Article

Analysis of Leonardo da Vinci's architectures through parametric modeling: a method for the digital reconstruction of the centrally planned churches depicted in Ms. B

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Abstract: Among many other themes, Leonardo da Vinci's Manuscript B contains several drawings of centrally planned churches, some of which are represented using a plan view paired with a bird's-eye view. The use of a bird's-eye instead of an elevation represents an innovative depiction technique, which allows to combine the immediacy of the perspective view with the measurability of the façades, and therefore to describe the three-dimensionality of the buildings. To understand the reasons behind the use of this original technique, the edifices' shape and classify them we decided to use 3D digital reconstruction techniques, for their ability to avoid misunderstandings in the reconstruction process and in the results. This article describes the method to create the digital models of sixteen churches. A Visual programming language (VPL) script was used as 3D base for modelling the churches achieved from a classification code expressing the aggregative rules of the churches. Then, the geometric process for the construction of the plan and its relationship with the elevation measures was studied for each church. Finally, this information was used for the completion of the 3D models, distinguishing more output variants each time there was an inconsistency between plan and perspective view, a variability of one architectural element or an uncertainty.

Keywords: digitization; 3D modeling; drawings; Renaissance; Leonardo da Vinci; Manuscript B; parametric.

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1. Introduction

Manuscript B was written by Leonardo da Vinci between 1487 and 1490 [1], during his Milanese period. It is preserved in the library of the *Institut de France* as a part of the Paris Manuscripts and contains drawings of several subjects, from flying and war machines to centrally planned churches. The architectural drawings contained in the manuscript are very dissimilar to other coeval architects' project drawings and differ from the drawings produced by architects illustrating codices and treatises [2]. In fact, among the centrally planned churches depicted in the manuscript sixteen buildings were represented by Leonardo using an original technique, a pairing of a plan and a perspective view, instead of the traditional technique made of a plan coupled with an elevation [3]. Moreover, these churches are characterized by some elements so far not well investigated due to the very small size of the drawings and to the inability of traditional analysis techniques (graphic, calligraphic, historical, …) to give an accurate explanation of their aim, of the representation technique used and of subject's depicted features.

This paper aims to investigate the advantages that 3D parametric modeling can offer to the analysis of these small drawings and to define an objective method for conducting this process. The development of a tailored technique for the analysis of these edifices through their hypothetical

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reconstruction is particularly useful since it could also be extended to other Renaissance case studies. In fact, the issues that one must consider when dealing with the reconstructive process of the centrally planned churches of Ms. B are a superset of the problems that one usually faces in any similar process dealing with unbuilt architecture documented just by drawings.

The study is part of a fruitful line of research which, starting from the 80s of the last century, has addressed the issue of formulating reconstructive hypotheses, both in the architectural and archaeological fields, of disappeared or destroyed buildings, sites and cities or only designed and therefore never built. The archaeologist John D. Wilcock recognized the great potential of the computer-generated reconstructions of historical monuments for use in his field of research back in 1973 [4]. One of the first scientific digital reconstructions of a historical building comes from archeology, when between 1985 and 1989 Mathrafal, an early medieval castle complex in Wales, Great Britain destroyed in the 13th century, was digitally reconstructed [5]. A milestone in the history of the digital reconstruction of historical architecture with art-historical objects is the project on Cluny III realized by Manfred Koob [6]. The result was a four-minute film that captured both the exterior and the interior of the church. At the end of the last century, within a wide range of initiatives, we saw the institution of Cultural Virtual Reality Laboratory (CVRLab) at the University of California in Los Angeles (UCLA), founded by Bernard Frischer and Diane Favro, which lead for example to the production of the extensive project of Rome Reborn [7,8]. The use of information technologies showed how they can offer new possibilities for investigation, not only as they allow the modeling of buildings in their entirety and the simulation of environmental or dynamic conditions (i.e. light, materials etc.), but above all because they allow researchers to investigate the formal and compositional rules that characterize some architectures. Exemplary in this context is the fruitful legacy generated by Stiny and Mitchel's studies on the grammar of form in the case of the Palladian villas [9], which gave rise to the strand of the reconstruction as *instauratio* suggested by Howard Burns during the years 1990 [10], and to subsequently arrive to the semantic modeling developed from the end of the last century [11,12].

Besides, the development of increasingly refined and sophisticated parametric and procedural modeling systems has given rise, within this broad riverbed, a specific field of research which, by exploiting the possibility made available by these design and modeling tools, tries to give responses to the many points of interest that characterize this theme: e.g., the possibility of exploring multiple alternative solutions as well as allowing the formal study of classical architecture [13,14], defining a method for reconstruction in the form of a three-dimensional analysis and construction rules [15,16,17], assuring transparency of analytical methods and criteria used throughout the reconstructive process, showing related level of uncertainty obtained, etc. [18].

Therefore, many reasons, that will be now pointed out in detail, make the subject ideal to be studied and classified through a 3D reconstruction. First, the churches depicted in the drawings were never realized, so, it is particularly useful to translate them in a 3D form to make them more immediate to understand. In fact, the digital reconstruction of unbuilt architecture constitutes a powerful instrument for the analytical process since it increases our ability to understand the features of the drawings and makes us able to notice more elements [19]. Three-dimensional models therefore assume the value of a "research method, transfer of knowledge and new forms of memory" [20], showing a great potential thanks to a profound interpretation of the sources and an extensive understanding of the object obtained creating a hypothetical reproduction [21]. On the other hand, the field of hypothetical reconstruction of unbuilt architecture, which was explored through several case studies [20, 22, 23] has some peculiarities, for instance a possible lack of information for some parts and a varying degree of accuracy in the sources, that should be addressed when trying to define a clear digitization methodology [20].

In this case, the 3D reconstruction is possible thanks to the depiction technique used by Leonardo, which uses a plan view paired with a perspective view. The latter is a bird’s-eye view that almost resembles a cavalier perspective, so that, if we take a cube as a reference, we have two faces that are parallel to the picture frame and that are consequently in true form [24]. In this way, Leonardo takes advantage of the capacity of axonometric representation to give immediate information about the

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volumetric layout and it combines it with the plan view to specify the interior distribution of space. This enables us to measure the plan and the façade of the edifice and to also know the disposition of volumes.

These two elements give us enough information regarding the exterior of the buildings but require us to make hypotheses regarding the interiors. In fact, this representational approach entails a lack of information regarding the interior development of the elements in height. These uncertainties imply the need of making multiple assumptions about the interior spaces of the churches. Thus, to maintain a rigorous approach, it is necessary to use coeval or related architectural references as examples. This necessity derives also from another important characteristic of the drawings: they are extremely limited in their dimensions, since they are contained in folios that measure approximately 16 cm x 23 cm, so they do not contain any architectural detail. Moreover, the churches' drawings are not accompanied by any information regarding their dimension and scale. The production of multiple possible models from fragmentary pieces of information constitutes a core topic in digital heritage research [21], making this case study particularly useful to understand the casuistry of sources that need to be considered.

Another feature emerging when considering the whole corpus of drawings is that a set of recurring compositive rules is clearly visible in it. In fact, the churches' layouts have several common elements and the corpus appears to be an exploration of all the aggregative possibilities of a given form around a central space, or, as it was defined by Francesco Paolo di Teodoro, a "geometric play" [25]. Moreover, the plans appear to be a variation on the theme of the octagon and octagonal stars [26]. This opens to the possibility of creating a complete classification of the layouts, which was tried in the past by two contributions that will be briefly presented [27,28] but, most importantly, this makes the architectures particularly suitable for a parametric modeling.

In order to use a parametric modeling method based upon Leonardo's rules and not upon the authors' subjective rules (a typical error in the historical reconstruction using 3D techniques) the digital reconstruction starts from an analysis of the aggregative features of the complete set of drawings, aimed at extrapolating all the recurrences. Then, the extracted rules were used to define a script potentially able to automatically model all the possible combinations of shapes following those rules, and therefor also the churches of Manuscript B.

The result of this procedure involves two different levels. First, as will be presented in Section 2, the parametric modeling becomes a means for extracting and reasoning on the geometrical variations and aggregative rules behind the whole *corpus*. A similar approach, aimed at studying the possible parametric variations of Palladio's Villa *La Rotonda*, was carried out in the past [13]. The aggregative study, other than a goal, is also useful to take advantage of all the recurrences between the churches in the 3D modeling process. Starting from it, we will define a system for the classification of the churches, based on the combination of volumetric elements around a central, octagonal space. The churches' layouts, in fact, share a common taste for the combination of elements, almost like a geometric play. This makes it possible to use the results of the aggregative study as the theoretical basis for the definition of a script for the first stage in the three-dimensional modeling. The nature of the layouts, in fact, is ideal for a parametric and procedural study, that could potentially resemble all the churches starting from the same rules. This step was achieved using Grasshopper, a visual programming language for Rhino, and customizing some components through GhPython.

Then in Section 3, the churches are considered one by one: first, the drawings will be analyzed in their proportions, deconstructing the plan view in its components to define a possibility for the geometric process behind its creation. A relation between the geometric elements in plan and elevation will be searched in the related bird's-eye views. Then, this information will be used to set specific values of height and width in the 3D parametric model. In many cases, the presence of inconsistencies between plan and perspective views will require us to make multiple hypotheses. Then, the 3D parametric model becomes the means to explicit the multiple possibilities for the drawings' interpretation and the inconsistencies, that the digital representation process well clarifies and shows. As it was anticipated, when dealing with the churches one by one, the absence of an interior view, the small dimension of the drawings, and the absence of a dimensional scale require us

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to make also hypotheses about certain elements for completing the digitization. Therefore, we previously conducted a thorough analysis of the historical context of the manuscript to identify the architectural examples that could be used as references for the assumptions that have to be made for the architectural details, for the interiors and for the building's dimension.

Finally, the obtained results are described and discussed.

2. Analysis and parametric modeling of the corpus of drawings as a whole

In this section the first part of the method, the one that considers the whole corpus of drawings, is described in its phases.

First, the aggregative rules behind the churches' designs are extrapolated, considering the attempts made in the past in this direction. Those aggregative rules will be used to classify the churches' layouts. In order to do so, every feature is catalogued as a variable which is defined by a certain value so that the design of every church can be defined by the entirety of them and vice versa. In fact, the result of this process will be a classification code made up by all the values characterizing the variables.

Second, this identification code is used as a base for the creation of a Grasshopper script that embeds the possibilities of each variable. This script is able to create basic 3D models for all the churches through procedural modelling, since the values of the variables involve a precise set of geometric operations.

2.1. A proposal for a new classification method based on an identification code

The churches of Ms. B are based on a common set of aggregative and dimensional rules. However, defining those rules uniquely is not easy, as the classification of the elements of the buildings can be done according to different points of view. In fact, different authors proposed different classifications based on their idea about the final purpose of Leonardo's drawings in Ms. B and the kind of approach - planimetric or volumetric - they carry out in the analysis of drawings.

Among the various studies that considered the churches of Ms. B, e.g. the ones made by Ludwig H. Heydenreich [29], Luigi Firpo [30] and Carlo Pedretti [31], which are certainly useful to analyze Leonardo's *ouvre*, we focus here on the works developed by Jean Paul Richter [27] and Jean Guillaume [28], who analyzed the churches through their classification, collecting in a schematic way their features. These classification methods are particularly useful to understand what are the core elements that define the churches layouts, and then become the starting point for the development of a new proposal.

The classification made by Richter [27] is based on the theory that the drawings of churches were aimed at the realization of a treatise on domes and are not referred to the construction of any building. For this reason, his study is focused on the planimetric classification of the connection between domes and the remaining part of the construction, rather than the aggregative rules behind the composition of the churches (Figure 1). In his work J. P. Richter does not pair plan and perspective views, considering them as separate designs. The churches inserted in the various groups then are taken both from plan and perspective views, so that, in case of inconsistency between them two drawings referred to the same building could be classified in different groups. Moreover, in this study the author considers the sacred edifices drawn by Leonardo in his entire production, and not only the ones depicted in Ms. B. Some of those churches are not centrally planned, so the first division he operates is between churches "formed on the plan of a Greek cross" and churches coming from longitudinal plan schemes. According to our aims, only the first macro-group was considered and analyzed, as can be seen in Figure 1, since it is the only one that involves the churches depicted in Ms.B.

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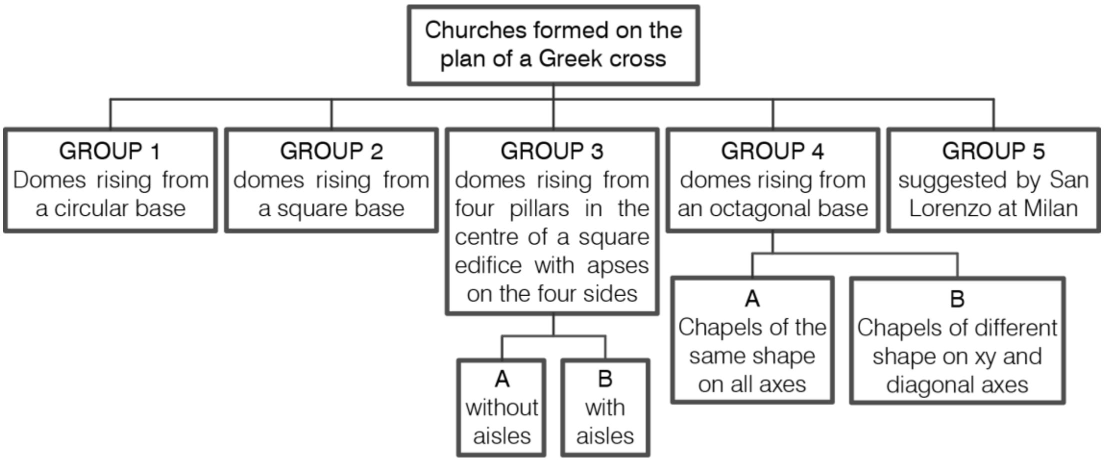

Figure 1. This scheme sums up the classification proposed by J. P. Richter for the first macro-group (centrally planned). The "churches formed on the plan of a Greek cross" are then divided in five groups. The first four groups depend on the shape on which the dome rises, while the fifth does not follow the same logic, merging the churches that show affinities with San Lorenzo at Milan. Group 3 and four are further divided into two subdivisions according to the features of the peripheral elements.

The second classification considered was carried out by Jean Guillaume in 1987 [28], in the occasion of the exhibition that took place in the Musée des Beaux-arts of Montreal about Leonardo architect and engineer. In this case, the point of view of the author is different and focuses more about the elements and their aggregations around the central space. Moreover, Guillaume tries, after an analysis in plan (Figure 2), to carry out a volumetric approach that considers the information contained in the perspective views drawn by Leonardo (Figure 3).

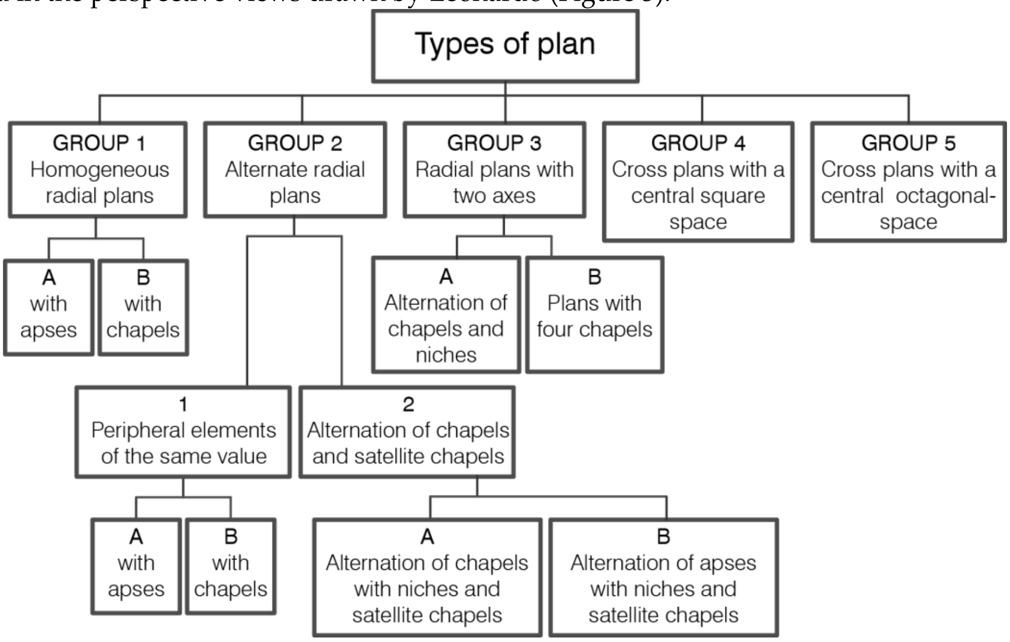

Figure 2. The scheme sums up the classification of plans proposed by J. Guillaume. First, the plans are divided in five groups according to the disposition of elements around the central space (groups 1-3) and to the shape of the central space (groups 4-5). Then some of the groups are further divided according to the feature of the peripheral elements, distinguishing between chapels and apses/niches.

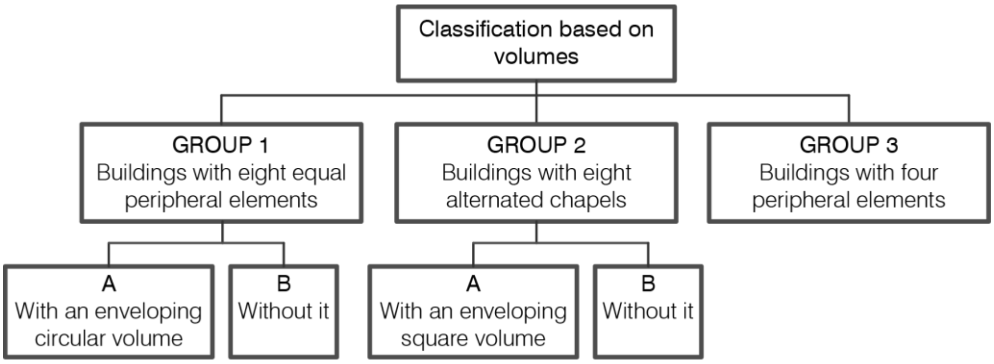

Figure 3. In this scheme the classification based on volumes proposed by J. Guillaume is described. The buildings are divided in three groups according to the alternation, evenness or partial absence of the peripheral elements. Then the first two groups are further divided according to the presence or absence of an enveloping volume.

Richter and Guillaume's works, starting from the same churches, defined two different methods for their classification into groups. As can be seen in the previous schemes, these methods are both based on a subsequential subdivision into groups, that is a hierarchical tree structure. However, even though both of them show several elements of interest, the use of this kind of logical structure to classify this *corpus* shows some problems due to the limited number of churches in the manuscript combined with the great amount of combinatorial possibilities for their layout. First, there will inevitably be branches containing only one or few elements, and second, using of this kind of subsequent subdivisions, and thus imposing a hierarchy, implies giving more importance to the variation of certain features over others.

Thus, the idea is to use a classification code, based on the association of values to the defined variables (Figure 4), rather than grouping the churches in a tree logical structure. The decision to operate in this way comes also from the observation of a characteristic of these layouts: they are, in fact the recombination of a group of elements around a central space, like in a geometric play, and they can thus be studied through a script (at least in a simple way, as we will see). Therefore, the use of a classification code has a further advantage: it notably simplifies the process of creation of a parametric model able to resemble the churches.

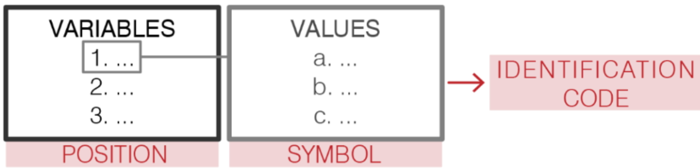

Figure 4. The scheme describes the structure of the classification method that we propose. An ordered list of variables (1,2,3, etc.) is selected by analyzing the corpus of churches, then, for each variable the possible values are defined (a, b, c, etc.) and described with an abbreviation. The sequence of variables and values constitutes the identification code.

On the other hand, the classifications made by Richter and Guillaume are useful to define what are the core characteristics of the churches' layouts that we consider necessary to fully describe the layout itself. For instance, the distinction used by both of them between alternate or equal peripheral elements on the XY/diagonal axes (Figures 1, 2 and 3), or the distinction made by Guillaume between the churches with and without an enveloping volumes (Figure 3), are an example of two important elements that should be featured in our identification code. The features that we consider in our classification method constitute the "variables", while the possible configurations that a variable can have constitutes the "values" (Figure 4). E.g. considering the previous examples, in the first case the

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variables are represented by “shape of the chapels in the XY axes” and “shape of the chapels on the diagonal axes”, while the values that each of them can assume, as we will describe, are many and are represented by all the shapes in the casuistry of the churches in the manuscript. The information about their equality or their difference, thus, is directly inferred from the values assigned to these two variables. Regarding the second example instead, the variable is constituted by the “presence of an enveloping volume” while the possible values that it can assume are two: yes (y) or no (n).

We will now list in detail all the variables that we decided to consider. However, before proceeding with the selection of the variables and with the definition of the identification code, it was necessary to operate a first distinction between two macro-groups of edifices that could not be defined through the same set of variables:

- Churches that have a central octagonal space
- Four-lobed churches

It is interesting to notice that the second group is itself part of a category of chapels that Leonardo uses as peripheral elements for the churches of the first group, so, the analysis of the layout of the second group is in fact part of the first one. The difference is that in one case the four-lobed layout is used as an independent system that stands alone as an edifice, while in the other it becomes the shape of the chapel of a bigger system.

Therefore, the idea is to start from the first group of churches and define a set of variables (in our case both topological and geometrical) that can define uniquely their layout. Then for each variable, all the possibilities that Leonardo explores are listed: these are the values of our variable. Afterwards, we decided an arbitrary order for the variables to be listed and defined a conventional symbol/initial for each value. In this way our classification system is completely explicated through an identification code.

2.1.1. Churches with a central octagonal space

We will now list, in order, the variables and their possibilities. The scheme is presented graphically in Figure 5.

The first variable is represented by the type of octagonal space, Regular (R) or Irregular (I). Therefore, a church with a central space shaped in plan as a regular octagon will have as a first part of the identification code "R/...".

The second variable depends on the disposition of chapels or niches around the central space. This disposition can be along a circle (C) or along a square (Q). So, if the same church of the previous example has the chapels placed along a square, its code will become "R/Q/...".

The third variable depends on the morphology of the chapels on the XY axes (A-) and on the diagonal axes (D-). In this way we can classify both the churches that have chapels of the same shape on all axes and chapels of different shape on XY and diagonal axes (i.e. the distinction made by J.P. Richter in Group 4, A and B that we described in Figure 1). This definition also incorporates the distinctions made by J. Guillaume in the five types of plan (Figure 2) and in the three types of volumes (Figure 3).

The possible values, thus the shapes that can be assumed are several and vary from the simple ones to the ones obtained through a combination of the firsts. The simple shapes are represented by octagonal (-O), circular (-C), semicircular (-SC), square (-Q), and rectangular (-R) chapels. From the combination of the previous shapes we obtain chapels that are made by the combination of more circles (-CC), by the combination of a square and circles (four lobed chapel) (-QC), by a rectangular shape combined with a semicircular one (-RSC). Thus, following the previous example which can also be seen in Figure 5, a church with chapels made up by a rectangle combined with a circle on the XY axes and circular chapels on the diagonal ones will be classified as "R/Q/A-RSC D-C/...". As we anticipated, the four lobed chapel (labeled -QC) will be better explained and subdivided in a further classification (A, B, C) in the next subsection, since Leonardo also uses it as an independent edifice in Ms. B, f. 93 v.

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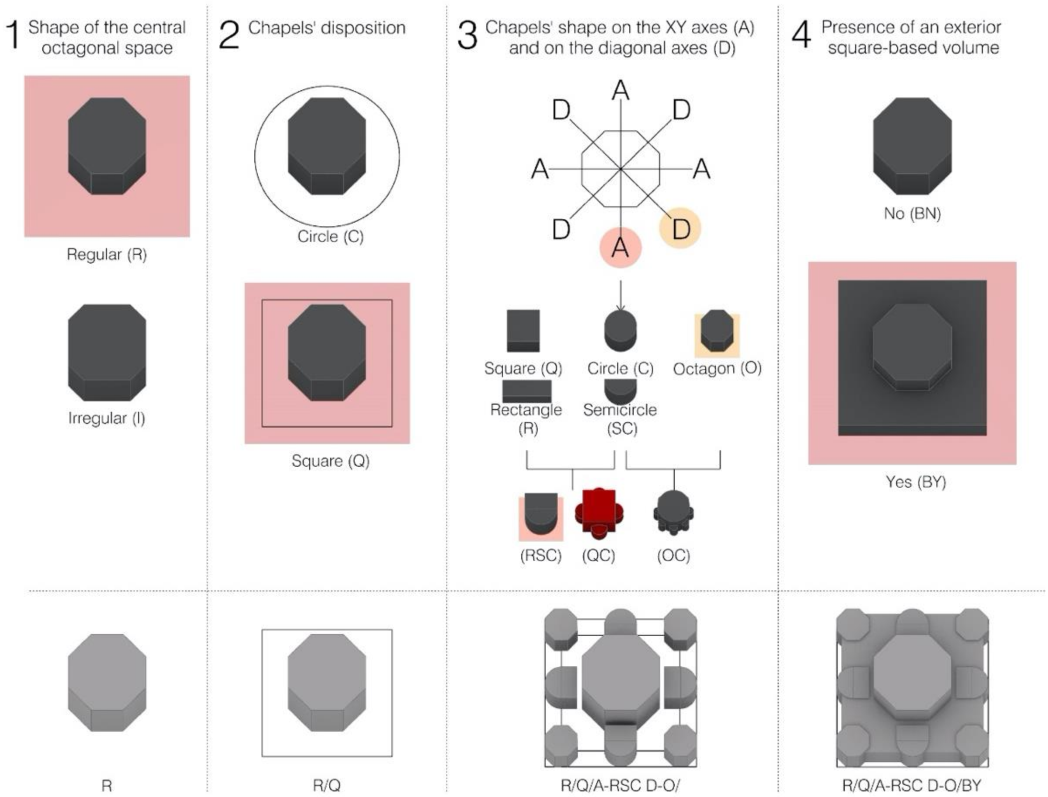

Figure 5. The image represents the possible values for each of the four variables and an example (at the bottom). Regarding the third variable, which involves the chapels' shape on the two axes (D and A), one of the possible composite shapes is represented by the chapel labeled "QC" (highlighted in red). This chapel configuration coincides with the four-lobed church's layout that will be described and classified in Subsection 2.1.2.

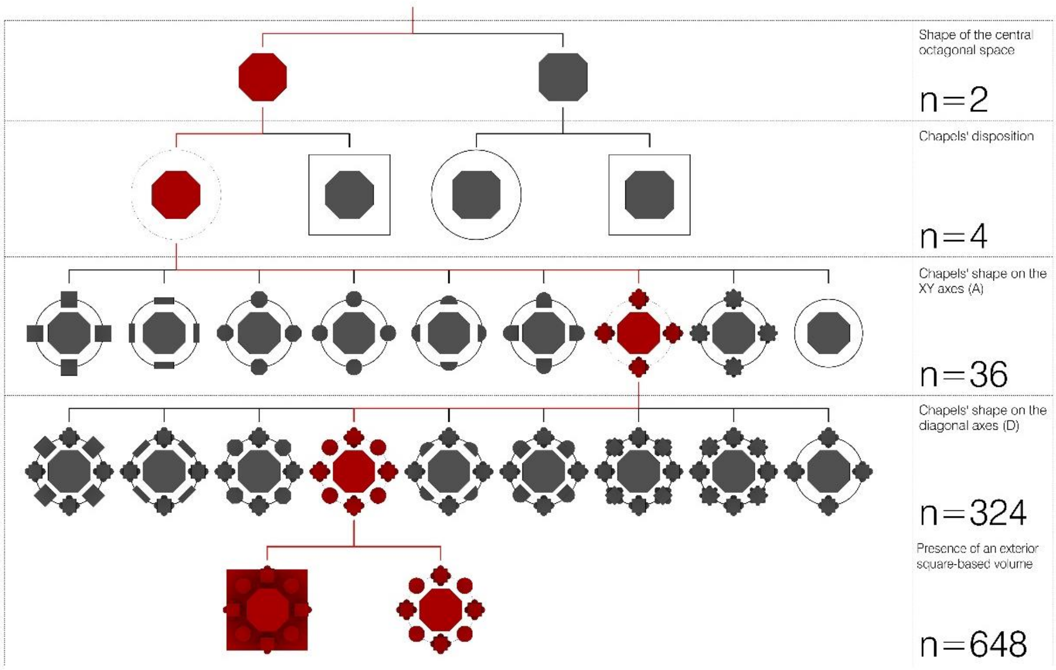

Figure 6. The figure represents the possible combinations of elements. From the fourth level of the tree only one branch is explored each time, due to the number of elements in the level, which can be seen on the right.

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The last variable depends on the presence (-Y) or absence (-N) of an exterior square-based volume (B-) that envelops some of the chapels. This distinction is made according to that made by J. Guillaume in his classification based on volumes (Figure 3). Therefore, the previous example, if characterized by an enveloping volume, will be defined as "R/Q/A-RSC D-C/BY".

Following this classification method, a total of 648 combinations can be obtained (Figure 6).

2.1.2. Four-lobed churches (QC chapel)

As we have seen, one of the possible shapes for the chapels is the one that comes from the combination of a square with four semicircular apses, named QC in the diagram seen before. Unlike what was done before by Richter and Guillaume, we will try to define a unitary discussion for all the chapels that are four-lobed, including the ones that have a dome rising from four pillars, from a square base or from an octagonal one. The idea is to try and include all these three possibilities in the same reasoning and use it in order to develop a unitary parametric model, as we will see in the next section. The elements to consider in our classification are two: the first one is the type (A, B, or C) and the second one is the ratio D/L (Figure 6).

Type A is the one where there are four pillars inside the main square space. There are different possibilities for its realization in detail, since the ceiling in the corners can assume different shapes (groin vault, sailing vault or flat), but this will not be considered in this low Level of Detail (LoD) aggregative analysis (while it will be fundamental in the later stage of three-dimensional reconstruction). The reference here is the *Sacello Carolingio* in the Church of *Santa Maria presso San Satiro*, at Milan.

Type B instead has the four spherical pendentives leaning on the sides of the square central space and can be referred, for example, to *Cappella Pazzi*, *Santa Maria delle Grazie* and the *Sacrestia Vecchia* in the church of *San Lorenzo* at Florence.

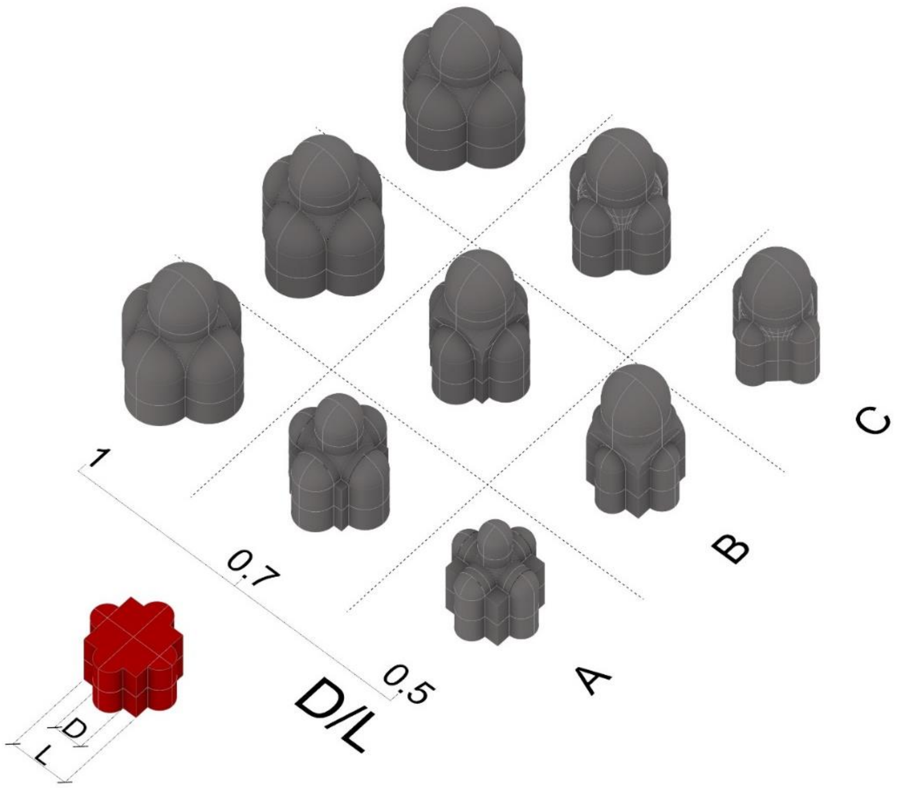

Figure 6. This figure explains the possible configuration that a four-lobed church or chapel can have, depending on the value of the ratio D/L with ranges from 0.5 to 1, and to the type A, B or C.

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In Type C, the endpoints of the semicircular apses are joined to form an octagon and its oblique sides become the basis for the pendentives, as it is done in *Cappella Chigi* at Rome.

The ratio D/L gives us information about the diameter (D) of the apses in comparison with the length (L) of the sides of the central square space. In this case we decided to consider for it a range of values from 0.5 (since we cannot find any example below it in both the references and Ms. B drawings) to 1. A value of D/L equal to 1 is in fact the borderline case, where Type B and Type C coincide, as can be seen in Figure 6. Nevertheless, the most correct way to consider it would be Type B, since topologically the central space is a square and not an octagon and in Type A the pilasters would end in the corners of the square space, bringing us back to Type B.

2.2. Definition of a parametric model based on the identification code

Given all the previous observation we built a parametric model (embedding some Python code in specific nodes in Grasshopper) that could potentially resemble all the churches of the manuscript, at least in as simple volumes. The idea is to turn our classification system into a system of toggles and sliders that can construct the basic shape of the church just choosing between them. This way, not only we could resemble the basic shape of a church of the manuscript by selecting the values identified in its identification code, but we could also resemble all the possible churches' layouts that are defined by the same set of aggregative rules.

The creation of this tool was fully obtained for the QC chapels, whose level of relatively low complexity makes it possible to easily create a script able to model all their possibilities. In particular, the D/L value is defined through a slider, as well as the height of the elements, while a toggle regulates the choice between the types A, B and C (Figure 7).

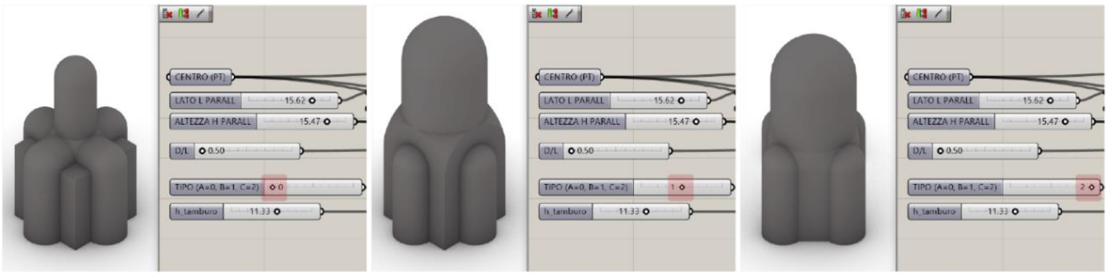
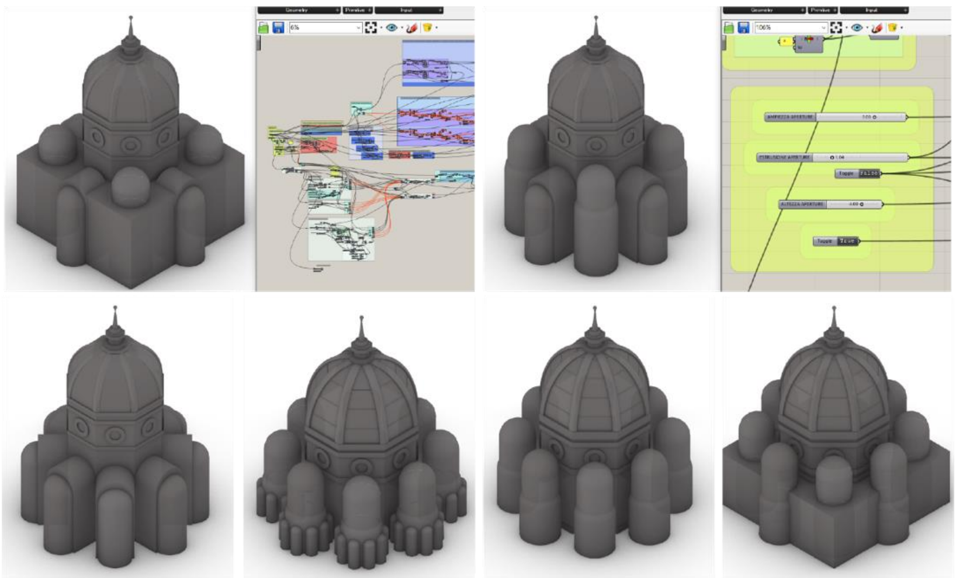


Figure 7. Representation of the functioning of the script and the modeled results for types A, B and C (from left to right, highlighted in red in the toggle).



(**a**)

Figure 8. The results obtained from the script for the churches with an octagonal central space.

On a theoretical level this could be done also for the churches with a central octagonal space, but the computational complexity in this case is higher and it would require a deeper research and accuracy in the development of the script. An attempt was made and, for now it can resemble all the churches that have square, circular, rectangular chapels or chapels made up by a rectangle and a semicircle (-RS chapels). The script also allows to regulate the distance of the chapels, the features of the central drum and the type of dome: pavilion (pointed or round, umbrella or spherical).

Being the first step, the architectural details were kept in a low LoD, sticking mainly to the level of information contained in the drawings, that sometimes appear to be small and schematic. However, in the model we defined it is possible to regulate the height and protrusion of various architectural elements, like the lantern, the moldings and the windows' frame and diameter on the drum. This was particularly useful in the subsequent phase of manual modeling in order to speed up and simplify the process.

3. Analysis and 3D modeling of each church

In this section the second phase of digitization is described. Following the analysis made in the previous step, the churches were considered one by one and the possible geometrical process for drawing them was researched. This was done by drawing the fundamental lines of the drawing and trying to redraw them through a geometrical process in a CAD environment.

Lastly, the base model produced with the parametric approach was defined in its details, combining it with the considerations made in the plan and elevation analysis. It was often necessary to produce more solutions for each church, either for a lack of information or for inconsistencies between plan and perspective views.

3.1. Plan and elevation analysis

Carlo Pedretti was the first to study the proportions of one of Leonardo da Vinci's buildings with the study he conducted on the drawing no. 238 v of the Academy of Venice, deducing the floor plan from an external perspective view [32].

On the other hand, Schofield studied the relation between the octagonal star scheme related to the Pell series and Silver Ratio and the church depicted in f. 95v of Ms. B [33]. It is interesting to notice that Leonardo seems to reason frequently on eight-pointed stars in Ms. B, as he uses this geometric construction both by itself and inside the plan of a church (f. 95 v, Ms. B).These considerations are especially useful because this geometric construction is not only present in f. 95 v but is widely used in many buildings of the manuscript, that are often constructed starting from a sequence of octagons. The geometrical figure of the octagon is strictly related to Pell numbers and Silver Ratio. In fact, given a regular octagon with a unitary side, the in-radius of the octagon equals $1+\sqrt{2}$, which is the Silver Ratio. The Silver Ratio can be obtained also dividing two consecutive numbers of the Pell Series and those numbers can also be retrieved drawing an eight-pointed star from the first octagon and then joining the points.

Then, a systematic study of the drawings in Ms. B was made by Jean Guillaume for the exhibition "*Léonard de Vinci, ingenieur et architecte*" for the Montreal Museum of Fine Arts [28,34]. Guillaume's study, in addition to reconstructing in three-dimensions some of the churches of Ms. B, developed a complex categorization of the buildings, focused on the chapels and on the apses that surround the central octagonal space.

Francesco P. Di Teodoro then made his contribution in the study of the proportions both of plans and elevations [25]. One of the elements noticed in his article is that many of the churches in Ms. B are impossible to model with a perfect consistency between plan and perspective view. This problem is probably related to the fact that the drawing Leonardo makes in Ms. B are - in Di Teodoro's opinion - intended as personal notes and don't present yet the characteristics of real-life projects, but this is also frequent when dealing with the digital reconstruction of drawn architecture regarding section and plan view [33]. The inconsistencies between plan and bird's-eye view will be constantly

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considered in our work, since we will evaluate it for each church and try to understand the reasons behind these differences in the drawings.

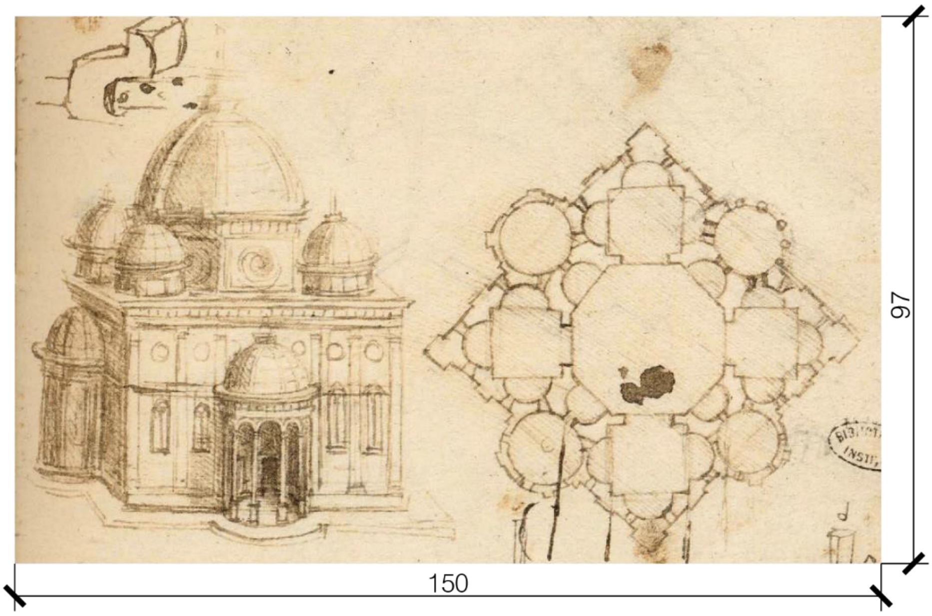

Figure 9. The church depicted in f.22 r with the dimension of the drawing in millimeters.

The proportional and geometrical analysis described in this section considers these contributions. The approach is thus aimed at systematically comprehending the proportions and the recurring elements. For this purpose, understanding the previous classifications of the churches was fundamental as it constituted the starting point for the geometrical analysis and for the addition of features to the 3D model.

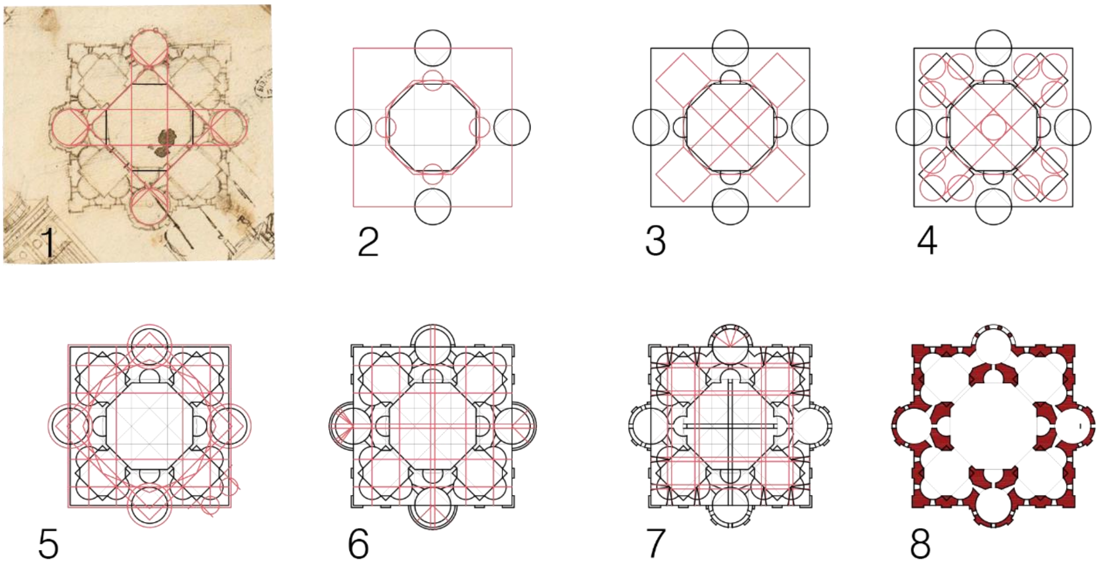

Figure 10. Geometrical process for the reconstruction of the plan starting from the central octagon.

In order to better explain the process, the case of the church depicted in f. 22r (Figure 9) is reported and described.

The process starts with the representation of the plan and its deconstruction, in order to define the geometric process that Leonardo could have used to design the church (Figure 10). In this case, starting from the central octagon and lengthening its sides and diameters (i.e. following the construction of the octagonal star of the Pell series) it is possible to find the extremes of the circular

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chapels' diameters (1). With a succession of geometrical operations (2-4) the other chapels' shapes are defined. Then, based on the geometries defined it is possible to trace the interior and exterior openings (5-7) and to finally hatch the masonry portions (8).

Then, the elevation is analyzed, trying to find a relation between the dimensions of the elements in the plan and their height (Figure 11). In order to do so, first it is necessary to scale the bird's-eye view drawing based on the plan. The fact that the perspective views are almost cavalier projection represents a useful characteristic to derive the measures of objects, since the façade is almost an orthographic elevation. In the example reported here, the height of the different elements was put in relation with the width of the square in plan and with the octagon. Moreover, the total height of the edifice can be related to the golden ratio of the square itself.

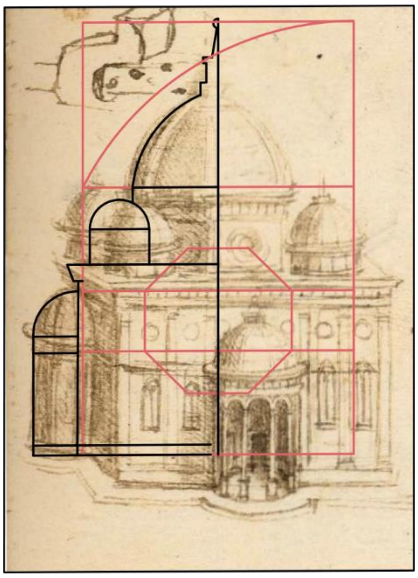

Figure 11. Relation between the geometry in plan and the height of the elements in the elevation.

3.2. Final modeling of multiple results

At this stage, all the churches were digitized one by one using a manual approach in a 2D and 3D CAD environment, starting from a proportional study and using the results of the parametric approach, explored in the previous section. In fact, once defined the classification code and the proportions between the elements' heights and width, it was possible to insert them in the Grasshopper script and to bake a low LoD model to start from.

Then, for each inconsistency between plan and perspective views a different 3D model was created. Then, more variants were modeled also in the case where Leonardo draws different solutions in the same drawing, varying the architectural elements. Moreover, the absence of a section view sometimes makes it impossible to have certainties about the structure of the interior. To overcome this issue, in case of uncertainty, more solutions have been explored, referring to the architectural examples that Leonardo might have seen during and before the writing of Ms. B.

For instance, regarding the example of f. 22 r, it was necessary to consider three elements of variability, each one corresponding to one of the categories described above:

a) The inconsistency between plan and perspective view regarding the width of the pilasters and the position of the double arched windows (Figure 12).

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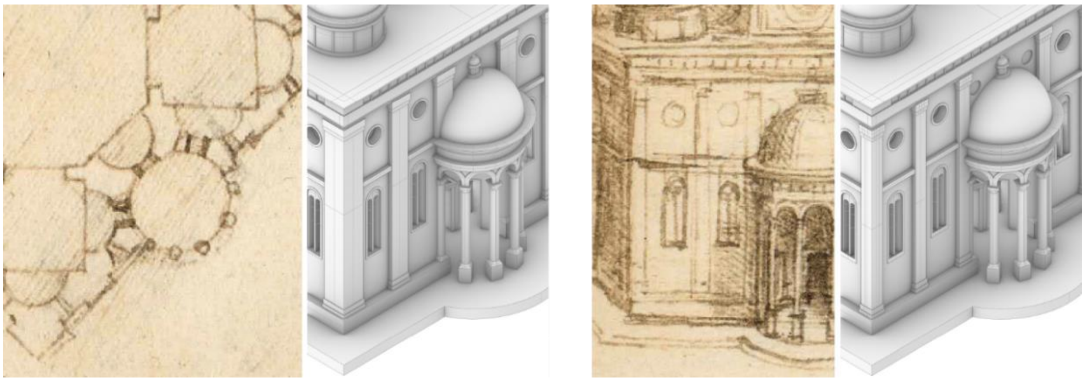

Figure 12. Highlight of the inconsistency between plan and perspective views and relative results in the 3D model (see the different width of the pilasters and the different position of the double-arched windows).

b) The difference between the right-hand-side and the left-hand-side of the façade in the height of the double-arched windows (Figure 13).

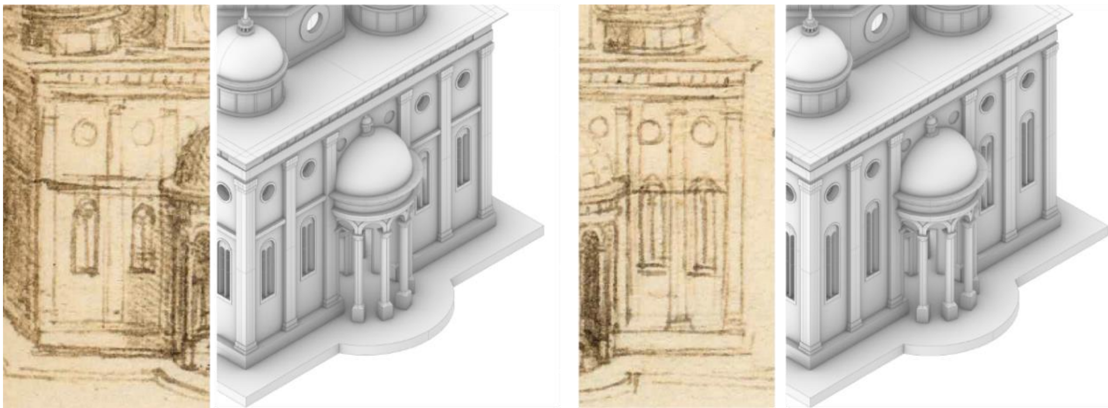

Figure 13. Comparison between the different height of the double-arched windows in the façade.

c) The interior shape of the QC chapels in plan, since it is not clear whether they should be classified as type B (i.e. a square central space with niches in its sides and the drum leaning on spherical pendentives) or as type C (i.e. an octagonal space with niches on the major sides of the octagon and a drum hat leans on pendentives, modeled according to the reference of *Cappella Chigi* in *S.M. del Popolo* at Rome) (Figure 14).

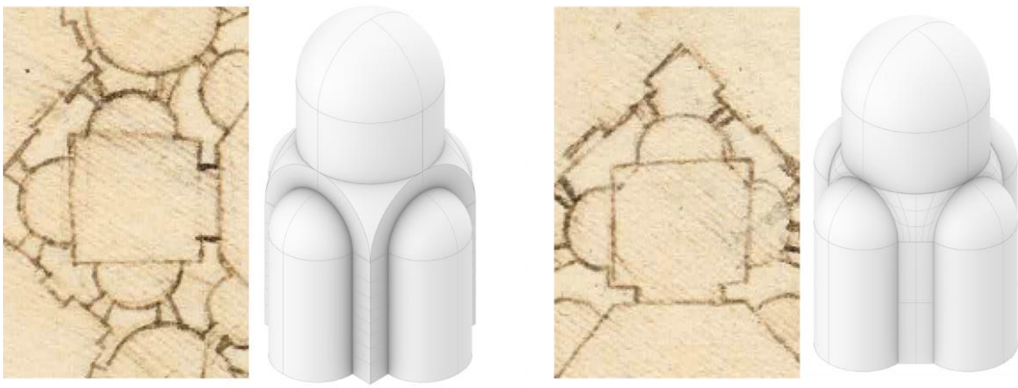

Figure 14. Comparison between the two variants modeled for the interior of the chapels: type B (on the left) and type C (on the right).

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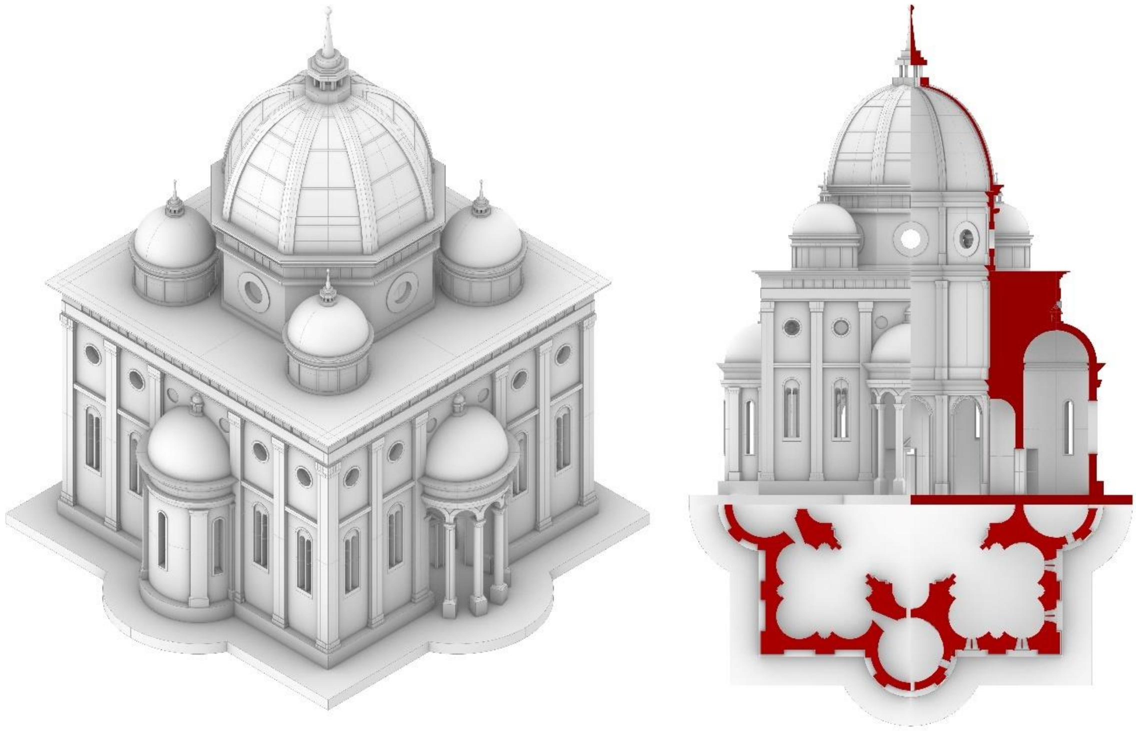
Therefore, the overall number of solutions produced in this case is eight (i.e. 2^3), each one related to the modification of one of the elements listed above. This example is emblematic of the great number of results that it was necessary to create in order to digitize sixteen churches. One of these solutions can be seen in Figure 15.



(**a**) (**b**)

Figure 15. (**a**) Axonometry and (**b**) plan, section and elevation of one of the eight solutions modeled for the church in f. 22r.

For all the solutions, we attempted to make a hypothesis for the dimension of the building, based on the architectural elements' width. In particular, the typical dimensions of the different kind of openings - single and double arched windows, circular windows, interior and exterior doors - that were collected in a preliminary study of coeval and relevant architectural references, were used to scale the 3D object and try to find the solution that could make the dimensions of all these elements realistic at the same time.

3. Results

The work described in Section 2 led to a classification method and a tool for modeling the churches in a low LoD, that helps in the visualization of the aggregative rules behind the churches. Moreover, the script is useful to appreciate the great number of combinations that could respond to these rules. The highest difficulty found in this process was the computational time: it would be necessary to redesign the script with a higher efficiency before adding the other chapels' shapes, but we think this could be an interesting element to develop in the future.

Then, in Section 3, the results are represented on one hand by the extrapolation of a possible geometric process for the reconstruction of the drawings and, on the other, by several possible 3D solutions for each church (Figure 16).

Another result emerging from the study of the drawings is that there is a recurrent use of Golden and Silver Ratio in the construction of plan views, and that the fundamental elements of this process is almost always related in some ways to the height of the parts in the perspective view. This showed the presence of a relatively complex geometrical construction also behind designs that are apparently very simple and sketchy. It is interesting to see how all these peculiarities could not have been noticed without a thorough work of combined analysis and deconstruction of the two views.

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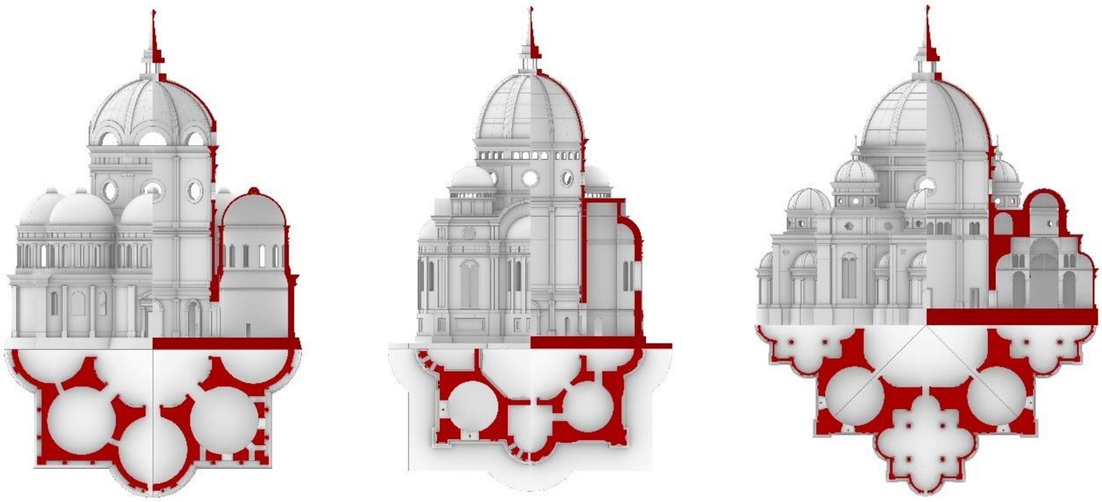

(a) (b) (c)

Figure 16. This figure represents three examples of the sixty-seven models realized, in plan, section and elevation: (**a**) f. 17v and 18r (A). (**b**) F. 17v and 18r (B). (**c**) F. 18v and 19r.

As we anticipated, one of the most interesting results that we could observe from the digitization work is that for the majority of the churches it was necessary to produce more than one three-dimensional solution (in particular, the total amount of solutions that were realized for the sixteen churches is equal to sixty-seven). This is related to two different kind of variables:

- Leonardo draws in different ways the same architectural element inside the same drawing;
- There are inconsistencies between plan and perspective views.

The first variable is deeply related to the fact that these drawing are personal annotations, and this proves that by drawing Leonardo is exploring solutions and testing them. The fact that these drawings were meant for personal annotations is also proven by the lack of accuracy in many of them, in fact, the more the churches are characterized by a simple layout, the more their representation is just schematic.

The second variable concerns both previous elements. In fact, as emerged from the digitization process, there are some inconsistencies that are evident and are related to the exploration of a different solutions, while others are due to problems of imprecision or schematism in the drawings.

Therefore, this practical work shows its utility also in gaining information about the drawings and drawing historical and artistic conclusions and hypotheses.

As regards the layout of churches, we could notice how Leonardo does not converge, through the pages of the manuscript, on a solution. Instead, he explores multiple possibilities in a nearly combinatory work. This, along with the other results previously pointed out, weighs in favor of the theories that want this speculation not to be meant for the design of any church. It is in fact more likely, in our opinion, that a work of this kind could constitute the basis for a later creation of a treatise on centrally planned churches.

Lastly, all these results enable us to make some considerations about the representation techniques used in the manuscript. The use of a plan and a bird's-eye view is in contrast with what was claimed by L. B. Alberti [35] and Raffaello [36] regarding the representation tools for architecture and we will try to give an explanation to this difference. First, it is interesting to notice that the method used by Leonardo could in turn show some affinities to the drawing instruments listed by Vitruvio: *ichnographia, orthographia and scaenographia* [37]. In fact, Leonardo's bird's-eye views could be considered as a combination of the two last instruments, since they show a façade which is almost an orthographic view of the elevation (*orthographia*) together with a perspective view that gives information about the volumes (*scaenographia*), and those are paired with a plan view (*ichnographia*). Of course, as it was claimed, the perspective of which Vitruvio writes about might be different to that of Leonardo and it may be a central perspective with the observer placed at a lower height. However, with this disposition, Leonardo manages to add also information about the roof in the same drawing.

It is reasonable to think that the graphical decision of Leonardo ultimately must be related to the purpose of the drawings, and thus to the information they are able to convey, and also to the features of the edifices. First of all, the bird's-eye view enables to obtain a synthetic overview of the whole, with attention to the way the volumes are spatially assembled, and this is particularly fitting for the recurrent layout of the churches, which is made by the aggregation of elements around a central space. Then, as we anticipated, the drawing is also able to give information about the roof and about the proportional measures in the façade, while the plan associated is fundamental to know about the circulation inside the edifice. Moreover, the final effect is very similar to that of a model placed on a table, as was noticed by Pedretti [31] and we know that the combined use of a plan view with a wooden model was not something rare in terms of architectural communication in Renaissance. Moreover also L. B. Alberti claimed the importance of the creation of a model in order to be aware of the problems of a project [35], and our analysis showed once again that Leonardo was using the drawings as a reasoning instrument.

To conclude, the representation technique developed by Leonardo appears to be on one hand extremely effective for rapidly annotating ideas and testing solutions, and on the other a powerful tool to convey immediate volumetric information along with measurable, orthographic views.

3. Conclusions

In Section 2 we analyzed on one hand J. P. Richter's work, pointing out some problems that could be related to an approach of classification that is focused on the analysis of the domes, and on the other hand, that of J. Guillaume, who instead carried out a study that starts from the aggregation of elements. From considerations about these works we developed a personal proposal for the classification of churches based on a code that assigns a letter for each variable in the definition of the layout. This approach is highly compatible with parametric methods of modeling, even though it was only used to define the fundamental elements of the model due to the computational complexity that rises when trying to create a single, comprehensive script for all the churches. This constitutes an interesting element to develop in future researches.

Then, in Section 3 we described the methodology used in order to derive proportional and geometrical information from the drawings and the way we used that information to model multiple reconstruction hypotheses. The need of producing more than one model for each church confirms the importance, when dealing with the reconstruction of drawn architecture, to lean on instruments for the synoptic visualization of results and of uncertainty and precision visualization. This could constitute another element of interest for a future research.

Lastly, as it was underlined in the results, the work proved itself to be a powerful means for noticing elements otherwise difficult to see in the *corpus* of drawing. Thus, it was particularly useful for deriving historical and artistic conclusions based on objective elements and it could also be extended to the work of others Renaissance architects, since the representation techniques that they used are a subset of the representation system used by Leonardo and their language is embedded in the same cultural environment characterized by similar architectural elements.

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