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Article

Relative-Entropy Variational Principle for Semiclassical Gravity with Finite-Resolution Boundaries

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Abstract

We propose a causal-diamond formulation of semiclassical gravity where a finite-resolution boundary regulator (Coherency Screen) supplies the edge structure for a local Wheeler–DeWitt description. Dynamics are defined by an informational principle: for each diamond O , the action is the relative entropy $S_{\text{rel}}(\rho_O \parallel \sigma_O[\lambda])$ between the physical state and a reference family on a fixed algebra. In the modular/KMS regime, the vacuum is at entanglement equilibrium; the leading dynamics become a linear-response problem governed by the Hessian of relative entropy (Kubo–Mori metric). This Hessian organizes deformations into tensor, vector and scalar sectors, yielding Einstein stiffness, Yang–Mills susceptibilities and mass gaps. The resulting local EFT is organized by a heat-kernel expansion (identifying the leading R^2 operator) and is compatible with a spinorial transport structure. Edge-mode counting and Newton's constant G fix the resolution scale at $M_s \sim 3 \times 10^{13}$ GeV. Identifying M_s with stiffness saturation places the high-curvature regime in a plateau universality class, predicting a tensor-to-scalar ratio $r \sim 10^{-3}$. We further discuss how this boundary logic constrains gauge and mass sectors, suggesting discrete coupling relations and a geometric hierarchy for charged leptons. The construction yields correlated, falsifiable targets tied to a single scale.

Keywords: semiclassical gravity; spectral action; relative entropy; causal diamonds; fine-structure constant; holographic boundary; edge modes; non-commutative geometry; holographic principle

1. Conceptual Foundations

We propose a finite-resolution framework for defining local quantum subsystems in semiclassical gravity, organized around the Wheeler–DeWitt (WDW) constraint [1]:

$$H\Psi = 0$$

where the global state Ψ is constrained rather than time-evolved and the operator H encodes the gravitational constraints and diffeomorphism invariance. Physically, this implies that observables must be relational.

We adopt this framework in a weak sense to enforce background independence without requiring a global solution. Dynamics is treated as local statistical inference on causal diamonds $O(p, q)$ [2], which serve as the fundamental units of accessible correlations [3]. In the small-diamond regime, modular flow supplies the intrinsic local clock [4,5].

In this context, geometry does not evolve in external time but is inferred from entropic boundary correlations [6–9], a view recently advanced in [10–13]. This inference uses relative entropy as a local mismatch functional: on each diamond, it quantifies the distinguishability between the actual boundary-completed state and its geometric reference.

We begin by identifying the minimal architecture required for local, diffeomorphism-invariant physics. We identify the causal diamond (mandated by operational consistency) and treat its boundary as a finite-resolution register (boundary completion), which stores the minimal edge data needed to define a subregion algebra [14]. We first establish that such a finite-capacity completion is a necessity

for local subsystems. We then derive the specific topology of this hardware and the entropic cost of representing smooth continuum structure on discrete boundaries [15,16]. Finally, we show how this topology fixes a channel multiplicity N (effective depth) [17] and, upon calibration to Newton's constant G , a physical bandwidth M_s .

With (N, M_s) fixed, the theory becomes highly constrained: the same vacuum structure that yields semiclassical gravity also sets the scale of gauge/matter response, so disparate observables across the hierarchy become linked rather than independently adjustable (since we fixed both the stiffness normalization and the physical cutoff controlling the EFT coefficients [18,19]), as illustrated in Appendices A–D.

We formalize the minimal architecture as an axiomatic basis in the next section.

1.1. Axiomatic Basis: Minimal Architecture

We are not starting from a preferred lattice or field content; instead, we approach quantum gravity as a problem of local inference. We adopt the following minimal ingredients to define subsystems, clocks and regulated observables without having to assume a background structure:

Principle A: Operational Regions

Locality is defined operationally by the minimal covariant unit: a finite causal diamond [2]. This geometry uniquely fixes the boundary interface where subsystem-defining information resides [14].

P1 (Non-factorization): In gravity, the diffeomorphism-invariant physical Hilbert space cannot factorize across spatial subregions, $\mathcal{H}_{\text{phys}} \neq \mathcal{H}_A \otimes \mathcal{H}_B$ [14]. In gravity, constraints prevent naive factorization; explicit boundary data are therefore required to strictly define the very notion of interiority and a unique subregion algebra [Forced].

P2 (Causal Diamonds): A finite experiment is a closed send–return loop between events p (prepare/send) and q (receive/read out), with $q \in J^+(p)$. The minimal covariant operational region is the causal diamond $O(p, q) \equiv J^+(p) \cap J^-(q)$ [2]. Because $O(p, q)$ is defined purely by causal relations, it fixes the subsystem region without choosing a foliation [Forced]. For minimality we restrict to the simply connected local closure: the maximal spatial slice is a 3-ball B^3 with boundary cut S^2 ; nontrivial topology would introduce extra discrete labels and is excluded by the minimal-architecture premise [Closure].

P3 (Boundary Completion): Since the physical Hilbert space does not factorize across regions (P1), a well-defined subregion algebra \mathfrak{A}_O requires boundary completion. We implement this algebraically by adjoining edge observables that generate a nontrivial center $\mathcal{Z}(\mathfrak{A}_O)$, encoding the gluing or charge data. Equivalently represented by an extended Hilbert space with edge modes, this center ensures that restriction and gluing operations are well-defined: compatible reduced descriptions are enforced by matching \mathcal{Z} on intersections [14,20] [Forced].

Principle B: Finite Bandwidth Modular Information

A boundary-defined subsystem requires finite capacity to avoid unbounded information density [21]. In the small-diamond/KMS regime, modular flow supplies the intrinsic clock of this register [4,5], rendering time local and emergent.

P4 (Modular Locality): In the small-diamond (local Rindler) regime, the vacuum state restricted to \mathfrak{A}_O is, to leading order, KMS with respect to a geometric modular flow: the modular Hamiltonian is well-approximated by the local boost generator (Bisognano–Wichmann/Unruh). Scale separation $\delta \ll \ell_O \ll R_{\text{curv}}$ controls this approximation; we do not assume modular locality outside this regime. The dimensionless modular parameter τ provides an intrinsic ordering of correlations (local clock) for subsystem; with standard boost normalization, KMS periodicity is 2π [4,5,22,23] [Forced].

P5 (Finite Capacity): A boundary-defined subsystem requires finite operational capacity. In the absence of a resolution limit, the density of distinguishable boundary states diverges, preventing a stable restriction to the local algebra \mathfrak{A}_O . We therefore postulate a physical regulator δ (operationally identified with the stretched-horizon thickness or vacuum impedance) and fix the hardware bandwidth

$M_s \equiv \delta^{-1}$ [Forced]. This cutoff renders the screen a finite-capacity register, ensuring a well-defined reduced description consistent with the Bekenstein bound [15,21]. For later spectral accessibility estimates, we adopt the compact homogeneous internal closure $\mathcal{M}_{\text{int}} \cong S^3 \times S^5$ as minimal working model [Closure].

Principle C: Topological Quantization and Minimal Transport

Consistency forces boundary charge sectors to be topologically quantized rather than continuously tunable [24]. Minimal isotropic routing then fixes the transport rule [25], with smooth forces emerging as the effective continuum description.

P6 (Gauge Topology): We model boundary charge sectors by a bulk Chern–Simons theory whose boundary reduction is a WZW current algebra [24,26]. On a bounded region, Gauss-law constraints generate edge currents as the gauge-invariant completion of the subregion algebra [14]. The associated level $k \in \mathbb{Z}$ quantizes these currents, replacing continuous field normalization with a discrete integer. This sets the fundamental stiffness unit that determines the matching-scale inverse coupling $\alpha^{-1}(M_s)$ in linear response [Closure].

P7 (Network Architecture): Minimal local screen architecture in 3D is constrained by two requirements: **Routing:** A local transport layer must exist and be isotropic in three spatial dimensions, with inversion symmetry (no built-in handedness) [Forced]. We realize this with the minimal inversion-symmetric spanning set $\{\pm x, \pm y, \pm z\}$, representing three independent antipodal pairs, fixing minimal coordination $z = 6$ and an octahedral (L_1) routing rule [Closure]. This establishes three independent spatial transport axes, over which isotropic accessibility counts sum linearly [Closure]. Routing is implemented in local orthonormal frames, as a global smooth tangent frame on S^2 is obstructed (Hairy Ball Theorem) [Forced]. Similar discrete-isotropy constructions define relativistic dynamics in QCA frameworks [25,27,28].

Payload: To admit local spinorial payloads (matter), the boundary algebra must lift local rotations from $SO(3)$ to its universal cover $SU(2)$ to admit projective representations [Forced]. The spinor lift selects $SU(2)$ as the minimal rotation payload; since $SU(2)$ is diffeomorphic to S^3 , an S^3 factor is the natural compact building block. We implement this minimal topological extension as a \mathbb{Z}_2 Ising-type twist sector, contributing a quantum dimension $d_\sigma = \sqrt{2}$ [29], imposing a topological entropy of $\ln \sqrt{2}$ (the half-bit cost of the spinor lift) [Closure].

Corollary (Port Symmetry $|\Gamma|$): The signed-axis routing implies a port redundancy of order $|\Gamma| = 2^3 = 8$, corresponding to independent sign configurations of the three transport axes. In regimes where only commuting phases remain stable (cannot resolve curvature), these sign labels act as redundant port labels, so phase volumes are quotiented by $|\Gamma|$ [Closure].

Corollary (Tip Anomaly κ): Near the tips (p, q) of the causal diamond, the screen cross-section becomes sub-resolution (P5), forcing a geometric mismatch between smooth modular flow and discrete routing [Forced]. We parameterize this irreducible overhead by a dimensionless scalar κ [Closure].

Corollary (Spectral Pixel): Axioms P6 and P7 specify a single unit: at finite resolution, a pixel is characterized by coupled spectral data $(\mathfrak{A}, \mathcal{H}, D)$. The local gauge algebra \mathfrak{A} (P6) acts on the payload space \mathcal{H} (P7) through the transport D , making gauge and matter operationally inseparable. The entropic Hessian probes orthogonal pixel deformations (Section 2) [Closure].

1.2. Screen Architecture, Resolution and Connectivity

Given Axioms P1–P7, this section defines the screen resolution scale and the effective capacity.

Location and Operational Definition

In quantum gravity, a local subsystem is not automatically defined, because the physical Hilbert space does not factorize across regions (P1) [14,30]. To define a consistent subregion algebra, we require boundary completion (P3) via edge modes [14,20] (independent of AdS/CFT). Therefore, the screen is not optional in our context; it is in fact the minimal structure required to define the very notion of interiority (interior subregion algebra). The screen is not a passive skin but an active register that must

distinguish and update states when excitations cross the diamond boundary; we treat these crossings as discrete update events (null arrivals) along the null generators.

A query–response loop between two events p and q defines the causal diamond $O(p, q)$ (P2), which is bounded by a null surface (lightlike boundary) [2]. The screen is the boundary completion on the causal diamond bifurcation surface (maximal-area waist), topologically S^2 , defined covariantly for each operational window rather than as a fixed background surface. The screen is not a fixed object in the vacuum, but a relational boundary. Its stability across experiments arises from the consistency of overlapping diamonds.

Flux Convention

The screen interface is the waist S^2 defined by diamond topology (Axiom P2). When we translate a screen response coefficient into standard gauge parameters $\alpha \equiv g^2/4\pi$, we apply the usual 4π flux normalization associated with unit-sphere interface.

Finite Resolution and Hardware Scales

Locality requires finite boundary capacity (P5). Because continuum QFT has divergent subregion entanglement and edge-mode contributions, the screen requires a regulator that makes microstates differing only below L_s indistinguishable to its algebra [21,31].

We implement this via a stretched-horizon regulator at proper distance δ from the waist [32], defining the coherency scale:

$$M_s \equiv \delta^{-1}, \quad L_s \equiv M_s^{-1}.$$

In the wedge limit, the vacuum state is KMS with respect to the boost generator (Bisognano–Wichmann) [4]. At the regulator boundary, proper time t relates to this dimensionless modular parameter τ via the redshift factor $t \approx \delta\tau$ [5]. Identifying the minimum resolvable proper time with the hardware scale ($t_{\min} \equiv \delta$) naturally forces the modular cutoff to unity (resolved modular update):

$$\varepsilon \equiv \tau_{\min} = 1.$$

Finite bandwidth implies aliasing: any continuum field description below L_s is a reconstruction, not additional physical distinguishability. This resolution limit means the continuous S^2 waist acts operationally as a finite set of distinguishable addresses (pixels) [15].

Capacity Decomposition

This cutoff forces a physical distinction between breadth and depth. We define:

Address: One distinguishable spatial location on the boundary at resolution L_s .

Breadth (N_{surf}): The number of addresses on a waist of area A , given by $N_{\text{surf}} \equiv AM_s^2$. This scales with geometry.

Depth (N): The effective multiplicity of independent response sectors per address. This is defined by exponentiating the additive entropy budget of the boundary completion ($N = e^{S_{\text{vac}}}$) (not a literal integer Hilbert dimension).

Intuitively, N counts the effective number of distinct internal states (spin, charge, edge modes) available at a specific address. The \mathbb{Z}_2 twist (P7) supplies structural overhead rather than an independent channel. We therefore count addressable payload depth as $n_{\text{ch}} \equiv N/\sqrt{2}$.

Physically, these channels act as parallel structural elements. Gravitational stiffness is extensive: like strands in a cable, many channels in parallel renormalize the aggregate response (scaling with total channel count), distinguishing the macroscopic Planck stiffness from the intensive single-channel bandwidth (M_s) [17,18].

Assuming uniform bandwidth across payload channels, the per-update energy granularity is given as $E_{\text{pix}} \equiv M_s/n_{\text{ch}}$, an intensive energy unit for single-pixel saturation.

Connectivity and Routing

To enable 3D transport, the architecture requires a transport topology (router). Following Axiom P7, minimal isotropic routing in three spatial dimensions is generated by the signed basis $\{\pm x, \pm y, \pm z\}$, closed under inversion and permutation, implying coordination $z = 6$. This is the simplest connectivity required to treat all three spatial dimensions symmetrically. Connectivity decomposes into lateral adjacency on the screen and transverse coupling to the bulk.

This discrete structure induces a geometric conflict: the router defines a step-counting metric with an L_1 -type unit ball (octahedral neighborhood), whereas the continuum isotropic limit corresponds to an L_2 ball (sphere).

Intuitively, the bulk geometry implies spherical (L_2) symmetry, while the discrete register enforces octahedral (L_1) routing. This mismatch imposes an unavoidable digitization cost. Routing is implemented in local orthonormal frames, as a global smooth tangent frame on S^2 is topologically obstructed.

The choice of coordination $z = 6$ (Axiom P7) is dictated by transport minimality. A tetrahedral lattice ($z = 4$) lacks the antipodal symmetry required to support parity-symmetric derivatives (Dirac propagation). Conversely, higher coordinations ($z \geq 12$) introduce spectral redundancies that shift the effective response coefficients, requiring additional ad-hoc projections to recover standard limits.

This mismatch is negligible at scales $\gg L_s$ (away from the tips) but becomes significant when the transverse size approaches the resolution scale, necessitating a tip anomaly correction [33].

Implication for Dynamics

With the architecture ($O(p, q)$, M_s , routing) fixed, the variational problem is well-posed: we must find the geometric reference $\sigma_O[g]$ that is statistically indistinguishable from the fixed boundary data ρ_O on the local algebra. We require a functional that (i) vanishes if the reference matches the data and (ii) is monotonic under restriction to ensure compatible descriptions on diamond overlaps (Axiom P3). The canonical measure satisfying these requirements is the relative entropy $S_{\text{rel}}(\rho_O \parallel \sigma_O[g])$ [10,34], which we identify as the coherency action in Section 2.

Because the reference family lives on this fixed algebra and regulator, the channel capacity N is not a tunable parameter but is strictly determined by the screen topology (calculated next). Furthermore, the presence of the fixed boundary regulator allows the local response to be organized into a controlled small-perturbation expansion (via the heat-kernel spectral method [35]), naturally bridging the gap between information geometry and effective field theory.

1.3. Topological Budget: Deriving the Channel Multiplicity N

With the screen architecture set, we determine the depth N by summing the additive capacity costs implied by the minimal construction (P2–P5, P7). We distinguish three independent contributions (assuming minimality): the bulk surface topology (S^2), the discretization overhead at the diamond tips and the payload twist sector.

Capacity costs are additive (entropy S), while state counts are multiplicative. We map the total vacuum entropy budget to the channel multiplicity N via integral geometry and information theory:

$$S_{\text{vac}} = \sum_i S_{i}, \quad N \equiv \exp(S_{\text{vac}}).$$

This is the standard translation from additive log-capacity bookkeeping to multiplicative state-space size. Since N represents an effective capacity rather than a literal integer eigenvalue, non-integer values are physically admissible.

Bulk Topology (S_{bulk})

The minimal boundary-completion log-capacity is an additive, diffeomorphism-invariant, scale-free functional of the cut S^2 (P2) [36]. In two dimensions, the canonical minimal choice is the Euler characteristic density (Gauss-Bonnet [37]). Fixing the normalization to minimal unit weight:

$$S_{\text{bulk}} \equiv \oint_{S^2} R^{(2)} dA = 4\pi\chi(S^2) = 8\pi.$$

This value is fixed topologically by $\chi(S^2) = 2$ via Gauss–Bonnet [37] and is metric independent.

Tip Anomaly (κ)

The tips (p, q) are geometric bottlenecks where null generators meet and the transverse cross-section approaches the resolution scale L_s (P5). When the cross-section is sub-resolution, transport is implemented by the discrete router (P7), so each modular cycle necessarily includes a finite number of routing junction events (discrete routing decisions) at the tips. We define the dimensionless tip overhead κ as the normalized count of these unavoidable junction events per modular cycle (impedance overhead). Representing smooth L_2 -isotropic modular flow by an L_1 step-routing primitive then incurs an irreducible mismatch cost [4,34].

In the local KMS regime the modular flow has period 2π (P4), and minimal 3D isotropy partitions transport into three independent antipodal axes $\{\pm x, \pm y, \pm z\}$ (P7). Normalizing one unavoidable junction event per axis per modular cycle fixes:

$$\kappa = \frac{1}{3 \times 2\pi} = \frac{1}{6\pi}.$$

The tip overhead enters additively as a bookkeeping cost for maintaining smooth modular ordering across a discrete junction; it is not a reduction of the channel count.

Twist Contribution (S_{twist})

The payload closure (P7) necessitates a spinor lift (local $SO(3) \rightarrow SU(2)$ extension) to admit spinorial boundary representations. We model this minimal closure as a \mathbb{Z}_2 twist sector with quantum dimension $d_\sigma = \sqrt{2}$ [29]. The additive overhead in the entropy budget (cost of the spinor lift) is:

$$S_{\text{twist}} = \ln d_\sigma = \ln \sqrt{2},$$

This can be seen as a half-bit cost determined by the quantum dimension. Sensitivity scales as $N \propto d_\sigma$, hence $N_{\text{eff}} = \kappa N \propto \kappa d_\sigma$. Larger non-Abelian sectors are excluded as they exceed the minimal lift required by the payload premise.

Resulting Channel Multiplicity (N)

Summing bulk, tip and twist contributions yields the total vacuum entropy:

$$S_{\text{vac}} = S_{\text{bulk}} + \kappa + S_{\text{twist}} = 8\pi + \frac{1}{6\pi} + \ln \sqrt{2}.$$

Exponentiating this sum determines the topology-locked channel multiplicity per address:

$$N = \sqrt{2} \exp\left(8\pi + \frac{1}{6\pi}\right) \approx 1.226 \times 10^{11}.$$

Thus N quantifies the effective depth per address implied by the minimal screen topology, tip overhead and spinor-lift sector. These degrees of freedom are localized to the boundary completion and need not introduce additional light propagating fields in the infrared.

Defect Proxy (Ω_{def})

We also define an aggregate routing overhead per address, Ω_{def} . Given coordination $z = 6$, the total defect budget per node is:

$$\Omega_{\text{def}} \equiv z\kappa = \frac{6}{6\pi} = \frac{1}{\pi} \approx 0.318.$$

We use Ω_{def} strictly as a compact proxy for persistent per-address routing overhead; it is not, by itself, a standalone cosmological identification.

Architectural Lock and Audit: We verify the structural rigidity of the parameters. $S_{\text{bulk}} = 8\pi$ is the minimal additive valuation on the closed waist S^2 ; a 4π choice is excluded by gluing requirements (Axioms P2–P3). Similarly, $\kappa = 1/(6\pi)$ is fixed by the 2π modular cycle and three transport axes. This constant simultaneously sets the stiffness participation $N_{\text{eff}} = \kappa N$ (Section 1) and the leakage normalization in the GKSL/Lindblad dynamics [38–40] (Section 2). Varying it would decouple these physical effects, turning a single hardware constant into two distinct knobs (κ_{stiff} vs. κ_{leak}). As a geometric consistency check, the operational value matches the normalized Regge deficit $\delta_{\text{def}}/(2\pi)^2 = 1/(6\pi)$ [33], confirming that the discrete routing correctly approximates the curvature bottleneck.

With the hardware capacity N fixed by the minimal architecture, the remaining step is to calibrate the energy scale M_s against Newton's constant.

1.4. Constitutive Relation and Calibration

We now link the discrete channel structure to the macroscopic stiffness of spacetime. We use the reduced Planck mass $M_P \equiv (8\pi G)^{-1/2}$ throughout.

Constitutive Stiffness Relation

In the Einstein–Hilbert action [41], the coefficient of curvature is the inverse gravitational constant. In the linear-response regime, the stiffness of the boundary scales with the number of effective channels [17,18]. Dimensional analysis at bandwidth M_s implies a single-channel coefficient $\sim M_s^2$. Summing over N_{eff} channels yields:

$$M_P^2 = \sum_{a=1}^{N_{\text{eff}}} M_s^2 = N_{\text{eff}} M_s^2, \quad N_{\text{eff}} \equiv \kappa N.$$

The factor κ represents the coupling efficiency over the modular cycle, limited by sub-resolution routing at the tips; it is fixed by the router normalization (not tuned to match G). This allows us to recover the Einstein–Hilbert stiffness by summing over finite channels, rather than discretizing a manifold or quantizing the graviton. This spectral reconstruction of the action shares a conceptual lineage with the spectral action principle [42] and mirrors the species-bound logic, where gravitational strength is diluted by the number of active degrees of freedom [17].

Section 2 will derive this structure from the Hessian of relative-entropic action principle.

Numerical Calibration

We calibrate the single-channel bandwidth M_s by matching this constitutive relation to the measured Newton constant G . We first work with the reduced Planck mass to match the standard normalization of the Einstein–Hilbert action:

$$M_P \equiv (8\pi G)^{-1/2} \simeq 2.435 \times 10^{18} \text{ GeV}.$$

Substituting the constitutive law $M_P^2 = N_{\text{eff}} M_s^2$ and solving for the hardware scale yields:

$$M_s = \frac{M_P}{\sqrt{N_{\text{eff}}}} \approx 3.02 \times 10^{13} \text{ GeV}.$$

M_s is the single-channel bandwidth (UV cutoff) implied by the physical resolution δ .

This scale M_s represents the physical bandwidth limit of the algebra. This derived scale aligns with intermediate scales often invoked in neutrino seesaw models [43] and plateau inflation [44], fixing the vacuum cutoff near the onset of these sectors.

Scale Inversion (L_P vs L_s)

The constitutive law implies the relation between the derived Planck length L_P and the physical hardware resolution $L_s = M_s^{-1}$:

$$L_P = \frac{L_s}{\sqrt{N_{\text{eff}}}}.$$

In this framework L_s is the physical resolution scale, while L_P is the effective stiffness scale implied by parallelization. Coarsening the vacuum geometry to the finite scale L_s removes the need to take $L_P \rightarrow 0$ as a physical resolution scale without sacrificing the macroscopic rigidity required by GR.

Consistency Check: Area Law

We verify that this calibration preserves the Bekenstein–Hawking entropy structure [45,46]. For a horizon of area A , the standard entropy (in natural units) is $S_{\text{BH}} = A/4G$. Substituting $1/G = 8\pi M_P^2$:

$$S_{\text{BH}} = 2\pi M_P^2 A = 2\pi(N_{\text{eff}} M_s^2) A = 2\pi N_{\text{surf}} N_{\text{eff}},$$

where $N_{\text{surf}} \equiv A/L_s^2$ represents the number of surface pixels; the entropy factors as surface addresses \times effective depth. Converting to bits ($S/\ln 2$) and taking $A_{\text{cosm}} \sim 4\pi H_0^{-2}$ gives $\sim 10^{122}$ bits.

This consistency check is non-trivial. Although the result follows algebraically from the constitutive lock $M_P^2 = N_{\text{eff}} M_s^2$, it carries a distinct physical implication: the standard horizon entropy is realized as surface addresses (N_{surf}) times effective depth (N_{eff}). The required information density is achieved via channel multiplicity, rather than by shrinking the physical pixel size to L_P . This recovers the Bekenstein–Hawking scaling using the finite hardware resolution L_s , without requiring the surface resolution itself to be L_P .

Separation of Scales

This calibration links the macroscopic rigidity scale M_P to the resolution scale M_s through the derived depth N . With M_s fixed, we define a per-address energy budget:

$$E_{\text{pix}} \equiv \frac{M_s}{n_{\text{ch}}}, \quad n_{\text{ch}} \equiv \frac{N}{\sqrt{2}},$$

where the $\sqrt{2}$ factor treats the twist sector as structural overhead rather than data-carrying payload, so n_{ch} counts addressable payload channels.

With the vacuum sector locked, we confront these inputs with observations. We test N and M_s against cosmological boundary conditions (Appendix A) and electroweak saturation scales (Appendix B). Using the same hardware parameters, we derive gauge couplings via intensive susceptibilities (Appendix C) and anchor lepton generations via topological accessibility (Appendix D). Throughout, N controls the extensive enhancement of gravitational stiffness, while M_s sets the intensive bandwidth (matching scale) governing gauge and mass response of the boundary algebra.

1.5. Emergent Time

In a Wheeler–DeWitt setting ($H\Psi = 0$), time is a relational label obtained by conditioning on an internal clock (Page–Wootters) [3]. For a partition $\mathcal{H}_{\text{total}} = \mathcal{H}_C \otimes \mathcal{H}_R$, the conditional state is $|\psi(t)\rangle_R = \langle t|_C|\Psi\rangle$. In the small-diamond regime, modular flow supplies a canonical local ordering of correlations (P4) [4,5]. We take the modular parameter τ as the intrinsic ordering variable (local clock). Proper time t is then an emergent coarse-grained reparameterization, valid when many modular updates are aggregated (with $t \sim \delta\tau$ at the cutoff surface).

This aligns with the modular viewpoint emphasised in noncommutative geometry, where the modular automorphism group plays the role of intrinsic evolution on an algebra [47]. Semiclassical Wheeler–DeWitt solutions admit a WKB form $\Psi[g] \approx \mathcal{A}[g]e^{iS_{\text{eff}}[g]}$. In the regime where modular flow is geometric, the phase functional S_{eff} motivates the relative-entropic coherency principle.

The key point is that the time reference is strictly local to the chosen diamond. In the screen picture, modular time is the ordering of distinguishable boundary updates (a finite information resource). Although updates are discrete at resolution M_s , coarse-graining over the deep channel bundle yields an effectively continuous time parameter at observational scales.

With the hardware configuration $(N, \kappa, M_s, \varepsilon)$ fixed, these become locked inputs for the dynamical response analysis in Section 2.

2. Coherency Action and Dynamics

With the screen architecture established in Section 1, we turn to dynamics, viewing it not as the time-evolution of a global state Ψ , but as the emergence of effective laws governing accessible correlations on a causal diamond [7,9,16]. We treat dynamics as an inference problem [48]: the reduced boundary state ρ_O is the primary data; spacetime geometry enters as the optimal reference description of that information. This perspective aligns with the thermodynamic paradigm of gravity [6,8] and recent developments in modified entropic cosmology [10–13,49]. For fixed boundary data ρ_O on a finite-resolution screen [28], we vary a semiclassical reference family $\sigma_O[\lambda]$ and select the model that minimizes relative entropy [34,49,50].

Much like hydrodynamics organizes microscopic dynamics into constitutive laws, the coherency framework packages quantum corrections into effective response coefficients at the matching scale M_s , reinterpreting them as the response of the observation layer (the lens) rather than the object [6].

Geometry is treated not as a fundamental field to quantize, but as the most economical description of boundary correlations. Because the algebra and resolution scale M_s are fixed, the variational space is locked and the response is constrained. Einstein gravity appears as a natural stiffness sector [6,51], while gauge forces and particle masses arise as intrinsic susceptibilities of the pixel algebra. The same physical scale sets the matching point for these couplings and the onset of saturation, linking the effective dynamics across sectors within a single boundary description [18,42,52].

2.1. Entropic Variational Principle

The action of the system is the informational cost of representing the boundary state. We define the coherency action as the relative entropy between the fixed physical state ρ_O and the geometric reference family $\sigma_O[\lambda]$ [34]:

$$S_{\text{coh}}[\lambda; O] := S_{\text{rel}}(\rho_O \| \sigma_O[\lambda]) \geq 0.$$

We define the reference family $\sigma_O[\lambda]$ as the manifold of maximum-entropy states on the fixed boundary-completed algebra \mathfrak{A}_O :

$$\alpha_O[\lambda] = \frac{e^{-K_O[\lambda]}}{\text{Tr}(e^{-K_O[\lambda]})}.$$

Here $K_O[\lambda]$ is a modular generator parameterized by local background fields $\lambda = (g_{\mu\nu}, A_\mu, \dots)$ coupled to the operators.

Relative entropy is a cost functional. In the KMS setting it functions as a modular free energy ($S_{\text{rel}} = \Delta\langle K \rangle - \Delta S$) and ensures overlap consistency via monotonicity; this is the Araki relative entropy of the boundary von Neumann algebra [34,50,53,54].

Imposing stationarity, the physical configuration minimizes the divergence between the reference model and the boundary correlations. Since the vacuum ρ_O is a modular equilibrium state (P4), the first law of entanglement $\delta S_{\text{ent}} = \delta\langle K \rangle$ ensures that first-order variations vanish ($\delta S_{\text{coh}}|_{\lambda_0} = 0$); excitations and restoring forces are therefore controlled by second-order variations (Hessian). Technically, this corresponds to a weak Wheeler–DeWitt stance [1]: the constraint generates local relational dynamics on finite operational windows [3].

The variational problem is locked by the axioms: the region O , the boundary-completed algebra \mathfrak{A}_O [14] and the physical cutoff $\delta = M_s^{-1}$ (with $\varepsilon = 1$) are fixed, while ρ_O is determined by the global constraint. Only the reference generator $K_O[\lambda]$ is varied within the maximum-entropy family [34,48,50]. In contrast to formulations that rely on auxiliary multipliers to enforce consistency, we embed these constraints directly into the hardware. Since the vacuum represents an equilibrium state, first-order variations vanish by stationarity. In the continuum limit, the aggregate entropic cost recovers the familiar Lagrangian density [35,41]:

$$S_{\text{coh}} \approx \int d^4x \sqrt{-g} \mathcal{L}_{\text{coh}}(x).$$

Remark on variational domain: We vary g at fixed container $(O, \mathfrak{A}_O, \delta)$ where: (i) for each g , the reference state $\sigma_O[g] = e^{-K_O[g]} / \text{Tr} e^{-K_O[g]}$ (KMS/modular regime) is defined on the same boundary-completed algebra \mathfrak{A}_O , so $\delta g_{\mu\nu}$ deforms the modular Hamiltonian $K_O[g]$ and its correlators without altering the underlying degrees of freedom; (ii) the diamond structure is fixed relationally by invariant events (p, q) rather than coordinate choice; and (iii) admissible variations are restricted to those that preserve the regulator placement at fixed proper distance $\delta = M_s^{-1}$ from the bifurcation surface.

Geometric Hessian and Kubo–Mori Response

We extract the effective laws of the screen from the second variation (Hessian) of the coherency action $S_{\text{coh}} = S_{\text{rel}}(\rho_O \| \sigma_O[\lambda])$ on the fixed cutoff algebra \mathfrak{A}_O . We organize deformations into three sectors: external metric deformations $\delta g_{\mu\nu}$ probing $\langle TT \rangle$ (stiffness/gravity), internal phase deformations δA_μ probing $\langle JJ \rangle$ (susceptibility/gauge), and scalar occupancy deformations $\delta\lambda$ probing $\langle MM \rangle$ (mass gap).

In the modular KMS vacuum σ_0 , first-order variations vanish at the locally optimal reference λ_0 . For an exponential reference family $\sigma(\lambda) \propto e^{-K(\lambda)}$, the Hessian of relative entropy equals the Kubo–Mori (quantum Fisher) metric [22,34,55]:

$$\delta S_{\text{coh}}|_{\lambda_0} = 0 \quad \implies \quad H_{IJ}(x, y) := \left. \frac{\delta^2 S_{\text{coh}}}{\delta\lambda^I(x) \delta\lambda^J(y)} \right|_{\lambda_0} \equiv G_{IJ}^{\text{KM}}(x, y).$$

Deformations couple through $\delta K = \int d^4x \sqrt{-g} \delta\lambda^I(x) \mathcal{O}_I(x)$, and the response kernel is the connected Kubo–Mori correlator:

$$G_{IJ}^{\text{KM}}(x, y) = \int_0^1 ds \langle \delta\mathcal{O}_I(x) \sigma_0^s \delta\mathcal{O}_J(y) \sigma_0^{1-s} \rangle_{\text{conn}}.$$

Because G_{IJ}^{KM} is a two-point kernel, scale separation $L_s \ll \ell_O \ll R_{\text{curv}}$ implies quasi-locality and admits a local derivative expansion [35,41,52]. In a Lorentz-invariant, charge-neutral vacuum, symmetry and Ward identities suppress mixing between operators of different spin/charge at leading derivative order, so the Hessian is block-diagonal.

Physical content of the Hessian

In the KMS regime, the Hessian is a positive response kernel: its principal deformation directions (eigenmodes) measure the associated stiffness scales (inverse susceptibilities). In the quasi-local derivative expansion, the low-momentum stiffnesses fix the effective parameters: M_P^2 in the tensor block, $1/g^2$ in the vector block and mass-gap scales in the scalar block. Positivity guarantees stability in every physical direction; null directions, when present, encode redundancies/constraints.

We apply this Hessian framework to the tensor, vector and scalar sectors in the following sections.

2.2. Spectral Expansion and EFT Coefficients

We use the spectral trace [52,55] as a compact way to write the near-equilibrium coherency cost, not as an extra postulate. In the small-diamond KMS/modular regime, the first law of entangle-

ment $\delta S_{\text{ent}} = \delta \langle K \rangle$ implies stationarity at the vacuum equilibrium, so the leading nontrivial term in $S_{\text{rel}}(\rho_O \| \sigma_O[\lambda])$ is its quadratic response: the Hessian (second functional derivative) of relative entropy. Schematically, this is given by the Kubo–Mori (quantum Fisher) form [22,34]:

$$S_{\text{coh}}[\lambda; O] \simeq \frac{1}{2} \int d^4x d^4y \delta\lambda^I(x) G_{IJ}^{\text{KM}}(x, y) \delta\lambda^J(y) + \mathcal{O}(\delta\lambda^3),$$

where $\delta\lambda$ collects the geometric/gauge/matter sources entering $K_O[\lambda]$. With $L_s \ll \ell_O \ll R_{\text{curv}}$, the Kubo–Mori kernel G^{KM} is quasi-local, so this quadratic form reduces to a local derivative expansion in curvature and field strengths with coefficients set at the fixed bandwidth M_s . We encode that same expansion with a Laplace/Dirac-type operator $D_{\mathcal{A}}^2(g, A, \dots)$ on the fixed boundary-completed structure, so that the heat-kernel coefficients of $\text{Tr} f(D_{\mathcal{A}}^2/M_s^2)$ match the local EFT series implied by the Kubo–Mori response.

To implement this encoding, we define a spectral trace using a positive operator $D_{\mathcal{A}}^2$ that packages the relevant geometric and gauge data and a smooth spectral window function f that suppresses modes above M_s [42,52,56]:

$$S_{\text{coh}} \approx \text{Tr} f\left(\frac{D_{\mathcal{A}}^2}{M_s^2}\right).$$

For smooth f , standard heat-kernel asymptotics apply [35,47,52,57]. The trace also serves as a mode-counting tool for the resolvable degrees of freedom on the internal manifold via integrated spectral density. Viewing f as a superposition of heat kernels via the Laplace transform:

$$f(x) = \int_0^\infty dt \tilde{f}(t) e^{-tx} \quad \Rightarrow \quad \text{Tr} f\left(\frac{D_{\mathcal{A}}^2}{M_s^2}\right) = \int_0^\infty dt \tilde{f}(t) \text{Tr} e^{-tD_{\mathcal{A}}^2/M_s^2}.$$

The short-time asymptotics of $\text{Tr} e^{-tD^2}$ then yield the familiar local EFT series:

$$\text{Tr} f(D_{\mathcal{A}}^2/M_s^2) \sim \int d^4x \sqrt{-g} [c_0 M_s^4 + c_2 M_s^2 R - c_F F_{\mu\nu}^2 + c_{R^2} R^2 + \dots].$$

Locality and symmetry fix the operator basis (R, F^2, R^2) . The non-universal content resides in the coefficients c_k , which depend on the detailed spectral data $(\mathcal{A}, D_{\mathcal{A}})$ and on the moments of the cutoff window f . Edge completion and finite resolution contribute additional boundary terms to these coefficients [35,57].

Gravity represents the aggregate stiffness of the entire channel bundle (sum of the tension of parallel strands); it is extensive, receiving a channel enhancement $c_2 \propto N_{\text{eff}}$. Conversely, gauge and matter terms are intensive: they encode phase/occupancy susceptibilities of the pixel algebra.

In the small-diamond/KMS regime, ρ_O is KMS with respect to the modular flow generated by K_O , so imaginary modular time is periodic with period 2π and supplies a thermal circle S^1 [4,54]. Axiom P2 identifies the spatial part as two B^3 halves glued across the waist, $B^3 \cup_{S^2} B^3 \simeq S^3$. The corresponding minimal compact manifold for the spectral/heat-kernel trace is therefore $S^3 \times S^1$ [35]. Since the screen is a glued interface (P3) and transport is discrete (P7), the heat-kernel coefficients include interface and defect terms, in addition to the smooth bulk invariants. With a physical cutoff $\delta = M_s^{-1}$ (P5) and a fixed algebra (P3), this series is a controlled expansion rather than a removable regulator artifact [18].

Quasi-locality at L_s implies that, for fixed-amplitude fields on an operational domain \mathcal{D} , the integrated cost factorizes into a geometric volume times an intensive coupling. The total capacity of the boundary-completed algebra \mathfrak{A}_O , when traced over the modular history S^1 , is the physical source of the integrated 4-volume of the operational domain.

In this view, spacetime volume counts resolved boundary updates. For a gauge sector, this implies:

$$S_{\text{gauge}}[\mathcal{D}] \sim \int_{\mathcal{D}} d^4x \sqrt{-g} \frac{1}{4g_{\text{YM}}^2} F^2 \Rightarrow \text{Vol}_4(\mathcal{D}) \cdot (1/g_{\text{YM}}^2) \cdot \langle F^2 \rangle_{\mathcal{D}}.$$

The Einstein coefficient is extensive (enhanced by the aggregate spectral trace of the channel bundle), while gauge and mass parameters are intensive response properties defined at the matching scale M_s . This correspondence explains why the 4D volume observed in GR is simply the integrated update count of a 2D boundary register. This factorization into volume times intensive coupling underlies the gauge and mass sector cost estimates derived in the following sections.

2.3. Tensor Sector: Gravity as Extensive Stiffness

This section identifies the leading low-momentum stiffness of the tensor block and matches it to the Einstein coefficient M_p^2 . In the low-curvature regime $|R| \ll M_s^2$, the reaction of the screen to small deformations is governed by linear response [6]. In the local Rindler limit, the modular Hamiltonian is generated by the local boost vector field ξ^μ (Bisognano–Wichmann/Unruh) and admits a natural decomposition into bulk and boundary contributions [4]:

$$K_O[g] \simeq K_O^{\text{bulk}}[g] + K_O^{\text{edge}}[g], \quad K_O^{\text{bulk}}[g] \simeq 2\pi \int_{\Sigma_O} d\Sigma^\mu \xi^\nu T_{\mu\nu}.$$

Physically, $K_O^{\text{edge}}[g]$ represents the diffeomorphism-invariant completion required by the boundary conditions (P1–P3) [14]. In the semiclassical regime, the variation of this edge term reproduces the geometric entropy functional of the cut (area/Wald-type) [21,45]. This identification is controlled by scale separation $\delta \ll \ell_O \ll R_{\text{curv}} \sim |R|^{-1/2}$ with $|R| \ll M_s^2$; thus K_O is effectively local in the EFT regime.

Considering perturbations about the vacuum in the local Rindler regime, we use the established result that imposing the entanglement first law, $\delta S_{\text{ent}} = \delta \langle K_O \rangle$, together with a geometric entropy functional, yields the Einstein equation at leading order [6,8,51,58]. As the first-law constraint cancels all linear variations at equilibrium, the leading dynamics are governed by the quadratic (Hessian) response around the vacuum. We apply this logic on the fixed cutoff algebra \mathfrak{A}_O .

Because the boundary entropy is strictly geometric in our construction ($N_{\text{surf}} \propto AM_s^2$) [2], thermodynamic consistency of the modular vacuum implies that the low-curvature response lies in the Einstein universality class. Identifying gravity as the aggregate rigidity of these N_{eff} parallel channels yields the constitutive relation [9,17]:

$$M_p^2 = N_{\text{eff}} M_s^2.$$

This corresponds to the extensive coefficient of the tensor sector: $\delta g_{\mu\nu}$ couples to $T^{\mu\nu}$, and diffeomorphism invariance selects R , with $M_p^2 \sim N_{\text{eff}} M_s^2$.

In the validity regime $|R| \ll M_s^2$, the resulting effective field equations take the standard form:

$$M_p^2 G_{\mu\nu} = T_{\mu\nu}^{(\psi)} + T_{\mu\nu}^{(A)} + \mathcal{O}(M_s^{-2}).$$

The $\mathcal{O}(M_s^{-2})$ remainder collects higher-derivative stiffness corrections (such as the R^2 sector), fixed by the same physical resolution scale.

2.4. Vector Sector: Gauge as Intrinsic Susceptibility

The vector response is controlled by the intensive spectral coefficient c_F , which defines the matching-scale coupling used in Appendix C.

Gauge Susceptibility

Because the leading response is quadratic (governed by the Kubo–Mori kernel [22]), we model perturbation as a weak background connection A_μ coupling to the conserved edge current J^μ required by the CS/WZW structure [20,59] (P6) through the modular generator

$$\delta K[A] = \int d^4x \sqrt{-g} A_\mu(x) J^\mu(x).$$

The corresponding quadratic coherency cost is then

$$\delta^2 S_{\text{coh}} = \frac{1}{2} \int d^4x d^4y A_\mu(x) G_{JJ}^{\mu\nu}(x, y) A_\nu(y),$$

where G_{JJ} is the Kubo–Mori kernel of the KMS vacuum (P4) on the fixed cutoff algebra \mathfrak{A}_O (P3). Ward identities enforce gauge invariance, so the leading local operator extracted from this response is $F_{\mu\nu} F^{\mu\nu}$. The matching-scale coupling $g(M_s)$ is read off as the coefficient of the canonical term $-\frac{1}{4} F^2$ in the quasi-local derivative expansion of $\delta^2 S_{\text{coh}}$. This identifies the vector-block stiffness with $1/g^2$ (or α^{-1}), quantized by integer level k .

This quadratic response defines the entropic susceptibility $\chi \equiv \partial_\lambda^2 S_{\text{rel}}$ [22,34,55]. Finite capacity fixes the resolved modular interval $\tau \in [\varepsilon, 2\pi - \varepsilon]$ (P5). In the modular/KMS regime, the edge-current correlator on the modular circle takes the universal form $\langle J(\tau) J(0) \rangle = \frac{k}{4} \sin^{-2}(\tau/2)$ (standard cylinder normalization) [4,59]. Integrating the geometric kernel $\sin^{-2}(\tau/2)$ over the resolved history $[\varepsilon, 2\pi - \varepsilon]$ yields the hardware-determined susceptibility factor:

$$I_{\text{mod}} = \int_\varepsilon^{2\pi-\varepsilon} d\tau \sin^{-2}(\tau/2) = 4 \cot(\varepsilon/2).$$

Finite resolution fixes $\varepsilon = \tau_{\text{min}} = 1$ (Section 1), so this factor is hardware-determined ($I_{\text{mod}} = 4 \cot(1/2)$), not tuned.

The overall unit of current J_μ is fixed once by matching the quadratic response to the canonical EFT normalization of $-\frac{1}{4} F^2$ at the matching scale M_s . After this single normalization choice, sector-to-sector differences enter only through k (P6) and through the charged sectors active at M_s .

In linear response, independent charge sectors contribute additively to the quadratic cost; a sector is active at M_s if it produces resolved edge-current fluctuations within the modular bandwidth set by $\varepsilon = 1$ (P5). With the synthesis of S^2 flux convention (P2), finite capacity (P5), gauge topology (P6) and linear response, the screen yields the inverse coupling:

$$\alpha^{-1}(M_s) = 4\pi k,$$

where $k \in \mathbb{Z}$ represents the effective edge level determined by P6.

This treats couplings as a property of the observation layer, not as the underlying dynamics. The screen fixes the matching value at M_s through its quantized edge current normalization. Ordinary RG running below M_s proceeds as usual.

Minimal Algebra and Current Normalization

The minimal inventory of gauge sectors is consistent with the finite algebra $\mathcal{A}_F = \mathbb{C} \oplus \mathbb{H} \oplus M_3(\mathbb{C})$ used in spectral-action formulations of the Standard Model [60,61]. In the screen picture this decomposition is read operationally (phase/orientation/mixing roles of the pixel register). For non-abelian factors, stiffness is measured by standard Lie-algebra data such as the dual Coxeter number ($h^\vee(SU(3)) = 3$, $h^\vee(SU(2)) = 2$). For the minimal gauge inventory adopted here, the commuting phase sector has rank 4; the Cartan fixed point is therefore T^4 .

We have shown that the framework naturally recovers the SM gauge inventory and identifies the non-abelian stiffness with h^\vee . Appendix C implements this identification quantitatively to obtain gauge couplings.

2.5. Scalar Sector: Matter and Mass as Scalar Response (Occupancy Hessian)

While gauge dynamics probe the operational history on the base manifold $S^3 \times S^1$ (P2, P4), fermion masses are controlled by internal accessibility (P5). We model this via a minimal compact fiber (homogeneous closure of the conformal frame bundle $\mathcal{M}_{\text{int}} \cong S^3 \times S^5$ (P5) [62,63]), where accessibility scales with the independent transport axes (P7).

We model the mass sector by coupling a scalar occupancy deformation to the gauge-invariant density operator $M = \bar{\psi}\psi$. This sector is less universal than the gauge response. In the symmetry decomposition of the Kubo–Mori Hessian H_{IJ} , the density operator $\bar{\psi}\psi$ is a Lorentz scalar, so mass resides in the scalar block through the correlator $\langle MM \rangle$.

Axiom P7 requires the transport layer to carry a \mathbb{Z}_2 grading, so the minimal payload admits spinor representations [59]. Locality then singles out Dirac-type transport as the canonical propagator [25], with a covariant derivative that couples the payload to both geometric address and gauge phase:

$$D_\mu \psi = \partial_\mu \psi + \underbrace{\frac{1}{4} \omega_\mu^{ab} \gamma_{ab} \psi}_{\text{gravity (address)}} - \underbrace{i A_\mu^a T^a \psi}_{\text{gauge (phase)}} .$$

On a bounded region, charged operators are not gauge-invariant without their edge completion (P3/P6). The minimal localized charged excitation on \mathfrak{A}_O is therefore a dressed composite. In the minimal closure we treat it as dipole-like, with pair energy $2m_f$; any additional overhead is absorbed into a factor $c_{\text{dress}} \geq 1$ (we take $c_{\text{dress}} = 1$ unless stated) [Closure].

We now encode mass as a scalar occupancy deformation of the modular generator.

Occupancy Deformation

We introduce a scalar source $\delta\lambda$ through a modular deformation:

$$\delta K_M \sim \int d^4x \sqrt{-g} \delta\lambda(x) \bar{\psi}(x)\psi(x),$$

equivalently $K_O \mapsto K_O + \int \sqrt{-g} \lambda \bar{\psi}\psi$. The Hessian then probes the density response $\langle MM \rangle$. Unlike the gauge sector (where Ward identities protect against a mass term at leading order) the scalar sector admits a relevant gap once localized defect/anchor sectors select a vacuum compatible with $m \neq 0$:

$$S_{\text{mass}} \approx - \int d^4x \sqrt{-g} m \bar{\psi}\psi \quad (+ \mathcal{O}(M_s^{-2})).$$

In this picture, matter excitations correspond to discrete topological defects of the boundary algebra [29]. Finite capacity (Axiom P5) bounds the number of such defect modes, which behave as fermionic two-state occupations in the minimal spinor-lift sector ($n \in \{0, 1\}$); this provides an operational origin of Pauli exclusion.

In static response, the scalar block fixes mass-gap scales through the inverse susceptibility χ^{-1} [22,55], which we estimate by spectral mode counting on the internal Laplacian [35,57]; Analogous to tensor and vector blocks, the mass gap represents the scalar stiffness (inverse susceptibility, $m \sim \chi^{-1}$). Because the physical cutoff renders the mode count finite, this stiffness is controlled by the accessible spectral density Ω (Appendix D).

In discrete modular time, a local mass gap is the on-site phase accumulated per resolved update; with scale set by E_{pix} , we parameterize this as $\theta_f \simeq m_f / E_{\text{pix}}$, constrained to the principal domain $|\theta_f| \leq \pi$.

Generational Topology and Dilution Principle

Finite resolution organizes the spectrum into modes with distinct effective accessible volumes Ω . If a mode explores a larger volume, its local intensity (and therefore its overlap with a localized anchor) is diluted. With normalization $|\psi|^2 \sim 1/\Omega$, the induced mass gap scales as:

$$m \propto \frac{m_{\text{anchor}}}{\Omega}.$$

Finite resolution (P5) and the modular clock (P4) imply a finite exploration budget. Near the diamond tips the geometry becomes sub-resolution, so the resolved boundary algebra behaves effectively as an open system with a coherence lifetime t_{loss} (tip leak time). Exploration proceeds within the internal gauge bundle (P6) and is characterized by a mixing time t_{mix} , set by the minimal routing primitive (P7): the modular interval required to spread a mode over the accessible phase space. Accessibility is classified by modular resolvability $\eta \equiv t_{\text{loss}}/t_{\text{mix}}$, ordering the regimes by stability.

Generations emerge as fixed points of modular dynamics, where the stability ratio η determines the accessibility class Ω and thus the mass m . This hierarchy follows a group-theoretic stratification of the internal manifold: the stable limit ($\eta \gg 1$) resolves the global topology G (Generation 1); the transient regime ($\eta \approx 1$) restricts accessibility to the tangent neighborhood of the local generator algebra \mathfrak{g} (Generation 2); and the prompt limit ($\eta \ll 1$) confines the state to the internal fiber's commuting phase torus T (Generation 3). This $G \rightarrow \mathfrak{g} \rightarrow T$ descent is a spectral filtration where accessibility becomes a constrained resource: stable modes resolve the infrared composite scale m_p , whereas prompt modes (decaying before hadronic dressing can manifest) default to the ultraviolet pixel budget E_{pix} . A 4th generation of matter is excluded as the filtration $G \rightarrow \mathfrak{g} \rightarrow T$ admits no further stable substructures.

Appendix D implements this filtration quantitatively and derives the lepton spectrum from it.

Finally, once masses are understood as occupancy stiffness in the scalar Hessian, finite resolution introduces an upper ceiling for localized excitations set by the per-address energy E_{pix} . Appendix B applies this logic quantitatively, treating the Higgs and Top Quark as near-ceiling modes.

2.6. Saturation and Bandwidth Limits

As curvature approaches the physical bandwidth scale ($R \sim M_s^2$), the screen can no longer resolve finer geometric gradients, so a nonlinear modification becomes necessary. The leading correction compatible with the bandwidth ceiling (Axiom P5) is quadratic in curvature (R^2). We take R^2 as the leading saturation correction by minimal closure: isotropic compression first saturates the scalar (trace) stiffness mode. This transitions the effective action to:

$$S_{\text{grav}} = \int d^4x \sqrt{-g} \left[\frac{M_P^2}{2} R + \lambda R^2 + \dots \right].$$

This structure places the high-curvature regime in the Starobinsky plateau universality class [44]. We adopt the minimal closure $\lambda = M_P^2/(12M_s^2)$, linking saturation directly to the hardware scale. Because the R^2 coefficient arises from the same channel bundle and spectral function as the Einstein term, its coupling is not an independent free parameter. It is locked to N_{eff} and M_s via the ratio:

$$\lambda = \frac{M_P^2}{12M_s^2}, \quad \mathcal{L}_{\text{eff}} = \frac{M_P^2}{2} \left(R + \frac{1}{6M_s^2} R^2 \right).$$

In this view, inflation is simply the elastic limit of the vacuum (onset of stiffness saturation when curvature attempts to exceed hardware resolution) [64]. This fixes the plateau targets for (n_s, A_s) in terms of M_s (Appendix A). By Axiom P5 (finite capacity), the spectrum has a fundamental ceiling (Appendix B). Conversely, identifying the cosmic horizon as the infrared bandwidth limit allows us to derive the late-time vacuum acceleration scale from the same boundary logic (Appendix D).

2.7. The Unified Coherency Action

Combining the tensor (gravity), vector (gauge) and scalar (mass) Hessian sectors with the first saturation correction (R^2), we obtain a unified coherency action. This result is valid as a derivative expansion for $|R| \lesssim M_s^2$, with the R^2 term becoming dynamically relevant as $R \rightarrow M_s^2$:

$$S_{\text{eff}} \approx \int d^4x \sqrt{-g} \left[\frac{M_P^2}{2} \left(R + \frac{1}{6M_s^2} R^2 \right) + \bar{\psi}(i\gamma^\mu D_\mu - m)\psi - \frac{1}{4g_{\text{YM}}^2} F^2 + \dots \right]$$

Here $F^2 \equiv F_{\mu\nu}^a F^{a\mu\nu}$ and m is the mass gap generated by the defect structure. The remainders are higher-dimension operators suppressed by M_s , organized by the heat-kernel expansion.

This single action unifies three distinct response behaviors of the screen:

Macroscopic Rigidity (Extensive): Gravity is stiff because it is built from a large bundle of channels ($M_P^2 \propto N_{\text{eff}}$).

Local Response (Intensive): Gauge susceptibility and occupancy stiffness are internal material properties of the pixel algebra.

Resolution Ceiling (Saturation): The system has a physical bandwidth. The R^2 term enforces the resolution scale M_s .

These regimes are not separate theories stitched together; they are the same fixed hardware evaluated at different information densities.

2.8. Dynamical Evolution and Information Flow

To propagate the state ρ_O across a sequence of nested diamonds (e.g., during evaporation) and obtain a backreacting radiation flux, we require a dynamical update rule. At finite resolution (P5), this is an open process. In the adiabatic regime, we approximate the sequence of inferential updates by a continuous evolution in modular time τ generated by a GKSL (Lindblad) equation [39,40]:

$$\frac{d\rho}{d\tau} = -i [K_{\text{mod}}(\tau), \rho] + \sum_{a,\omega} \gamma_a(\omega; \tau) \left(L_{a,\omega} \rho L_{a,\omega}^\dagger - \frac{1}{2} \{L_{a,\omega}^\dagger L_{a,\omega}, \rho\} \right)$$

Here, ρ is the reduced state on the diamond algebra [30], while the modular Hamiltonian K_{mod} generates the intrinsic modular flow (P4). The parameter τ acts as the local clock of this reduced algebra. In contrast, t denotes the coarse-grained proper time of an external observer, related by the redshift $t \simeq \delta\tau$ at the cutoff surface (Section 1) [5]. Thus, d/dt is a derivative along the emergent relational clock of the operational window, not a fundamental time evolution of the global state Ψ .

The unitary term represents the coherent modular drift; the dissipator (composed of jump operators $L_{a,\omega}$ and decay rates γ_a) encodes irreversible information leakage. We identify $L_{a,\omega}$ with the modular components of resolved excitations: edge currents (P6) and gauge-invariant dressed spinorial (dipole-like) excitations (P7/P3), localized at the tip interface. The GKSL form guarantees complete positivity and trace preservation ($\text{Tr}\rho = 1$). To preserve the local KMS equilibrium, we impose modular covariance, $e^{iK_{\text{mod}}S} L_{a,\omega} e^{-iK_{\text{mod}}S} = e^{-i\omega S} L_{a,\omega}$, and KMS detailed balance, $\gamma_a(-\omega; \tau)/\gamma_a(\omega; \tau) = e^{-2\pi\omega}$.

Finite resolution acts as a coarse-graining filter on the diamond algebra. The stable pointer algebra is the fixed-point subalgebra $\mathfrak{A}_{\text{eff}} = \text{Fix}(\mathcal{E})$. In the low-coherence limit, this structure loses ordering and becomes commuting (Cartan sector).

The dissipation strength is set by the tip anomaly (P7) $\gamma_a(\omega; \tau) = \kappa \hat{\gamma}_a(\omega/M_s; \tau)$, where $\hat{\gamma}_a$ suppresses unresolved frequencies $\omega \gtrsim M_s$. The dissipative contribution to the modular-energy flux, $d\langle K_{\text{mod}} \rangle / d\tau|_{\mathcal{D}} = \text{Tr}(K_{\text{mod}} \mathcal{D}[\rho])$, acts as the radiated power in modular time, updating the background geometry (M, A).

UV-Independence of the Macroscopic Law

In the adiabatic limit, tip-dominated dissipation implies a local power $P_{\text{loc}} \sim \kappa M_s^2 \mathcal{C}$. Applying the redshift from the cutoff ($\sqrt{g_{tt}} \approx \kappa_H / M_s$, with surface gravity κ_H) yields the macroscopic power:

$$P_\infty \approx P_{\text{loc}} (\sqrt{g_{tt}})^2 \sim \kappa \mathcal{C} M_s^2 \left(\frac{\kappa_H}{M_s} \right)^2 = \kappa \mathcal{C} \kappa_H^2 = \frac{\kappa \mathcal{C}}{16 M^2} = \frac{\mathcal{C}}{96 \pi M^2}$$

Matching this to the semi-classical Hawking formula $P_\infty = \alpha_{\text{sc}}(T_H) M^{-2}$ [46] implies the structural benchmark $\alpha_{\text{sc}}(T_H) \approx \kappa \mathcal{C}(T_H) / 16 = \mathcal{C} / 96 \pi$. Using the energy balance $P_\infty = -d(Mc^2)/dt$, we obtain the screen evaporation law:

$$\frac{dM}{dt} = -\frac{\hbar c^4}{G^2} \frac{\kappa \mathcal{C}(T_H)}{16} \frac{1}{M^2} = -\frac{\hbar c^4}{G^2} \frac{\mathcal{C}(T_H)}{96 \pi} \frac{1}{M^2}$$

The parameter $\mathcal{C}(T_H)$ represents the species-integrated emissivity at temperature T_H .

UV-Safety and Lorentz Invariance

Other discrete theories fail to hide their lattice scale in the infrared. We instead impose M_s as an operational boundary bandwidth and let modular/KMS redshift plus CPTP contractivity wash it out of macroscopic laws, leaving only dimensionless response coefficients [41]. This makes the finite-resolution screen UV-safe. Because the hardware scale M_s cancels out of the macroscopic law, the framework avoids the lattice curse common to discrete models; the discretization scale does not leave a preferred-frame imprint, thereby preserving Lorentz invariance in the infrared.

Also because M_s cancels, the leading dynamics are determined by the topological routing cost κ and the active-channel factor \mathcal{C} . In this picture, the Hawking flux is determined by how the screen leaks information at its ports, not by the details of the discretization.

Information Preservation and Page-Curve Turnover

The information paradox is reinterpreted here as a bandwidth problem. Because the algebra \mathfrak{A}_O is fixed and has finite capacity (P5), information is redistributed rather than destroyed. The apparent loss in semiclassical gravity is an artifact of $M_s \rightarrow \infty$, which assumes an infinite-density register.

To track global information, we embed the effective evolution into a moving-partition closure on a fixed total space $\mathcal{H}_{\text{tot}} = \mathcal{H}_{\text{scr}} \otimes \mathcal{H}_{\text{rad}}$. The shrinking horizon is modeled as a time-dependent reassignment of degrees of freedom from the active diamond algebra $\mathfrak{A}_{O(\tau)}$ to the radiation algebra $\mathfrak{A}_{\text{rad}}(\tau)$. The reassignment rate is tied to the decreasing pixel count $N_{\text{surf}}(\tau) = A(\tau) M_s^2$, ensuring that as the screen capacity diminishes, the state ρ_{tot} remains pure. This provides a well-posed Page-curve turnover criterion.

2.9. Relation to Continuum Approaches

Our formulation complements continuum entropic programs like Bianconi's [10,49] and related emergent gravity proposals [11,12]. We however impose a stricter structure by locking the operational hardware a priori. Interactions emerge as statistical responses of the screen, not independent inputs added to the geometry. Edge completion (P1–P3) and the physical cutoff $\delta = M_s^{-1}$ (P4–P5) anchor renormalization at a fixed scale, while the router–payload distinction (P7) makes the admissible transport part of the construction rather than an external assumption. This naturally enforces an extensive/intensive split between gravitational stiffness and local susceptibility.

Our construction also connects to spectral-action ideas in Non-Commutative Geometry (NCG), using the spectral trace to represent the mode-summed Kubo–Mori response [47,52]. Here, the cutoff is not a formal regulator Λ , but the operational bandwidth M_s fixed by the screen. As a result, the coefficients become matching-scale response properties rather than renormalization artifacts.

2.10. Recovery of Standard Limits

The effective action generates the standard equations of motion, with coefficients that are now structurally linked. For the gravitational sector, the equations include the standard higher-derivative contribution:

$$M_P^2 G_{\mu\nu} + \frac{M_P^2}{6M_S^2} H_{\mu\nu}^{(R^2)} = T_{\mu\nu}^{\text{eff}}, \quad H_{\mu\nu}^{(R^2)} = 2RR_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R^2 - 2\nabla_\mu\nabla_\nu R + 2g_{\mu\nu}\square R.$$

Einstein Regime (Extensive Linear Response): Stationarity of the coherency cost (dominated by the tensor Hessian) constrains the background geometry to the Einstein universality class [6,51]. The stiffness M_P^2 is not arbitrary but set by the aggregate channel capacity N_{eff} [17]: $M_P^2 G_{\mu\nu} \approx T_{\mu\nu}^{\text{eff}}$.

Dirac–YM Regime (Payload Transport): The spinor-lift grading (Axiom P7) mandates that matter be spinorial, while the vector and scalar Hessian blocks select Yang–Mills and Dirac kinetics as the minimal transport rules [25]:

$$(i\gamma^\mu D_\mu - m)\psi = 0, \quad \nabla_\mu F^{\mu\nu} = J^\nu.$$

These forms represent the transport logic of the boundary algebra and fix the kinematics. While the Einstein sector follows from known logic [6,16], the simultaneous recovery of minimal Dirac/Yang–Mills transport from screen constraints is, to our knowledge, new in this setting. This advance is enabled by resolving the boundary microstructure (internal pixel algebra) rather than only its thermodynamic macrostate, allowing us to treat GR (from entanglement) and Dirac (from transport) as sibling outputs of the same inference engine.

Microscopic Arrow of Time: Finite resolution enforces structural irreversibility. Near the diamond tips, the transverse cross-section becomes sub-resolution at scale δ (Axiom P5), forcing an effective split $\mathcal{A}_{\text{full}} \rightarrow \mathcal{A}_{\text{res}} \otimes \mathcal{A}_{\text{sub}}$. Tracing out sub-pixel modes ($\rho_{\text{res}} = \text{Tr}_{\text{sub}}\rho_{\text{full}}$) induces a Completely Positive Trace-Preserving (CPTP) map Φ . Since relative entropy is contractive, $S_{\text{rel}}(\Phi(\rho)\|\Phi(\sigma)) \leq S_{\text{rel}}(\rho\|\sigma)$, the coherency action is monotonic along the induced open evolution. Physically, integrating out \mathcal{A}_{sub} yields a Feynman–Vernon noise kernel ($\text{Im}(I_{\text{eff}}) > 0$) [40,65]. Most theories have to assume an arrow of time; here it is a direct consequence of finite-resolution boundaries.

3. Geometric Origin of Constants and Cosmological Dynamics

This chapter serves as a compact summary of the framework quantitative predictions. The derivations themselves are not carried out here; they are provided in the Appendix A, where each sector applies the same two ingredients established in Sections 1 and 2: a fixed screen architecture defined by the seven axioms (P1–P7) with minimal natural closures, and a single dynamical engine (identifying the physical action as relative entropy, governed by the linear response of a boundary-completed algebra in the modular/KMS regime).

The objective of this chapter is to demonstrate, in a central location, the numerical implications of these ingredients. In the proposed framework, the fundamental parameters of nature (particle masses, coupling strengths, cosmological parameters) are not arbitrary numbers that must be fitted to data. Once the channel multiplicity (N) is fixed by pure entropic geometry and the resolution (M_S) is calibrated by our sole dimensional input (G), these quantities appear as the necessary operating settings of the screen itself.

The scope of the predictions may appear unusually broad. In standard practice, these sectors are treated as distinct domains. However, we present these results here as a single Result Ledger because they are derived from the same fixed hardware. In the framework, the same pair (M_S, N) that limits ultraviolet curvature during inflation also dictates the infrared leakage of the vacuum and the pixel budget of the Higgs. To split these results into separate reports would obscure the mechanism, transforming derived necessities into a series of unexplained numerical coincidences.

To ensure transparency, the ledger (Table 1) explicitly lists the specific Axioms (P1–P7) and the minimal closures for each result. Furthermore, note that each appendix concludes with model compression audits that itemize the inputs, closures, outputs and falsifiers for the specific sector. From just one dimensional input (G) and seven natural geometric axioms, we recover 26 observational targets across sixty orders of magnitude, suggesting that the entire physical hierarchy is the output of a single, unified architecture.

Table 1. Result Ledger.

Quantity	Prediction	Observation	Deviation	Class	Derivation Basis
Appendix A: Cosmology: Capacity Saturation and Entropic Response					
Coherence \mathcal{N}_*	$18\pi \approx 56.55$	56.98	0.76%	[I,P]	P4, P5, P7 [C1, C2]
Scalar amplitude A_s	2.08×10^{-9}	2.10×10^{-9}	1.0%	[M,P]	P4, P5, P7 [C1, C2]
Spectral tilt n_s	0.9646	0.9649	0.03%	[M,P]	Derived from \mathcal{N}_*
Tensor ratio r	0.0038	< 0.036	Consistent	[B,P]	Derived from \mathcal{N}_*
Running α_{run}	-6.3×10^{-4}	-0.005 ± 0.007	Consistent	[M,P]	Derived from \mathcal{N}_*
Scaloron mass M_R	3.02×10^{13} GeV	3.00×10^{13} GeV	0.7%	[L,D]	P5 [C1]
Stiffness λ_{R^2}	5.42×10^8	5.44×10^8	0.4%	[L,D]	P5, P7 [C1]
Vacuum Ω_Λ	$4/6 \approx 0.667$	0.689 ± 0.006	3.2%	[M,P]	P3, P5, P7 [C3]
Struct. growth S_8	0.816	0.76–0.84	Consistent	[M,E]	P4, P5, P7 [C4, C5]
Acceleration floor a_0	1.04×10^{-10}	$\sim 1.2 \times 10^{-10}$	13%	[H,E]	P4
Appendix B: Electroweak Saturation and Mass Generation					
Higgs mass m_H	125.7 GeV	125.25 ± 0.17	0.36%	[M,P]	P3, P5, P6, P7 [C6]
VEV v	246.3 GeV	246.22	0.03%	[M,P]	P4, P5, P7 [C7]
Top mass m_t	174.2 GeV	172.69 ± 0.30	0.87%	[M,P]	P3, P5, P6, P7 [C8]
Top Yukawa y_t	1.00	0.99 ± 0.01	1.0%	[L,D]	Derived from m_t, v
Higgs quartic λ_H	0.130	0.126	3.2%	[L,D]	Derived from m_H, v
Appendix C: Gauge Couplings as Entropic Stiffness					
Fine-structure α_0^{-1}	137.035999216	137.035999206	10^{-11}	[M,P]	P2–P7
UV EM α_{em}^{-1}	113.1	113.4	0.2%	[I,P]	P5, P6, P7
UV Strong α_s^{-1}	37.7	38 ± 2	1.0%	[S,P]	P5, P6, P7
Coupling Ratio \mathcal{R}	3.00	3.03	1.0%	[S,D]	P6, P7
Weak mix $\sin^2 \theta_W$	0.375	0.375 ± 0.005	Consistent	[S,P]	P6, P7
Appendix D: Lepton Mass Spectrum via Spectral Filtration					
Proton mass m_p	942 MeV	938 MeV	0.4%	[M,P]	P5, P6 [C9]
Ratio m_p/m_e (μ)	1836.12	1836.15	10^{-5}	[M,P]	P5, P7 [C10]
Electron mass m_e	0.511009	0.510999	10^{-5}	[M,D]	P5, P7 [C10] [m_p]
Muon mass m_μ	104.25 MeV	105.66	1.3%	[M,P]	P5, P7 [C11] [m_p]
Tau mass m_τ	1.788 GeV	1.777	0.6%	[M,P]	P5, P7 [C12, C13] [E_{pix}]
Tau lifetime τ_τ	3.30×10^{-13}	2.90×10^{-13}	14%	[M,D]	P6 [C14]

3.1. Result Ledger (Model Predictions vs. Observations)

Class [X, Y]: Entries carry [X, Y], where X specifies the comparator class (M=Measured, I=Inferred, S=SM/RGE, B=Bound, H=Heuristic) and Y specifies the audit class (P=Primary/Direct, D=Dependent/Algebraic, E=Exploratory).

Derivation Basis: Lists the axioms (P#) and Minimality Closures (C#) used. Entries using an empirical anchor for precision are also given in square brackets.

Axioms (P1–P7): P1 (Non-factorization); P2 (Causal Diamonds); P3 (Boundary Completion); P4 (Modular Locality); P5 (Finite Capacity); P6 (Gauge Topology); P7 (Network Architecture, Routing, Payload)

Closures (minimality assumptions): [C1] No-hierarchy Saturation ($\xi = 1$); [C2] Minimal Decoherence ($c_{\text{exit}} = 1$); [C3] Minimal Leakage ($L = 1$); [C4] Distinguishability Threshold ($\Delta S \approx 1$); [C5] Dimensional Dilution ($v_{\text{eff}} = 1/4$); [C6] Minimal Structural Entropy ($\Delta S_H = 4 \ln 2$); [C7] Uniform Phase RMS; [C8] Minimal Dressing ($c_{\text{dress}} = 1$); [C9] Effective Running ($\langle b_0 \rangle$) + Cavity Geometric Factor ($3\pi/2$); [C10] Minimal Internal Manifold ($S^3 \times S^5$); [C11] Minimal Tangent Algebra ($\mathfrak{su}(2)$); [C12] Minimal Rank 4 Inventory (T^4); [C13] Signed-Axis Redundancy ($\Gamma = 8$); [C14] Standard Weak Scaling ($C_\tau \approx 5$).

3.2. Derivation Summary and Insights

The architectural foundation is established in Section 1. Rather than tuning independent sectors for inflation, forces and mass, we start from a single operational object: a causal diamond equipped with a boundary-completed algebra (edge completion) to make local subsystems well-defined [2,14,20], a finite bandwidth M_s and a channel multiplicity N . As such, we did not try to quantize the graviton; instead, we discretize the gravitational stiffness (and thus information capacity) of the boundary [6,18]. We determined the constants (N, M_s) in Section 1 using established entropic geometry to fix the topology and capacity of the screen [7,10,16]. Those two constants carry most of the load: the single-channel bandwidth M_s sets saturation and activation thresholds, while the channel multiplicity N sets collective averaging and stiffness [17]. As a result, the same architecture simultaneously controls inflationary amplitudes, coupling strengths and mass gaps.

The dynamical engine in Section 2 relies on a specific identification: we treat the physical action not as a postulate, but as a relative-entropic mismatch functional [34,58]. By identifying the Hessian of this relative entropy (Kubo–Mori metric) with the system linear response [22,50], we derive the matter sectors from geometry. Operationally, this Hessian naturally organizes into orthogonal tensor, vector and scalar blocks: tensor stiffness fixes M_p , vector stiffness fixes couplings ($1/g^2$) and scalar susceptibility fixes mass gaps. Finite resolution also makes the modular update an open-system coarse-graining [39,40], so fixed-point structure and spectral counting become physical properties rather than artifacts. The heat-kernel expansion then provides the map from these response kernels to effective parameters at the matching scale [35].

Cosmology: Capacity Saturation and Entropic Response (Appendix A)

In the ultraviolet, finite resolution makes high curvature a saturation problem: as $R \rightarrow M_s^2$, the response cannot grow indefinitely and the effective dynamics falls into the plateau universality class. In our language, the inflaton is not a new field; it is the scalar response mode of the stiffness sector as curvature approaches the bandwidth ceiling, with M_s setting the scalaron scale through the minimal saturation identification. The scalar amplitude becomes a hardware diagnostic: the curvature readout averages over N parallel channels, so the variance scales as $1/N$, turning $A_s \sim 10^{-9}$ into a channel-count statistic. The resulting plateau relations then fix $(n_s, r, \alpha_{\text{run}})$ once \mathcal{N}_* is set by a coherence budget tied to the router.

In the deep infrared, the same open-system machinery is used: leakage redshifts to an H^2 channel. This reflects the redshift-cancellation mechanism: the microscopic scale drops out, leaving only a dimensionless utilization ratio (active channels vs. ports), so the vacuum fraction reduces (in the minimal closure) to $4/6$. Late-time growth suppression and acceleration floor are treated as coherence and noise limits of the same boundary, with phenomenology closures stated transparently.

Electroweak Saturation and Mass Generation (Appendix B)

The key split is extensive versus intensive response. The Planck scale reflects collective stiffness of many channels in parallel ($M_p^2 \propto N_{\text{eff}} M_s^2$); the electroweak scale reflects the intensive single-pixel budget $E_{\text{pix}} \sim M_s/N$. Here, the Higgs plays the role of the screen's electroweak register, the minimal boundary structure needed to define and normalize the $SU(2)$ charge basis. The top mass is simply the first fermionic excitation that saturates a single-pixel unitary step (Nyquist phase domain) once dressing is imposed. This yields a compact set of matching-scale locks (m_H, v, m_t) and derived relations such as $y_t \approx 1$ and λ in the observed range, without introducing new mass parameters at the screen

level. This recasts the hierarchy problem in geometric terms: gravity is weak because it is extensive (shared by N channels), while electroweak thresholds are intensive (per pixel).

Gauge Couplings as Entropic Stiffness (Appendix C)

Gauge couplings are read as vector-sector stiffness: the quadratic coherency cost of a background connection on the fixed algebra. Two ingredients make this rigid. First, the finite modular history fixes the integration window. Concretely, the universal modular kernel integrated over the resolved history $[\varepsilon, 2\pi - \varepsilon]$ produces a fixed susceptibility factor (with $\varepsilon = 1$ set by hardware), so the result is a protocol feature of the screen rather than a regulator choice. Second, topological quantization (P6) replaces continuous normalization freedom by integer levels k . Once the continuum normalization is fixed, microscopic details collapse into integer data (levels and inventory counts), which is why coupling ratios become discrete locks rather than adjustable parameters. The same spectral heat-kernel machinery used for EFT coefficients then provides a global static estimate of $\alpha^{-1}(0)$ as a spectral sum on the fixed history manifold.

Lepton Mass Spectrum via Spectral Filtration (Appendix D)

Generations arise as stable accessibility limits under finite-coherence coarse-graining. The generation barrier is algebraic: under the open modular update, the stable fixed-point algebra abelianizes as coherence shrinks, so accessible structure reduces in the canonical sequence $G \rightarrow \mathfrak{g} \rightarrow T$. This gives a precise criterion and organizes the hierarchy as the standard group-theoretical Lie filtration realized operationally. In the global limit, accessibility reduces to a volume count on the minimal internal closure and yields the rigid $6\pi^5$ ratio; in the tangent limit it reduces to generator content; in the commuting limit it reduces to Cartan phase volume modulo router redundancy. Where an empirical anchor is used (notably m_p), it is marked explicitly and used to precision-test a dimensionless geometric ratio rather than to tune the architecture.

Unification Across Scales

The ledger is best read as a controlled change of regime with information density. At low curvature, the extensive channel bundle yields Einstein stiffness. At intermediate densities, the intensive pixel budget partitions into electroweak thresholds. Near the bandwidth ceiling, stiffness saturates and produces plateau inflation. In the deep infrared, maintenance and decoherence effects of an open boundary dominate. The same pair (M_s, N) therefore links UV saturation (inflation), mid-scale thresholds (E_{pix}) and IR maintenance (leakage) without introducing separate sector scales. The compactness of the results comes from consistency: instead of tuning the model for each problem, we commit to a single architecture and let it dictate the physics across all scales.

4. Conclusions and Outlook

This paper proposes a finite-resolution Coherency Screen as the minimal boundary ontology needed to define local physics in a diffeomorphism-invariant quantum theory. We work in a Wheeler–DeWitt setting ($\hat{H}\Psi = 0$) where the global state is constrained rather than time-evolved. In such a framework, the central operational problem is not to "quantize the graviton", but to make a local subsystem well-defined when gravitational Hilbert spaces do not factorize cleanly across subregions.

4.1. Architectural Foundation: Discretizing Capacity

The architectural foundation is established in Section 1. Rather than tuning independent sectors for inflation, forces, and mass, we start from a single operational object: a causal diamond equipped with a boundary-completed algebra to make local subsystems well-defined.

The fundamental discretization in this framework is not a lattice of spacetime points. It is a count of independent coherency channels carried by the regulated boundary degrees of freedom. In the spirit of Heisenberg on the archipelago of Helgoland, we discretize the gravitational stiffness, not coordinates, rendering the gravitational variational principle well defined.

The key master equation is the topology-locked channel multiplicity (screen depth):

$$N = \sqrt{2} \exp\left(8\pi + \frac{1}{6\pi}\right) \approx 1.226 \times 10^{11}$$

Concretely, N is the effective number of independent coherency channels (parallel strands) available per screen pixel to store and transmit the boundary edge data that keeps the vacuum geometry stable. The constitutive relation connects the macroscopic stiffness M_P to the microscopic bandwidth M_s via this channel count:

$$M_P^2 = N_{\text{eff}} M_s^2 = (\kappa N) M_s^2$$

This fixes the fundamental resolution scale of the screen to:

$$M_s = \frac{M_P}{\sqrt{\kappa N}} \approx 3.02 \times 10^{13} \text{ GeV}$$

This result encodes a clear interpretation: M_s is the single-channel bandwidth (physical resolution), while M_P is the collective stiffness enhanced by parallelization across N_{eff} channels. Gravity is extensive in channel depth; gauge response is intensive and does not scale with N . This recasts the hierarchy problem in geometric terms: gravity is weak because it is extensive (diluted over N channels), while electroweak thresholds are intensive (set by the single-pixel energy E_{pix}).

4.2. Axiomatic Basis

The framework is built from seven screen-natural axioms that formalize what is needed for local physics in constrained gravity: non-factorization, operational locality on causal diamonds, edge completion, modularity (KMS), finite capacity, gauge topology (quantized edge structure) and discrete isotropy (minimal transport). In the screen context, these are not ad hoc additions; they are the minimal operational requirements to define subsystems, clocks and regulated observables without having to assume a background structure.

4.3. Dynamical Engine and Hessian Response

The dynamical engine in Section 2 relies on a specific identification: we treat the physical action not as a postulate, but as a relative-entropic mismatch functional. By identifying the Hessian of this relative entropy (Kubo–Mori metric) with the system linear response, we derive the matter sectors directly from geometry.

Operationally, this Hessian naturally organizes into orthogonal blocks: tensor stiffness fixes M_P , vector stiffness fixes couplings ($1/g^2$) and scalar susceptibility fixes mass gaps. Furthermore, finite resolution makes the modular update an open-system coarse-graining described by GKSL (Lindblad) evolution. This ensures that fixed-point structures and spectral counting become physical properties rather than regulator artifacts. This dissipative mechanism provides a concrete leakage channel in the deep IR, with the tip anomaly setting the structural scale of dissipation utilized in Appendix A.

4.4. Cross-Sector Locking and Model Rigidity

Once Newton's constant G fixes the overall stiffness scale, the remaining outputs are determined by discrete screen structure and regulator choices.

Section 3 summarizes this in the Results Ledger: 26 observational targets spanning cosmological statistics (Appendix A), electroweak saturation (Appendix B), gauge couplings (Appendix C) and particle masses (Appendix D).

A useful way to express this rigidity is to adopt coherency units by setting $M_s = 1$. In these units, the structure is fixed by discrete screen data and the variational protocol; conventional units are recovered by the single conversion set by G . This is a highly non-trivial reduction of parametric freedom: a broad set of observables becomes a consistency web rather than independent inputs.

At low curvature, the extensive channel bundle yields Einstein stiffness. At intermediate densities, the intensive pixel budget partitions into electroweak thresholds. Near the bandwidth ceiling, stiffness saturates and produces plateau inflation.

In this view, the inflationary scalar amplitude is simply a hardware diagnostic: the curvature readout averages over N parallel channels, so the variance scales as $1/N$, turning the observed amplitude ($A_s \sim 10^{-9}$) into a simple channel-count statistic. Similarly, particle generations arise not from arbitrary replication, but because the stable fixed-point algebra abelianizes as coherence shrinks, forcing the accessible structure to descend the group-theoretical Lie filtration sequence $G \rightarrow \mathfrak{g} \rightarrow T$.

For each prediction, the ledger states the specific Axioms (P1–P7) and minimal Closures used. Furthermore, each topic appendix concludes with a Model Compression Audit to prevent parameter choices from being disguised as structural predictions.

4.5. Context and Outlook

This work builds on established entropic routes to gravity, edge-mode treatments of local algebras and spectral methods for organizing the relative-entropic action. Our contribution is to treat the boundary hardware as fixed upstream and to use a single relative-entropic variational protocol, implemented through the Hessian of second variations (Kubo–Mori response), to link tensor, vector and scalar response across scales, producing correlated outputs rather than sector-by-sector inputs.

We conclude with a final reflection: In this framework, the perceived reality is the most efficient geometric description of boundary correlations. It is not an extra structure added to quantum mechanics; it is the reference description that minimizes the coherency cost under the fixed state space and bandwidth of a finite-resolution screen. Instead of tuning the model for each problem, we commit to a unique architecture and let it dictate the physics across all scales. The resulting physics is therefore not a matter of arbitrary tuning, but a consequence of what a finite-capacity, boundary-completed quantum theory can represent consistently. In this sense, the observed universe emerges as the coherent expression of a unified operational information architecture.

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Appendix A. Cosmology: Capacity Saturation and Entropic Response

This appendix demonstrates that the same finite-resolution screen governs cosmology at both extremes of curvature, recasting standard cosmological ingredients as screen response limits fixed by (N, κ, M_s) and some minimal natural closures.

In the UV, geometric stiffness saturates the information capacity (inflationary plateau); in the deep IR, the finite horizon surface enforces maintenance costs (vacuum fraction) and coherence limits (growth suppression and acceleration floor). Inputs such as H_0 and T_{CMB} are treated as boundary conditions of our local patch, while the functional forms and dimensionless locks are traced to the fixed hardware (N, κ, M_s) and the dynamical engine of Section 2. Whenever the architecture fixes a quantity only up to an $\mathcal{O}(1)$ factor, we adopt the minimal unity normalization and mark it as a closure. Significant departures from unity would amount to adding new intermediate structure beyond the minimal screen.

Appendix A.1. Geometric Stiffness and Activation (UV Saturation)

In Section 2, we showed that the screen remains transparent in the intermediate scales ($R \ll M_s^2$), naturally reproducing General Relativity at low curvature (where routing impedance is negligible). However, finite bandwidth (P5) imposes a hard resolution limit on both geometry and EFT. As the curvature scale approaches the pixel resolution ($R \rightarrow M_s^2$), the system runs out of distinguishable microstates. The response saturates, locking the dynamics into the Starobinsky plateau universality class [44]. The effective action in the saturation regime is:

$$S_{\text{eff}} \approx \int d^4x \sqrt{-g} \left[\frac{M_P^2}{2} R + \lambda_{R^2} R^2 \right].$$

The standard plateau dictionary defines the mass scale M_R via $\lambda_{R^2} \equiv M_P^2 / (12M_R^2)$. We identify the onset of stiffness saturation with the spectral aperture limit M_s up to an order-unity factor ξ : $M_R \equiv \xi M_s$ with $\xi \sim \mathcal{O}(1)$. We adopt the minimal no-hierarchy assumption $\xi = 1$ (saturation occurs exactly at the pixel limit) [Closure]. We obtain:

$$M_R = M_s \simeq 3.02 \times 10^{13} \text{ GeV}$$

(Observed: $3.00 \times 10^{13} \text{ GeV}$ [66] (Starobinsky-consistent scale); Relative Deviation: 0.7%)

Using $M_P^2 = N_{\text{eff}} M_s^2 = \kappa N M_s^2$ with $\kappa \equiv 1/6\pi$ and channel multiplicity N , the R^2 stiffness coefficient becomes:

$$\lambda_{R^2} = \frac{M_P^2}{12M_R^2} = \frac{\kappa N}{12\xi^2} \xrightarrow{\xi=1} \frac{\kappa N}{12} \approx 5.42 \times 10^8$$

(Observed: 5.44×10^8 [66] (inferred); Relative Deviation: 0.4%)

Appendix A.2. Coherence Duration (\mathcal{N}_*)

We determine the duration of the inflationary phase by analyzing the stability of the coherent state against routing impedance. The discrete octahedral router (P7) induces an irreducible mismatch κ per modular step $\Delta\tau = 1$ (P5 resolved modular update). With two entangled tips, coherence is sustained until the accumulated mismatch reaches a critical distinguishability threshold c_{exit} along each of the $D = 3$ transport axes:

$$m_* \approx \frac{2Dc_{\text{exit}}}{\kappa}.$$

To convert to e-folds, we use the plateau relation $H \simeq M_R/2$ and $t \simeq \delta\tau$ at the cutoff surface (Section 1), so one modular step increments $\Delta\mathcal{N} = H\Delta t \approx (M_R/2)(\delta\Delta\tau) \approx \xi/2$. We set the distinguishability threshold $c_{\text{exit}} = 1$ as the minimal operational definition of decoherence [67] [Closure]. Thus:

$$\mathcal{N}_* \approx m_* \Delta\mathcal{N} \approx \frac{\xi c_{\text{exit}} D}{\kappa} = \frac{D}{\kappa} = 18\pi \approx 56.55$$

(Observed: 56.98 [66] (n_s -inferred); Relative Deviation: 0.8%)

This implies inflation ends when coherence is exhausted, not when a scalar field is tuned to roll.

Appendix A.3. Primordial Observables (A_s, n_s, r, α_s)

The primordial amplitude is a capacity diagnostic: the coarse-grained curvature readout averages over N parallel channels, so the variance scales as $1/N$. Substituting the screen parameters into the standard plateau amplitude:

$$A_s = \frac{M_R^2 \mathcal{N}_*^2}{24\pi^2 M_P^2} \approx \frac{\xi^4 c_{\text{exit}}^2 D^2}{24\pi^2 \kappa^3 N} = \frac{81\pi}{N} \simeq 2.08 \times 10^{-9}$$

(CMB-inferred: $2.10 \pm 0.03 \times 10^{-9}$ [66]; Relative Deviation: 1.0%)

With $\mathcal{N}_* \approx 56.55$, the standard plateau consistency relations yield the following spectral parameters (spectral index n_s , tensor-to-scalar ratio r , running of spectral index α_s):

$$n_s \approx 1 - \frac{2}{\mathcal{N}_*} = 1 - \frac{1}{9\pi} \approx 0.9646$$

(Observed: 0.9649 ± 0.0042 [66]; Relative Deviation: 0.03%)

$$r \approx \frac{12}{\mathcal{N}_*^2} \approx 0.0038$$

(Constraint: $r < 0.036$ [68]; Consistent)

$$\alpha_s \approx -\frac{2}{\mathcal{N}_*^2} \approx -6.3 \times 10^{-4}$$

(Constraint: $\alpha_s = -0.0045 \pm 0.0067$ [66]; Consistent)

Appendix A.4. Cosmological Constant as Entropic Horizon Leakage

In the deep IR, the screen does not saturate; instead, the horizon surface becomes informationally sparse [2]. We derive the cosmological constant not as a bulk vacuum energy summation (which diverges), but as the steady-state horizon leakage cost required to balance the open modular update (Section 2) [69,70].

For a flat universe we have $\rho_{\text{crit}} = 3M_p^2 H^2$. As derived in Section 2, the local GKSL dissipation rate [39,40] redshifts to a macroscopic power $P_\infty \propto \kappa_{\text{tip}} \mathcal{C} H^2$ (\mathcal{C} is the active-channel factor introduced in Section 2), where the lattice scale M_s cancels out. Balancing this leakage over a Hubble 4-volume implies an effective density $\rho_\Lambda \propto H^2$. Since $\rho_{\text{crit}} \propto H^2$, the vacuum fraction Ω_Λ reduces to a dimensionless coefficient fixed by router topology:

$$\Omega_\Lambda(H) \equiv \frac{\rho_\Lambda}{\rho_{\text{crit}}} = L \times \left(\frac{\mathcal{C}(T_H)}{N_{\text{cap}}} \right)$$

We adopt the minimal closure $L = 1$ [Closure], interpreting the vacuum energy as exactly the maintenance cost with no overhead. The is fixed by two architectural integers:

Topological Capacity ($N_{\text{cap}} = 6$): The six signed ports $\{\pm x, \pm y, \pm z\}$ of the P7 router set the minimal number of independent leakage channels.

Active Inventory ($\mathcal{C}_0 = 4$): In the deep IR, only massless long-range fields contribute to leakage channels: the photon (2 polarizations) and the graviton (2 polarizations), yielding $\mathcal{C}_0 = 2 + 2 = 4$.

$$\Omega_{\Lambda,0}^{\text{pred}} = \frac{4}{6} \approx 0.667$$

(Observed: 0.689 ± 0.006 [71]; Relative Deviation: 3.2%)

This result unifies geometry (ρ_{crit} factor 3), topology ($N_{\text{cap}} = 6$) and thermodynamics ($\mathcal{C}_0 = 4$) into a single structural lock. Additional massless long-range species would increase \mathcal{C}_0 and shift the predicted band upward, making this result strictly falsifiable.

Appendix A.5. IR Regime: Horizon Coherence and S_8

While the preceding results derive from the structural properties of the screen, the following analyses represent a phenomenological extension, separable from the core entropic-variational construction. As noted in the Result Ledger of Section 3, these late-time dynamic effects (S_8, a_0) are classified as [Exploratory]. Here, we model the onset of decoherence with minimal distinguishability, treating dark matter phenomenology as a coherence impedance effect. However, this extension remains strictly constrained by the same horizon architecture: the successful recovery of the cosmic acceleration scale without fine-tuning confirms that the channel multiplicity (N) remains the correct effective measure of degrees of freedom in the IR.

As the causal patch grows toward the deep IR, the discrete transport on the screen becomes susceptible to entropic drift. We define the effective clustering strength via the tensor-sector Kubo–Mori stiffness $\mathcal{K}(O_a)$ of a horizon-sized diamond O_a :

$$f(a) \equiv \frac{G_{\text{eff}}(a)}{G_N} = \frac{\mathcal{K}(O_0)}{\mathcal{K}(O_a)}.$$

The system maintains GR-like stiffness ($f \approx 1$) only while the screen transport remains coherent. We define the onset of decoherence a_* by the standard information-theoretic limit of distinguishability: the suppression activates when the accumulated relative-entropy drift over one Hubble time reaches the distinguishability threshold $\Delta S(a_*) \approx 1$ [Closure].

Beyond this threshold, the update map becomes lossy ($f(a) < 1$). This suppression satisfies relative-entropic contractivity, consistent with monotonic information loss along the RG flow. We model the dilution of participating channels by the inverse spacetime dimension ($d = 4$) available for entropic drift, yielding the minimal benchmark $\nu_{\text{eff}} = 1/d = 1/4$ [Closure]:

$$N_{\text{part}}(a) \propto \left(\frac{H(a)}{H(a_*)} \right)^{\nu_{\text{eff}}}.$$

This suppression appears as a finite-bandwidth effect in the S_8 parameter. Using the standard Λ CDM growth index approximation Ω_m^γ with $\gamma \approx 0.55$:

$$S_8^{\text{pred}} \approx S_8^{\text{Planck}} \times f(1)^{0.55} \approx 0.816$$

(Observed: 0.76–0.84 [66,72]; Consistent)

Appendix A.6. Galactic Regime: Acceleration Floor (a_0)

Axiom P4 implies a thermal horizon background $T_H \simeq \hbar H / 2\pi k_B$ [6,73,74]. An inertial acceleration signal a becomes operationally indistinguishable from the horizon noise when its Unruh temperature T_U [5] drops below T_H . The crossover $T_U \sim T_H$ fixes an acceleration floor: $a_0(z) \propto cH(z)$. Unlike static modifications of gravity, this floor is dynamic and evolves with redshift. Using the shared 2π KMS normalization:

$$a_0 = \frac{1}{2\pi} cH_0 \approx 1.04 \times 10^{-10} \text{ m s}^{-2}$$

(Observed: $\approx 1.2 \times 10^{-10} \text{ m s}^{-2}$ [75,76]; Consistent)

Appendix A.7. Model Compression Audit

Inputs (architecture): (N, κ, M_s) with $N \approx 1.2 \times 10^{11}$; $\kappa = 1/6\pi$; $M_s \approx 3 \times 10^{13} \text{ GeV}$.

Closures (minimality): No-hierarchy saturation ($\zeta = 1$); unit distinguishability ($c_{\text{exit}} = 1$); unit leakage coefficient ($L = 1$); unit distinguishability threshold ($\Delta S \approx 1$); minimal spacetime dimensional dilution ($\nu_{\text{eff}} = 1/d = 1/4$).

Outputs: Inflationary plateau ($n_s \simeq 0.965, r \simeq 0.004, \alpha_s \simeq -6 \times 10^{-4}, A_s \simeq 2.1 \times 10^{-9}, \mathcal{N}_* \simeq 56.6$); stiffness ($\lambda_{R^2} \simeq 5.4 \times 10^8$); vacuum fraction ($\Omega_{\Lambda,0} \simeq 0.67$); growth suppression ($S_8 \simeq 0.82$); acceleration floor ($a_0 \simeq 1.0 \times 10^{-10}$).

Falsifiers: Violation of plateau consistency (e.g. $r \ll 10^{-3}$); vacuum fraction significantly outside the 4/6 topological band; absence of late-time growth suppression ($S_8 \approx S_8^{\text{Planck}}$); redshift-independent acceleration floor ($a_0 \not\propto H(z)$).

Appendix B. Electroweak Saturation and Mass Generation

In this appendix, we demonstrate that the mass scales of the electroweak sector (m_H, v, m_t) are not arbitrary free parameters, but constrained by finite-resolution boundary physics: register cost (Higgs), phase-noise floor (VEV), and unitary bandwidth saturation (top quark). The relations are defined

at the screen matching scale E_{pix} . To compare our bare parameters with experimental data, we use standard SM scheme conversions [71].

Appendix B.1. Unitary Pixel Budget (E_{pix})

In the modular/KMS regime (P4), local dynamics are controlled by linear response via the Kubo–Mori metric [22]. The physical resolution $\delta = M_s^{-1}$ (P5) sets the shortest modular update $\varepsilon = 1$, fixing a finite unitary bandwidth M_s on the boundary algebra (P3).

A pixel carries an effective channel depth N (Section 1). Axiom P7 adds a \mathbb{Z}_2 spinor-lift overhead with quantum dimension $d_\sigma = \sqrt{2}$. This supplies spin structure but is not an independent payload channel; the addressable payload depth is $n_{\text{ch}} = N/\sqrt{2}$. In linear response, stiffness is additive. With no preferred channel, the unitary pixel budget E_{pix} is defined by the minimal equipartition assumption as the energy scale per resolved payload channel per modular update:

$$E_{\text{pix}} \equiv \frac{M_s}{n_{\text{ch}}} = \frac{\sqrt{2}M_s}{N} \approx 348.4 \text{ GeV}$$

Heavier excitations would require multi-pixel encoding. E_{pix} sets the single-pixel activation scale used below.

Appendix B.2. Higgs Boson: Register Cost

The Higgs field sets the scale (metric) of the internal charge space. While gauge freedom removes three Goldstone modes at long distances, the screen boundary description must represent the full $SU(2)$ doublet (four real components) to define the vacuum state and normalize gauge charges (P3/P6). We define the Higgs mass m_H as the minimal entropic cost to sustain this 4-component vacuum structure against modular noise:

Component Count ($N_H = 4$): The full real-component count of the doublet (SM inventory).

Bit Cost ($\ln 2$): The minimal nonzero distinguishability increment at the cutoff (one resolved modular step $\varepsilon = 1$ splits the Hilbert space into two sectors). This is consistent with the $\{0, 1\}$ occupancy used for defect modes (Section 2) and the $\ln \sqrt{2}$ half-bit twist overhead (Section 1).

This yields a total structural entropy $\Delta S_H = 4 \ln 2$ [Closure]. This corresponds to the minimal possible quantization of one bit per component.

Following the boundary-physics analog of Landauer’s bound [77] (where the thermal noise kT is replaced by the pixel bandwidth E_{pix}), the mass is the energy required to maintain this register:

$$m_H = \frac{E_{\text{pix}}}{\Delta S_H} = \frac{E_{\text{pix}}}{4 \ln 2} \approx 125.7 \text{ GeV}$$

(Observed: $125.25 \pm 0.17 \text{ GeV}$ [71]; Relative Deviation: 0.36%).

Appendix B.3. Vacuum Expectation Value: Noise Floor

The Vacuum Expectation Value (VEV) v represents the magnitude of the Higgs condensate (the radial order parameter). At resolution δ , the system cannot stably lock the absolute phase against an external reference (P4/P5), making the phase effectively random (maximum entropy) [48].

As a result, interference cancels the linear average, leaving the root-mean-square fluctuation as the stable observable. The natural per-update amplitude scale is E_{pix} . We assume the phase is uniformly distributed $\phi \in [0, 2\pi)$ [Closure]:

$$v^2 = \langle (E_{\text{pix}} \cos \phi)^2 \rangle_\phi = \frac{1}{2} E_{\text{pix}}^2 \quad \rightarrow \quad v = \frac{E_{\text{pix}}}{\sqrt{2}} \approx 246.3 \text{ GeV}$$

(Observed: 246.22 GeV [71]; Relative Deviation: 0.03%).

Appendix B.4. Top Quark: Bandwidth Saturation

Fermions cannot exist as isolated local observables (P6) [14]; with edge completion (P3), the minimal local excitation is a gauge-invariant dressed dipole. Because the screen resolution is discrete ($\epsilon = 1$), local modular evolution is implemented as a discrete update on a pixel algebra, represented by a unitary step operator U_{pix} . Its eigenvalues are pure phases $e^{i\theta}$, resolvable without aliasing within the principal domain $\theta \in (-\pi, \pi]$. With the pixel bandwidth E_{pix} representing the energy scale of this update, a mass gap m_f corresponds to a phase rotation $\theta \approx m_f/E_{\text{pix}}$ per step (analogous to discrete Dirac-walk angles [78]). Representability of the full dipole (pair energy $2m_f$) requires the total rotation angle to fit within the principal domain. We adopt the Nyquist saturation closure $c_{\text{dress}} = 1$ [Closure] (minimal dressing depth): $2m_f c_{\text{dress}} \lesssim E_{\text{pix}}$.

In this limit, the saturation estimate yields:

$$m_t \approx \frac{1}{2} E_{\text{pix}} \approx 174.2 \text{ GeV}$$

(Observed: $172.69 \pm 0.30 \text{ GeV}$ [71]; Relative Deviation: 0.87%).

A sub-percent dressing overhead is expected for a gauge-invariant excitation that cannot sit exactly on the aliasing boundary.

Appendix B.5. Structural Locks and Vacuum Stability

The predictions for m_t and m_H carry intrinsic uncertainties from scheme conversion and finite width ($\sim 1 \text{ GeV}$ for top, $\sim 0.2 \text{ GeV}$ for Higgs) [79]. Within these margins, our values are consistent with experiment. Physically, the vacuum does not select these masses; it simply clips any excitation that attempts to exceed the bit-depth of the screen (E_{pix}). This saturation condition fixes the dimensionless couplings (specifically the top Yukawa coupling y_t and the Higgs self-coupling λ) as geometric ratios:

$$y_t \equiv \frac{\sqrt{2} m_t}{v} \approx 1$$

(Inferred: 0.99 ± 0.01 [71]; Relative Deviation: $\sim 1\%$)

$$\lambda \equiv \frac{m_H^2}{2v^2} = \frac{1}{16(\ln 2)^2} \approx 0.130$$

(Inferred: 0.126 ± 0.001 [71]; Relative Deviation: 3.2%)

This value places the vacuum right on the edge of stability, consistent with standard RG analyses where λ runs toward a near-critical small value in the 10^{11} – 10^{13} GeV range [80] (within current parameter uncertainties). This does not discard the Standard Model; instead, it provides the missing boundary conditions for its inputs, which then evolve according to standard renormalization flows.

Appendix B.6. Model Compression Audit

Inputs (architecture): Hardware scale M_s ; channel depth N ; spinor overhead $d_\sigma = \sqrt{2}$ (P7) $\rightarrow E_{\text{pix}} = \sqrt{2} M_s / N$.

Closures (minimality): $\Delta S_H = 4 \ln 2$; uniform phase $\phi \in [0, 2\pi)$ for RMS VEV; minimal dressing depth $c_{\text{dress}} = 1$.

Outputs: $m_H \approx 125.7 \text{ GeV}$; $v \approx 246.3 \text{ GeV}$; $m_t \approx 174.2 \text{ GeV}$; derived locks $y_t \approx 1$; $\lambda \approx 0.130$ at the matching scale (vacuum metastability).

Falsifiers: Persistent violation of these relations after standard scheme conversion/RG evolution; discovery of a fermion heavier than the top; measured Higgs quartic incompatible with the predicted boundary condition.

Appendix C. Gauge Couplings as Entropic Stiffness

This appendix derives SM gauge couplings as response properties of the boundary state, not as free parameters. We define a gauge coupling by the entropic stiffness of the coherency functional: the Kubo–Mori (quantum Fisher) response of the vacuum reference state [22] to a background gauge deformation in the modular/KMS regime (Axiom P4) [4,54], evaluated on the fixed boundary algebra (Axiom P3) at finite resolution (Axiom P5) [14,21]. Topological quantization of the edge current algebra fixes an integer level k (Axiom P6) [24], while the conventional 4π factor enters through flux normalization on the diamond waist S^2 (bifurcation surface defined in Axiom P2). Gauge couplings follow from the vector (current) sector of the Kubo–Mori Hessian, while masses follow from the scalar (density) sector (Appendix D).

Appendix C.1. Entropic Stiffness Quantization

We first establish the quantization rule that turns the vector-sector response into a definite gauge coupling at the matching scale. In Section 2, we fixed the measurement protocol: the coupling is read off from the quadratic coherency cost of a weak background gauge deformation in the KMS regime matched to the canonical normalization of $-\frac{1}{4}F^2$.

Microscopically, the stiffness arises from integrating the universal modular noise kernel $\langle \mathcal{J}(\tau)\mathcal{J}(0) \rangle \propto \sin^{-2}(\tau/2)$ over the resolved modular history $[\varepsilon, 2\pi - \varepsilon]$, yielding the susceptibility factor $I_{\text{mod}} = 4 \cot(\varepsilon/2)$ (Section 2).

Using this protocol with the fixed modular window $\varepsilon = 1$ (P5) and integer edge level k (P6), we obtain the master relation:

$$\alpha^{-1}(M_s) = 4\pi k.$$

This relation acts as a dictionary between the discrete edge currents and the continuous field. The factor 4π is not a prediction of the screen tiling, but simply the standard geometric conversion factor for flux through a sphere ($\int_{S^2} d\Omega = 4\pi$). By fixing the continuum definition to the standard form $\alpha \equiv g^2/4\pi$ and the canonical normalization $-\frac{1}{4}F^2$, we force all microscopic details of the screen (such as tiling or coordination) to be absorbed into the integer level k . Thus, 4π fixes the continuum flux convention at the S^2 interface, while k is the quantized edge level (with microscopic normalization absorbed into the matching at M_s).

This holds at leading derivative order in the regime $\delta \ll \ell_O \ll R_{\text{curv}}$ and $|p| \ll M_s$. For an Abelian factor, independent charged sectors contribute additively to the quadratic cost, so the effective level k is an additive inventory over the charged sectors active at M_s .

Appendix C.2. Strong Sector Prediction ($k = 3$)

For the strong interaction with gauge group $SU(3)_c$, the problem is reduced to fixing one integer:

$$\alpha_s^{-1}(M_s) = 4\pi k_s$$

Non-Abelian gauge fields carry charge. Physically, the vacuum stiffness is dominated by self-interaction (anti-screening) [81], rather than the sum over matter charges characteristic of the Abelian sector. The edge current algebra must therefore be normalized by an intrinsic property of the $SU(3)$ current algebra itself. We identify the stiffness level k_s with the canonical integer that measures the adjoint self-interaction normalization, the dual coxeter number h^\vee ($k_s = h^\vee$) [82]. For $SU(N)$, $h^\vee = N$. For $SU(3)_c$ we have $k_s = 3$ and the matching-scale coupling follows immediately:

$$\alpha_s^{-1}(M_s) = 4\pi \cdot 3 = 12\pi \simeq 37.7$$

(SM-inferred at M_s : $\alpha_s^{-1} \simeq 38 \pm 2$ [71,83] + running; Relative Deviation: $\sim 1\%$)

Appendix C.3. Electromagnetic Prediction ($k = 9$) and Consistency Check

We now apply the stiffness quantization $\alpha^{-1}(M_s) = 4\pi k$ to the electromagnetic sector. Stiffness is driven here by matter screening ($\sum Q^2$) of all long-range charged sectors active at M_s . Summing the boundary inventory fixed in Section 1 (3 quark/lepton generations + 1 Higgs channel):

Quarks: 3 generations \times 3 colors (u, d) $\implies 9 \times [(2/3)^2 + (-1/3)^2] = 5$

Leptons: 3 generations (e, ν) $\implies 3 \times [(-1)^2 + 0^2] = 3$

Higgs: One complex charged component in the $SU(2)$ doublet $\implies 1 \times 1^2 = 1$

The total electromagnetic stiffness is the integer sum $k_{\text{em}} = 5 + 3 + 1 = 9$. Substituting this into the master formula:

$$\alpha_{\text{em}}^{-1}(M_s) = 4\pi \times 9 = 36\pi \simeq 113.1$$

We compare this geometric prediction against SM couplings extrapolated to the matching scale $M_s \simeq 3 \times 10^{13}$ GeV. Using the unbroken-basis identity with 1-loop SM values ($\alpha_1^{-1} \approx 42.6, \alpha_2^{-1} \approx 42.4$) [83]:

$$\alpha_{\text{SM}}^{-1}(M_s) = \alpha_2^{-1} + \frac{5}{3}\alpha_1^{-1} \approx 113.4$$

(Prediction: 113.1; Relative Deviation: $\sim 0.2\%$)

This convergence yields an effective mixing angle $\sin^2 \theta_W(M_s) \approx \alpha_{\text{em}}(M_s)/\alpha_2(M_s) \approx 113.1^{-1}/42.4^{-1} \approx 0.375$, recovering the canonical projection factor $3/8$ (standard hypercharge normalization) defined by the ratio $\sum T_3^2 / \sum Q^2$ over fermion generation [84].

Internal Consistency Check (Integer Lock)

Because both gauge sectors share the same 4π flux normalization, the ratio of inverse couplings depends only on the integers k ($k_s = 3$):

$$\frac{\alpha_{\text{em}}^{-1}(M_s)}{\alpha_s^{-1}(M_s)} = \frac{k_{\text{em}}}{k_s} = \frac{9}{3} = 3$$

(SM-inferred ratio at M_s : ≈ 3.03 ; Relative Deviation: 1%)

This integer lock is a clean falsifiable prediction at the matching scale. Any additional electrically charged sector active below M_s would shift k_{em} and spoil this integer relation.

Appendix C.4. Infrared Static Response: Fine-Structure Constant

We now derive the low-energy fine-structure constant $\alpha(0)$ (Thomson limit). Unlike the UV coupling at M_s (which counts local particle states via algebra), the static IR coupling measures the integrated susceptibility of the entire vacuum manifold. We define this observable rigorously using the Euclidean path integral: the static inverse coupling is the coefficient of the quadratic gauge kinetic term ($F_{\mu\nu}^2$) in the effective action [35] defined by the Laplace-type kinetic operator Δ_A on the operational history manifold ($S^3 \times S^1$), distinct from the internal fiber accessibility that governs masses (Appendix D) [35]. In this interpretation, $\alpha^{-1}(0)$ is the renormalized $\omega \rightarrow 0$ Kubo–Mori susceptibility of the boundary state, with microscopic fluctuations already packaged into the effective coefficient [22,34].

Spectral-Volume Correspondence

To evaluate this functional without explicitly performing RG running, we utilize the standard spectral representation. Just as hydrodynamics replaces microscopic collisions with effective constitutive laws, this expansion packages microscopic quantum loops into effective geometric invariants [41]. In the static limit, response coefficients are controlled by the same spectral data that determines the heat-kernel trace, so these invariants directly encode the resummed vacuum response.

Spectral Expansion and Selection Rules

We determine the static inverse coupling by summing the minimal scale-free contributions to the integrated F^2 response permitted by the heat-kernel structure on the history manifold [35]. To avoid ad hoc inventory, we distinguish:

Controlled inputs: The Kubo–Mori/Hessian response definition; heat-kernel structure for bulk/interface/defect contributions.

Architectural inputs: Minimal operational closure $S^3 \times S^1$ (P2, P4); \mathbb{Z}_2 spinor lift and $z = 6$ router (P7); fixed regulator $\varepsilon = 1$ (P5).

Selection principle: Leading contributions must be Extensive Measures (scaling with history four-volume). Finite-resolution corrections must be Monotone Penalties: decreasing with both bulk history volume (better averaging) and boundary capacity (better resolution). This excludes ratios like Vol/Cap^2 and isolates the inverse product $(\text{Vol} \cdot \text{Cap})^{-1}$ as the leading admissible correction.

In modular normalization, operator-dependent prefactors are fixed once by UV matching at M_s ; the static estimate therefore reduces to dimensionless history invariants on the fixed history manifold.

Summing the spectral coefficients under these constraints:

Bulk volume ($4\pi^3$) (controlled spectral invariant): Fundamental four-volume of $S^3 \times S^1$. Given the minimal history closure $S^3 \times S^1$ (P2, P4), this term is structurally fixed as the geometric volume of the vacuum manifold.

Twist sector ($\pi^2 + \pi$) (architectural input): The architecture of the screen (Axiom P7) imposes a \mathbb{Z}_2 routing identification. This identification induces a twisted sector in the heat-kernel trace [35], contributing the half-measures of the spatial and temporal components, $\pi^2 + \pi$, accounting for the capacity of the twisted paths. Given the \mathbb{Z}_2 spinor lift (P7), standard heat-kernel theory mandates this term as the unavoidable spectral capacity of the twisted sector.

Gluing penalty ($-1/32\pi^4$) (selection principle): By the monotone-penalty criterion, the leading scale-free interface suppression is proportional to $(\text{Vol}_{\text{bulk}} \cdot \text{Cap}_{S^2})^{-1}$, with $\text{Cap}_{S^2} = \oint_{S^2} R^{(2)} dA = 8\pi$ fixed in Section 1 (Gauss–Bonnet) [37]. Because the bipartite history requires matching the trace across the bifurcation interface (Axiom P3), we retain the minimal contribution consistent with gluing methods [85,86]: this defines the joint phase space of the glued configuration, yielding a negative correction of $-1/(4\pi^3 \cdot 8\pi) = -1/32\pi^4$. Intuitively, this term represents the spectral cost at the screen waist. Given the bipartite diamond structure (P3) and the monotone selection principle, this term is the unique admissible interface penalty determined by boundary capacity.

Defect correction ($+1/64\pi^6$) (architectural input): Finally, we account for the geometric bottlenecks at the diamond tips. Heat kernels on such Regge-type singularities acquire localized contributions proportional to the curvature deficit [33,86]. Because these six routing directions represent independent constraints, we assign one modular-phase suppression factor $(2\pi)^{-1}$ per independent routing direction; independence makes suppression multiplicative, yielding the localized spectral correction $(2\pi)^{-6} = +1/64\pi^6$. Given the discrete $z = 6$ routing architecture (P7), this term is forced by the intersection of six independent phase-space constraints, representing the tip anomaly of the diamond.

The parameter $\varepsilon = 1$ is fixed by Axiom P5 (via $\tau_{\min} = 1$ defined in Section 1) and is not tunable. Variations $\varepsilon \rightarrow 1 + \delta$ shift the regulator surface and thus the matching scale M_s at $\mathcal{O}(\delta)$, representing a hardware scheme change rather than a fit parameter.

Prediction

$$a^{-1}(0) = 4\pi^3 + \pi^2 + \pi - \frac{1}{32\pi^4} + \frac{1}{64\pi^6} \approx 137.035\,999\,216$$

(Observed: 137.035 999 206(11) [87]; Relative Deviation $\approx 7 \times 10^{-11}$ ($< 1\sigma$)).

Observation and Falsifiability

The two most precise measurements of α ([87] and [88]) currently disagree by $> 5\sigma$. Our approach discriminates between them:

LKB 2020 (Rb) [87]: 137.035 999 206(11) \rightarrow Agreement ($< 1\sigma$)

Berkeley 2018 (Cs) [88]: 137.035 999 046(27) \rightarrow Disfavored ($> 5\sigma$)

Our approach supports the LKB 2020 result [87] (which also resolves the electron $g - 2$ tension in favor of the Standard Model), also acting as a clean falsifier.

Geometric Origin of Strength

The accuracy of this result comes from rigidity, not tuning. The screen fixes the subsystem, the clock and the transport rule, so the coupling is read globally as a coarse-grained vacuum stiffness rather than perturbative loop series. This dependence is structurally rigid: because the inputs are discrete topological invariants rather than continuous parameters, the prediction admits no tuning. Any modification to the routing or charge inventory would destroy the agreement entirely.

Appendix C.5. Model Compression Audit

Inputs (architecture): Hardware scale M_s ; fixed regulator $\varepsilon = 1$ (via $\tau_{\min} = 1$).

Closures (minimality): Modular/KMS regime (P4); integer level k (P6); history $S^3 \times S^1$ (P2, P4); \mathbb{Z}_2 spinor lift and $z = 6$ router (P7); boundary capacity $\text{Cap}(S^2) = 8\pi$ (Axiom P2, Gauss–Bonnet).

Outputs: UV relation $\alpha^{-1}(M_s) = 4\pi k$; strong $\alpha_s^{-1} = 12\pi$ ($k_s = 3$); EM $\alpha_{\text{em}}^{-1} = 36\pi$ ($k_{\text{em}} = 9$); Integer lock $\alpha_{\text{em}}^{-1}/\alpha_s^{-1} = 3$; IR static prediction $\alpha^{-1}(0) \approx 137.035999216$ (spectral sum).

Falsifiers: Any new charged sector below M_s (breaks integer lock); confirmation of Berkeley (Cs) α [88] over LKB (Rb) [87].

Appendix D. Lepton Mass Spectrum via Spectral Filtration

This appendix derives the charged-lepton mass spectrum (m_e, m_μ, m_τ) from the finite-resolution screen architecture (Axioms P1–P7), with finite bandwidth (Axiom P5) providing the central cutoff, and the linear-response framework (Hessian of relative entropy) established in Section 2. Imposing a physical bandwidth implies that accessibility is a resource rather than a formal limit [15,21].

We show that the three-generation structure of matter follows from the canonical accessibility limits of the Lie-theoretic filtration of symmetry ($G \rightarrow \mathfrak{g} \rightarrow T$).

Appendix D.1. Dressed Hadronic Anchor m_p

The hardware scale M_s sets the maximum frequency. The observable unit of mass for stable matter is the proton (m_p) [89]. We must show that the hardware naturally generates this unit through confinement physics [81].

The confinement scale Λ_{QCD} is generated by running the strong coupling from M_s down to the pole, driven by the coupling $\alpha_s^{-1}(M_s) = 12\pi$ (Appendix C) [41]. Using a 1-loop beta function ($\langle b_0 \rangle \approx 7.25$ (effective running)):

$$\Lambda_{\text{QCD}} \approx M_s \cdot \exp\left[-\frac{2\pi(12\pi)}{\langle b_0 \rangle}\right] \approx 200 \text{ MeV}$$

We model the proton as the ground-state of a confinement cavity $R \sim \Lambda_{\text{QCD}}^{-1}$, so m_p is set by the lowest semiclassical orbital mode [90] [Closure]:

$$m_p^{\text{mech}} \approx \frac{3\pi}{2} \Lambda_{\text{QCD}} \approx 942 \text{ MeV}$$

(Observed: 938 MeV [71]; Relative Deviation $\sim 0.4\%$)

This establishes m_p as a natural dressed anchor generated by UV data, up to standard running uncertainties. For the precision lepton ratios below, we use the empirical m_p as the precise reference for the geometric anchor.

Appendix D.2. Mass as Spectral Susceptibility

Section 2 defines local dynamics via the Hessian of relative entropy. In the linear-response limit (Kubo-Mori formalism [22]), the static susceptibility χ of the vacuum is proportional to the integrated spectral density [91] of the Laplacian Δ_{int} [35,57]:

$$\chi \sim \sum_n \frac{|\langle 0|\mathcal{O}|n\rangle|^2}{E_n} \implies m \propto \chi^{-1} \propto \frac{m_{\text{ref}}}{\Omega}.$$

Here, Ω represents the number of resolvable modes on the manifold \mathcal{M}_{int} (via Weyl's law [91], linking continuous volume to discrete bandwidth). Larger accessible volume lowers the gap; restricted accessibility raises it.

The finite-resolution screen implies a loss of fine-grained information (coarse-graining) (Section 2). Let \mathcal{E}_t denote the resulting symmetry-respecting coarse-graining semigroup [39] on the observables and let $\mathfrak{A}_{\text{eff}} = \text{Fix}(\mathcal{E})$ be its stable fixed-point algebra.

Let $\eta \equiv t_{\text{coh}}/t_{\text{mix}}$ order the regime. Operationally, t_{mix} is set (up to order-unity factors) by Δ_{int}^{-1} , the inverse spectral gap of the relevant mixing generator. These regimes are distinguished by their accessibility class in the open modular setting, where mass and coherence time are co-determined properties of the eigenstructure, quantified via the Kubo–Mori susceptibility [22,55]. As η decreases, the fixed-point algebra $\mathfrak{A}_{\text{eff}}$ abelianizes under the coarse-graining. Non-commuting information is lost first, leaving a commuting pointer algebra [92].

Any compact Lie group G possesses a unique canonical hierarchy that decoheres in three stages: (1) global topology (manifold G), (2) local non-abelian curvature (algebra \mathfrak{g}) and (3) a maximal commuting skeleton (Cartan torus T). The 3-sphere S^3 is isomorphic to the Lie group $SU(2)$. Once we have $SU(2)$ (via P7), modular flow (P4) and finite bandwidth (P5), this filtration is the only way the structure can decay.

Intuitively, imagine viewing a spinning sphere ($SU(2)$) with worsening focus (η). At high res ($\eta \gg 1$), the topology is resolved ($G \rightarrow \text{Electron}$). At medium res ($\eta \sim 1$), the shape blurs, leaving only local tangent directions ($\mathfrak{g} \rightarrow \text{Muon}$). Finally, at low res ($\eta \ll 1$), non-abelian motion averages out, leaving only static, commuting charges ($T \rightarrow \text{Tau}$).

For simple Lie groups like $SU(2)$, this filtration is structurally exhaustive, as no other natural subgroups exist between these levels. We are simply observing the canonical decomposition series of the geometry (P5), filtered by modular flow (P4). Under coarse-graining, these layers peel away in order of complexity:

Global Regime ($\eta \gg 1$): Non-abelian observable content remains accessible (G).

Tangent Regime ($\eta \sim 1$): Only local generator data remains coherent (\mathfrak{g}).

Commuting Regime ($\eta \ll 1$): The algebra reduces to a Maximal Abelian Subalgebra (MASA) (Cartan phase sector T , geometric fiber).

Standard field theory assumes all particles see the same vacuum geometry; the screen framework recognizes that short-lived excitations are subject to a holonomy barrier that precludes the resolution of global topology prior to decay.

The three-generation structure thus follows from the canonical accessibility limits of a Lie group: the standard Lie-theoretic filtration of symmetry ($G \rightarrow \mathfrak{g} \rightarrow T$) mapped onto finite coherence windows. This hierarchy determines exactly which subalgebras survive as stable limits under coarse-graining.

Coarse-graining induces a nested reduction of accessible algebras as η decreases (non-abelian \rightarrow local generators \rightarrow commuting phases). We use the asymptotic limits $\eta \gg 1$ and $\eta \ll 1$ for counting; the intermediate regime is treated as a controlled proxy.

We write $m_{\text{ref}} = E_{\text{pix}} R_{\text{IR}}$. Here R_{IR} is determined by whether the gauge-invariant dressing channel is available on $\mathfrak{A}_{\text{eff}}$ (P3/P6). In the global/tangent regimes ($\eta \gtrsim 1$), dressing is available ($R_{\text{IR}} = m_p/E_{\text{pix}}$). In the commuting regime ($\eta \ll 1$), non-abelian dressing is obstructed ($R_{\text{IR}} = 1$).

Appendix D.3. Lepton Cascade

We apply the filtration to $\mathcal{M}_{\text{int}} \cong S^3 \times S^5$ (P5) and the P7 router. The logical chain is natural:

$$\text{3D Space (P7)} \xrightarrow{\text{Rotation}} SO(3) \xrightarrow{\text{Quantum}} SU(2) \xrightarrow{\text{Topology}} S^3 \text{ (P5)}.$$

Since $S^3 \cong SU(2)$ (the group of unit quaternions), this manifold naturally carries the Lie structure required by the isotropic router (P7), making the filtration $G \rightarrow \mathfrak{g} \rightarrow T$ strictly applicable.

Define the spectral volume by $\Omega(\eta) = \text{Tr}(\Pi_{\text{axes}} \otimes \Pi_{\text{int}})$. The same accessibility Ω underlies all three generations; in the three limits it reduces to a volume count (global), a generator count (tangent) and a Cartan phase-volume count (commuting).

Generation 1: Electron (Global Limit)

Regime: Stable ($\eta \rightarrow \infty$). Resolves full global symmetry G .

P7 fixes $\text{Tr}(\Pi_{\text{axes}}) = 3$ (isotropic transport). We adopt the minimal internal closure $\mathcal{M}_{\text{int}} \cong S^3 \times S^5$ [Closure], which yields $\text{Vol}(S^3 \times S^5) = 2\pi^5$. The leading spectral term depends on the volume. Under these axioms, the accessibility is structurally fixed:

$$\Omega_e = 3 \cdot 2\pi^5 = 6\pi^5, \quad m_e = \frac{m_p}{6\pi^5} \approx 0.511009 \text{ MeV} \quad (m_p/m_e = 6\pi^5 \approx 1836.12)$$

(Observed: 0.510999 MeV, $m_p/m_e \approx 1836.15$ [71]; Relative Deviation $\approx 10^{-5}$)

Generation 2: Muon (Tangent Limit)

Regime: Transient ($\eta \sim 1$). Resolves local algebra \mathfrak{g} .

Accessibility reduces to local generator content. Operationally, this regime resolves generator directions but not global identifications. The minimal compact algebra controlling the local frame is $\mathfrak{so}(3) \cong \mathfrak{su}(2)$ [82] [Closure]. Thus:

$$\Omega_\mu = 3 \times \dim(\mathfrak{su}(2)) = 9, \quad m_\mu = \frac{m_p}{9} \approx 104.25 \text{ MeV}$$

(Observed: 105.66 MeV [71]; Relative Deviation $\sim 1.3\%$)

A percent-level deviation can be expected here, as Ω_μ is a tangent-regime proxy.

Generation 3: Tau (Commuting / Cartan Fiber)

Regime: Prompt ($\eta \ll 1$). Fixed-point algebra abelianizes to T .

The accessible phase volume is the Cartan torus T^4 (rank 4, fixed by the minimal gauge inventory established in Section 2 and Appendix C). The router symmetry $\Gamma \cong (\mathbb{Z}_2)^3$ (Axiom P7) acts as a port-label redundancy on the commuting phases, so $|\Gamma| = 8$ (axis permutations correspond to physical rotations and are not quotiented):

$$\Omega_\tau = \frac{\text{Vol}(T^4)}{|\Gamma|} = \frac{(2\pi)^4}{8} = 2\pi^4, \quad m_\tau = \frac{E_{\text{pix}}}{2\pi^4} \approx 1.788 \text{ GeV}$$

(Observed: 1.777 GeV [71]; Relative Deviation $\sim 0.6\%$)

In this commuting regime, the excitation is pixel-bound, so E_{pix} sets the anchor. The mass estimate is a static (susceptibility/accessibility) statement; the lifetime check below is a separate dynamical consistency test.

Appendix D.4. Lifetime Check

The prompt regime implies a short weak lifetime. Using scaling $\tau \propto m^{-5}$ with channel factor $C_\tau \approx 5$ (counting lepton and color doublets):

$$t_\tau \approx t_\mu \left(\frac{m_\mu}{m_\tau} \right)^5 \frac{1}{C_\tau} \approx 3.3 \times 10^{-13} \text{ s}$$

(Observed: $2.9 \times 10^{-13} \text{ s}$ [71])

This supports the assignment of the Tau to the commuting ($\eta \ll 1$) regime.

Appendix D.5. Model Compression Audit

Inputs (architecture): $M_s, E_{\text{pix}}; m_p$ (derived from M_s via confinement, then used as geometric anchor only).

Closures (minimality): Cavity resonance $3\pi/2$ for proton mass (lowest semiclassical orbital mode); $\mathcal{M}_{\text{int}} \cong S^3 \times S^5$ (minimal internal closure, P5); $\mathfrak{su}(2)$ tangent algebra (minimal compact generator); T^4 (minimal rank 4 gauge inventory, Section 2); $\Gamma \cong (\mathbb{Z}_2)^3$ (signed-axis redundancy, P7 corollary).

Outputs: $m_e; m_\mu; m_\tau; t_\tau$.

Falsifiers: Failure of geometric integers ($6\pi^5, 9, 2\pi^4$); discovery of a 4th charged lepton generation.

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