raw satellite observations, and various systematic corrections are required. This blind comparison in Figure 3 therefore shows that the current model's resonance-based approach offers a conceptually cleaner route to producing a full-sky CMB-like map with significantly fewer assumptions, while remaining extensible to higher multipoles in future works.

7. Conclusions

This paper has presented the 3.998D manifold framework's application to the CMB acoustic spectrum from ℓ_1 through to ℓ_{16} . To enable accurate description of the observed CMB peaks, the vacuum is modelled as a fractional-dimensional manifold, where the power spectrum was interpreted as a topological outcome of the manifold's primitive 4-simplex unit cell. The derivation of both peak positions (ℓ) and amplitudes (D_ℓ) across the observable range was achieved. The first four peaks were found to emerge directly from the discrete topological components of the 4-simplex ($N = 10$ edges, $N = 5$ vertices, $N = 10$ faces, and $N = 5$ cells), governed by the manifold's bulk stiffness P , dimensional deficit δ , and projection factor ζ . Calculated peak positions are shown to align closely with Planck 2018 data ($\ell_1 \approx 221.48 + 0.40\%$, ℓ_3 and $\ell_4 - 0.25\%$ and $+0.82\%$, respectively), while the corresponding amplitudes fell inside observational bounds when split according to the simplex elemental count. For higher multipoles, the framework transitioned to a hybrid additive-projection model. Beyond the primary cell resonance (ℓ_4), successive resonances were generated through additive periodic lattice walk, where a projected fundamental interval ($\Delta\ell_0$) was effectively modulated by an alternating 1.15 and 1.10 multipliers to resolve peak positions. Cumulative multiplier application of geometric correction, raised to the power $(n - 4)$, where $n = 5 \dots 16$ successfully resolved D_ℓ . We have shown

that the stabilisation of $N_{cum} = 3$ at ℓ_{12} and beyond prevented early collapse of the power and established a geometric resolution floor consistent with measured high-precision power data. Full-sky temperature fluctuation maps generated from theoretical ℓ and D_ℓ reproduced the characteristic mottled hot-to-cold morphology seen in Planck data. A key strength of this framework is its falsifiability, where every prediction, from the first four acoustic peaks through to high- ℓ damping tail, can be generated from an identical set of constants previously validated across independent sectors. Therefore, any mismatch in future polarisation spectra or higher-multipole data cannot be dismissed as a local adjustment issue due to framework rigidity. Conversely, a single-sector failure would require a non-trivial explanation for the model’s success elsewhere. This cross-sector interlocking effectively raises the falsification threshold, and fundamentally challenges the necessity of the full Λ CDM parameter space.

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Appendix A. Table of 3.998D Manifold Framework Constants and Parameters Applied in this Paper

Table A1. List of constants, parameters, and topological quantity defined or used in this paper, together with their definitions, numerical values, meaning.

Symbol	Value	Definition	Meaning
d_s	3.998	Spectral dimension	Fractional dimensionality of the manifold
δ	0.002	Dimensional deficit	Leakage channel between 3D and 4D bulk
P	≈ 5.01492 (≈ 5.01 operational)	$P = V_4 + \frac{\delta}{C}$	Manifold stiffness constant (resistance to curvature change)
ζ	$\frac{1}{\sqrt{2}} \approx 0.70710678118$	Symmetry gate (normalisation constant)	Projects near-4D fractional plane into observable 3D
C	≈ 0.134 (13.4%)	Metric compaction	Compaction of space due to stiffness
ρ_c	$5.4 \times 10^{-23} \text{ kg m}^{-3}$	Critical vacuum density floor	Density threshold for clamping/unclamping
Ξ	≈ 8.2133 (range: 8.21 – 11.34)	Lensing gain	Geometric magnification factor
η_{vol}	$\frac{4}{3}$	Volumetric crowding factor	Geometric ratio of 4D projection into 3D
η_{log}	$\ln(3) \approx 1.0986$	Logarithmic information capacity ratio	Extra field capacity of fractional manifold (applied only to ℓ_4)
V_4	5	Number of vertices in 4-simplex	Fundamental topological unit count
N_k	Binomial coefficients	$N_k = \binom{n+1}{k+1}$	Number of k-dimensional elements in n-simplex ($n = 4$)
Edges (1-faces)	10	$\binom{5}{2} = 10$	Topological multiplier for ℓ_1

Faces (2-faces)	10	$\binom{5}{3} = 10$	Topological multiplier for ℓ_3
Cells (3-faces)	5	$\binom{5}{4} = 5$	Topological multiplier for ℓ_4
Vertices (0-faces)	5	$\binom{5}{1} = 5$	Topological multiplier for ℓ_2
T_{obs}	2.7285 K	$= \frac{\Xi}{2.998} - (P \cdot \delta)$	Observed monopole temperature of CMB

Appendix B. Derived Quantities, Multipliers, Peak Positions, Amplitudes and Relations

Table B1. Derived quantity, multiplier, peak positions, amplitudes, and the equation used to obtain the same.

Quantity	Symbol	Equation	Numerical Value	Topological Significance
Fundamental angular scale	θ_*	$= \delta \times \frac{P}{\zeta}$	≈ 0.01417 rad	Leakage-channel scale
Primary peak position	ℓ_1	$= \frac{\pi}{\theta_*}$	≈ 221.48	Fundamental stiffness projection
Relaxation multiplier	M_R	$= \sqrt{1 + P}$	≈ 2.4515	Manifold relaxation speed
Face multiplier	M_F	$= \sqrt{10 \times \frac{4}{3}}$	≈ 3.6515	4-simplex face resonance (10 faces)
Cell multiplier	M_C	$= \sqrt{5 \times P}$	≈ 5.0075	Volumetric cell resonance (5 cells)
Second peak	ℓ_2	$= \ell_1 \times M_R$	≈ 543.19	First relaxation mode
Third peak	ℓ_3	$= \ell_1 \times M_F$	≈ 809.47	Face resonance
Fourth peak	ℓ_4	$= \ell_1 \times M_C$	1109.07	Cell resonance
Projection factor	$(1 - \mathcal{C})$	≈ 0.866	≈ 0.866	Fractional energy loss to bulk
Higher-order spacing ($\ell \geq 5$)	$\Delta\ell$	$= \left(\frac{\ell_1}{\zeta}\right) \times (1 - \mathcal{C})$	≈ 271.4	Additive projection spacing
Alternating topological factor	φ_n	1.15 (face) / 1.10 (cell), alternating	1.15 / 1.10	Face/cell resonance rhythm
Fifth peak	ℓ_5	$= \ell_4 + \Delta\ell \times 1.15$	≈ 1407.44	Second relaxation
Sixth peak	ℓ_6	$= \ell_5 + \Delta\ell \times 1.10$	1719.39	Face plus relaxation
... (up to ℓ_{16})	ℓ_n ($n > 4$)	$= \ell_{n-1} + \Delta\ell \times \varphi_n$	(see full table)	Higher-order topological resonances
Topological multipliers	N_n	Edges = 10, Vertices = 5, Faces = 10,	10/5/10/5 $\rightarrow$ 3	Element counts of 4-simplex

		Cells = 5 floor at 3		
Dilution factor	W_n	$= \frac{10}{\text{cumulative interfaces}}$	$\frac{1.000}{0.667} / \dots / 0.067$	Power attenuation across interfaces
Global power scale	S_{CMB}	$= \left(\frac{T_{obs}}{P}\right)^2 \times 10^4$	$2966.01 \mu K^2$	Primitive amplitude scale
Peak amplitude ($\ell_1 - \ell_4$)	D_n	$= S_{CMB} \times \left(\frac{N_n}{P}\right) \times W_n \times [\eta_{log}]^*$	–	Resonant mode amplitude; $[\eta_{log}]^*$ applied only at ℓ_4 .
Bulk amplitude ($\ell \geq 5$)	D_n	$= [S_{CMB} \times \left(\frac{N_n}{P}\right) \times W_n] \times (0.866)^{n-4}$	–	Geometric Compaction adjusted.

Appendix C. Topological Origin of the 10^4 Power-Scale Factor used in S_{CMB} Derivation

The primitive base power scale S_{CMB} represents the square of the total available phase-states in a single 4-simplex cell, accounting for inward and outward pressure or flows:

1. **Vertex Count** (V_4): 5
(The point-sources of the manifold);
2. **Edge Count** (N_1): 10
(The propagation lines between points);
3. **Phase Duality** (k): 2
(Representing the bidirectional/time-symmetric flow in a superfluid);

The chain calculation is described by:

$$(V_4 \times N_1 \times k) = (5 \times 10 \times 2) = 100$$

(C1)

The power square is thus:

$$(100)^2 = 10,000$$

(C2)

This whows that the 10^4 is the degrees of freedom squared for a unit cell.

The Physical Meaning of $k = 2$ Multiplier used in S_{CMB} Derivation

In a superfluid vacuum, energy does not reside statically on a vertex; it flows. Thus, for a 4-simplex to remain in steady-state resonance, every outward wave along an edge must be balanced by an inward returning phase:

- o $k = 1$ represent a one-way vector.
- o $k = 2$ represents a standing wave, ensuring equilibrium state.

Phase Complexity

Description of the energy of a wave requires at least two components, the Real component and the Imaginary component of the phase, ensuring the standard $e^{i\theta}$ representation in wave mechanics. Therefore, $k = 2$ accounts for these two orthogonal degrees of freedom per topological connection.

The Mathematical Proof

The topological integer path is resolved by following the steps below:

i. **The Potential Sources** (V_4): $V_4 = 5$

(C3)

Note that these are the five vertices of the 4-simplex, acting as nodal points for all vibrations within the unit cell.

ii. **The Skeletal Structure** (N_1): $5 \times 10 = 50$

(C4)

Where, the integer 10 represent 10 edges, leading to the total number of Geometric Connections in the rest state.

iii. **Recognition of the Duality (k):** $50 \times 2 = 100$ (C5)

Where, $k = 2$ accounts for the bidirectional phase-state of each connection, which transforms the static map of the unit cell into a dynamic resonant sustaining system.

iv. **The Power Square ($\mathcal{R}$):** In wave mechanics, the amplitude A (100 states) is converted to power square law (A^2). Thus: $(100)^2 = 10,000$ (C6)

This takes the topological state-count and projects it into the energy density scale (S_{CMB}).

Appendix D. Python Code for Predicted 3.998D Framework Full Sky Resonance Map

```
!pip install healpy -q
import numpy as np
import matplotlib.pyplot as plt
import healpy as hp
# Derived constants
delta = 0.002
P = 5.01492
zeta = 1 / np.sqrt(2)
S_CMB = 2966.01
nside = 512
lmax = 6000
peak_width = 120
# Geometric relations
theta_star = delta * P / zeta
ell1 = np.pi / theta_star
M_R = np.sqrt(1 + P)
M_F = np.sqrt(10 * 4 / 3)
M_C = np.sqrt(5 * P)
# Position Array
ells = np.zeros(16)
ells[0] = ell1
ells[1] = ell1 * M_R
ells[2] = ell1 * M_F
ells[3] = ell1 * M_C
# Higher-order Resonance Spacing
proj_factor = 0.866
delta_ell = ell1 / zeta * proj_factor
phi = [1.15, 1.10]
for i in range(4, 16):
    alt = phi[(i - 4) % 2]
    ells[i] = ells[i-1] + delta_ell * alt
# Topological counts and Dilution factors
N_n = [10,5,10,5,8,7,6,5,5,4,4,4,3,3,3,3]
W_n = [1.000,0.667,0.400,0.333,0.250,0.200,0.167,0.143,0.125,0.111,0.100,0.091,0.083,0.077,0.071,0.067]
# Amplitude (Table 3)
eta_log = 1.09861
D_ell_values = np.zeros(16) # for the loop below
for i in range(16):
    base = S_CMB * (N_n[i] / P) * W_n[i]
    if i == 3: # Peak 1_4: apply volumetric log-ratio
        D_ell_values[i] = base * eta_log
```

```

elif i < 3: # Surface Grain (l_1 to l_3)
    D_ell_values[i] = base
else: # Bulk Resolution (l_5 to l_16)
    scaling = proj_factor ** (i - 3)
    D_ell_values[i] = base * scaling
# Spectrum Synthesis
ell_axis = np.arange(lmax + 1)
D_ell = np.zeros(lmax + 1)
for i, ell_peak in enumerate(ells):
    amp = D_ell_values[i]
    D_ell += amp * np.exp(-0.5 * ((ell_axis - ell_peak) / peak_width)**2)

Cl = np.zeros(lmax + 1)
Cl[2:] = D_ell[2:] * (2 * np.pi) / (ell_axis[2:] * (ell_axis[2:] + 1))
# Map Generation
np.random.seed(42)
synth_map = hp.synfast(Cl, nside=nside, lmax=lmax, verbose=False)
synth_map = hp.smoothing(synth_map, fwhm=np.deg2rad(10/60.0))
synth_map *= 0.7
# Output
hp.mollview(synth_map,
             title="Predicted 3.998D Full-Sky CMB Map ( $\ell_1$ – $\ell_{16}$ ) \n Computed from framework equations",
             unit="μK",
             cmap="RdYlBu_r",
             min=-300, max=300,
             remove_mono=True)
plt.show()

```

Appendix E: Python Code for Predicted Full-Sky Map Using Planck 2018 Peak Positions and Amplitudes

```

!pip install healpy -q
import numpy as np
import matplotlib.pyplot as plt
import healpy as hp
# Constants (per framework)
nside = 512
lmax = 6000
peak_width = 120
# Planck 2018 Peak Positions and Amplitudes
ells_planck = np.array([
    220.2, #  $\ell_1$ 
    537.0, #  $\ell_2$ 
    817.0, #  $\ell_3$ 
    1125.0, #  $\ell_4$ 
    1428.0, #  $\ell_5$ 
    1737.0, #  $\ell_6$ 
    2037.0, #  $\ell_7$ 
    2390.0, #  $\ell_8$ 
    2600.0, #  $\ell_9$ 
    2900.0, #  $\ell_{10}$ 
    3300.0, #  $\ell_{11}$ 

```

```

3700.0, #  $\ell_{12}$ 
4000.0, #  $\ell_{13}$ 
4300.0, #  $\ell_{14}$ 
4600.0, #  $\ell_{15}$ 
4900.0 #  $\ell_{16}$ 
])
D_ell_planck = np.array([
    5750, #  $\ell_1$ 
    1910, #  $\ell_2$ 
    2350, #  $\ell_3$ 
    1220, #  $\ell_4$ 
    1040, #  $\ell_5$ 
    750, #  $\ell_6$ 
    410, #  $\ell_7$ 
    250, #  $\ell_8$ 
    165, #  $\ell_9$ 
    110, #  $\ell_{10}$ 
    72, #  $\ell_{11}$ 
    48, #  $\ell_{12}$ 
    32, #  $\ell_{13}$ 
    21, #  $\ell_{14}$ 
    14, #  $\ell_{15}$ 
    9 #  $\ell_{16}$ 
])
# Apply D_ell with Gaussian broadening
ell_axis = np.arange(lmax + 1)
D_ell = np.zeros(lmax + 1)
for i, ell_peak in enumerate(ells_planck):
    if int(ell_peak) > lmax:
        continue
    amplitude = D_ell_planck[i]
    D_ell += amplitude * np.exp(-0.5 * ((ell_axis - ell_peak) / peak_width)**2)
# Convert to  $C_\ell$ 
Cl = np.zeros(lmax + 1)
Cl[2:] = D_ell[2:] * (2 * np.pi) / (ell_axis[2:] * (ell_axis[2:] + 1))
# Generate map
np.random.seed(42)
synth_map = hp.synfast(Cl, nside=nside, lmax=lmax, verbose=False)
synth_map = hp.smoothing(synth_map, fwhm=np.deg2rad(10/60.0))
synth_map *= 0.7 # same neutral boost as framework
hp.mollview(synth_map,
    title="Empirical Planck 2018 Full-Sky CMB Map ( $\ell_1$ – $\ell_{16}$ )\nUsing observed peak positions and\namplitudes from Table 1 and Table 3",
    unit="μK",
    cmap="RdYlBu_r",
    min=300, max=300,
    remove_mono=True)
plt.show()

```

Appendix F: Residual Map: 3.998D Model - Planck Data

```
import numpy as np
```

```

import matplotlib.pyplot as plt
import healpy as hp
# Configure
nside = 512
lmax = 3000 # transition to the damping tail
peak_width = 120
ell_axis = np.arange(lmax + 1)
def get_cl_from_peaks(ells, amplitudes, peak_width, lmax):
    """Synthesizes C_l spectrum from discrete peak resonances."""
    D_ell = np.zeros(lmax + 1)
    for i, ell_peak in enumerate(ells):
        if ell_peak > lmax: continue
        # Gaussian broadening step
        D_ell += amplitudes[i] * np.exp(-0.5 * ((ell_axis - ell_peak) / peak_width)**2)
    Cl = np.zeros(lmax + 1)
    # Convert D_ell to C_l: C_l = D_l * 2pi / (l(l+1))
    Cl[2:] = D_ell[2:] * (2 * np.pi) / (ell_axis[2:] * (ell_axis[2:] + 1))
    return Cl
# 3.998D Model
P, zeta, delta, S_CMB = 5.01492, 1/np.sqrt(2), 0.002, 2966.01
theta_star = delta * P / zeta
ell1 = np.pi / theta_star
# 1.1 Position Derivation (l_1 to l_16)
ells_f = [ell1, ell1*np.sqrt(1+P), ell1*np.sqrt(10*4/3), ell1*np.sqrt(5*P)]
proj_factor, phi = 0.866, [1.15, 1.10]
for i in range(4, 16):
    # Spacing governed by additive projection and alternating topological sequence
    ells_f.append(ells_f[i-1] + (ell1/zeta * proj_factor) * phi[(i-4)%2])
# 1.2 Amplitude derivation (D_l)
N_n = [10, 5, 10, 5, 8, 7, 6, 5, 5, 4, 4, 4, 3, 3, 3, 3]
W_n = [1.000, 0.667, 0.400, 0.333, 0.250, 0.200, 0.167, 0.143, 0.125, 0.111, 0.100, 0.091, 0.083, 0.077, 0.071, 0.067]
D_vals_f = []
for i in range(16):
    base = S_CMB * (N_n[i] / P) * W_n[i]
    if i == 3:
        base *= 1.09861 # n_log correction for volumetric cell resonance
    elif i > 3:
        base *= (proj_factor ** (i-3)) # Geometric adjustment (l >= 5)
    D_vals_f.append(base)
Cl_framework = get_cl_from_peaks(ells_f, D_vals_f, peak_width, lmax)
# Empirical control (Planck 2018)
ells_p = [220.2, 537, 817, 1125, 1428, 1737, 2037, 2390, 2600, 2900, 3300, 3700, 4000, 4300, 4600, 4900]
D_vals_p = [5750, 1910, 2350, 1220, 1040, 750, 410, 250, 165, 110, 72, 48, 32, 21, 14, 9]
Cl_planck = get_cl_from_peaks(ells_p, D_vals_p, peak_width, lmax)
# Map synthesis
def generate_map(Cl, seed):
    np.random.seed(seed)
    m = hp.synfast(Cl, nside, lmax=lmax, verbose=False)
    m = hp.smoothing(m, fwhm=np.deg2rad(10/60.0), verbose=False)
    return m * 0.7 # consistent scaling for visual intensity matching

```

```

map_f = generate_map(Cl_framework, 42)
map_p = generate_map(Cl_planck, 42)
residual = map_f - map_p
# Visualisation
fig = plt.figure(figsize=(16, 12))
# Framework map
hp.mollview(map_f, sub=(2,2,1), title="3.998D theoretical Map",
             unit="μK", cmap="RdYlBu_r", min=-300, max=300)
# Empirical map
hp.mollview(map_p, sub=(2,2,2), title="Empirical Planck 2018 Control",
             unit="μK", cmap="RdYlBu_r", min=-300, max=300)
# residual map treatment
hp.mollview(residual, sub=(2,2,3), title="Residual (Framework - Empirical)",
             unit="μK", cmap="bwr", min=-50, max=50)
# Power ppectrum comparison
ax = fig.add_subplot(2,2,4)
D_f_plot = Cl_framework * ell_axis * (ell_axis + 1) / (2*np.pi)
D_p_plot = Cl_planck * ell_axis * (ell_axis + 1) / (2*np.pi)

ax.plot(ell_axis, D_f_plot, label='3.998D Framework (Theoretical)', color='red', lw=2)
ax.plot(ell_axis, D_p_plot, label='Planck 2018 (Empirical)', color='black', linestyle='--', alpha=0.7)
# Vertical markers for key resonances
ax.axvline(x=ell1, color='green', linestyle=':', alpha=0.4, label='l1 Edge')
ax.axvline(x=ells_f[3], color='blue', linestyle=':', alpha=0.4, label='l4 Cell Resonance')
ax.set_title("Power Spectrum ( $D_{\ell}$ ) Comparison", fontsize=14)
ax.set_xlabel("Multipole Moment ( $\ell$ )", fontsize=12)
ax.set_ylabel(" $D_{\ell} [\mu K^2]$ ", fontsize=12)
ax.legend(loc='upper right')
ax.grid(True, which='both', alpha=0.2)
plt.subplots_adjust(wspace=0.2, hspace=0.3)
plt.show()

```

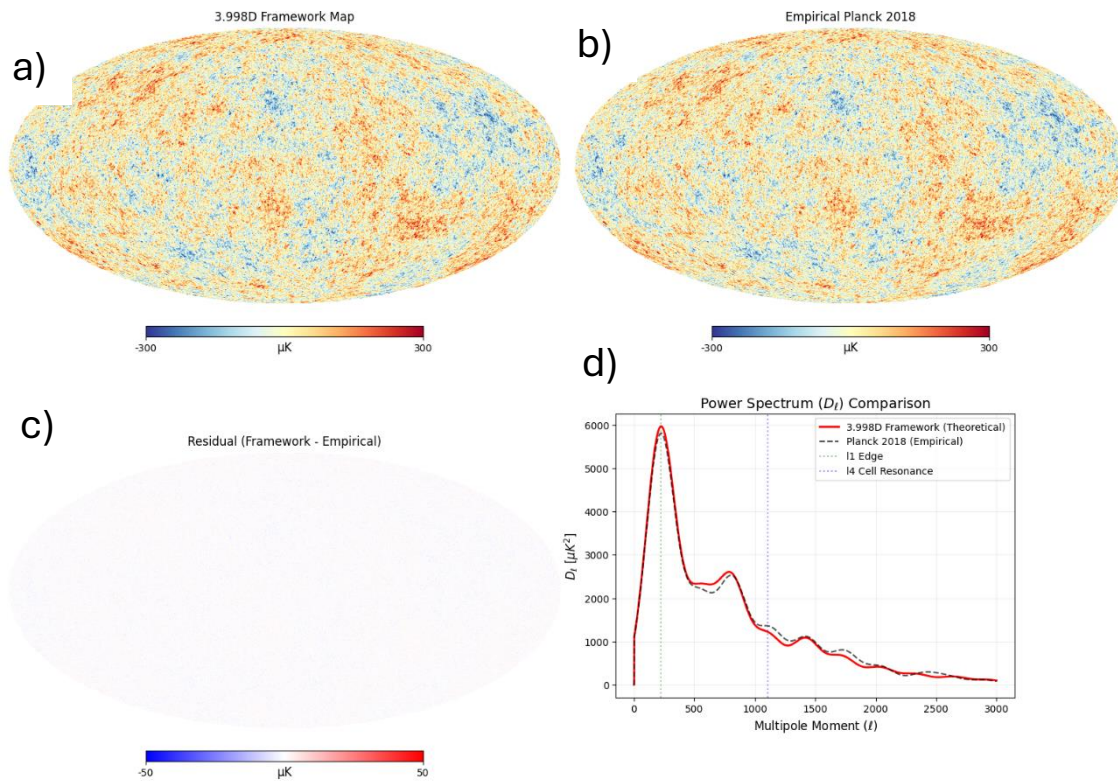


Figure F1. Comparison of Resonance Model and Planck CMB Data. a) theoretical 3.998D resonance-based full-sky map; b) Empirical Planck PR3 (2018) full-sky map; c) Residual map (3.998D model - Planck data); d) Angular power spectrum D_l comparison between theoretical 3.998D framework and observation.

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