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The Riemann Hypothesis Remains Unresolved: Fundamental Discrepancy in Computational Methods for the Widely Accepted Nontrivial Zeros of the Riemann Zeta Function, ζ(s)

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ABSTRACT: The long-standing mathematical problem, the Riemann Hypothesis, states that all nontrivial zeros of the Riemann zeta function ζ(s) lie on the line Re(*s*) =1/2. This study examines the computational methods used to determine nontrivial zeros, such as ½ + 14.13472514i, ½ + 21.02203964i, ½ + 25.01085758i, and others of ζ(*s*).The analysis finds that these methods are derived from the equation ξ(*s*) =(*s*/2) (*s*-1) $(\pi)^{-s/2}$ Γ(*s*/2) ζ(*s*), by assuming that zeros of functions ζ(*s*) and ξ(*s*) are identical. However, the graphical examination of the locations of zeros of both functions, suggests this assumption is incorrect, rendering the computational methods flawed. Consequently, the widely accepted zeros computed using these methods, might not actually be zeros of ζ(s). In addition, the formula for the number of zeros N(T) on the line s=1/2 in a specific interval [0, T], based on the same assumption is also invalid. The study offers a new perspective on the Riemann Hypothesis by highlighting potential flaws in existing methods used for computing zeros of ζ(s). This finding could contribute to ongoing efforts to resolve the hypothesis.

Keywords: Riemann Zeta function; Xi Function; Nontrivial zeros; Riemann Hypothesis

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1. Introduction

Bernhard Riemann [1] in his research report (1859) introduced two functions, the Riemann's zeta function, $\zeta(s)$ and the Xi function $\xi(t)$ defined as:

$$\zeta(s)=\sum_{n=1}^{\infty}\left(1/\mathrm{n}^{s}\right) \quad s=\sigma+it,\ \ \sigma,t\in\mathbb{R} \tag{1}$$

$$\xi(t)=\underline{\Pi(s/2)}\cdot(s-1)\pi^{-s/2}\zeta(s),\ \ s=\underline{1/2}+it \tag{2}$$

The definition (2) is the original definition of the function ξ derived by Riemann himself. However, in mathematics literature, authors [2], [3], and others use the definition:

$$\xi(s)=(s/2)\cdot\prod(s/2-1)\cdot(s-1)\pi^{-s/2}\zeta(s) \tag{3}$$

There exists a similar kind of relation between Dirichlet Eta function $\eta(s)$ and $\zeta(s)$ given by

$$\eta(s)=\left(1-2^{1-s}\right)\zeta(s) \tag{4}$$

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About the locations of zeros (nontrivial) of function $\zeta(\sigma+it)$, Riemann proposed a conjecture known as the Riemann Hypothesis "All nontrivial zeros of the function; $\zeta(s)$ lie on the line $\sigma=1/2$."

Riemann also proposed, the formula for the total number of zeros $\mathrm{N}(\mathrm{T})$ of function $\xi(t)$ or concurrently of $\zeta(s)$, situated on the line $\sigma=1/2$, in a specific interval $0<t<\mathrm{T}$ given by

$$\mathrm{N}(\mathrm{T})\approx(\mathrm{T}/2\pi)\left[\log_e(\mathrm{T}/2\pi)-1\right] \tag{5}$$

The above formula was proved by Van Mangoldt [4] in 1905, now is known as the Riemann-Mangoldt formula has the form;

$$\mathrm{N}(\mathrm{T})\approx(\mathrm{T}/2\pi)\left[\log_e(\mathrm{T}/2\pi)-1\right]+7/8 \tag{6}$$

The Authors [5] also derived above formula from Argument principle of Complex Analysis, assuming that zeros of functions $\xi(s)$ and $\zeta(s)$ are identical.

Table 1 shows the number of zeros $\mathrm{N}(\mathrm{T})$ computed by some leading authors on the line $\sigma=1/2$ in specific intervals $[0,\mathrm{T}]$. Remarkably, zeros computed by these all authors are ½ + 14.13472514i, ½ + 21.02203964i, ½ + 25.01085758i, and others, but with different accuracy of decimal places.

Table 1. The number of zeros of the function $\zeta(s)$ computed by leading authors:

Author	Year	Range of height (0 < *t* < T)	Number of zeros N(T)
B. Riemann [1]	1959	T≤ 26	3
J.P. Gram [6]	1903	T≤ 50	10
R. J. Backlund [7]	1914	T≤ 210	79
J.I. Hutchinson [8]	1925	T≤ 300.468	138
E.C. Titchmarsh [9]	1935	T≤ 1468	1,041
A.M. Turing [10]	1953	T≤ 25735.93	1,104
D. H. Lehmer[11]	1956	T≤ 9878.910	10,000
N.A. Meller [12]	1958	T≤ 4735	35,337
R.S. Lehman [13]	1966	T≤ 170571.36	250,000
R.P. Brent [14]	1979	T≤ 32,585,736.4	75,000,000
J. van de Lune, H. J. J. te Riele, D.T. Winter [15]	1986	T≤ 545439823.215	1,500,000,001
Gourdon, Xavier [16]	2004	T≤ 2381374874120.45508	10^{13}
M. Odlyzko [17]	1992	T≤ $2.\times10^{20}$	1.75×10^{8}
Dave Platt, Tim Trudgian [18]	2020	T≤ 3 000 175 332 800	12 363 153 437 138

Since 1959, there have been many but unsuccessful attempts to resolve the Riemann hypothesis. One such attempt has been- computing zeros of $\zeta(s)$ on the line $\sigma=1/2$ in a specific interval $0\le t\le \mathrm{T}$, and concludes that the Riemann hypothesis is valid (true) up to height T. The other theoretical attempt to prove the Riemann hypothesis has been to show at least one zero existing on a line $\sigma=1/n$, $n\in\mathbb{N}-\{1,2\}$ in the interval $(0,1/2)\cup(1/2,1)$. However, so far this attempt could not produce any marked result.

Methods used for Computing of Zeros of Function $\zeta(s)$:

So far, the methods used by authors [1] to [18], and others to compute zeros of $\zeta(s)$ are derived from equations (3) and (4) as the case employed. **Case I**: the solutions of equation (3) for $s=\underline{1/2}+it$ are the zeros of $\xi(s)$, and $\xi(s)=0\Leftrightarrow\zeta(s)=0$. **Case II** (used rarely): the solutions of equation (4)

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for are the zeros of $\eta(s)$, and $\eta(s)=0 \Leftrightarrow \zeta(s)=0$. All the methods determine the zeros of $\zeta(s)$ using an associated function $Z(t)$, derived from equation (3), as follows:

$$\begin{aligned}\xi(1/2+it) &= (s/2)\cdot\prod(s/2-1)\cdot(s-1)\pi^{-s/2}\zeta(s), s=1/2+it \\ &= e^{\log\prod(s/2-1)}\cdot\pi^{-s/2}\cdot(s(s-1)/2)\zeta(s) \\ &= \left[e^{\mathrm{Re}\{\log\prod((s/2))-1)\}}\cdot\pi^{-1/4}\cdot(-t^2-1/4)/2\right]\cdot\left[e^{\mathrm{Im}\{\log\prod((s/2))-1)\}}\pi^{-it/2}\zeta(1/2+it)\right]\end{aligned} \quad (7)$$

Since on the line $\mathrm{Re}(s)=1/2$ or t-axis, the function $\xi(s)$ is real-valued, therefore, at the location of every zero, the function $\xi(1/2+it)$ changes its sign. Since, the first factor of the product on the right is a negative real number, therefore, the sign of $\xi(1/2+it)$ is opposite that of the second factor. In addition, the sign change of function $\xi(1/2+it)$ occurs when there is a change in the sign of the second factor. This second factor, that reveals a zero of $\xi(1/2+it)$ or that of $\zeta(1/2+it)$ is termed the $Z(t)$ function. For computing zeros of $\zeta(s)$, J.P. Gram, G.H. Hardy, and B. Riemann perceived function $Z(t)$ in different ways as follows:

Gram Law [7]:

$$\begin{aligned}\zeta(1/2+\mathrm{i}t) &= e^{-\mathrm{i}\theta(t)}Z(t) \\ &= Z(t)\cos\theta(t)-iZ(t)\sin\theta(t) \\ \mathrm{Im}\,\zeta(1/2+\mathrm{i}t) &= -Z(t)\sin\theta(t)\end{aligned} \quad (8)$$

Here, $\theta(t)=(t/2)\log[(t/2\pi)-1]-(\pi/8)+1/48t$ .At a zero, the function $\zeta(1/2+it)$ changes its sign that depends on the changes of sign of $Z(t)$, and at point $\theta(t)=n\pi$ right side of equation (7) is zeros. This point is termed as the Gram point. In Gram's Method, a zero $s=(1/2+it)$ exists at a point between two consecutive gram points.

Hardy Method: G.H. hardy [19] defines function $Z(t)$ as:

$$Z(t)=e^{i\theta(t)}\zeta(1/2+\mathrm{i}t)$$

Here, $\theta(t)=\mathrm{Im}[\log\Gamma(1/4+\mathrm{i}t/2)]-(t/2)\log\pi\le n\pi$

Riemann–Siegel Formula:

$$\begin{aligned}Z(t) &= e^{i\theta(t)}\sum_{n=1}^{[x]}\left(1/n^{(1/2+\mathrm{i}t)}\right)+e^{-i\theta(t)}\sum_{n=1}^{[x]}\left(1/n^{(1/2-\mathrm{i}t)}\right)+O\left(t^{-1/4}\right) \\ &= 2\sum_{n=1}^{[x]}\left(\cos(\theta-t\log n)/n^{-1/2}\right)+O\left(t^{-1/4}\right), x=\sqrt{t/2\pi}\end{aligned}$$

Riemann computed first three zeros of $\zeta(s)$ using above formula. Carl Ludwig Siegel [20] in 1932 traced this formula in Riemann's work.

The number of zeros $N(T)$ of function $\zeta(s)$ on the line s=1/2 in a specific range $0<t<T$ is the number of times; the graph of function $Z(t)$ intersects t-axis. Each point of intersection $(t_z,0)$ produces a zero $1/2+it_z$. All the authors in Table-1 have computed t_z, $1/2+it_z$, and $1/2+it_z$ as

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the zeros of functions $Z(t), \zeta(1/2+it)$, and $\xi(1/2+it)$ respectively. Thus, their calculations are based on the assumption that the functions $\xi(s)$ and $\zeta(s)$ have identical zeros.

The main objective of this paper is to show that the assumption that zeros of functions $\xi(s)$ and $\zeta(s)$ are identical, is incorrect; consequently, the computational methods derived from this assumption are flawed. Hence, the widely accepted zeros, such as ½ + 14.13472514i, ½ + 21.02203964i, ½ + 25.01085758i, and others, computed through these flawed methods, cannot be zeros of function $\zeta(s)$. Additionally, the formula (5) to calculate the number of zeros $N(T)$, based on the same assumption, is invalid for zeros of function $\zeta(s)$.

2. Results

Theorem I: Let $\eta(x)=(1-2^{1-x})\zeta(x), x\in\mathbb{R}$, $\eta(x)=\frac{1}{1^x}-\frac{1}{2^x}+\frac{1}{3^x}-\cdots+(-1)^{n-1}\frac{1}{n^x}+\cdots$, and $\zeta(x)=\frac{1}{1^x}+\frac{1}{2^x}+\frac{1}{3^x}+\cdots+\frac{1}{n^x}+\cdots$. Then functions $\eta(x)$ and $\zeta(x)$ cannot have identical zeros.

Illustration 1: Since, for $x\geq 6$, the value of each function is ≈ 1. So, we consider three short forms of the functions $\eta(x)$ and $\zeta(x)$ defined as:

$$\eta(x)_{comp.}=(1-2^{1-x})\left(\frac{1}{1^x}+\frac{1}{2^x}+\frac{1}{3^x}+\cdots+\frac{1}{n^{12}}\right),\ \eta(x)_{odd}=\frac{1}{1^x}-\frac{1}{2^x}+\frac{1}{3^x}-\cdots+\frac{1}{11^x},$$

$$\eta(x)_{even}=\frac{1}{1^x}-\frac{1}{2^x}+\frac{1}{3^x}+\cdots-\frac{1}{12^x},\ \text{and}\ \zeta(x)=\frac{1}{1^x}+\frac{1}{2^x}+\frac{1}{3^x}+\cdots+\frac{1}{12^x}$$

,

The graphs of functions $\eta(x)_{comp.}, \eta(x)_{odd}$ $\eta(x)_{even}$ $(1-2^{1-x})$, and $\zeta(x)$ are shown in Fig.1 (a). As shown in this figure, for $x<6$, $\eta(x)_{comp.}$ (solid line) and $\zeta(x)$ (point dotted line) are different functions. At an arbitrary point $x=14.135$, still different (graphs shown in inset), but their graphs are far separated parallel lines, that shows, functions $\eta(x)_{comp.}$ and $\zeta(x)$ never intersect at a point. Thus, the functions $\eta(x)_{comp.}$ and $\zeta(x)$ cannot have identical zeros.

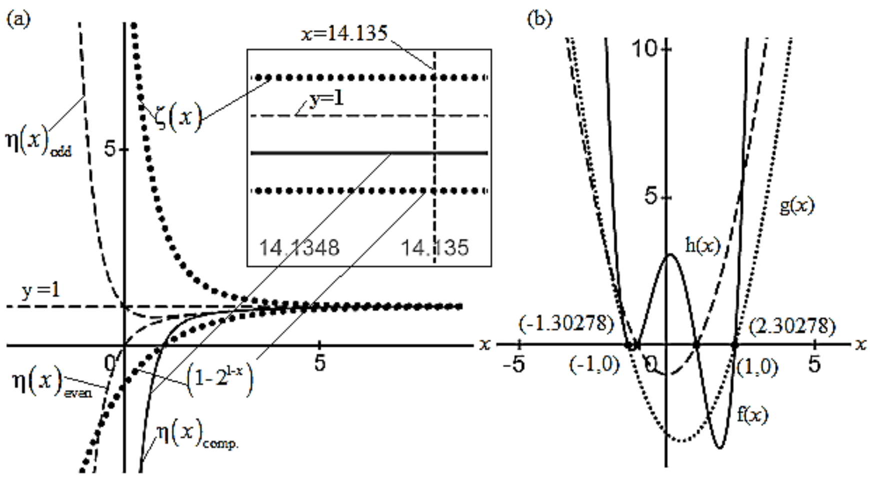

Fig.1. (a) Graphs of functions η(x) comp. and product functions, (b) Graphs of function f(x) and product functions.

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Illustration 2: Consider a polynomial function $f(x)$ defined as: $f(x)=(x^2-1)(x^2-x-3),\ x\in\mathbb{R}$. As shown in Fig. 1(b), graph of product function $f(x)=(x^2-1)(x^2-x-3)$ and factor function $g(x)=(x^2-x-3)$ intersects at *x*-axis, producing two zeros $x=-1.30278, 2.30278$, and function f(*x*) and $h(x)=(x^2-1)$ intersects at *x*-axis producing two zeros $x=\pm1$.

Illustrations-1 and **2** show, when product function and its factor functions are polynomial functions, zeros of product function are identical to the zeros of factor functions [See Fig 1(b)]. However, when one of the factor functions, is a function defined by series, the product function and factor function cannot have identical zeros [See Fig 1(a)].

Auxiliary Theorem II: The conclusion of equivalence of zeros of functions $\xi(s)$ and $\zeta(s)$ from the definition: $\xi(s)=(s/2)\cdot\prod(s/2-1)\cdot(s-1)\pi^{-s/2}\zeta(s)$ is incorrect.

Illustration: We write functions $(x/2)\cdot\prod_m(x/2-1)\cdot(x-1)\pi^{-x/2}=\Lambda_m(x), \zeta_n(x)=\sum_n^{\infty}(1/n^x)$ and define the functions: $\xi_{m,n}(x)=\Lambda_m(x)\zeta_n(x)\ x\in\mathbb{R}$. Since $\zeta_{n\geq6}(x)\approx1$, for ease, we consider, $\zeta_{18}(x)=\sum_{n=1}^{18}(1/n^x)$ .The first five ξ -functions are;

$$\xi_{1,18}(x)=(x/2)(\underline{x/2}-1)(x-1)\pi^{-s/2}\zeta_{18}(x)$$,

$$\xi_{2,18}(x)=(x/2)(\underline{x/2}-1)(\underline{x/2}-2)(x-1)\pi^{-s/2}\zeta_{18}(x)$$,

$$\xi_{3,18}(x)=(x/2)(\underline{x/2}-1)(\underline{x/2}-2)(\underline{x/2}-3)(x-1)\pi^{-s/2}\zeta_{18}(x)$$

$$\xi_{4,18}(x)=(x/2)(\underline{x/2}-1)(\underline{x/2}-2)(\underline{x/2}-3)(\underline{x/2}-4)(x-1)\pi^{-s/2}\zeta_{18}(x)$$

$$\xi_{5,18}(x)=(x/2)(\underline{x/2}-1)(\underline{x/2}-2)(\underline{x/2}-3)(\underline{x/2}-4)(\underline{x/2}-5)(x-1)\pi^{-s/2}\zeta_{18}(x)$$

The graphs of product functions $\xi_{m,n}(x)$ and factor functions $\Lambda_m(x)$ and $\zeta_n(x)$ are shown in Fig. 2(a), 2(b), 2(c), and 2(d).

In Fig. 2(a), curves of functions $\xi_{1,18}(x)$ and $\Lambda_1(x)$ meet at points (6.36708, 0.97524), (0, 0), (1, 0), and (2, 0). These functions have common zero at *x*= 0, 1, and 2. However, the graphs of functions $\xi_{1,18}(x)$ and $\zeta_{18}(x)$ nor of functions $\zeta_{18}(x)$ and $\Lambda_1(x)$ intersect.

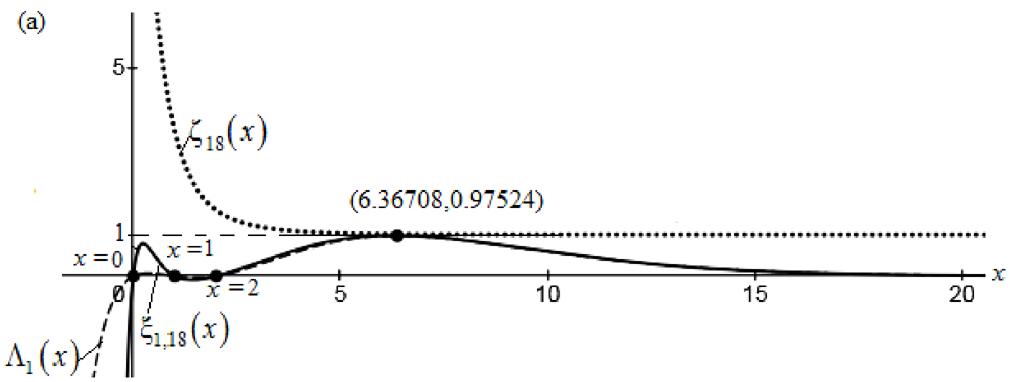

Fig. 2(a): Graphs of functions $\xi_{1,18}(x)$, $\Lambda_1(x)$, and $\zeta_{18}(x)$.

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In Fig. 2(b), graphs of functions $\xi_{2,18}(x)$ and $\Lambda_2(x)$ intersect at points (5.96503, 0.96597), (0,0), (0,1), (0,2), and (0,4), so have common zeros at x=0,1, 2 and 4. Graphs of functions $\xi_{2,18}(x)$ and $\zeta_{18}(x)$ meet at two points (6.06193, 1.01657), (13.56908, 1.00008), but these points do not produce zero/s of either function $\xi(x)$ or $\zeta(x)$.

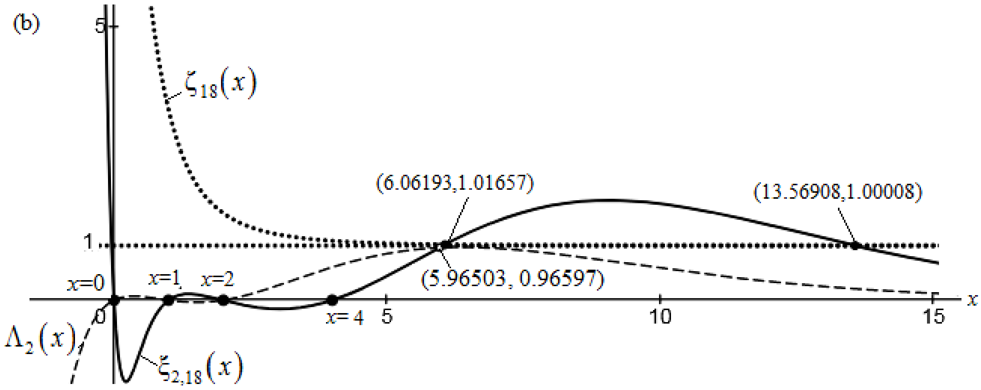

Fig. 2 (b): Graphs of functions $\xi_{2,18}(x)$, $\Lambda_2(x)$, and $\zeta_{18}(x)$.

In Fig. 2(c), the number of factors of function $\Lambda_3(x)$ is increased. The functions $\xi_{3,18}(x)$ and $\Lambda_3(x)$ start overlapping, and the functions $\xi_{3,18}(x)$ and $\Lambda_3(x)$ share five zeros at x= 0, 1, 2, 4, and 6. The function $\xi_{3,18}(x)$ shares points (7.29841, 1.006734), (20.07938, 1.000001 with function $\zeta_{18}(x)$. However, these points do not produce zero/s of either function. The function $\Lambda_3(x)$ shares points (7.30521, 1.0067), and (20.07938, 1.000001), with $\zeta_{18}(x)$ but these points are of no use in this study.

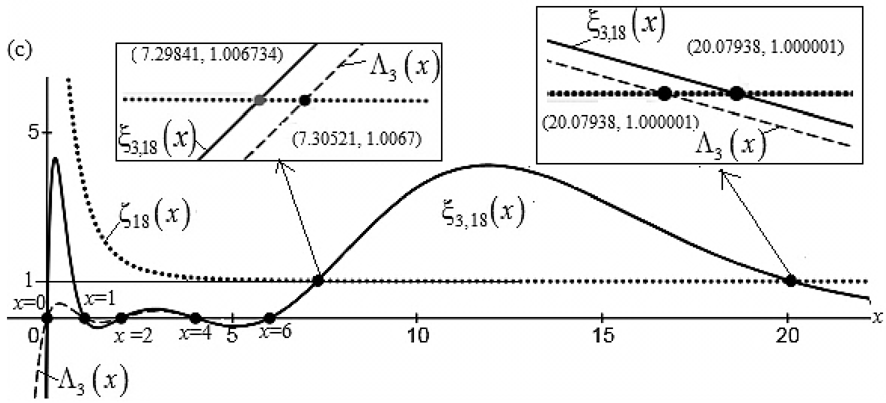

Fig. 2 (c): Graphs of functions $\xi_{3,18}(x)$, $\Lambda_3(x)$, and $\zeta_{18}(x)$.

In Fig. 2(d), on increasing the number of factors of function $\Lambda_4(x)$ further, functions $\xi_{4,18}(x)$ and $\Lambda_4(x)$ overlap and share six zeros at x= 0, 1, 2, 4, 6 and 8. The combine function of functions $\xi_{4,18}(x)$ and $\Lambda_4(x)$ shares points (8.78883, 1.002331) and (26.92861, 1) with the function $\zeta_{18}(x)$. That shows the functions $\xi(x)$ and $\zeta(x)$ may intersect at most at two points on the line $\zeta(x)=1$ or these functions have only two zeros on the line $\zeta(x)=1$.

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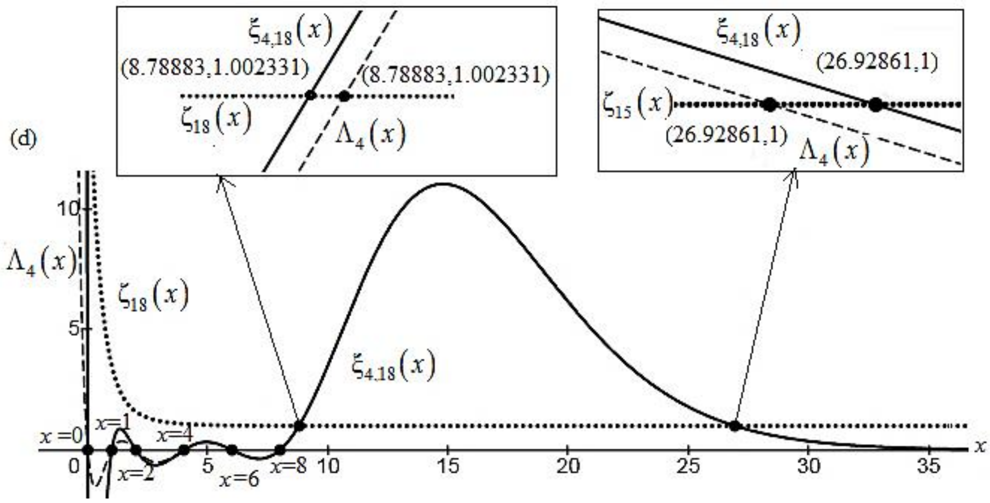

Fig. 2(d): Graphs of functions $\xi_{4,18}(x)$, $\Lambda_4(x)$, and $\zeta_{18}(x)$.

3. Discussion

The illustrations-1, 2 of auxiliary Theorems I; clearly differentiate behaviors of a product function and its factor functions regarding zeros of the type of functions: (i) when both; the product functions and factor functions are polynomial functions (of real variable) – in this case zeros of factor functions coincide with zeros of product functions. Riemann (1859) used this characteristic of real polynomial functions to the equation $\xi(s)=\Lambda(s)\zeta(s)$ to find zeros of function $\zeta(s)$. Later subsequent authors also used to derive the function Z(t). As shown graphically in illustration-1, this application is flawed and so the methods derived from this perception.

Moreover, the function $\zeta(s)$ is the function of complex variable defined by a series that cannot be treated like a polynomial function when verify its zero. For example; if $x=a$ is a zero of function $\zeta(x)$, then $\zeta(a)\neq 0$; rather, $\zeta(a)=\left(\frac{1}{1^a}+\frac{1}{2^a}+\frac{1}{3^a}+\cdots\right)\to 0$. Further, the illustration-2 of auxiliary Theorems II, clearly shows that in case of $\xi(x)=\Lambda(x)\zeta(x)$, if zeros of $\xi(s)$ and $\zeta(s)$ have identical zeros $a_1, a_2, \ldots$ (say), then the function $\zeta(x)$ should be a polynomial function, and that could be written as: $\zeta(x)=(x-a_1)\times(x-a_2)\times(x-a_2)\times\cdots$. However, function $\zeta(x)$ cannot be written in this factor form because the product on the right do not form a series. Thus, if one assumes that zeros of function $\xi(x)$ and $\zeta(x)$ are identical, then both function must be polynomials of real or complex variable. Further, the functions $\xi(x)$ and $\zeta(x)$ can have at most two zeros that lie on the line $\zeta(x)=1$. But, the authors listed in Table 1, have computed zeros count up to 10^{20} or more. The question is: From where did so many zeros originate? The answer is simple; the methods employed for computing zeros are flawed or since there exist more than two zeros of $\zeta(s)$, therefore, they (zeros) should be independent of function $\xi(s)$ or the function $\xi(s)$ should have different form.

It appears that the certain acceptance of ½ + 14.13472514i, ½ + 21.02203964i, ½ + 25.01085758i, and others as the zeros of function $\zeta(s)$ might be prompted by fact that the number of computed zeros in certain interval is the same as that predicted by the formula for $\mathrm{N}(\mathrm{T})$.

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In this article, in Theorem I, II, and illustrations 1, and 2 functions are considered of real variable but results are also valid for variable because $s = x + iy$ $x, y \in \mathbb{R}$ *can be written as* $s = x + i0 \in \mathbb{C}$.

An interesting outcome of this study: From the illustration of auxiliary Theorem II, an interesting observation leads a new theorem in the "Theory of Functions Defined by a Series." In the graphs, 2(a) to 2(d), we observe that the graph of product function $\xi(x)$ intersects the graph of factor function $\zeta(x)$ at points that lie on a line defined by the denominator of the largest term (here $1/1^x$) of the function $\zeta(x)$. The line is $\zeta(x) = 1^x$. If we take this line as the zero-line (or *x*-axis) for the functions $\xi(x)$ and $\zeta(x)$, that results in a theorem with the statement: "***The nontrivial zeros of a function defined by a series always lie on a line***" This line likely to be parallel to *x*- axis or *y*-axis. Further, investigation is required.

The invalidity of accepted zeros of $\zeta(s)$ has occurred due to misinterpretation of the equation $\xi(t) = \underline{\Pi(s/2)} \cdot (s-1)\pi^{-s/2}\zeta(s)$, $s = \underline{1/2} + it$, as discussed above: If one uses the original definition (2) of the function ξ and calculates zeros of $\zeta(s)$ corresponding to arbitrary zeros of function $\xi(t), t \in \mathbb{R}$, then zeros of functions $\xi(t)$ and $\zeta(s)$ are not only different but also lie on two mutually perpendicular lines, as shown in Fig. 3 (b).

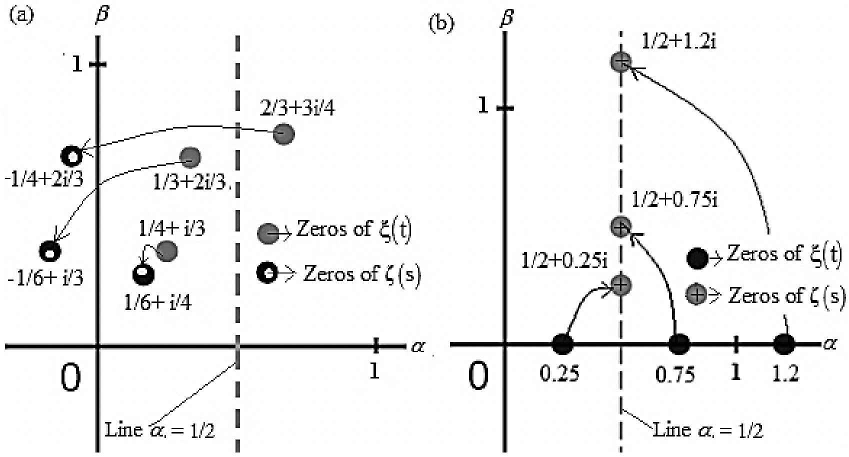

Fig. 3. Zeros of functions ξ (t) and ζ(s): (a) t is complex variable; (b) t is real variable.

The zeros of functions $\xi(t)$ and $\zeta(s)$ shown in Fig. 3 are calculated as: suppose $t = \alpha + i\beta$, as the root of $\xi(t)$. Now, if we assume $\xi(t) = 0 \Rightarrow \zeta(s) = 0$ then $s = (1/2 - \beta) + i\alpha$. However, the assumption $\xi(t) = 0 \Rightarrow \zeta(s) = 0$ is invalid as in the illustrations of Theorem I.

4. Conclusion

The methods used for computing nontrivial zeros of function $\zeta(s)$ are derived from assumption that the zeros of functions ξ and ζ are identical. This assumption is incorrect, rendering the methods flawed. Because this assumption forms the basis for the derivation of function $Z(t)$ that is used to compute zeros of $\zeta(s)$ by solving the equation $Z(t) = 0$, therefore, the widely accepted zeros, such as ½ + 14.13472514i, ½ + 21.02203964i, and others cannot be conclusively accepted as zeros of the function ζ(s). In addition, the validity of the formula $N(T) \approx (T/2\pi)[\log_e(T/2\pi) - 1]$, derived

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under the same assumption, is called into question. The root cause of these issues lies in a misinterpretation of the equation $\xi(t) = \underline{\Pi(s/2)\cdot(s-1)\pi^{-s/2}}\zeta(s)$, $s = \underline{1/2} + it$ in which authors consider functions ζ and ξ as the function of same variable. This fundamental oversight in the methodology for computing the nontrivial zeros of ζ(*s*) could be a significant factor in why the Riemann Hypothesis remains unresolved. Addressing these flaws and reexamining the assumptions underlying Z(*t*) and N(T) is essential for advancing our understanding of the zeta function and resolving this long-standing conjecture.

Additional Information:

We propose the complex numbers, (1/2 +35.28748i), (1/2 + 51.67246i), (1/2 + 67.98489i), (1/2 + 71.51417i), (78.21896i), and (1/2 +83.77055i) as the only nontrivial zeros of the function $\zeta(s)$ *in the interval 0< t <100.*

Declaration:

The Author does not have any compelling interest for writing this research article.

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