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Christian Kwame Kpotosu *, Stanley Amegbor , Bismarck Mifetu , Raymond Bessa Kwadzo Ezah

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Senior High School Students' Difficulties with Geometry Topics

Christian Kwame Kpotosu *, Stanley Amegbor, Bismarck Mifetu and Raymond Bessa Kwadzo Ezah

University of Cape Coast, Department of Mathematics and ICT Education, Ghana;
stanleyjackson525@gmail.com (S.A.); mifetubismark84@gmail.com (B.M.);
ezahbkraymond@gmail.com (R.B.K.E.)
* Correspondence: christian.kpotosu001@stu.ucc.edu.gh; Tel. +233543753646

Abstract: Students have historically fared poorly on the West African Examinations Council's (WAEC) mathematics exam. This study investigated senior high school students' difficulties with studying Geometry in the Cape Coast Metropolis, Central Region, Ghana. Using the descriptive survey design, data were collected from 300 Senior High School Two (SHS 2) students using simple random sampling. Students responded to the questionnaire and a 30-item multiple-choice achievement test on geometry. Four geometry concepts students perceived difficult to study are Circle theorems, Perpendicularity of Tangent and radius of a circle, Angle between Tangent and a chord and Tangents from an external point. The reasons for the difficulty in studying geometry topics are the unavailability of instructional materials, the geometry topics being complex, and teachers not teaching geometry concepts to the students' understanding. Students' gender did not influence the learning of concepts in geometry at a 0.05 level of significance. There was an insignificantly weak positive correlation between the students' performance and their perceived difficult geometry concepts. It was concluded that the Ministry of Education does not provide relevant instructional materials to the various schools to teach geometry concepts. Schools do not organise workshops and seminars for teachers on effectively teaching geometry concepts in mathematics.

Keywords: Chord; Circle theorems; Geometry; Geometry Difficulties; Geometry topics; Senior high schools; Tangents; Perpendicularity

Introduction

Mathematics is a mandatory course of study from elementary school through high school, but at the college level, it is an elective. According to Ayinla (2011), mathematics is the bedrock of all knowledge, with applications in every other field. Mathematics is also a language scientist use to convey concepts through common vocabularies and frameworks.

Geometry investigates the connections between geometric figures. These forms may be flat or three-dimensional. A plane is a geometric shape in which a straight line drawn between any two points on the surface passes entirely within the plane itself. However, a solid shape has boundaries that can't always be captured in a flat representation. Geometry refers to studying shapes and their attributes, which can be either flat or three-dimensional. The dimensions of a plane shape are two or more of the same, making it a two-dimensional shape like a square, rectangle, circle, polygon, triangle, and so on. A solid shape's length, width, and height (or base area and height) define its three dimensions. The cone, pyramid, sphere, cylinder, prism, cube, and cuboid are all examples of solid shapes.

Perceptions and beliefs about mathematics originate from past experiences, including both cognitive and affective dimensions (Ministry of Education [MoE], 2010), even though the mathematics curriculum is constructed on the constructivist's principles with an emphasis on opportunities for students to explore mathematical situations in the environment to make their discoveries (Aguilar, Rosas & Juan Zavaleta, 2012). A person's mathematical identity can be thought

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of from two perspectives: the cognitive domain, which pertains to one's understanding of mathematics, and the affective domain, where one's emotions and reactions to the subject matter come into play. The term is used in a broad sense to include all mental images, mental associations, beliefs, attitudes, and feelings that pertain to mathematics and mathematical education.

Both male and female students can immediately see the practical applications of geometry in fields like measuring and estimating. Lines, angles, curves, and shapes, along with their properties and relationships, are at the heart of Geometry. The systematic description and definition of the world is a skill both male and female students can gain from geometry. In addition, it facilitates the development of skills such as creative problem-solving, analytical thinking, and the ability to draw connections between mathematical concepts and the real world.

Students continue to dislike the geometrical concepts and, as a result, perform poorly on examinations, despite the usefulness of geometry in real-life situations. Students' performance has steadily declined over the years, and widespread failure on mathematics exams is a real phenomenon (Kafata & Mbetwa, 2016). According to Steele and Huhn (2018), students consistently demonstrate subpar performance on senior-year mathematics exercises and tests because they struggle with solving geometry tasks. One possible explanation for this failure is that these topics receive only cursory treatment in courses designed to prepare future Geometry teachers (Helena & Maria, 2015).

Tutak and Adams (2015) also discovered that many students do not grasp the fundamental geometrical concepts taught in mathematics courses, leading many to drop out without completing their formal education. Candidates' difficulties with mensuration, construction, and circle theorems were confirmed in every WAEC Chief Examiners' Report from 2007 through 2018. Students' test scores were not encouraging as a result of all of these factors.

Fabiyi (2017) investigated the mathematical concepts in geometry that Nigerian high school seniors found most challenging. Out of a total of 23, students found eight of them challenging to learn. This included construction, coordinate geometry, and circle theorems. He also discovered that students place a higher value on concepts they find difficult to learn in geometry than on those they rate as easy to learn. Similar findings were found by Adegun and Adegun (2013), who found that students found construction and locus, longitude, and latitude to be challenging.

In addition, many reasons have been proposed to explain the difficulty of learning geometry. Research shows that students have difficulty grasping geometric concepts in mathematics because of teachers' methods of instruction, geometric language, and a lack of visualising abilities (Chua et al., 2017; Berenger, 2017). Gender differences, lack of reasoning skills, lack of time, an insufficient school curriculum, and a lack of proof on the part of students also play a role (Uduosoro, 2011; Nigerian Educational Research and Development Council [NERDC], 2012). All of these are thought to impede students' ability to learn geometry. The unavailability of instructional materials and teachers' method of instruction was cited by Fabiyi (2017) as reasons for students' perception of geometry concepts as difficult.

To the same end, Telima (2011) commissioned a study titled Problems of Teaching and Learning of Geometry to pinpoint these issues and propose remedies. Most math teachers have a weak grasp of geometry, students have a similarly weak grasp of the subject, student attitudes toward learning are negative, and the classroom setting is not optimal for either teaching or learning. Analysis of the mathematical performance of Ghanaian students by Anamuah-Mensah, Mereku, and Asabere-Ameyaw (2004) revealed that Algebra, Measurement, and Geometry were the students' weak content areas.

From the foregoing literature, it can be seen that students have not been performing well in mathematics, especially in geometry. Hence, the present study sought to investigate the geometry concepts in mathematics perceived as difficult to study by senior high school students in the Cape Coast Metropolis, Ghana. Also, the causes of the difficulty in studying the concepts and the relationship between the perceived difficult geometry concepts and their performance in mathematics were considered.

It is hoped that the results of this study will aid in finding a solution to some basic educational problems in the learning and teaching of mathematics, especially geometry, in schools. The identified

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difficult geometry concepts and the plausible causes of the difficulty in studying the concepts would help the curriculum planners and developers to plan the curriculum in a way that will make students develop an interest in learning geometry. Curriculum implementers, including teachers and students, will find more interest in teaching and learning the concepts due to their numerous advantages.

Materials and Methods

The design used for the study is a descriptive survey. The study employed the descriptive survey design because the mode of data collection was done by issuing only questionnaires to the participants to respond to. The target population for this study was all second-year senior high school students in the Cape Coast Metropolis. Within the metropolis, only two single-sex schools (males) and two co-educational schools were selected to participate in this study. The main reason for using second-year senior high school students was that it is assumed that they had completed about 85% of the Geometry concepts in the senior high school curriculum at the time the study was conducted.

A multi-stage sampling technique was used to obtain the schools participating in the study. Two co-educational and two single-sex schools were selected using simple random sampling. In all, 300 senior high school two students during the 2018/2019 academic year were selected to form the sample for the study. There were 100 students from each of the schools selected.

Questionnaires and achievement tests were the main instruments used for data collection. The questionnaire was adapted from Fabiyi (2017) and modified by the researchers to suit this work concerning the purpose of the study, research questions and research hypotheses. The instrument consists of four sections. Section 'A' comprises respondents' personal information (age and sex). Section 'B' contains twenty-two concepts in geometry from the senior high school mathematics curriculum from which the students were to identify geometry concepts perceived as difficult to learn measured on a 4-point Likert Scale (Very Difficult (VD) = 1, Difficult (D) = 2, Easy (E) = 3, Very Easy (VE) = 4) and Not Yet Taught (NYT). Section 'C' contains 10 plausible causes of the perceived difficulties. The students should choose their level of agreement or disagreement, measured on a 4-point Likert Scale (Strongly Agree = 4, Agree = 3, Strongly Disagree = 2 and Disagree = 1). The achievement test was self-developed using questions from the West African Examinations Council's questions concerning Geometry concepts. There were thirty (30) objective test items on the achievement test.

For confidentiality, the instruments did not record the students' names to allay their fears of being exposed. The instruments were administered to the students in the schools selected for this study. The instrument was administered in the third week of May 2019. The researchers took a letter of introduction from the Department of Mathematics and I. C. T Education, the University of Cape Coast, to the various Heads of the selected Senior High Schools. This letter was later sent to the various Heads of Mathematics Departments. The mathematics teachers helped in the administration of the questionnaires. The questionnaire administration lasted for three days, a day for each school.

Data were analysed using descriptive statistical tools (i.e., the mean, standard deviation, frequency counts and percentages) and inferential statistical tools (i.e., the t-test and Pearson Correlation Coefficient). These analyses were done with the help of the Statistical Packages for Social Sciences (SPSS version 21).

Results

Results of Demographic Data of Respondents

Table 1 shows 300 respondents from the four senior high schools were involved in the research. There were 200 males representing 66.7%, and 100 females representing 33.3% of the total sample for the study. The age distribution of students reveals that there were 24 (8.0%) of the students below 16 years, 245 (81.7%) were between 16-18 years, 28 (9.3%) of them were between 19-21 years, and 3 (1.0%) were above 21 years.

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Table 1. Demographic characteristics of respondents.

	Scale	Frequency	Percentage %
Gender	Male	200	66.7
	Female	100	33.3
Age	Below 16 years	24	8.0
	16-18 years	245	81.7
	19-21 years	28	9.3
	Above 21 years	3	1.0

Geometry Concepts Perceived as Difficult to Study by Senior High School Students

From Table 2, out of the twenty-two geometry concepts, it was revealed that the students perceived four concepts as difficult to study. These are Circle theorems 156 (52.0%) students perceived it difficult; Perpendicularity of tangent and radius of a circle 138 (46.0%) students perceived it difficult, Angle between Tangent and a chord 146 (48.7%) students perceived it difficult, Tangents from an external point 141 (47.0%) students perceived it difficult.

Table 2. Geometry concepts perceived as difficult to study.

Geometry Concepts	Difficult concepts		Not Difficult concepts		Not yet taught	
	f	%	f	%	f	%
Angles	38	12.7	255	85.0	7	2.3
Lines	42	14.0	247	82.3	11	3.7
Theorems on triangles	77	25.7	199	66.3	24	8.0
Special triangles	79	26.3	194	64.7	27	9.0
Pythagoras's Theorem (Right-Angled triangle)	54	18.0	241	80.3	5	1.7
Properties of Quadrilaterals	63	21.0	218	72.7	19	6.3
Polygons	52	17.3	224	74.7	24	8.0
Area and Circumference of a circle	66	22.0	223	74.3	11	3.7
Circle Theorems	156	52.0	127	42.3	17	5.7
Perpendicularity of tangent and the radius of a circle	138	46.0	135	45.0	27	9.0
The angle between a tangent and a chord	146	48.7	116	38.7	38	12.7
Tangents from an external point	141	47.0	114	38.0	45	15.0
Length of an arc of a circle	90	30.0	170	56.7	40	13.3
Length of a chord of a circle	81	27.0	181	60.3	38	12.7
Perimeters of plane figures	69	23.0	199	66.3	32	10.7
Areas of sectors and segments	93	31.0	168	56.0	39	13.0
Area of quadrilaterals	71	23.7	196	65.3	33	11.0
Coordinates Geometry	83	27.7	190	63.3	27	9.0
Sine, Cosine and Tangent of right-angled triangles	94	31.3	191	63.7	15	5.0
The use of calculators to read sine, cosine and tangent angles	77	25.7	210	70.0	13	4.3
The inverse of trigonometric ratios	113	37.7	174	58.0	13	4.3
Angles of elevation and depression	123	41.0	165	55.0	12	4.0

The analysis further revealed that eighteen geometry concepts are perceived as easy to study by the students, with angles (85%), lines (82.3%) and Pythagoras's theorem (right-angled triangle) (80.3%) perceived as the easiest concepts to study. Properties of Quadrilaterals, Polygons, Area and Circumference of a circle, and using calculators to read sine, cosine and tangent angles are perceived as easy to study by 70% - 80% of students.

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Even though students perceived some geometry concepts as difficult to study, in general, the degree of their perceived difficult geometry concepts is lower than the ones they perceived as not difficult (easy).

Causes of Difficulty in Studying Geometry Concepts

From Table 3, the main reason students perceived geometry concepts as difficult to study is that there are insufficient instructional materials to teach geometry concepts. As many as 226 students representing 75.3%) agreed with this. This reason followed: Geometry topics are complex (66% in agreement), Teachers do not teach geometry concepts to students' understanding (55.7%), and Geometry in basic school was not taught to the students' understanding (55.3%).

Table 3. Causes of difficulty in studying geometry topics.

Rank	Statement	Agree		Disagree	
		f	%	f	%
1st	There are not enough instructional materials to teach geometry topics	226	75.3	74	24.7
2nd	Geometry topics are complex	198	66.0	102	34
3rd	The topics are very complex	187	62.4	113	37.6
4th	Teachers do not teach geometry concepts, to my understanding	167	55.7	133	44.3
5th	Geometry in basic school was not taught to my understanding	166	55.3	134	44.7
6th	Geometry topics are difficult to understand	161	53.7	139	46.3
7th	The time teacher spends on geometry topics is not enough	151	50.3	149	49.7
8th	I do not show interest in geometry topics	124	41.3	176	58.7
9th	Textbooks do not explain the geometry topics, to my understanding	123	41.0	177	59
10th	Geometry topics consume a lot of time during tests and examinations	83	27.7	217	72.3

It was further revealed that the least cause of students' difficulty in studying geometry concepts is that geometry topics consume a lot of time during tests and examinations. Two hundred and seventeen students representing 72.3%, indicated their disagreement with this item.

There is No Significant Difference in the Number of Geometry Concepts Perceived as Difficult to Study by Male and Female Students

From Table 4, the number of male students that perceived circle theorems as difficult is 92 (59.0%) against 64 (41.0%) female students. It can also be seen that, 84 (60.9%) males and 54 (39.1%) females perceived the perpendicularity of tangent and radius of a circle to be difficult. In all four cases, more male than female students perceived the concepts as difficult to study.

Table 4. Responses of male and female students on the four perceived difficult geometry concepts.

Difficult Concepts	Male		Female	
	F	%	F	%
Circle Theorems	92	59.0	64	41.0
Perpendicularity of tangent and radius of a circle	84	60.9	54	39.1
The angle between a tangent and a chord	91	62.3	55	37.7
Tangents from an external point	91	64.5	50	35.5

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A further test was done using the independent sample t-test to determine if there exists any difference between the number of geometry concepts perceived as difficult to study by male and female students. This is presented in Table 5.

Table 5. Results of t-Test and descriptive statistics for perceived difficult geometry concepts by sex.

Gender	N	Mean	Std. Deviation	Df	t-value	p-value
Male	200	8.45	3.524	298	0.220	0.8260
Female	100	8.35	3.534			

Results from Table 5 show that the mean score of males in their perceived difficult geometry concepts was 8.45, and that of females was 8.35. The mean and standard deviation were calculated using the total score of the four identified difficult concepts for all male and female students.

The t-calculated value was 0.22, and its p-value was 0.8260 at a 5% significant level. Since the p-value is greater than the 0.05 significant level, we fail to reject the null hypothesis (H_0). It is therefore concluded that there is no statistically significant difference between male and female students in their perception of difficult geometry topics in the SHS core Mathematics. Hence, we can conclude that gender does not influence students' perception of difficult geometry concepts.

There is no significant relationship between students' performance in geometry and their perceived difficulty with geometry concepts

Table 6 reveals that as many as 185 (61.6%) of the students obtained marks above 15 out of the maximum 30 marks for the test, with 22 (7.3%) scoring as high as 26 or more. From Table 2 (0.7%), students had test scores between 1-5 out of 30 marks. In all, 115 (38.3%) of the students had marks of 15 or below. The mean score was calculated to be 17.58, with a standard deviation of 5.91. This shows that, on average, the students performed quite well on the test.

Table 6. Range of achievement test scores of the students.

Score	Frequency	Percentage (%)
1-5	2	0.7
6-10	46	15.3
11-15	67	22.3
16-20	72	24.0
21-25	91	30.3
26-30	22	7.3
Total	300	100

Pearson Correlation (r) was calculated using the total test scores of the three hundred students and the total scores on their four perceived difficult geometry concepts to verify the degree of relationship between their performance and the perceived difficult concepts. The results are presented in Table 7.

Table 7. Relationship between students' performance and their perceived difficult geometry topics.

		Geometry Test Scores	Perceived Difficult Geometry Topics
Geometry Test Scores	Pearson Correlation	1	.043
	Sig. (2-tailed)		.462
	N	300	300
Perceived Difficult Geometry Topics	Pearson Correlation	.043	1
	Sig. (2-tailed)	.462	
	N	300	300

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Concerning the Pearson correlation (r) value of 0.043, the correlation was positively weak. This implies that students' perceived difficulty with geometry topics in core mathematics does influence their performance. Since the correlation coefficient is positive, there is a direct weak insignificant relationship between students' perception of difficult geometry topics and their performance in geometry.

An insignificant positive ($r = 0.043$, $p>0.05$) relationship exists between students' performance and their perceived difficult geometry topics in SHS core Mathematics. Thus, students tend to perform poorly in the topics they perceive to be difficult and well in the topics they perceive to be less difficult (easy). In other words, a student who perceived some topics to be difficult will perform poorly in those topics compared to a student who perceived the topics to be less difficult (easy).

Discussion

This study investigated the geometry topics perceived as difficult to study by Senior High Schools students. It was found that students perceived four geometry concepts as difficult to study. These are Circle theorems, Perpendicularity of Tangents and radius of a circle, Angle between Tangent and a chord, and Tangents from an external point. Students also perceived Angles, Lines and Pythagoras's theorem (right-angled triangle) as the easiest concepts to study. It was further revealed that 70% - 80% of the students perceived the Properties of Quadrilaterals, Polygons, Area and Circumference of a circle, and using calculators to read sine, cosine and tangent angles as easy to study.

Circle theorem in its entirety is considered a difficult concept to study because, after every year of writing the West African Secondary School Certificate Examinations, the WAEC chief examiners always identify circle theorem as one of the major weaknesses of students, whereas concepts such as Polygons, Area and Circumference of a circle, and the use of calculators to read sine, cosine and tangent of angles were identified as students' strengths. These findings align with WAEC Chief Examiners' Report (2016, 2017, 2018), which confirmed that candidates had challenges answering questions on circle theorems. Also, Fabiyi (2017) included circle theorems in his eight identified geometry concepts perceived as difficult by students. Anamuah-Mensah, Mereku & Asabere-Ameyaw (2004), in their analysis of Ghanaian students' performance in mathematics, also included Geometry as one of the students' weak content areas.

Furthermore, the main causes of difficulty in studying geometry concepts identified by students are: There are not enough instructional materials for teaching geometry concepts, Geometry topics are complex, Teachers do not teach geometry concepts to students' understanding, and Geometry in basic school was not taught to the students' understanding as the pressing reasons making the study of geometry concepts difficult. The issue of instructional materials to teach mathematics concepts has impeded students' understanding of mathematical concepts. Teaching and learning materials make the mathematics concepts easily understandable by students since teachers are expected to use the materials to represent the concepts, thereby not presenting the mathematics concepts abstractly for the students. This finding agrees with the findings by Fabiyi (2017) that students' reasons for perceiving geometry concepts as difficult include: Unavailability of instructional materials and teachers' method of instruction. The results conformed to the findings of Ubi, Odiong and Igiri (2018) when he concluded that non-availability and obsolescence of instructional materials as the factors that contribute to the learning of geometry in mathematics difficult. Similarly, Kafata and Mbetwa (2016) observed, on a general note, that secondary schools lack facilities and equipment for teaching.

Additionally, it was revealed that there is no statistically significant difference between male and female students' perceptions of the number of difficult geometry concepts at a 0.05 level of significance. Even though several studies see geometry as gender sensitive, this study revealed that gender does not play any role in the study of geometry concepts. This is because both male and female students see it as an equal responsibility to do well in mathematics; hence they do not succumb to any pressures that make them think geometry is for a particular kind of people. This conforms with the studies conducted by Harisman et al. (2017), who found that gender does not influence the study of geometry.

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Finally, the study further revealed an insignificantly weak positive relationship between students' performance on the achievement test and their perceived difficult geometry concepts. This means that the more students perceive geometry topics to be difficult, the more likely they will not perform well in geometry tests. Similarly, when students do not view geometry as difficult, they are likely to perform well on geometry tests. The findings conform to the findings of Chua et al. (2017), who found a weak significant relationship between geometry and students' overall performance. These perceptions are one way or the other attributed to the factors that students disclosed to be linked to their difficulty in learning geometry concepts. The prominent factor was inadequate instructional materials. Thus, when there are no instructional materials to teach the Geometry concepts, students are likely not to understand the concepts and hence develop disinterest in them, which is likely to lead to their inability to perform on those concepts. The Ministry of Education should provide relevant instructional materials to various schools for teaching of concepts in geometry. Schools should organize workshops and seminars for teachers on how to teach the difficult geometry concepts in mathematics effectively.

Conclusions and Recommendations

This study offers valuable insights into students' perceptions of challenging geometry concepts in Senior High Schools. Four geometry topics have been found to be particularly difficult for students, with circle theorems being consistently challenging over the years. In spite of various attempts to enhance performance in geometry, including analysing exam weaknesses and emphasising difficult concepts through research, students still face difficulties with specific topics. There are several factors that contribute to the challenge at hand, including a lack of adequate instructional materials, the presence of complex topics, and the use of ineffective teaching methods. It is worth noting that the perception of difficult geometry concepts was not significantly influenced by gender among students. Nevertheless, a slight positive correlation was observed between students' perceptions of difficulty and their performance on achievement tests, underscoring the significance of addressing students' perceptions to enhance learning outcomes.

It is therefore crucial for the Ministry of Education to give utmost importance to ensuring that schools have sufficient instructional materials for teaching geometry concepts. These resources should be designed to enhance comprehension and encourage active participation, encompassing physical materials like textbooks, diagrams, models, as well as digital tools. It would be beneficial for schools to arrange workshops and seminars to provide teachers with valuable techniques for instructing challenging geometry concepts. Training should prioritise creative teaching approaches, personalised learning techniques, and methods for simplifying complex ideas for students. It is important for curriculum developers to carefully evaluate and update the mathematics curriculum to ensure that it is in line with the cognitive development and instructional requirements of students in geometry. It is important to organise topics in a logical order, offer ample practice opportunities, and include real-world applications to make the content more relevant and engaging. Teachers should encourage the creation of collaborative learning environments that foster teamwork and cooperation among students. This will enable them to effectively tackle geometry problems, engage in meaningful discussions about key concepts, and provide mutual support in their learning journey. Collaborative methods such as peer tutoring, group projects, and cooperative learning activities have the potential to cultivate a more profound comprehension and boost students' self-assurance when it comes to challenging subjects.

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