

Weak and Strong Warm Logamediate Anisotropic Inflationary Universe Model

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This paper is given to the investigation of warm inflation using Modified Chaplygin gas in the background of locally rotationally symmetric Bianchi Identity type I. We find out the field equations and perturbations parameters such as; scalar power spectrum, scalar spectral index, scalar potential and tensor to scalar ratio under slow roll approximation. We find out these parameters in directional of Hubble parameter during the Logamediate inflationary regime in weak and strong case. These cosmological parameters shows that the anisotropic model is also compatible WMAP7 with recent observational data Planck 2018.

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I. INTRODUCTION

The standard universe model (hot big-bang cosmology) successfully explains the observations of cosmic microwave background (CMBR) but there are still some unresolved issues i.e origin of fluctuations, Horizon, Flatness and magnetic monopole. Inflation is very successfully resolved the theoretical and paradigm in addressing the shortcomings of standard model issues [1–8]. Scalar field as a primary ingredient of inflation provides the causal interpretation of the origin of Large scale structure (LSS) distribution and observed anisotropy of CMB [9, 10]. Inflationary standard models are classified into slow-roll and reheating epochs. In slow-roll period, potential energy dominates kinetic energy and all interactions between scalar (inflatons) and other fields are neglected, hence the universe inflates [11]. Subsequently, the universe enters into reheating period where the kinetic energy is comparable to potential energy. Thus, the inflaton starts an oscillation about minimum of its potential losing their energy to other fields that present in the theory [12]. After this epoch, the universe is filled with radiation. According to the current universe, the cold inflation is the ending stage of the inflating universe as compare to the warm inflation [13, 14]. The warm inflation is only a way that thermal radiation production and reheating epoch. The formation of Large scale structure (LSS) and also formation of initial fluctuation can be production of constant density by the thermal fluctuations can become the affects of dissipation. The Hubble parameter is also less then as comparative to decay rates, According to the process of microscope the thermalized particles can be produced. The radiation dominated phase is easily enters into the universe, when the inflating era can be stopped. Finally, the remaining matter particles is produced [15, 16]. In the scenario of warm inflation discussed many points in [17]. The motivation of warm inflation is completely different as compare to their result. In scenario of inflation era, the dissipative effects could be lead to a friction term in the equation of motion and also described the dissipative coefficient. In case of low, high and constant temperature regime particularly described in dissipation coefficient [18–29]. The dissipation coefficient is discuss the two cases weak $R \ll 1$ and strong $R \gg 1$ [30, 31]. In scenario of warm inflation era, the general form of dissipation coefficient can be written as;

$$\Gamma(T, \varphi) = C_\varphi \frac{T^z}{\varphi^{z-1}}$$

where T is the temperature of the thermal bath, φ is the scalar field, C_φ is a dissipation microscopic dynamics and z ia an integer term for the different specific values s.t $z = 3, 1, 0, -1$ for a low, high and constant temperature. The

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value of $z = 1$ is represent the high temperature (SUSY-case) $\Gamma \propto T$ and for $z = 0$ leads to normal temperature (exponentially decaying propagator in the SUSY case) is $\Gamma \propto \varphi$ and for $z = -1$ non SUSY case leads to decay rates is $\Gamma \propto \frac{\varphi^2}{T}$ and for $z = 3$ the most common form $\Gamma \propto \frac{T^3}{\varphi^2}$ leads to a most common form for considering logamediate model [32]-[35] According to the condition of warm inflation, its can be existence of thermal radiation and temperature $T \gg H$. The thermal fluctuations and quantum is proportional to T and H . According to chaplygin gas with exotic equation of state and with negative pressure can be described by,

$$\rho_{cg} = -\frac{\tilde{\chi}}{\rho_{cg}},$$

and

$$\rho_{g_{cg}} = -\frac{\tilde{\chi}}{\rho_{g_{cg}}^\lambda}.$$

This equation can be extended in the form of generalized chaplygin gas and $-\tilde{\chi}$ and λ is a constant parameter. For the value of $\lambda = 1$ thus it is converted to the original chaplygin gas. However, The chaplygin gas is also converted in the form modification of chaplygin gas by the equation of state,

$$p_{mcg} = \tilde{\zeta} \rho_{mcg} - \frac{\tilde{\chi}}{\rho_{mcg}^\lambda}. \quad (\text{I.1})$$

This paper is investigated can be obtained by, we discuss section 2 the basic formalism of warm inflation in the view of MCG. In next two sections 3,4 we discuss the weak and strong regime in the scenario of MCCG and also find out explicit expressions of inflaton and rate of decay as well as perturbation parameters s.t scale factor, tensor to scalar ratio, scalar power spectrum spectral index and also discuss the graphical behavior by the constraints of observational recent planck data 2018. In last section, section 5 we summarized the result.

II. MODIFIED CHAPLYGIN GAS INSPIRED INFLATION

In this section, we discuss the general form of modified chaplygin gas in the view of dissipative coefficient for the inflaton decay rate Γ and we have formalism of equation of state in scenario of MCCG;

$$p_{mcg} = \tilde{\zeta} \rho_{mcg} - \frac{\tilde{\chi}}{\rho_{mcg}^\lambda}, \quad (\text{II.1})$$

Where $\tilde{\zeta}$ and $\tilde{\chi}$ are two constant parameter and $0 \leq \lambda \leq 1$. Particularly, where P_{mcg} is represent the pressure and ρ_{mcg} is a energy density of chaplygin gas. We find the energy density of chaplygin gas according to equation of stress energy and use the scale factor a ,

$$\begin{aligned} \rho_{mcg} &= \left(\frac{\tilde{\chi}}{\tilde{\zeta} + 1} + \frac{\tilde{\zeta}}{\zeta^{(\tilde{m}+2)(\lambda+1)(\tilde{\zeta}+1)}} \right)^{\frac{1}{\lambda+1}} \\ &= \rho_{mcg0} \left(\tilde{\chi}_s + \frac{1 - \tilde{\chi}_s}{\zeta^{(\tilde{m}+2)(\tilde{\zeta}+1)(\lambda+1)}} \right)^{\frac{1}{\lambda+1}}, \end{aligned} \quad (\text{II.2})$$

Where $\tilde{\chi}_s = \frac{\tilde{\chi}}{\tilde{\zeta} + 1} \frac{1}{\rho_{mcg0}^{\lambda+1}}$. According to equation (2.2), we introduce some parameters $\tilde{\chi}_s$, $\tilde{\zeta}$ and λ and $\tilde{\zeta}$ are positive integration constant. specially, we use differential age of old galaxies such that oscillation peak parameter, Baryonic acoustic, SN Ia data and growth index for the different and specific values (for best-fit) are obtained by $\tilde{\chi}_s = 0.8252$, $\tilde{\zeta} = 0.0046$ and $\lambda = 0.1905$. The contribution of two equations energy density of mater $\rho_{\tilde{m}}$ and and energy density of radiation field ρ_φ in background of inflation,

$$\left(\frac{\tilde{\chi}}{\tilde{\zeta} + 1} + \rho_{\tilde{m}}^{(\lambda+\lambda\tilde{\zeta}+\tilde{\zeta}+1)} \right)^{\frac{1}{\lambda+1}} \rightarrow \left(\frac{\tilde{\chi}}{\tilde{\zeta} + 1} + \rho_\varphi^{(\lambda+\lambda\tilde{\zeta}+1+\tilde{\zeta})} \right)^{\frac{1}{\lambda+1}}. \quad (\text{II.3})$$

Consider a universe is a flat then the radiation field and inflation field φ are self interacting then we have written in the form of Friedmann equation as follows,

$$H_2^2 = \frac{\kappa}{1+2\tilde{m}} \left(\left[\frac{\tilde{\chi}}{1+\tilde{\zeta}} + \rho_\varphi^{(1+\lambda)(\tilde{\zeta}+1)} \right]^{\frac{1}{1+\lambda}} + \rho_\gamma \right), \quad (\text{II.4})$$

where H is known as a Hubble parameter is define $H = \frac{\dot{a}}{a}$ and $\kappa = 8\pi G$. According to modified Friedmann equation and we suppose that inflation field φ and radiation field in the scenario of flat universe, yielding

$$\dot{\rho}_\varphi + (\tilde{m} + 2)(\rho_\varphi + P_\varphi) = -\Gamma \dot{\varphi}^2 \Rightarrow \dot{\rho}_\varphi + (\tilde{m} + 2)\dot{\varphi} + \dot{V} = -\Gamma \dot{\varphi}^2, \quad (\text{II.5})$$

and

$$\dot{\rho}_\gamma + \frac{4}{3}(\tilde{m} + 2)H_2\rho_\gamma = \Gamma \dot{\varphi}^2, \quad (\text{II.6})$$

$$\tilde{\rho}_\varphi = \frac{\dot{\varphi}^2}{2} + V(\varphi) \quad (\text{II.7})$$

$$\tilde{P}_\varphi = \frac{\dot{\varphi}^2}{2} - V(\varphi) \quad (\text{II.8})$$

Here ρ_φ is known as the energy density and P_φ is known as the pressure, both the function are related to the same field. Where similarly term is, consequently, $V(\varphi)$ is a scalar potential. The condition of energy density of radiation field is $\rho_\varphi \gg \rho_\gamma$ then Eq.(2.4)

$$\begin{aligned} H_2^2 &\approx \frac{\kappa}{1+2\tilde{m}} \left(\left[\frac{\tilde{\chi}}{1+\tilde{\zeta}} + \rho_\varphi^{(1+\lambda+\lambda\tilde{\zeta}+\tilde{\zeta})} \right]^{\frac{1}{1+\lambda}} \right) \\ &= \frac{\kappa}{2\tilde{m}+1} \left[\frac{\tilde{\chi}}{\tilde{\zeta}+1} + \left(V(\varphi) + \frac{\dot{\varphi}^2}{2} \right)^{(\lambda+\lambda\tilde{\zeta}+1+\tilde{\zeta})} \right]^{\frac{1}{1+\lambda}}. \end{aligned} \quad (\text{II.9})$$

By solving these equations in terms of field Eqs.(2.5) and (2.9),

$$\begin{aligned} \dot{\varphi}^2 &= \frac{(2+4\tilde{m})(-\dot{H}_2)}{\kappa(\tilde{m}+2\tilde{\zeta}+\tilde{m}\tilde{\zeta}+2)(1+R)} \left[\frac{(1+2\tilde{m})H_2^2}{\kappa} \right]^{\frac{-\tilde{\zeta}}{1+\tilde{\zeta}}} \\ &\times \left[1 - \frac{\tilde{\chi}}{\tilde{\zeta}+1} \left(\frac{(1+2\tilde{m})H_2^2}{\kappa} \right)^{-(\lambda+1)} \right]^{\left(\frac{-\tilde{\zeta}-\lambda\tilde{\zeta}-\tilde{\zeta}}{(\tilde{\zeta}+1+\lambda\tilde{\zeta}+\lambda)} \right)}, \end{aligned} \quad (\text{II.10})$$

we characterized a new parameter as follows,

$$R \equiv \frac{\Gamma}{(\tilde{m}+2)(H_2)}. \quad (\text{II.11})$$

According to this condition $\dot{\rho}_\gamma \ll \frac{4}{3}(\tilde{m}+2)H_2\rho_\gamma$, by combining Eq. (2.6) and (2.10)

$$\begin{aligned} \tilde{\rho}_\gamma &= \frac{3\Gamma\dot{\varphi}^2}{4(\tilde{m}+2)H_2} = \frac{3\Gamma(-\dot{H}_2)}{2\kappa(H_2)(1+\tilde{\zeta})(1+R)} \\ &\times \left[\frac{(2\tilde{m}+1)H_2^2}{\kappa} \right]^{\frac{-\tilde{\zeta}}{1+\tilde{\zeta}}} \left[\frac{(2\tilde{m}+1)}{(\tilde{m}+2)^2} \right] \\ &\times \left[1 - \frac{\tilde{\chi}}{\tilde{\zeta}+1} \left(\frac{(1+2\tilde{m})H_2^2}{\kappa} \right)^{-(1+\lambda)} \right]^{\left(\frac{-\tilde{\zeta}-\lambda\tilde{\zeta}-\tilde{\zeta}}{(\tilde{\zeta}+1+\lambda\tilde{\zeta}+\lambda)} \right)}, \end{aligned} \quad (\text{II.12})$$

Particularly, the thermalized energy density is known as $\rho_\gamma = C_\gamma T^4$. According to Minimal Super symmetric standard model (MSSM), $C_\gamma = \frac{\Pi^2 g_*}{30}$, $g_* = 228.75$ and $C_\gamma = 70$ [36], by solving Eq.(2.12)

$$T = \left[\frac{3\Gamma(-\dot{H}_2)(1+2\tilde{m})}{2\kappa H_2 C_\gamma (\tilde{m}+2)^2 (1+\tilde{\zeta})(1+R)} \right]^{\frac{1}{4}} \left[\frac{(1+2\tilde{m})H_2^2}{\kappa} \right]^{\frac{-\tilde{\zeta}}{(4\tilde{\zeta}+4)}} \times \left[1 - \frac{\tilde{\chi}}{\tilde{\zeta}+1} \left(\frac{(1+2\tilde{m})H_2^2}{\kappa} \right)^{-(\lambda+1)} \right]^{\frac{-\tilde{\zeta}-\lambda\tilde{\zeta}-\lambda}{(4+4\tilde{\zeta}+4\lambda\tilde{\zeta}+4\lambda)}}, \quad (\text{II.13})$$

By Considering Eqs. (2.9), (2.10) and (2.13) ,

$$V = \left(\frac{(1+2\tilde{m})H_2^2}{\kappa} - \frac{\tilde{\chi}}{1+\tilde{\zeta}} \right)^{\frac{1}{(1+\tilde{\zeta}+\lambda+\lambda\tilde{\zeta})}} + \frac{\dot{H}_2}{\kappa(1+\tilde{\zeta})(1+R)} \times \frac{(1+2\tilde{m})}{(\tilde{m}+2)} \left[\frac{(1+2\tilde{m})(H_2^2)}{\kappa} \right]^{\frac{-\tilde{\zeta}}{\tilde{\zeta}+1}} \times \left[1 - \frac{\tilde{\chi}}{\tilde{\zeta}+1} \left(\frac{(2\tilde{m}+1)H_2^2}{\kappa} \right)^{-(1+\lambda)} \right]^{-\frac{(\tilde{\zeta}+\lambda(1+\tilde{\zeta}))}{(1+\tilde{\zeta})(1+\lambda)}}, \quad (\text{II.14})$$

The dissipative coefficient can be obtained as,

$$\Gamma^{\frac{4-z}{4}} = \left[\frac{3(1+2\tilde{m})(-\dot{H}_2)}{2\kappa(C_\gamma)H_2(\tilde{m}+2)^2(1+\tilde{\zeta})(1+R)} \right]^{\frac{z}{4}} \left[\frac{(1+2\tilde{m})(H_2^2)}{\kappa} \right]^{\frac{-z\tilde{\zeta}}{4(1+\tilde{\zeta})}} \times C_\varphi \varphi^{1-z} \left[1 - \frac{\tilde{\chi}}{1+\tilde{\zeta}} \left(\frac{(1+2\tilde{m})H_2^2}{\kappa} \right)^{-(\lambda+1)} \right]^{-\frac{z(\tilde{\zeta}+\lambda(1+\tilde{\zeta}))}{(\tilde{\zeta}+1)(4+4\lambda)}}, \quad (\text{II.15})$$

In this era, the scalae factor of logamediate inflationary model is given by,

$$a(t) = \exp^{f(\ln t)^g} \quad f > 0 \text{ and } g > 1 \quad (\text{II.16})$$

where g and f are two dimensionless constant parameter [37]. The next three sections, we will discuss the two cases weak dissipative regime and strong dissipative regime.

III. THE WEAK DISSIPATIVE REGIME

In this case, the weak ($R \ll 1$) function can be converted in number of scalar field's φ , by using Eq. (2.10) and (2.16), we get

$$\varphi(t) - \varphi_0 = \frac{\tilde{\tau}[t]}{\tilde{\omega}}, \quad (\text{III.1})$$

Where φ_0 is a constant term of this integration , and $\tilde{\omega}$ is a constant term and $\tilde{\tau}[t]$ is a function of comic time is given by, (with condition $\varphi_0 = 0$)

$$\tilde{\omega} = \sqrt{2}(1+\tilde{\zeta})^{\frac{3}{2}} \sqrt{(\tilde{m}+2)} \left(\frac{1}{f*g} \right)^{\frac{1-\tilde{\zeta}}{2(1+\tilde{\zeta})}} \left(\frac{\kappa}{1+2\tilde{m}} \right)^{\frac{1}{2(1+\tilde{\zeta})}}$$

$$\begin{aligned}
\tilde{\tau}[t] = & \frac{2(1+\tilde{\zeta})^2 \text{Gamma} \left[\frac{1+3\tilde{\zeta}+g-\tilde{\zeta}g}{2(1+\tilde{\zeta})}, \frac{-\tilde{\zeta} \text{Log}[t]}{1+\tilde{\zeta}} \right] (\ln[t])^{\frac{(1-\tilde{\zeta})(1-g)}{2(1+\tilde{\zeta})}}}{\tilde{\zeta}} \\
& \times \left(\frac{-\tilde{\chi} t^{\frac{-2(1+\tilde{\zeta})}{(1+\tilde{\zeta})(1+\lambda)}} (\tilde{\zeta} + \lambda + \tilde{\zeta}\lambda)}{(1+\tilde{\zeta})(1+\lambda)} \right) \\
& \times \text{Gamma} \left[1 - \frac{(-1+g)(3+4\lambda+\tilde{\zeta}(5+4\lambda))}{2(1+\tilde{\zeta})} \right] \\
& - \frac{(2+3\tilde{\zeta}+2\lambda+2\tilde{\zeta}\lambda) \text{Log}[t]}{2(1+\tilde{\zeta})} (\ln[t])^{\frac{1+3\tilde{\zeta}+g-\tilde{\zeta}g}{2+2\tilde{\zeta}}} \\
& \times \left(\frac{-(2+3\tilde{\zeta}+2\lambda+2\tilde{\zeta}\lambda) \text{Log}[t]}{1+\tilde{\zeta}} \right)^{-1+\frac{(-1+g)(3+4\lambda+\tilde{\zeta}(5+4\lambda))}{2(1+\tilde{\zeta})}}
\end{aligned}$$

Under the slow slow roll approximation, we formulate the Gamma function and also we find out the scalar potential in terms of scalar field φ by using $\frac{\dot{\varphi}^2}{2} < V(\varphi)$, we get

$$V(\varphi) \approx \left[\left(\frac{f^2(2\tilde{m}+1)g^2}{\kappa((\tilde{\tau}^{-1}[\tilde{\omega}\varphi])^2(\ln(\tilde{\tau}^{-1}[\tilde{\omega}\varphi]))^{(2-2g)})} \right)^{1+\lambda} - \frac{\tilde{\chi}}{\tilde{\zeta}+1} \right]^{\frac{1}{(1+\tilde{\zeta})(1+\lambda)}}, \quad (\text{III.2})$$

we can written as constant dissipative coefficient in terms of φ as follows,

$$\begin{aligned}
\Gamma(\varphi) = & \left[\frac{3(1+2\tilde{m})}{\kappa(C_r)(2+2\tilde{\zeta})(\tilde{m}+2)^2(\ln(\tilde{\tau}^{-1}[\tilde{\omega}\varphi]))} \right]^{\frac{z}{4-z}} \\
& \times \left(\frac{(1+2\tilde{m})f^2g^2}{\kappa((\tilde{\tau}^{-1}[\tilde{\omega}\varphi])^2(\ln(\tilde{\tau}^{-1}[\tilde{\omega}\varphi]))^{2(1-g)})} \right)^{\frac{-z\tilde{\zeta}}{(1+\tilde{\zeta})(4-z)}} C_{\varphi}^{\frac{4}{4-z}} \varphi^{\frac{4(1-z)}{4-z}} \\
& \times \left[1 - \frac{\tilde{\chi}}{1+\omega} \left(\frac{\kappa((\tilde{\tau}^{-1}[\tilde{\omega}\varphi])^2(\ln(\tilde{\tau}^{-1}[\tilde{\omega}\varphi]))^{2(1-g)})}{(1+2\tilde{m})f^2g^2} \right)^{1+\lambda} \right]^{\frac{-z(\tilde{\zeta}+\lambda(1+\tilde{\zeta}))}{(1+\tilde{\zeta})(1+\lambda)(4-z)}}, \quad (\text{III.3})
\end{aligned}$$

In a cosmological time, the number of e-folds N is interpolated between two different time initial t_1 and final t_2 as follows,

$$\begin{aligned}
N = & \left(\frac{\tilde{m}+2}{3} \right) \int_{t_1}^{t_2} H_2 dt \\
= & \frac{f(\tilde{m}+2)}{3} \left[(\ln(\tilde{\tau}^{-1}[\tilde{\omega}\varphi_2]))^g - (\ln(\tilde{\tau}^{-1}[\tilde{\omega}\varphi_1]))^g \right], \quad (\text{III.4})
\end{aligned}$$

According to inflationary scenerio, anisotropic model can be proposed by [38, 39]. Slow roll parameter determine's the degree of the anisotropy. Anisotropy during inflation cannot be completed neglected because slow roll parameter is factual known as order of a percent. Dimensionless slow roll parameter can be expressed ϵ and η by [40]. These parameters presented in function of φ defined as,

$$\epsilon = - \left(\frac{3}{\tilde{m}+2} \right) \frac{\dot{H}_2}{H_2^2}, \quad (\text{III.5})$$

and

$$\eta = - \left(\frac{3}{\tilde{m}+2} \right) \frac{\ddot{H}_2}{\dot{H}_2 H_2}, \quad (\text{III.6})$$

In inflationary scenario in terms of scalar filed slow roll parameter, it is a nearest of early and possible stage of $\epsilon = 1$,

$$\varphi_1 = \frac{1}{\omega} \left[\tau \exp \left(\frac{3}{\tilde{m}+2} \frac{1}{fg} \right)^{\frac{1}{g-1}} \right], \quad (\text{III.7})$$

There are four ways in which scalar perturbations can be represented i.e scalar spectral indices and scalar tensor power spectra n_s, n_T and $P_R[k], P_T[k]$ According to standard scalar field, the scalar density perturbations can be written as in the form of of $P_R^{\frac{1}{2}} = (\frac{\tilde{m}+2}{3}) \frac{H_2}{\dot{\varphi}} \delta\varphi$ is derive by $\delta\varphi^2 \simeq (\frac{m+2}{3}) H_2 T$ [41]-[43]. The scalar power spectrum is obtained by Eqs. (2.10) ,(2.13) and (2.15)

$$P_R = F_1 \varphi^{\frac{1-z}{4-z}} t^{\frac{(3z-11)(1+\tilde{\zeta})+2\tilde{\zeta}(z-3)+2(3-z)(1+\tilde{\zeta})}{(1+\tilde{\zeta})(4-z)}} (\ln[t])^{\frac{(g-1)[(11-3z)(1+\tilde{\zeta})+2\tilde{\zeta}(3-z)]+(g-1)(z-3)(1+\tilde{\zeta})}{(1+\tilde{\zeta})(4-z)}} \times \left[1 - \frac{\tilde{\chi}}{1+\tilde{\zeta}} \left(\frac{(1+2\tilde{m})H_2^2}{\kappa} \right)^{-(\lambda+1)} \right]^{\frac{(3-z)(\tilde{\zeta}+\lambda(1+\tilde{\zeta}))}{(\tilde{\zeta}+1)(4-z)(1+\lambda)}}, \quad (\text{III.8})$$

This function is converted in the form of scalar field φ ,

$$P_R = F_1 \varphi^{\frac{1-z}{4-z}} (\tilde{\tau}^{-1}[\tilde{\omega}\varphi_2])^{\frac{(3z-11)(1+\tilde{\zeta})+2\tilde{\zeta}(z-3)+2(3-z)(1+\tilde{\zeta})}{(1+\tilde{\zeta})(4-z)}} \times (\ln(\tilde{\tau}^{-1}[\tilde{\omega}\varphi_2]))^{\frac{(g-1)[(11-3z)(1+\tilde{\zeta})+2\tilde{\zeta}(3-z)]+(g-1)(1+\tilde{\zeta})(z-3)}{(1+\tilde{\zeta})(4-z)}} \times \left[1 - \frac{\tilde{\chi}}{1+\tilde{\zeta}} \left(\frac{(1+2\tilde{m})H_2^2}{\kappa} \right)^{-(\lambda+1)} \right]^{\frac{(3-z)(\tilde{\zeta}+\lambda(1+\tilde{\zeta}))}{(\tilde{\zeta}+1)(4-z)(1+\lambda)}}, \quad (\text{III.9})$$

where F_1 is a constant term then,

$$F_1 = (\frac{m+2}{3})^3 \left(\frac{\kappa(1+\tilde{\zeta})(\tilde{m}+2)}{2(1+2\tilde{m})} \right) \left(\frac{3(1+2\tilde{m})C_\varphi}{2\kappa C_\gamma (\tilde{m}+2)^2(1+\tilde{\zeta})} \right)^{\frac{1}{4-z}} \times \left[\frac{1+2\tilde{m}}{\kappa} \right]^{\frac{(3-z)\tilde{\zeta}}{(4-z)(1+\tilde{\zeta})}} (fg)^{\frac{(11-3z)(1+\tilde{\zeta})+2\tilde{\zeta}(3-z)+(z-3)(1+\tilde{\zeta})}{(1+\tilde{\zeta})(4-z)}} P_R = F_2(\tilde{\tau}(G(N)))^{\frac{1-z}{4-z}} (G[N])^{\frac{(3z-11)(1+\tilde{\zeta})+2\tilde{\zeta}(z-3)+2(3-z)}{(1+\tilde{\zeta})(4-z)}} \times (\ln(G[N]))^{\frac{(g-1)[(11-3z)(1+\tilde{\zeta})+2\tilde{\zeta}(3-z)]+(g-1)(z-3)}{(1+\tilde{\zeta})(4-z)}} \times \left[1 - \frac{\tilde{\chi}}{1+\tilde{\zeta}} \left(\frac{(1+2\tilde{m})H_2^2}{\kappa} \right)^{-(\lambda+1)} \right]^{\frac{(3-z)(\tilde{\zeta}+\lambda(1+\tilde{\zeta}))}{(\tilde{\zeta}+1)(4-z)(1+\lambda)}}, \quad (\text{III.10})$$

where δ_2 and $G[N]$ are defined by $F_2 = F_1 \omega^{\frac{z-1}{4-z}}$ and $G[N] = [\exp(\frac{3N}{g(\tilde{m}+2)} + (\frac{3}{\tilde{m}+2})fg)^{\frac{g}{g-1}}]^{\frac{1}{g}}$. The scalar spectral index n_s is defined as $n_s = 1 + \frac{d \ln P_R}{d \ln \kappa}$, and by combining Eqn. (3.1) and (3.11), we get

$$n_s = 1 + \frac{(3z-11)(1+\tilde{\zeta})+2(1+\tilde{\zeta})(3-z)+2\tilde{\zeta}(3-z)}{fg(1+\tilde{\zeta})(4-z)(\ln[t])^{g-1}} + \frac{(g-1)[(11-3z)(1+\tilde{\zeta})+2\tilde{\zeta}(3-z)+(z-3)(1+\tilde{\zeta})](g-1)}{fg(1+\tilde{\zeta})(4-z)(\ln[t])^g} + n_2 + n_3, \quad (\text{III.11})$$

Where n_2 and n_3 are terms of above equation can be obtained by

$$n_2 = \frac{1-z}{4-z} \sqrt{\frac{2(1+2\tilde{m})}{fg\kappa(\tilde{m}+2)(1+\tilde{\zeta})}} \left(\frac{(1+2\tilde{m})(fg)^2}{\kappa} \right)^{\frac{-\tilde{\zeta}}{1+\tilde{\zeta}}} \times (\ln[t])^{\frac{(1-g)(1+3\tilde{\zeta})}{2}} t^{\frac{-\tilde{\zeta}}{1+\tilde{\zeta}}} \times \left[1 - \frac{\tilde{\chi}}{1+\tilde{\zeta}} \left(\frac{(1+2\tilde{m})f^2g^2(\ln[t])^{2(g-1)}}{\kappa t^2} \right)^{-(\lambda+1)} \right]^{\frac{-(\tilde{\zeta}+\lambda(1+\tilde{\zeta}))}{2(\tilde{\zeta}+1)(1+\lambda)}}$$

and

$$n_3 = \left(\frac{-2\tilde{\chi}}{(1+\tilde{\zeta})^2} \right) \left(\frac{3-z}{4-z} \right) (\tilde{\zeta} + \lambda(1+\tilde{\zeta})) \frac{(\kappa/(2\tilde{m}+1))^{1+\lambda}}{(\tilde{\alpha}g)^{3+2\lambda}} \\ \times \left(\frac{1}{\ln[t]} \right)^{(3+2\lambda)(g-1)} (t)^{2(1+\lambda)} \left[1 - \frac{\tilde{\chi}}{1+\tilde{\zeta}} \left(\frac{\kappa(t)^{2(1-g)}}{(2\tilde{m}+1)\tilde{f}^2g^2} \right)^{1+\lambda} \right]^{-1}$$

The scalar spectral index in the form of N

$$n_s = 1 + \frac{(3z-11)(1+\tilde{\zeta}) + 2(1+\tilde{\zeta})(3-z) + 2\tilde{\zeta}(3-z)}{fg(1+\tilde{\zeta})(4-z)(\ln(G[N]))^{g-1}} \\ + \frac{(g-1)[(11-3z)(1+\tilde{\zeta}) + 2\tilde{\zeta}(3-z) + (z-3)(1+\tilde{\zeta})](g-1)}{fg(1+\tilde{\zeta})(4-z)(\ln(G[N]))^g} \\ + n_2 + n_3, \quad (\text{III.12})$$

Where n_2 and n_3 are defined as

$$n_2 = \frac{1-z}{4-z} \sqrt{\frac{2(1+2\tilde{m})}{fg\kappa(\tilde{m}+2)(1+\tilde{\zeta})}} \left(\frac{(1+2\tilde{m})(fg)^2}{\kappa} \right)^{\frac{-\tilde{\zeta}}{1+\tilde{\zeta}}} \\ \times (\ln(G[N]))^{\frac{(1-g)(1+3\tilde{\zeta})}{2}} (G(N))^{\frac{-\tilde{\zeta}}{1+\tilde{\zeta}}} \\ \times \left[1 - \frac{\tilde{\chi}}{1+\tilde{\zeta}} \left(\frac{(1+2\tilde{m})f^2g^2(\ln(G[N]))^{2(g-1)}}{\kappa(G[N])^2} \right)^{-(\lambda+1)} \right]^{\frac{-(\tilde{\zeta}+\lambda(1+\tilde{\zeta}))}{2(\tilde{\zeta}+1)(1+\lambda)}}$$

and

$$n_3 = \left(\frac{-2\tilde{\chi}}{(1+\tilde{\zeta})^2} \right) \left(\frac{3-z}{4-z} \right) (\tilde{\zeta} + \lambda(1+\tilde{\zeta})) \frac{(\kappa/(2\tilde{m}+1))^{1+\lambda}}{(fg)^{3+2\lambda}} \\ \times \left(\frac{1}{\ln(G[N])} \right)^{(3+2\lambda)(g-1)} (G[N])^{2(1+\lambda)} \\ \times \left[1 - \frac{\tilde{\chi}}{1+\tilde{\zeta}} \left(\frac{\kappa(\ln(G[N]))^{2(1-g)}}{(2\tilde{m}+1)\tilde{f}^2g^2} \right)^{1+\lambda} \right]^{-1}$$

The tensor perturbations is written as in form of standard scalar inflation Ref.([44]), we may compute the tensor to scalar ratio is $r = \frac{P_g}{P_R}$, we obtained,

$$P_g = 8\kappa \left(\frac{H_2}{2\pi} \right)^2 \left(\frac{\tilde{m}+2}{3} \right), \quad (\text{III.13})$$

$$r(\varphi) = \frac{2(\tilde{m}+2)\kappa f^2 g^2}{3\pi^2 F_1} \varphi^{\frac{\tilde{\zeta}-1}{4-z}} t^{\frac{(11-3z)(1+\tilde{\zeta})+2(1+\tilde{\zeta})(3-z)+2\tilde{\zeta}(z-3)+2(1+\tilde{\zeta})(4-z)}{(1+\tilde{\zeta})(4-z)}} \\ \times (\ln[t])^{\frac{(1-g)[(11-3z)(1+\tilde{\zeta})+2\tilde{\zeta}(z-3)+(3-z)(1+\tilde{\zeta})+2(4-z)(1+\tilde{\zeta})]}{(1+\tilde{\zeta})(4-z)}} \\ \times \left[1 - \frac{\tilde{\chi}}{1+\tilde{\zeta}} \left(\frac{\kappa t^2 (\ln[t])^{2(1-g)}}{(2\tilde{m}+1)\tilde{f}^2g^2} \right)^{1+\lambda} \right]^{\frac{(z-3)(\tilde{\zeta}+\lambda(1+\tilde{\zeta}))}{(1+\tilde{\zeta})(4-z)(1+\lambda)}} \quad (\text{III.14})$$

Tensor to scalar ratio in terms of number of e-folds N ,

$$\begin{aligned}
 r = & \frac{2(\tilde{m} + 2)\kappa f^2 g^2}{3\pi^2 F_1} (\tilde{\tau}(G[N]))^{\frac{z-1}{4-z}} (G[N])^{\frac{(1-3z)(1+\tilde{\zeta})+2(1+\tilde{\zeta})(3-z)+2\tilde{\zeta}(z-3)+2(1+\tilde{\zeta})(4-z)}{(1+\tilde{\zeta})(4-z)}} \\
 & \times (\ln(G[N]))^{\frac{(1-g)(11-3z)(1+\tilde{\zeta})+2\tilde{\zeta}(z-3)+(3-z)(1+\tilde{\zeta})+2(4-z)(1+\tilde{\zeta})}{(1+\tilde{\zeta})(4-z)}} \\
 & \times \left[1 - \frac{\tilde{\chi}}{1 + \tilde{\zeta}} \left(\frac{\kappa(G[N])^2 (\ln(G[N]))^{2(1-g)}}{(2\tilde{m} + 1)\tilde{f}^2 g^2} \right)^{1+\lambda} \right]^{\frac{(z-3)(\tilde{\zeta}+\lambda(1+\tilde{\zeta}))}{(1+\tilde{\zeta})(4-z)(1+\lambda)}} \quad (III.15)
 \end{aligned}$$

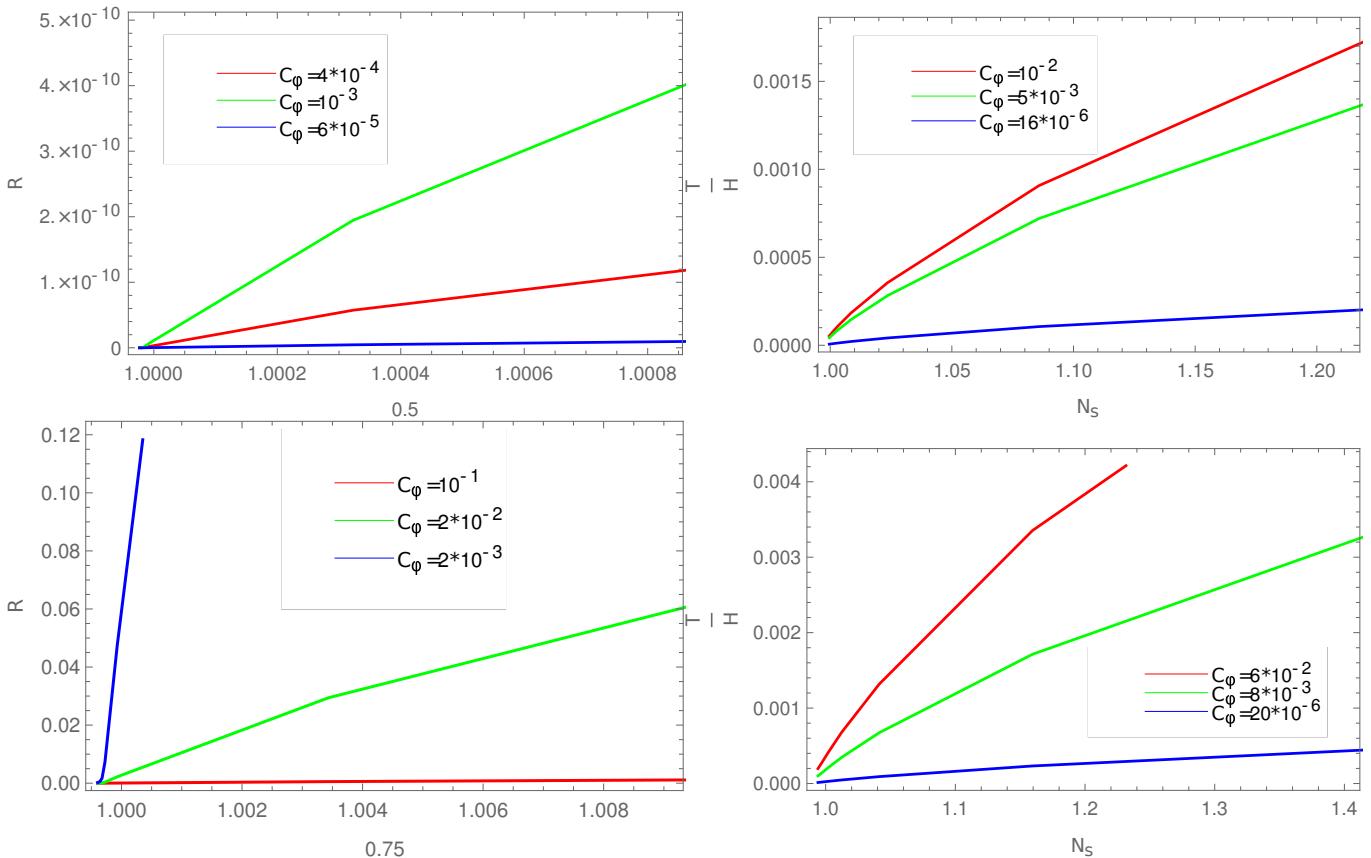


Figure 1,2,3,4 plot of left R versus n_s and right plot T versus n_s $\tilde{\zeta} = 0.0046$, $\tilde{\chi} = 0.8289$, $\tilde{m} = 1$, $g = 15$, $f = 0.9805$, $\kappa = 1$

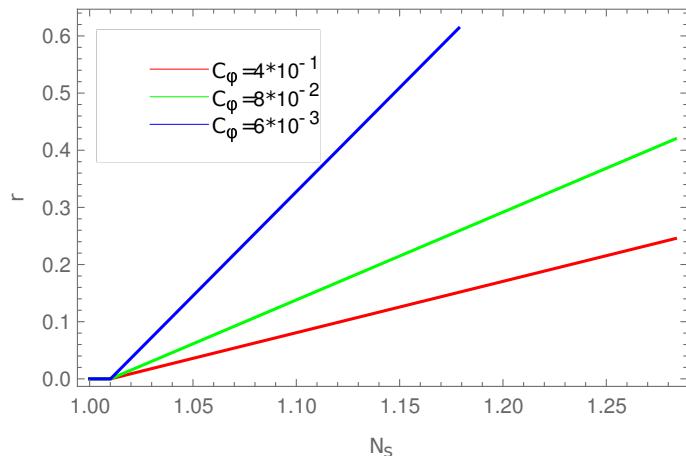


Figure 5 plot of n_s versus r $\tilde{\zeta} = 0.0046$, $\tilde{\chi} = 0.8289$, $\tilde{m} = 1$, $g = 15$, $f = 0.9805$, $\kappa = 1$

IV. THE STRONG DISSIPATIVE REGIME

In this section, we analyze the strong dissipative regime $\Gamma \ll (\tilde{m} + 2)H_2$ in the scenario of inflationary model and discuss two special cases s.t $z = 3$ and $z \neq 3$. We formulated the solution of scalar field as the function of cosmic time by combining Eq. (2.10) and (2.15),

$$\check{\varphi}(t) - \check{\varphi}_0 = \exp\left(\frac{\check{\tau}[t]}{\check{\omega}}\right) \quad (\text{IV.1})$$

The term $\check{\omega}$ and function $\check{\tau}[t]$ are defined as,

$$\begin{aligned} \check{\tau} &= \frac{2^{\frac{1}{8}} \times 3^{\frac{3}{8}}}{(\tilde{m} + 2)^{\frac{-3}{8}} \times C_{\varphi}^{\frac{-1}{2}} \times C_{\gamma}^{\frac{-3}{8}}} \left(\frac{1}{fg}\right)^{\frac{5+3\tilde{\zeta}}{8(1+\tilde{\zeta})}} \left(\frac{\kappa}{1+2\tilde{m}}\right)^{\frac{7+8\tilde{\zeta}}{8(1+\tilde{\zeta})}} \\ \check{\tau}[t] &= (\ln[t])^{\frac{(g-1)(\tilde{\zeta})}{8(1+\tilde{\zeta})}} \frac{-\tilde{\chi}t^{-2(1+\lambda)}(\tilde{\zeta} + \lambda + \tilde{\zeta}\lambda)}{(1+\lambda)(1+\tilde{\zeta})} (2)^{-\frac{(-1+g)(11+16\lambda+\tilde{\zeta}(13+16\lambda))}{4(1+\tilde{\zeta})}} \\ &\times \text{Gamma}[1 - \frac{(-1+g)(11+16\lambda+\tilde{\zeta}(13+16\lambda))}{4(1+\tilde{\zeta})}] \\ &\times \frac{(9+8\lambda+2\tilde{\zeta}(5+4\lambda))\ln[t]}{4(1+\tilde{\zeta})} (\ln[t])^{\frac{3+5\tilde{\zeta}+5g+3\tilde{\zeta}g}{8(1+\tilde{\zeta})}} \\ &\times \left(\frac{-(9+8\lambda+2\tilde{\zeta}(5+4\lambda))\ln[t]}{(1+\tilde{\zeta})}\right)^{-\frac{(-1+g)(11+16\lambda+\tilde{\zeta}(13+16\lambda))}{4(1+\tilde{\zeta})}} \\ &\times \left(\frac{\kappa t^2(\ln[t])^{2(1-g)}}{(fg)^2(1+2\tilde{m})}\right)^{1+\lambda} \\ &+ \left(\frac{8(1+\tilde{\zeta})^2}{(1+2\tilde{\zeta})}\right) (\ln[t])^{\frac{(5+4\tilde{\zeta})(g-1)}{8(1+\tilde{\zeta})}} \left(-\frac{\ln[t] + 2\tilde{\zeta}\ln[t]}{4(1+\tilde{\zeta})}\right)^{-\frac{(5+3\tilde{\zeta})(g-1)}{8(1+\tilde{\zeta})}} \\ &\times \text{Gamma}\left[\frac{(3+5\tilde{\zeta}+5\tilde{\zeta}+3\tilde{\zeta}g)}{8(1+\tilde{\zeta})}, -\frac{(1+2\tilde{\zeta})(\ln[t])}{4(1+\tilde{\zeta})}\right] \end{aligned}$$

The Hubble parameter as function of inflations field for $z = 3$ by using equation's (2.15) and (4.1)

$$H_2(\check{\varphi}) = \frac{fg}{(\check{\tau}^{-1}[\check{\omega} \ln \check{\varphi}]) (\ln(\check{\tau}^{-1}[\check{\omega} \ln \check{\varphi}]))^{1-g}} \quad (\text{IV.2})$$

In strong case, the scalar potential $V(\check{\varphi})$ is given by

$$V(\check{\varphi}) \approx \left[\left(\frac{(1+2\tilde{m})f^2g^2}{\kappa(\check{\tau}^{-1}[\check{\omega} \ln \check{\varphi}])^2 (\ln(\check{\tau}^{-1}[\check{\omega} \ln \check{\varphi}]))^{2(1-g)}} \right)^{1+\lambda} - \frac{\chi}{1+\tilde{\zeta}} \right]^{\frac{1}{(1+\tilde{\zeta})(1+\lambda)}} \quad (\text{IV.3})$$

We analyze the constant dissipative coefficient for special case $z = 3$ by using eqns (2.1) and (4.1), we obtain the result as;

$$\begin{aligned} \Gamma(\check{\varphi}) &= F_3(\tilde{\tau}(G(N)))^{-2} (G(N))^{\frac{-3}{2(1+\tilde{\zeta})}} (\ln(G(N)))^{\frac{3(g-1)(1-\tilde{\zeta})}{4(1+\tilde{\zeta})}} \\ &\times \left[-\frac{\chi}{\zeta+1} \left(\frac{\kappa(G(N))^2(\ln(G(N)))^{(-2g+2)}}{f^2(1+2\tilde{m})g^2} \right)^{1+\lambda} + 1 \right]^{\frac{-3(\zeta+\lambda(1+\zeta))}{4(1+\zeta)(1+\lambda)}} \end{aligned} \quad (\text{IV.4})$$

Where $F_3 = C_{\check{\phi}} \left[\frac{3(1+2\tilde{m})fg}{2\kappa C_\gamma(1+\zeta)(\tilde{m}+2)} \right]^{\frac{3}{4}} \left(\frac{(1+2\tilde{m})f^2g^2}{\kappa} \right)^{\frac{-3\zeta}{4(1+\zeta)}}$ is a constant term. By using eqn (2.1) and (4.1), the interaction between the quantity of e-folds N is found as,

$$\begin{aligned} N &= \left(\frac{\tilde{m}+2}{3} \right) \int_{t_1}^{t_2} H_2 dt \\ &= \frac{(\tilde{m}+2)f}{3} \left[(\check{\tau}^{-1}[\check{\omega} \ln \check{\psi}_2])^g - (\check{\tau}^{-1}[\check{\omega} \ln \check{\psi}_1])^g \right] \end{aligned} \quad (\text{IV.5})$$

According to the early universe, the thermal fluctuations provide the main source of scale density in scenario of inflationary. For the first time, the power spectrum is introduced by [45], when the friction coefficient in the inflation equation of motion depends on temperature. If the constant dissipative coefficient increases with temperature then the always increases the scalar perturbations. We discuss the high temperature case by ([46, 47]). For the strong dissipative regime $R \equiv \frac{\Gamma}{(\tilde{m}+2)H_2} > 1$. Where $\langle \delta\psi \rangle$ is the scalar field fluctuation and can be written as $\langle \delta\psi \rangle^2 \simeq \frac{\kappa_F T}{2\pi^2}$. The new function is known as wave number and it can defined κ_F this function is also will be equal to $\kappa_F = \sqrt{(\frac{\tilde{m}+2}{3})\Gamma H_2} = (\tilde{m}+2)\sqrt{\frac{H_2}{3}}$ after following at these Eqs. (2.1) ,(2.12) and (2.14), this equations will be equal to the scalar perturbation,

$$\begin{aligned} P_R &= \left(\frac{\tilde{m}+2}{3} \right)^{\frac{5}{2}} \frac{H^{\frac{5}{2}} \Gamma^{\frac{1}{2}} T}{2\pi^2 \dot{\check{\psi}}} = \left(\frac{\tilde{m}+2}{3} \right)^{\frac{5}{2}} \frac{\kappa(1+\zeta)C_{\check{\phi}}^{\frac{3}{2}} \phi^{-3}}{4\pi^2(1+2\tilde{m})} \left(\frac{1+2\tilde{m}}{\kappa} \right)^{\frac{-3\zeta}{8(1+\zeta)}} \\ &\times H_2^{\frac{3(2+\zeta)}{4(1+\zeta)}} (-\dot{H}_2)^{\frac{3}{8}} \left[\frac{3(1+2\tilde{m})}{2\kappa C_\gamma(\tilde{m}+2)(1+\zeta)} \right]^{\frac{11}{8}} \\ &\times \left[1 - \frac{\chi}{1+\zeta} \left(\frac{(1+2\tilde{m})H_2^2}{\kappa} \right)^{-(1+\lambda)} \right]^{\frac{-3(\zeta+\lambda(1+\zeta))}{8(1+\zeta)(1+\lambda)}} \end{aligned} \quad (\text{IV.6})$$

Another form of scalar perturbation can be written as in scalar $\check{\phi}$ perturbation $\check{\phi}$;

$$\begin{aligned} P_R &= F_4(\check{\tau}^{-1}[\check{\omega} \ln \check{\phi}])^{-\frac{3(3+2\tilde{\zeta})}{4(1+\tilde{\zeta})}} (\ln(\check{\tau}^{-1}[\check{\omega} \ln \check{\phi}]))^{\frac{3(g-1)(5+3\zeta)}{8(1+\zeta)}} \check{\phi}^{-3} \\ &\times \left[1 - \frac{\chi}{1+\zeta} \left(\frac{\kappa(\check{\tau}^{-1}[\check{\omega} \ln \check{\phi}])^2 (\check{\tau}^{-1}[\check{\omega} \ln \check{\phi}])^{2(1-g)}}{(1+2\tilde{m})f^2g^2} \right)^{1+\lambda} \right]^{\frac{-3(\zeta+\lambda(1+\zeta))}{2(2+2\zeta)(2+2\lambda)}} \end{aligned} \quad (\text{IV.7})$$

where

$$\begin{aligned} F_4 &= \left(\frac{\tilde{m}+2}{3} \right)^{\frac{5}{2}} \frac{\kappa C_{\check{\phi}}^{\frac{3}{2}}(1+\zeta)}{(1+2\tilde{m})4\pi^2} \left(\frac{1+2\tilde{m}}{\kappa} \right)^{\frac{-3\zeta}{8(1+\zeta)}} \\ &\times (fg)^{\frac{2(3(5+3\zeta)}{8(1+\zeta)}} \left[\frac{3(1+2\tilde{m})}{2(\tilde{m}+2)\kappa C_\gamma(1+\zeta)} \right]^{\frac{11}{8}}} \end{aligned}$$

The power spectrum written as a number of e-folds for $z = 3$

$$\begin{aligned} P_R &= F_4(G[N])^{-\frac{3(3+2\tilde{\zeta})}{4(1+\tilde{\zeta})}} (\ln(G[N]))^{\frac{3(g-1)(5+3\zeta)}{8(1+\zeta)}} \exp \left(\frac{-3}{\check{\omega}} (\check{\tau}(G[N])) \right) \\ &\times \left[1 - \frac{\chi}{1+\zeta} \left(\frac{\kappa(G[N])^2 (G[N])^{2(1-g)}}{(1+2\tilde{m})f^2g^2} \right)^{1+\lambda} \right]^{\frac{-3(\zeta+\lambda(1+\zeta))}{4(1+\zeta)(2+2\lambda)}}, \end{aligned} \quad (\text{IV.8})$$

Considering the Eqs. (4.7) and (4.5) ;

$$n_s = 1 - \frac{3(3+2\tilde{\zeta})}{4fg(1+\zeta)(\ln[t])^{g-1}} + \frac{3(g-1)(5+3\zeta)}{8fg(1+\zeta)(\ln[t])^g} + n_1 + n_2 \quad (\text{IV.9})$$

where n_1 and n_2 are the next two terms of spectral index

$$\begin{aligned}
 n_1 = & -3 \left(\frac{2(1+2\tilde{m})}{\kappa(1+\zeta)} \right)^{\frac{1}{2}} \left[\frac{3(1+2\tilde{m})}{2\kappa C_\gamma(\tilde{m}+2)(1+\zeta)} \right]^{-\frac{3}{8}} \frac{(fg)^{\frac{-3-4\tilde{\zeta}}{8(1+\zeta)}}}{C_\varphi^{\frac{1}{2}}} \\
 & \times \left(\frac{(2\tilde{m}+1)}{\kappa} \right)^{\frac{-\zeta}{8(1+\zeta)}} \left(\ln(\tilde{\tau}^{-1}[\check{\omega} \ln \check{\varphi}]) \right)^{\frac{(1-g)(3+4\tilde{\zeta})}{8(1+\zeta)}} \left((\tilde{\tau}^{-1}[\check{\omega} \ln \check{\varphi}])^{\frac{1+2\tilde{\zeta}}{4(1+\zeta)}} \right. \\
 & \times \left. \left[1 - \frac{\chi}{1+\zeta} \left(\frac{(\ln(\tilde{\tau}^{-1}[\check{\omega} \ln \check{\varphi}]))^{2(1-g)} ((\tilde{\tau}^{-1}[\check{\omega} \ln \check{\varphi}])^2 \kappa)}{(1+2\tilde{m}) f^2 g^2} \right) \right]^{-\frac{-(\zeta+\lambda)(1+\zeta)}{8(1+\zeta)(1+\lambda)}} \right) \tag{IV.10}
 \end{aligned}$$

and

$$\begin{aligned}
 n_2 = & \frac{3(\zeta+\lambda(1+\zeta))}{4(1+\zeta)} \left(\frac{\tilde{\chi}}{1+\tilde{\zeta}} \right) \left(\frac{(\frac{\kappa}{1+2\tilde{m}})^{1+\lambda}}{(fg)^{3+2\lambda}} \right) (\tilde{\tau}^{-1}[\check{\omega} \ln \check{\varphi}])^{2(1+\lambda)} \\
 & \times (\ln(\tilde{\tau}^{-1}[\check{\omega} \ln \check{\varphi}]))^{(3+2\lambda)(1-g)} \\
 & \times \left[1 - \frac{\chi}{1+\zeta} \left(\frac{\kappa((\tilde{\tau}^{-1}[\check{\omega} \ln \check{\varphi}])^2 (\tilde{\tau}^{-1}[\check{\omega} \ln \check{\varphi}])^{2(1-g)})}{(1+2\tilde{m}) f^2 g^2} \right) \right]^{-1} \tag{IV.11}
 \end{aligned}$$

Similarly, can be written as number of e-folds of spectral index as given ,

$$\begin{aligned}
 n_s = & 1 - \frac{3(3+2\tilde{\zeta})}{4fg(1+\tilde{\zeta})(\ln(G[N]))^{g-1}} + \frac{3(g-1)(5+3\zeta)}{8fg(1+\zeta)(\ln(G[N]))^g} \\
 & + n_1 + n_2 \tag{IV.12}
 \end{aligned}$$

where

$$\begin{aligned}
 n_1 = & -3 \left(\frac{2(1+2\tilde{m})}{\kappa(1+\zeta)} \right)^{\frac{1}{2}} \left[\frac{3(1+2\tilde{m})}{2\kappa C_\gamma(\tilde{m}+2)(1+\zeta)} \right]^{-\frac{3}{8}} \frac{(fg)^{\frac{-3-4\tilde{\zeta}}{8(1+\zeta)}}}{C_\varphi^{\frac{1}{2}}} \\
 & \times \left(\frac{(2\tilde{m}+1)}{\kappa} \right)^{\frac{-\zeta}{8(1+\zeta)}} (\ln(G[N]))^{\frac{(1-g)(3+4\tilde{\zeta})}{8(1+\zeta)}} (G[N])^{\frac{1+2\tilde{\zeta}}{4(1+\zeta)}} \\
 & \times \left[1 - \frac{\chi}{1+\zeta} \left(\frac{(\ln(G[N]))^{2(1-g)} (G[N])^2 \kappa}{(1+2\tilde{m}) f^2 g^2} \right) \right]^{-\frac{-(\zeta+\lambda)(1+\zeta)}{8(1+\zeta)(1+\lambda)}} \tag{IV.13}
 \end{aligned}$$

and

$$\begin{aligned}
 n_2 = & \frac{3(\zeta+\lambda(1+\zeta))}{4(1+\zeta)} \left(\frac{\tilde{\chi}}{1+\tilde{\zeta}} \right) \left(\frac{(\frac{\kappa}{1+2\tilde{m}})^{1+\lambda}}{(fg)^{3+2\lambda}} \right) (G[N])^{2(1+\lambda)} \\
 & \times (\ln(G[N]))^{(3+2\lambda)(1-g)} \\
 & \times \left[1 - \frac{\chi}{1+\zeta} \left(\frac{\kappa(G[N])^2 (\ln(G[N]))^{2(1-g)}}{(1+2\tilde{m}) f^2 g^2} \right) \right]^{-1} \tag{IV.14}
 \end{aligned}$$

The tensor perturbations can be written as $\check{\varphi}$

$$\begin{aligned}
 r = & (\frac{\tilde{m}+2}{3}) \frac{2\kappa(fg)^2}{\pi^2 F_4} (t)^{\frac{(1-2\tilde{\zeta})}{4(1+\zeta)}} (\ln[t])^{\frac{(g-1)(1+7\tilde{\zeta})}{8(1+\zeta)}} \\
 & \times \check{\varphi}^3 \left[1 - \frac{\chi}{1+\zeta} \left(\frac{\kappa t^2 (\ln[t])^{2(1-g)}}{(1+2\tilde{m}) f^2 g^2} \right)^{1+\lambda} \right]^{\frac{3(\tau+\lambda)(1+\zeta)}{8(1+\zeta)(1+\lambda)}} \tag{IV.15}
 \end{aligned}$$

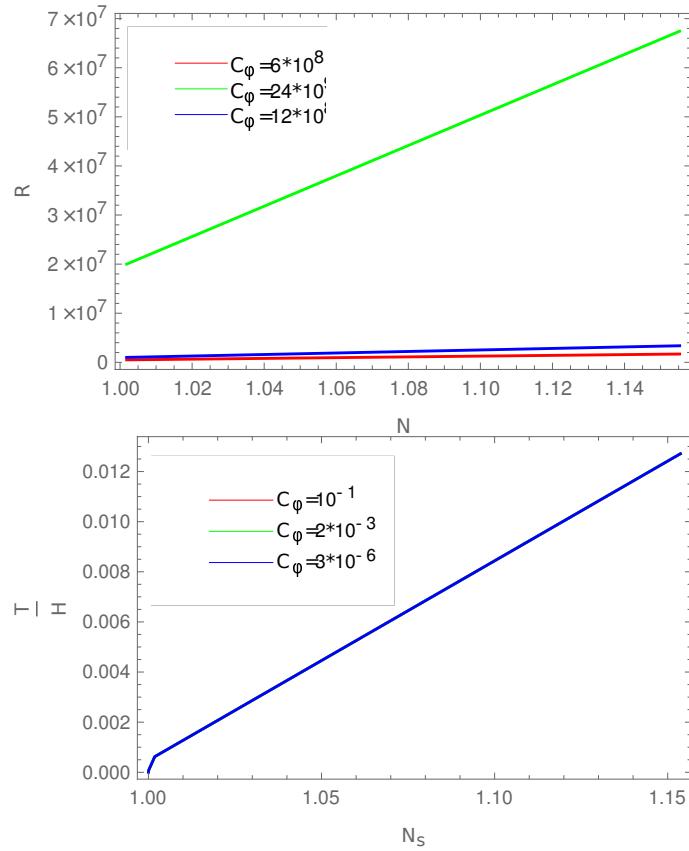


FIG. 1: plot of left R verses n_s and right plot T verses n_s $\tilde{\zeta} = 0.0046$, $\tilde{\chi} = 0.8289$, $\tilde{m} = 1$, $g = 20$, $f = 0.9805$, $\kappa = 1$

As the weak regimen the tensor to scalar ratio in terms of number of e-folds becomes

$$\begin{aligned}
 r &= \left(\frac{\tilde{m}+2}{3}\right) \frac{2\kappa(fg)^2}{\pi^2 F_4} (G[N])^{\frac{(1-2\zeta)}{4(1+\zeta)}} (\ln(G[N]))^{\frac{(g-1)(1+7\zeta)}{8(1+\zeta)}} \\
 &\times \check{\psi}^3 \left[1 - \frac{\chi}{1+\zeta} \left(\frac{\kappa(G[N])^2 (\ln(G[N]))^{2(1-g)}}{(1+2\tilde{m})f^2 g^2} \right)^{1+\lambda} \right]^{\frac{3(\zeta+\lambda(1+\zeta))}{8(1+\zeta)(1+\lambda)}} \tag{IV.16}
 \end{aligned}$$

V. THE STRONG DISSIPATIVE REGIME

In this case, we discuss the scalar field for a special case of a strong regimen for $z \neq 3$, we obtained

$$\hat{\phi}(t) - \hat{\phi}_0 = \frac{\hat{\tau}_n[t]}{\hat{\omega}_n} \tag{V.1}$$

where $\hat{\phi}$ is another new scalar field which is defined as $\phi[t] = \frac{2}{3-z} \hat{\phi}^{\frac{3-z}{2}}$ and also $\hat{\omega}_n$ and $\hat{\tau}_n[t]$ both function written as,

$$\hat{\omega}_n = \frac{3^{\frac{z}{8}}}{2^{\frac{4+z}{8}}} \frac{C_\phi^{\frac{1}{2}}}{C_\gamma^{\frac{n}{8}}} \left(\frac{1}{1+\tilde{\zeta}} \right)^{\frac{z-4}{8}} \left(\frac{1}{\tilde{m}+2} \right)^{\frac{z}{8}} \left(\frac{1}{fg} \right)^{\frac{8-z+\tilde{\zeta}z}{8(1+\tilde{\zeta})}} \left(\frac{\kappa}{1+2\tilde{m}} \right)^{\frac{4-z}{8(1+\tilde{\zeta})}}$$

and

$$\begin{aligned}
\hat{\tau}_n[t] = & \left(\frac{4(1+\tilde{\zeta})}{(2(\tilde{\zeta}-1)+z)} \right) \left(\frac{2(1-\tilde{\zeta})+z}{4(1+\tilde{\zeta})} \right)^{-\frac{(g-1)(8+z(\tilde{\zeta}-1))}{8(1+\tilde{\zeta})}} \\
& \times \text{Gamma} \left[1 + \frac{(g-1)(8+z(\tilde{\zeta}-1))}{8(1+\tilde{\zeta})}, -\frac{z+2(\tilde{\zeta}-1)\ln[t]}{4(1+\tilde{\zeta})} \right] \\
& - \left(\frac{\tilde{\chi}\kappa(z-4)(\tilde{\zeta}+\lambda(1+\tilde{\zeta}))}{(1+2\tilde{m})(fg)^2(1+\lambda)(6+z+8\lambda+2\tilde{\zeta}(5+4\lambda))^3} \right) \\
& \times (2)^{3-\frac{8+(-1+\tilde{\zeta})z-16(1+\tilde{\zeta})\lambda+g(8+z-\tilde{\zeta}z+16\lambda+16\tilde{\zeta}(1+\lambda))}{8(1+\tilde{\zeta})}} \\
& \times \text{Gamma} \left[\frac{(16(1+\lambda)+z)-g(8+z+16\lambda)}{4(1+\tilde{\zeta})} \right. \\
& \left. + \frac{\tilde{\zeta}(24-z+16\lambda+f(z-16(1+\lambda)))}{8(1+\tilde{\zeta})}, \frac{(6+z+8\lambda+2\tilde{\zeta}(5+4\lambda))\ln[t]}{4(1+\tilde{\zeta})} \right] \\
& \times (\ln[t])^{-\frac{8+(-1+\tilde{\zeta})z+g(8-\tilde{\zeta}z-16+z)+z}{8(1+\tilde{\zeta})}} \\
& \times \left(-\frac{(6+z+8\lambda+2\tilde{\zeta}(5+4\lambda))\ln[t]}{1+\tilde{\zeta}} \right)^{\frac{8+(-1+\tilde{\zeta})z-16(1+\tilde{\zeta})\lambda+g(8+z-\tilde{\zeta}z+16\lambda+16\tilde{\zeta}(1+\lambda))}{8(1+\tilde{\zeta})}} \\
& \times \left(\frac{\kappa(\ln[t])^{(2-2g)}}{(1+2\tilde{m})(fg)^2} \right)^\lambda
\end{aligned}$$

According to this case, The definition of Hubble parameter is define,

$$H_2(\hat{\phi}) = \frac{fg}{((\hat{\tau}^{-1}[\hat{\omega}\hat{\phi}])(\ln(\hat{\tau}^{-1}[\hat{\omega}\hat{\phi}]))^{1-g})} \quad (\text{V.2})$$

For special second case $z \neq 3$, The Potential in this form

$$V(\hat{\phi}) \approx \left[\left(\frac{(1+2\tilde{m})f^2g^2}{\kappa(G[N])^2(\ln(G[N]))^{2(1-g)}} \right)^{1+\lambda} - \frac{\tilde{\chi}}{1+\tilde{\zeta}} \right]^{\frac{1}{(1+\tilde{\zeta})(1+\lambda)}} \quad (\text{V.3})$$

For this case, the dissipative coefficient after the solved can be written as,

$$\begin{aligned}
\Gamma(\hat{\phi}) = & F_n \hat{\phi}^{-2} (\tau_n^{-1}[\varpi_n \ln \hat{\phi}])^{\frac{2z\tilde{\zeta}(1-g)-z(2-g)(1+\tilde{\zeta})}{4(1+\tilde{\zeta})}} \\
& \times \left[-\frac{\tilde{\chi}}{\tilde{\zeta}+1} \left(\frac{\kappa(\tau_n^{-1}[\varpi_n \ln \hat{\phi}])^{2(1-g)}}{f^2(2\tilde{m}+1)g^2} \right)^{1+\lambda} + 1 \right]^{\frac{-z(\tilde{\zeta}+\lambda(1+\tilde{\zeta}))}{(1+\tilde{\zeta})(4\lambda+4)}} \quad (\text{V.4})
\end{aligned}$$

and the constant term, where $F_z = C_{\hat{\phi}} \left[\frac{3(1+2\tilde{m})\tilde{\chi}g(1-g)}{2(\tilde{m}+2)\kappa C_\gamma(1+\tilde{\zeta})} \right]^{\frac{z}{4}} \left(\frac{(1+2\tilde{m})f^2g^2}{\kappa} \right)^{\frac{-z\tilde{\zeta}}{4(1+\tilde{\zeta})}}$.

For this case, The number of e-folds ,we get

$$N = f \left(\frac{m+2}{3} \right) \left[(\tau_n^{-1}[\varpi_n \ln \hat{\phi}_2])^g - (\tau_n^{-1}[\varpi_n \ln \hat{\phi}_1])^g \right]. \quad (\text{V.5})$$

In special second case for $z \neq 3$, The Power Spectrum can be found that

$$\begin{aligned}
P_R = & F_{1n} \hat{\phi}^{\frac{3(1-z)}{2}} (\ln[t])^{\frac{(g-1)[(6-3z)(1+\tilde{\zeta})-8(1+\tilde{\zeta})]}{8(1+\tilde{\zeta})}} \left(\frac{1}{t} \right)^{\frac{2(6-3z)(1+2\tilde{\zeta})-8(1+\tilde{\zeta})}{8(1+\tilde{\zeta})}} \\
& \times \left[1 - \frac{\tilde{\chi}}{1+\tilde{\zeta}} \left(\frac{\kappa t^2 (\ln(t))^{2(1-g)}}{(2\tilde{m}+1)f^2g^2} \right)^{1+\lambda} \right]^{\frac{(6-3z)(\tilde{\zeta}+\lambda(1+\tilde{\zeta}))}{8(1+\tilde{\zeta})(4-z)(1+\lambda)}} \quad (\text{V.6})
\end{aligned}$$

Where,

$$F_{1n} = \left[\frac{(\tilde{m} + 2)}{3} \right]^{\frac{5}{2}} \frac{\kappa(1 + \tilde{\zeta})}{2(1 + 2\tilde{m})} C_{\tilde{\phi}}^{\frac{3}{2}} \left(\frac{1 + 2\tilde{m}}{\kappa} \right)^{\frac{(6-3z)\tilde{\zeta}}{8(1+\tilde{\zeta})}} \left[\frac{3(1 + 2\tilde{m})}{2\kappa C_{\gamma}(\tilde{m} + 2)(1 + \tilde{\zeta})} \right]^{\frac{3z+2}{8}} \\ \times (fg)^{\frac{[(1+3\tilde{\zeta})(6-3z)-8(1+\tilde{\zeta})]}{8(1+\tilde{\zeta})}}. \quad (V.7)$$

Consequently, The power spectrum is defined number of e-folds, we get

$$P_R = F_{1n}(\tilde{\tau}_n(G[N]))^{\frac{3(1-z)}{2}} (ln(G[N]))^{\frac{(g-1)[(6-3z)(1+3\tilde{\zeta})-8(1+\tilde{\zeta})]}{8(1+\tilde{\zeta})}} \left(\frac{1}{G[N]} \right)^{\frac{2(6-3z)(1+2\tilde{\zeta})-8(1+\tilde{\zeta})}{8(1+\tilde{\zeta})}} \\ \times \left[1 - \frac{\tilde{\chi}}{1 + \tilde{\zeta}} \left(\frac{\kappa(G[N])^2 (ln(G[N]))^{2(1-g)}}{(2\tilde{m} + 1)f^2 g^2} \right)^{1+\lambda} \right]^{\frac{(6-3z)(\tilde{\zeta}+\lambda(1+\tilde{\zeta}))}{8(1+\tilde{\zeta})(4-z)(1+\lambda)}} \quad (V.8)$$

Where γ_n is a constant term and is defined by $\gamma_n = (\frac{1}{\delta_n})^{\frac{3-3z}{2}}$ and also written as of scalar spectrum index in scalar field

$$n_s = 1 + \frac{(g-1)[(6-3z)(1+3\tilde{\zeta})-8(1+\tilde{\zeta})]}{8fg(1+\tilde{\zeta})(ln[t])^g} \\ - \frac{2(6-3z)(1+2\tilde{\zeta})-8(1+\tilde{\zeta})}{8fg(1+\tilde{\zeta})(ln[t])^{g-1}} + n_{1n} + n_{2n}, \quad (V.9)$$

where, The terms n_{1n} and n_{2n} are

$$n_{1n} = \left(\frac{3(1-z)}{2C_{\tilde{\phi}}^{\frac{1}{2}}} \right) \left(\frac{2(1 + 2\tilde{m})}{\kappa(1 + \tilde{\zeta})} \right)^{\frac{1}{2}} \left[\frac{3(1 + 2\tilde{m})fg}{2C_{\gamma}\kappa(\tilde{m} + 2)(1 + \tilde{\zeta})} \right]^{\frac{-z}{8}} \\ \times \left(\frac{(1 + 2\tilde{m})f^2 g^2}{\kappa} \right)^{\frac{\tilde{\zeta}(z-4)}{8(1+\tilde{\zeta})}} \tilde{\phi}^{\frac{z-3}{2}} (ln[t])^{\frac{(g-1)[2\tilde{\zeta}(z-4)-\tilde{m}(1+\tilde{\zeta})]}{8(1+\tilde{\zeta})}} \\ \times (t)^{\frac{(\tilde{m}-4)(1-\tilde{\zeta})}{8(1+\tilde{\zeta})}} \left[1 - \frac{\tilde{\chi}}{1 + \tilde{\zeta}} \left(\frac{\kappa t^2 (ln[t])^{2(1-g)}}{(1 + 2\tilde{m})\tilde{\alpha}^2 g^2} \right)^{1+\lambda} \right]^{\frac{(z-4)(\tilde{\zeta}+\lambda(1+\tilde{\zeta}))}{8(1+\tilde{\zeta})(1+\lambda)}} \quad$$

and

$$n_{2n} = - \frac{(6-3z)\tilde{\chi}}{4(1 + \tilde{\zeta})} \frac{[\tilde{\zeta} + \lambda(1 + \tilde{\zeta})]}{(1 + \tilde{\zeta})(fg)^{3+2\lambda}} \\ \times (\kappa/(1 + 2\tilde{m}))^{1+\lambda} (t)^{2(1+\lambda)} \left(\frac{1}{ln[t]} \right)^{(g-1)(3+2\lambda)} \\ \times \left[1 - \frac{\tilde{\chi}}{1 + \tilde{\zeta}} \left(\frac{\kappa(t)^2 (ln[t])^{2(1-g)}}{(1 + 2\tilde{m})f^2 g^2} \right)^{1+\lambda} \right]^{-1}.$$

In this case $z \neq 3$ and the scalar spectral index which can be expressed in number of e-folds,

$$n_s = 1 + \frac{(g-1)[(6-3z)(1+3\tilde{\zeta})-8(1+\tilde{\zeta})]}{8fg(1+\tilde{\zeta})(ln(G[N]))^g} \\ - \frac{2(6-3z)(1+2\tilde{\zeta})-8(1+\tilde{\zeta})}{8fg(1+\tilde{\zeta})(ln(G[N]))^{g-1}} + n_{1n} + n_{2n}, \quad (V.10)$$

where n_{1n} and n_{2n} are

$$\begin{aligned} n_{1n} = & \left(\frac{3(1-z)}{2C_{\tilde{\phi}}^{\frac{1}{2}}} \right) \left(\frac{2(1+2\tilde{m})}{\kappa(1+\tilde{\zeta})} \right)^{\frac{1}{2}} \left[\frac{3(1+2\tilde{m})fg}{2C_{\gamma}\kappa(\tilde{m}+2)(1+\tilde{\zeta})} \right]^{\frac{z}{8}} \\ & \times \left(\frac{(1+2\tilde{m})f^2g^2}{\kappa} \right)^{\frac{\tilde{\zeta}(z-4)}{8(1+\tilde{\zeta})}} \tilde{\phi}^{\frac{z-3}{2}} (\ln(G[N]))^{\frac{(g-1)[2\tilde{\zeta}(z-4)-\tilde{m}(1+\tilde{\zeta})]}{8(1+\tilde{\zeta})}} \\ & \times (G[N])^{\frac{(\tilde{m}-4)(1-\tilde{\zeta})}{8(1+A)}} \left[1 - \frac{\tilde{\chi}}{1+\tilde{\zeta}} \left(\frac{\kappa(G[N])^2(\ln(G[N]))^{2(1-g)}}{(1+2\tilde{m})\tilde{\alpha}^2g^2} \right)^{1+\lambda} \right]^{\frac{(z-4)(\tilde{\zeta}+\lambda(1+\tilde{\zeta}))}{8(1+\tilde{\zeta})(1+\lambda)}} \end{aligned}$$

and

$$\begin{aligned} n_{2n} = & -\frac{(6-3z)\tilde{\chi}}{4(1+\tilde{\zeta})} \frac{[\tilde{\zeta}+\lambda(1+\tilde{\zeta})]}{(1+\tilde{\zeta})(fg)^{3+2\lambda}} \\ & \times (\kappa/(1+2\tilde{m}))^{1+\lambda} (G[N])^{2(1+\lambda)} \left(\frac{1}{\ln(G[N])} \right)^{(g-1)(3+2\lambda)} \\ & \times \left[1 - \frac{\tilde{\chi}}{1+\tilde{\zeta}} \left(\frac{\kappa(G[N])^2(\ln(G[N]))^{2(1-g)}}{(1+2\tilde{m})f^2g^2} \right)^{1+\lambda} \right]^{-1}. \end{aligned}$$

In the second case, similarly we also find the tensor-to-scalar ratio,

$$\begin{aligned} r = & \frac{2\kappa(fg)^2}{\pi^2 F_n} \left(\frac{\tilde{m}+2}{3} \right) \tilde{\phi}^{\frac{3(z-1)}{2}} \\ & \times (\ln[t])^{\frac{(1-g)[(6-3z)(3\tilde{\zeta}+1)-16(1+\tilde{\zeta})]}{8(1+\tilde{\zeta})}} (t)^{\frac{2(1+2\tilde{\zeta})(6-3z)-16(1+\tilde{\zeta})}{8(1+\tilde{\zeta})}} \\ & \times \left[1 - \frac{\tilde{\chi}}{1+\tilde{\zeta}} \left(\frac{\kappa(t)^2(\ln[t])^{-2(g-1)}}{(1+2\tilde{m})f^2g^2} \right)^{1+\lambda} \right]^{\frac{(3z-6)(\tilde{\zeta}+\lambda(1+\tilde{\zeta}))}{8(1+\tilde{\zeta})(1+\lambda)}}, \end{aligned} \quad (V.11)$$

Similarly, this equation can also be written as number of e-folds

$$\begin{aligned} r = & \frac{2\kappa(fg)^2}{\pi^2 F_n} \left(\frac{\tilde{m}+2}{3} \right) \tilde{\phi}^{\frac{3(z-1)}{2}} \\ & \times (\ln(G[N]))^{\frac{(1-g)[(6-3z)(3\tilde{\zeta}+1)-16(1+\tilde{\zeta})]}{8(1+\tilde{\zeta})}} (G[N])^{\frac{2(1+2\tilde{\zeta})(6-3z)-16(1+\tilde{\zeta})}{8(1+\tilde{\zeta})}} \\ & \times \left[1 - \frac{\tilde{\chi}}{1+\tilde{\zeta}} \left(\frac{\kappa(G[N])^2(\ln(G[N]))^{-2(g-1)}}{(1+2\tilde{m})f^2g^2} \right)^{1+\lambda} \right]^{\frac{(3z-6)(\tilde{\zeta}+\lambda(1+\tilde{\zeta}))}{8(1+\tilde{\zeta})(1+\lambda)}}. \end{aligned} \quad (V.12)$$

VI. CONCLUSION

In the present work we have studied the warm inflationary dynamics by modified Chaplygin gas in the background of rotationally symmetric Bianchi identity I. We formulated the inflationary expansion by the process of constant dissipative coefficient $\Gamma = C_{\phi} T^z / \varphi^{z-1}$ where, $z = -1, 0, 1, 3$. We are considering weak and strong dissipative regime and find out the several inflaton decay rates. Under the slow roll approximation we formulated the scalar power spectrum, scalar power index and tensor to scalar ratio subsequently. According to essential condition of warm inflation $T \gg H_2$ and this always satisfied the weak ($R \ll 1$) and strong ($R \gg 1$) dissipative regime. The limit of dissipative parameter C_{ϕ} for upper and lower is satisfied the condition of warm inflation and the conditions of decay rates. The parameters r and n_s does not compose any constraints data on the contrary the Planck data, on the considering two- marginalized constraints at 68 and 95 C.L. However, The recent planck observational data is

compatible for tensor to scalar ratio r . According to recent observational planck data, the strong dissipative regime for special case $z = 3$ the conditions of model evolves under the this regime and for two-dimensional marginalized constraints on the parameter r and n_s for the constant dissipative parameter C_ϕ by the set of upper and lower limit. Finally the values of $z = -1$ and $z = 0$ are not satisfied the condition warm inflation for the case of strong dissipative and recent data and the plot of r verses n_s can't draw in a strong case, since the predicted the value spectral index is always greater than unity. It is interesting, the recent observational data is also compatible with our inflationary dynamic model for specific value of tensor to scalar ratio $r \sim 0$. We conclude that the warm logamediate inspired modified chaplygin gas with rotationally symmetric Bianchi Identity I with current planck 2018 data for all inflation decay ratios for different parameterized z with evolves under constant dissipative regime.

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