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Cristina Margarita Gómez-Sarabia and [Jorge Ojeda-Castaneda](#) *

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Article

Foucault-Barker Mask: Nonconventional Schlieren Technique

Cristina M. Gómez-Sarabia ¹ and Jorge Ojeda-Castañeda ^{2,*}

¹ Artes Digitales y Gestión Empresarial, DICIS, University of Guanajuato, México

² Emeritus, Electronics Department, DICIS, University of Guanajuato, México

* Correspondence: jojedacas@ugto.mx

Abstract: We present a theoretical framework for designing optical masks useful for implementing Schlieren techniques. We revisit the use of effective transfer functions, for emphasizing the role of masks symmetries. We unveil a statistical model describing phase gradients as randomly located prisms. We disclose the use of two similar coding masks for implementing a Schlieren technique, based on optical autocorrelations. One mask covers the source, and the other mask acts as a spatial filter. We exploit the properties of the Barker sequences, for achieving highly peaked noncyclic autocorrelations. This operation leads to the design of an automatic sensor for phase gradients.

Keywords: Schlieren techniques; Foucault test; Barker sequences; optical autocorrelations

1. Introduction

For current applications in optical microscopy and optical testing, researchers have developed techniques for rendering visible transparent structures [1–4]. Among others, we note Foucault's remarkable invention of the knife-edge.

This optical technique initiated experimental methods for testing optical elements. visualizing the departures of sphericity of mirrors and lenses [5].

According to Krehl and Engemann [6], independently of Foucault, Toepler was the first person who visualized shock waves, by employing a knife-edge [7].

Renitz also noted that there is an early version of the knife-edge [8]. Furthermore, Renitz has also recognized previous experimental results from Wiesel and Marat [9].

Zernike presented the phase contrast technique for microscopy as an improved version of the knife-edge technique [10]. Independently of Zernike, Lyot proposed a phase contrast technique, used for visualizing polishing inhomogeneities in a coronagraph [11].

Due to the contributions of Schardin [12] and Burton [13], the use of the knife-edge is now recognized as a member of the Schlieren techniques [14–17].

Here, for implementing nonconventional Schlieren techniques, we unveil the use of 1-D optical masks, coded either with the Barker sequences [18], or with the pseudorandom sequences [19].

For developing our current discussion, in Section 2, we revisit the basics of an effective transfer function, which was initially proposed by Menzel [20,21]. This concept was further developed by one of us [22–25]. This discussion considers, as input pattern, a transparent grating that has weak phase variations.

For emphasizing the usefulness of the transfer function, in Section 3, we present a statistical model, which describes the presence of a randomly located prism.

In Section 4, we discuss the usage of a coded source. We note that there is a link between the Schlieren techniques and the mathematical expression for the ambiguity functions, as employed in RADAR engineering [26–30]. We disclose the use of optical autocorrelations for implementing Schlieren techniques.

In Section 5, we illustrate the relevance of coding the source, by numerically evaluating the autocorrelation of pseudorandom sequences and of the Barker sequences. And in Section 6, we summarize our contribution.

2. Effective Transfer Function

In Figure 1, we depict the schematics of an optical processor, which is used for describing our proposal.

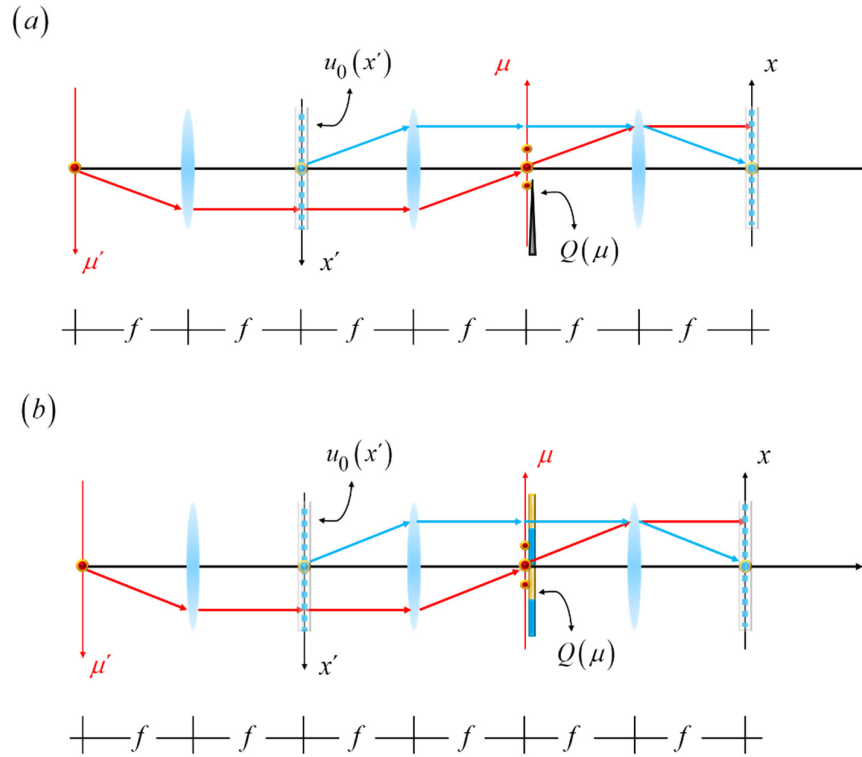


Figure 1. Schematics of the optical setup under consideration. The spatial filter is located pupil at the axis, denoted with the Greek letter μ . This axis is the geometrical optics conjugate of the source axis, μ' . In (a) a knife edge acts as the spatial filter. In (b) a coded mask substitutes the knife edge.

In Figure 1, the input pattern is placed in the axis denoted as x' . The input pattern is a sinusoidal phase grating, with period d . Its complex amplitude transmittance is

$$t(x', y') = \exp \left[i 2 \pi \left(\frac{a}{\lambda} \right) \sin \left(2 \pi \frac{x'}{d} \right) \right]. \quad (1)$$

In Equation (1) the amplitude of the phase delays is denoted with the lower-case Latin Letter a . And the upper-case Greek letter λ denotes the wavelength. Next, we consider that the above grating has phase variations that are weak, in the sense that $\left(\frac{a}{\lambda} \right)^2 \ll \left(\frac{a}{\lambda} \right)$.

Under this assumption, the complex amplitude transmittance can be approximated by the initial two terms of its Taylor expansion. Hence, we consider that the input patterns have the following complex amplitude transmittance

$$u_0(x', y') = 1 + i 2 \pi \left(\frac{a}{\lambda} \right) \sin \left(2 \pi \frac{x'}{d} \right). \quad (2)$$

The Fourier spectrum of Equation (2) is

$$U_0(\mu, \nu) = \delta(\mu) \delta(\nu) + \pi \left(\frac{a}{\lambda} \right) \left[\delta\left(\mu - \frac{1}{d}\right) - \delta\left(\mu + \frac{1}{d}\right) \right] \delta(\nu). \quad (3)$$

In Equation (3), the Greek letters, μ and ν denote the spatial frequencies at Fourier domain. We note that the mathematical expression in Equation (3) represents the complex amplitude distribution impinging on the optical mask. If we denote as $Q(\mu, \nu)$ the amplitude transmittance of the optical mask, then after the mask the amplitude transmittance reads

$$U(\mu, \nu) = Q(\mu, \nu) U_0(\mu, \nu). \quad (4)$$

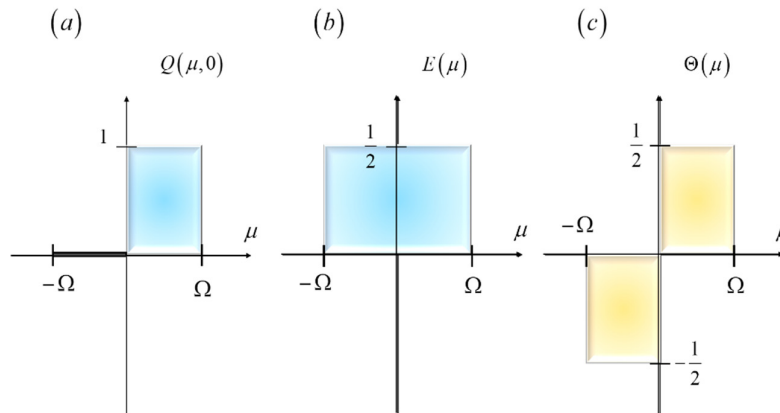
By taking an inverse Fourier transformation of Equation (4), we obtain the complex amplitude distribution at the image plane. That is,

$$u(x, y) = Q(0, 0) + \pi \left(\frac{a}{\lambda} \right) \left[Q\left(\frac{1}{d}, 0\right) \exp\left(i 2 \pi \frac{x}{d}\right) - Q\left(-\frac{1}{d}, 0\right) \exp\left(-i 2 \pi \frac{x}{d}\right) \right]. \quad (5)$$

Here, it is convenient to recognize that $Q(\mu, 0)$ can be expressed in a unique manner as the sum of two functions. The function $E(\mu) = E(-\mu)$ is an even function. And the function $\Theta(\mu) = -\Theta(-\mu)$ is an odd function. For further details, please see Appendix A.

$$Q(\mu, 0) = E(\mu) + \Theta(\mu). \quad (6)$$

In Figure 2, we depict the even component, $E(\mu)$, and the odd component, $\Theta(\mu)$, of the amplitude transmittance of the function $Q(\mu, 0)$.



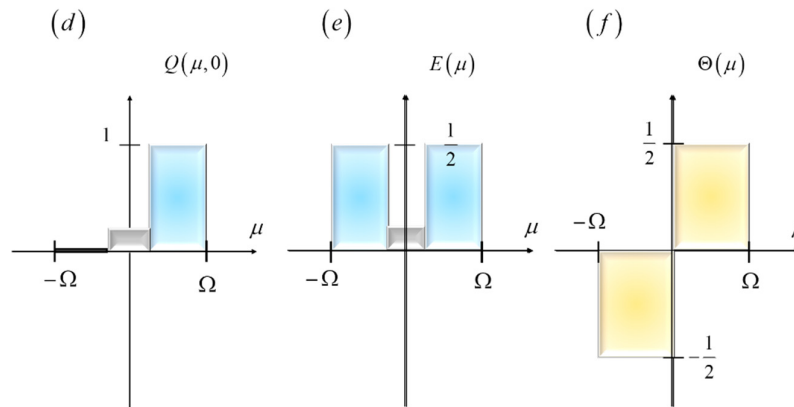


Figure 2. Amplitude transmittance of the mask $Q(\mu, 0)$, as well as its even component and odd components. In (a) we depict the amplitude transmittance of the knife edge. In (b) we show its even component. In (c) we show its odd component. Next, the amplitude transmittance of the modulation contrast mask, as in reference [31]. In (d) we illustrate its amplitude transmittance. In (e) we show its even component. And in (c) we display its odd component.

If we substitute Equation (6) in Equation (5) we have that

$$u(x, y) = Q(0, 0) + \left(2\pi \frac{a}{\lambda}\right) \Theta\left(\frac{1}{d}\right) \cos\left(2\pi \frac{x}{d}\right) + i \left(2\pi \frac{a}{\lambda}\right) E\left(\frac{1}{d}\right) \sin\left(i 2\pi \frac{x}{d}\right). \quad (7)$$

The definition of the normalized image irradiance distribution is

$$I(x, y) \triangleq \frac{|u(x, y)|^2}{|Q(0, 0)|^2}. \quad (8)$$

From Equations (7) and (8) we obtain the normalized image irradiance distribution

$$I(x, y) = 1 + \left(\frac{2\pi a}{\lambda}\right) \left(\frac{2}{E(0)} \Theta\left(\frac{1}{d}\right)\right) \cos\left(2\pi \frac{x}{d}\right). \quad (9)$$

Now, it is relevant to make the following comparison between Equations (2) and (9). The complex amplitude distribution at the input plane, in Equation (2), is mapped as an image irradiance distribution, in Equation (9). The initial sinusoidal variation is transferred as a cosinusoidal variation, with modulation that is equal to

$$H\left(\frac{1}{d}\right) \triangleq \frac{2}{E(0)} \Theta\left(\frac{1}{d}\right). \quad (10)$$

In other words, there is a spatial frequency mapping, which can be associated with an effective transfer function, which is sampled at the spatial frequency $1/d$. This remarkable simple result is conveniently depicted in Figure 3.

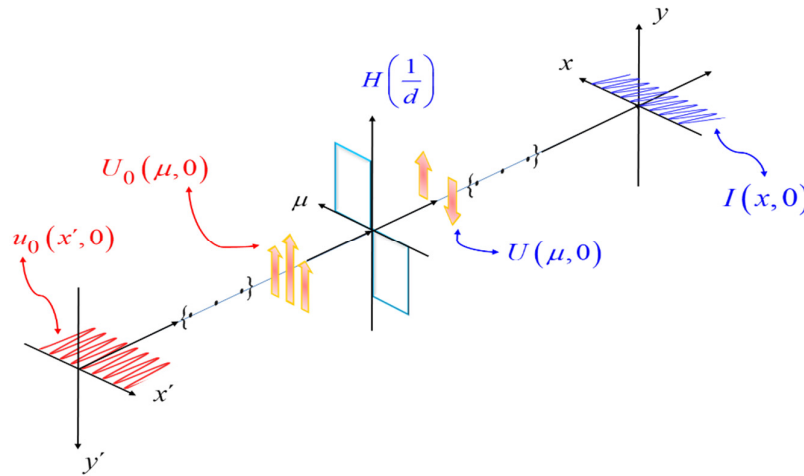


Figure 3. Pictorial portraying the spatial frequency transfer of an initial amplitude sinusoidal distribution, into the final irradiance cosinusoidal distribution. At the center of the schematics, we depict the spatial frequency windows described in Equation (10).

Furthermore, it is apparent from Equation (10) that the action of effective transfer function can be enhanced if the value at the origin of the even component, $E(0)$, is less than unity. This observation is useful for proposing improvements to two previous Schlieren techniques, which are depicted in Figure 2. For Wolter's phase edge, the coded mask has the following amplitude transmittance

$$Q(\mu, \nu) = \begin{cases} -1 & \text{if } -\Omega < \mu < 0 \\ 1 & \text{if } 0 < \mu < \Omega \end{cases}. \quad (11)$$

In Equation (11), the Greek letter Ω denotes the cut-off spatial frequency. In Figure 4a we plot Equation (11). Now, for the modulation contrast technique [31], the amplitude transmittance is

$$Q(\mu, \nu) = \begin{cases} 0 & \text{if } -\Omega < \mu < -\varepsilon\Omega \\ \alpha & \text{if } -\varepsilon\Omega < \mu < \varepsilon\Omega \\ 1 & \text{if } \varepsilon\Omega < \mu < \Omega \end{cases}. \quad (12)$$

In Equation (12), the Greek letters α and ε denote two real numbers with values lower than unity. The letter α represents an attenuation coefficient at the center of the mask. And the letter ε indicates the length covered by the central section, as compared with the length covered by the mask.

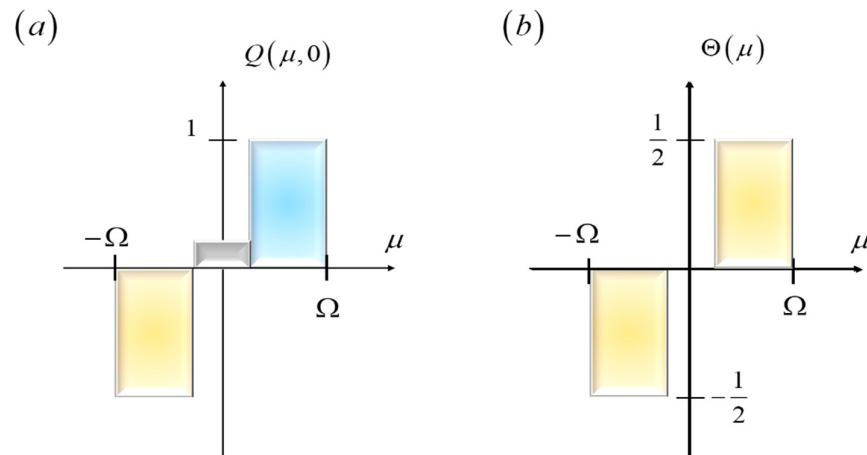


Figure 4. Amplitude transmittance of an improved Schlieren technique. In (a) the amplitude distribution incorporates the light gathering advantage described in Equation (11). This mask also incorporates the

advantage of reducing the amplitude transmittance, at the center of the mask, $E(0)$, as in Equation (12). In (b) the odd component of the proposed mask.

In Figure 4, we plot the amplitude for an improved version, which incorporates a transmission equals minus unity; as in Wolter's phase edge. And it also incorporates a central attenuation coefficient, as in the modulation contrast technique. Its amplitude distribution reads

$$Q(\mu, \nu) = \begin{cases} -1 & \text{if } -\Omega < \mu < -\varepsilon\Omega \\ \alpha & \text{if } -\varepsilon\Omega < \mu < \varepsilon\Omega \\ 1 & \text{if } \varepsilon\Omega < \mu < \Omega \end{cases}. \quad (13)$$

Next, we note that some authors use, in a lax manner, a heuristic approach for describing phase gradients. This description considers that a phase gradient is a randomly located prism. To our knowledge, this heuristic approach has not been described in terms of scalar diffraction. For gaining physical insight, on the heuristic approach, we present next a diffraction-based description.

3. Randomly Located Prism

In Equation (14) we denote the amplitude transmittance of a prism that has a random location. That is,

$$t(x', y'; X) = 1 + \left[\exp \left(i 2 \pi \left(\frac{a}{\lambda} \sin(\theta) \right) \right) - 1 \right] \delta(x' - X). \quad (14)$$

In Equation (14) the random variable X takes values that obey to a probability density function $p(X)$. Its Fourier transform pair is the characteristic function

$$\Phi_X(\mu) = \int_{-\infty}^{\infty} p(X) \exp(-i 2 \pi \mu X) dX. \quad (15)$$

Under the assumption of weak phase delays, we can approximate Equation (14) as follows

$$u_0(x', y'; X) = 1 + i 2 \pi \left(\frac{a}{\lambda} \sin(\theta) \right) \delta(x' - X). \quad (16)$$

Its Fourier spectrum reads

$$U_0(\mu, \nu; X) = \delta(\mu) \delta(\nu) + i \left(2 \pi \frac{a}{\lambda} \sin(\theta) \right) \exp(-i 2 \pi \mu X) \delta(\nu). \quad (17)$$

After passing through the coding mask, $Q(\mu, \nu)$, the Fourier spectrum reads

$$U(\mu, \nu; X) = Q(\mu, \nu) U_0(\mu, \nu; X). \quad (18)$$

By substituting Equation (17) in Equation (18) we obtain the explicit expression

$$U(\mu, \nu; X) = Q(0, 0) \delta(\mu) \delta(\nu) + i \left(2 \pi \frac{a}{\lambda} \sin(\theta) \right) Q(\mu, 0) \exp(-i 2 \pi \mu X) \delta(\nu). \quad (19)$$

Next, as discussed in Appendix B, we take the ensemble average of Equation (19). The average Fourier spectrum is

$$\begin{aligned}\langle U(\mu, \nu) \rangle &= Q(0, 0) \delta(\mu) \delta(\nu) + \\ &+ i \left(2\pi \frac{a}{\lambda} \sin(\theta) \right) Q(\mu, 0) \Phi_X(\mu) \delta(\nu).\end{aligned}\quad (20)$$

Then, we recognize the convenience of defining the transfer function

$$H(\mu) \triangleq Q(\mu, 0) \Phi_X(\mu). \quad (21)$$

The characteristic function is assumed to be an even function. Then, the even component of $H(\mu)$ has the following expression

$$E_H(\mu) = E(\mu) \Phi_X(\mu). \quad (22)$$

And the odd even component of $H(\mu)$ has the following form

$$\Theta_H(\mu) = \Theta(\mu) \Phi_X(\mu). \quad (23)$$

Consequently, Equation (20) can be written as

$$\begin{aligned}\langle U(\mu, \nu) \rangle &= Q(0, 0) \delta(\mu) \delta(\nu) + \\ &+ i \left(2\pi \frac{a}{\lambda} \sin(\theta) \right) [E_H(\mu) + \Theta_H(\mu)] \delta(\nu).\end{aligned}\quad (24)$$

As discussed in Appendix B, the average complex amplitude distribution is

$$\begin{aligned}\langle u(x, y) \rangle &= Q(0, 0) - \left(2\pi \frac{a}{\lambda} \sin(\theta) \right) \mathcal{H}_H(x) + \\ &+ i \left(2\pi \frac{a}{\lambda} \sin(\theta) \right) e_H(x).\end{aligned}\quad (25)$$

And as noted in Appendix B, the average irradiance distribution is

$$\langle I(x, y) \rangle = 1 - \left(4\pi \frac{a}{\lambda} \sin(\theta) \right) \mathcal{H}_H(x). \quad (26)$$

From Equation (26) and from Figure 5, we observe two relevant features. The presence of a randomly located prism is imaged as a uniform average irradiance distribution, plus a term that contains the phase modulation caused by the prism.

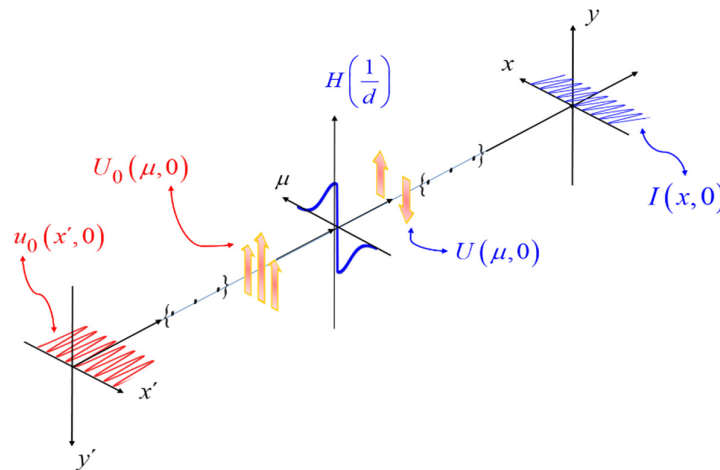


Figure 5. Same as Figure 3, but now the effective transfer function is $\Theta_H(\mu)$, as in Equation (20).

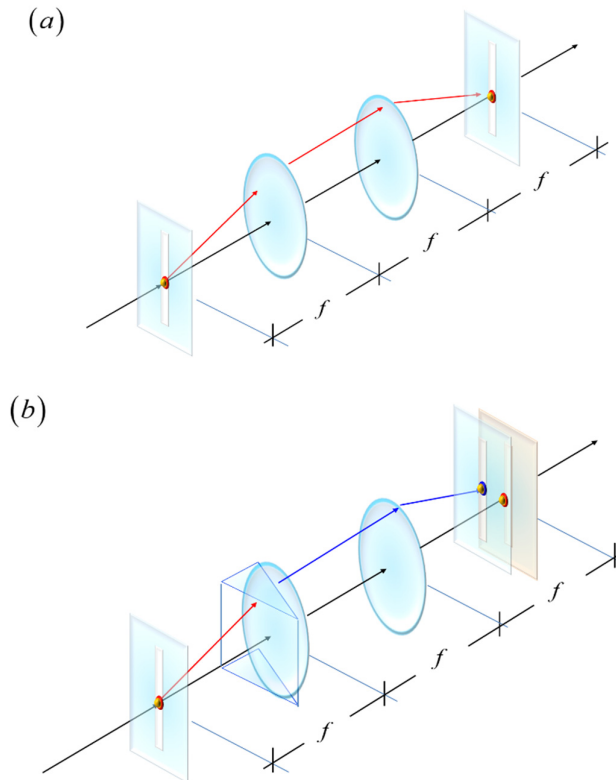
In other words, the phase gradient is rendered visible, as an irradiance variation, due to the presence of the function $\mathcal{G}_H(x)$. Hence, like Equation (10), it is convenient to define the following effective transfer function

$$H\left(\frac{1}{d}\right) \triangleq \Theta_H\left(\frac{1}{d}\right). \quad (27)$$

From Equations (23) and (27), we claim that the effective transfer function is the product of a characteristic function times the odd component of coding mask. And consequently, as depicted in Figure 5, the current effective transfer function has now an apodizing effect. To our knowledge, this is a novel formulation.

4. Coded Source: An Imaging Process

We note the existence of heuristic descriptions for considering a transparent structure as a set of randomly located prisms. This heuristic description ignores any discussions on scalar diffraction. For rectifying this limitation, next we develop an imaging process, which considers a coded source. As part of our rationale, we consider first a simple case. As depicted in Figure 6, we consider the source as a slit. At its conjugated image, (μ, ν) , we place a second slit that acts as the spatial filter.



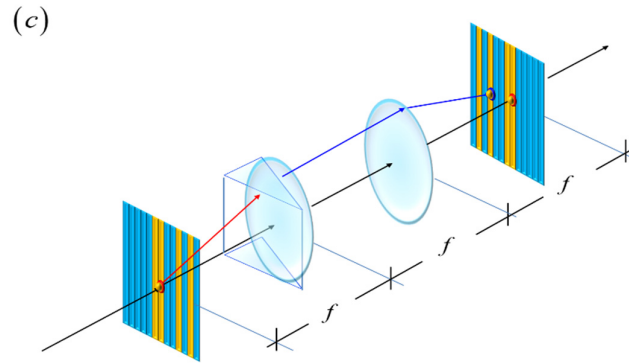


Figure 6. The use of a coded source, in conjunction with a coded spatial filter. In (a) the source is a slit. At its conjugate image, the spatial filter is also a slit. In (b) we depict the impact of a phase gradient. In (c) two coded masks are used for implementing a mask product.

In the absence of phase gradients, the two slits coincide. And the light throughput achieves its maximum value. On the other hand, if a phase gradient is present then the image of the source is laterally shifted (say by the amount ζ / λ). This mismatch generates a drop in light throughput. That is,

$$T = \left| \int_{-\infty}^{\infty} \text{rect}\left(\frac{\mu}{\Omega}\right) \text{rect}\left(\frac{\mu - \frac{\zeta}{\lambda}}{\Omega}\right) d\mu \right|^2. \quad (28)$$

In Equation (28), the Greek letter omega denotes the width of the slit used to cover the source. If one evaluates Equation (28), then one obtains

$$T = \left[1 - \frac{|\zeta|}{2\Omega\lambda} \right] \text{rect}\left(\frac{\zeta}{4\Omega\lambda}\right). \quad (29)$$

It is apparent from Equation (29) that the light throughput varies slowly with the variable ζ , except for a very narrow slit. However, the light throughput is reduced if one uses narrow slits. Hence, we search next for other options. To conclude, we consider that the source is coded with multiple slits. Then, at the source plane the amplitude transmittance is

$$U_0(\mu', \nu') = Q(\mu'). \quad (30)$$

In Equation (30) the Greek letters μ' and ν' denote spatial frequency coordinates at the source plane. At the input plane, the coordinates we employ are x' and y' . At this plane, the amplitude distribution is the Fourier transform of Equation (30). That is,

$$u_0(x', y') = q(x') \delta(y'). \quad (31)$$

In Equation (31) the function $q(x')$ is the 1-D Fourier spectrum of the coded source. Next, we consider that a prism alters the amplitude distribution in Equation (28). Then, the new amplitude distribution reads

$$u(x', y') = \exp\left(-i 2\pi \frac{\zeta}{\lambda} x'\right) u_0(x', y'). \quad (32)$$

In Equation (32) we consider the presence of a weak phase gradient. Its deflection angle is denoted with the Greek letter ζ . The amplitude distribution, at the pupil aperture, is the inverse Fourier transform of Equation (32). That is,

$$U(\mu, \nu) = Q\left(\mu - \frac{\zeta}{\lambda}\right). \quad (33)$$

Next, we consider that the amplitude distribution, in Equation (33), impinges on a second coded mask. For this application, the second mask acts as the spatial filter. Its amplitude transmittance is $Q^*(\mu)$. Then, just after the mask, the amplitude transmittance is

$$\Psi(\mu, \nu) = Q^*(\mu) U(\mu, \nu). \quad (34)$$

It is straightforward to substitute Equation (33) in Equation (34), for obtaining

$$\Psi(\mu, \nu) = Q^*(\mu) Q\left(\mu - \frac{\zeta}{\lambda}\right). \quad (35)$$

As expected, at the pupil aperture, the amplitude distribution is a product between a lateral shifted version of the coded source, times the complex conjugate transmittance of the coded mask. And consequently, at the image plane the amplitude distribution is

$$\psi(x, y; \zeta) = \int_{-\infty}^{\infty} Q^*(\mu) Q\left(\mu - \frac{\zeta}{\lambda}\right) \exp(i 2 \pi x \mu) d\mu. \quad (36)$$

The mathematical expression in Equation (36) is like the definition of the ambiguity function, as is used in RADAR engineering. This is a previously unknown feature of the Schlieren masks. However, any further discussion on this topic is beyond our current scope. We observe that Equation (36), at the origin ($x=0, y=0$), is the light throughput T in Equation (28). That is,

$$|\psi(0, 0; \zeta)|^2 = \left| \int_{-\infty}^{\infty} Q^*(\mu) Q\left(\mu - \frac{\zeta}{\lambda}\right) d\mu \right|^2. \quad (37)$$

From Equation (37) we state the following working hypothesis. By coding the source as well as the spatial filter with the same sequence, its autocorrelation usefully detects phase gradients. This working hypothesis leads us to the following proposal.

5. Optical Visualization and Automatic Detection

Since a pseudorandom sequence or a Barker sequence have sharp autocorrelations, then they can be useful for automatically detecting phase gradients. For elaborating on this working hypothesis, we note that either the pseudorandom sequences, or the Barker sequences have a finite number of elements, say L . This integer number L is denoted as the sequence length. In electronics, the sequences are commonly employed as cyclic sequences. However, in optics, due to the finite size of the mask the coding sequences are noncyclic. Next, we elaborate on this difference.

The pseudorandom sequences exhibit a high peak over an otherwise uniform value, as depicted in Figure 7. In this pictorial, we show two examples of the pseudorandom sequences. In these sequences the amplitude transmittance has values that are equal to unity, or equal to zero. In Figure 7a, the length is $L = 7$. And in (b) the length is $L = 15$. These examples were selected for making comparisons with the results of other authors [32].

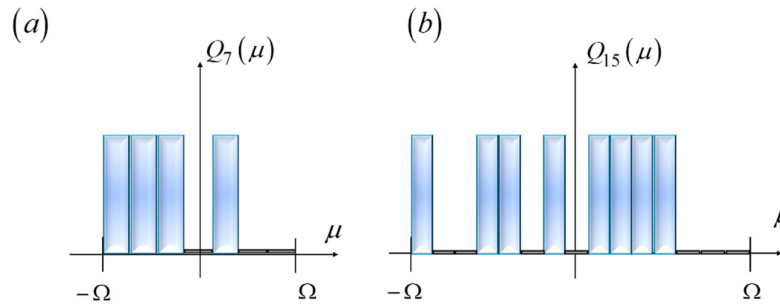


Figure 7. Amplitude transmittance of the mask, $Q_L(\eta)$, which is coded with a pseudorandom sequence of length L . In (a) the length is $L = 7$. In (b) the length $L = 15$.

In Figure 8, we show the cyclic autocorrelations associated with the sequences in Figure 7. The autocorrelations exhibit peaks that overshoot a uniform value.

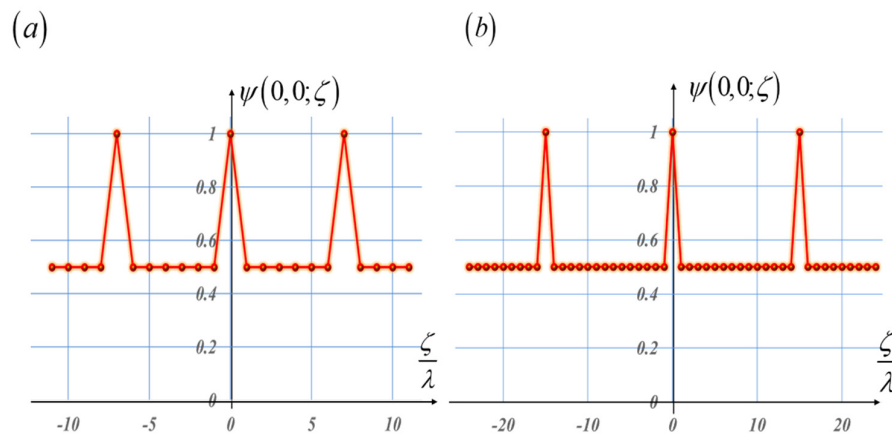


Figure 8. Cyclic autocorrelations associate with the pseudorandom sequences in Figure 7. The cyclic autocorrelations exhibit peaks that overreach a uniform value.

In Figure 9, we show that the Barker sequences of length $L = 7$ and $L = 13$. Now, the amplitude transmittance has values equal to unity, or equal to minus unity.

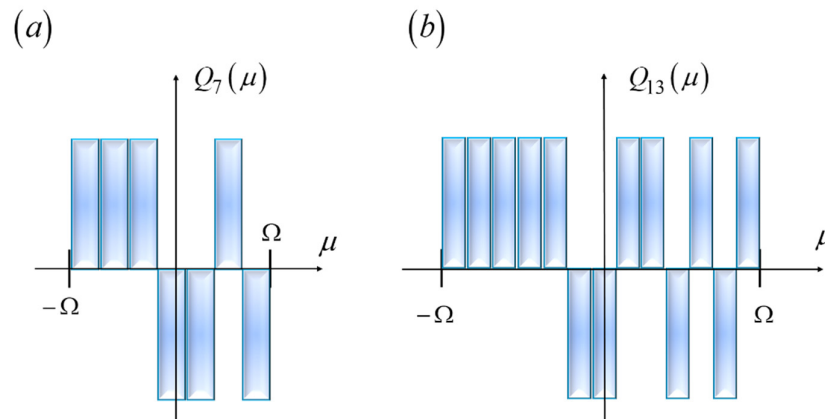


Figure 9. Amplitude transmittance of the mask, $Q_L(\eta)$, which is coded with a Barker sequence of length L . In (a) the length is $L = 7$. In (b) the length $L = 13$.

In Figure 10, we plot the cyclic autocorrelation associated with the sequences in Figure 9. It is apparent from Figure 10 that the autocorrelations also have peaks that overrun the values of the adjacent sidelobes. The peaks baselines narrow down from the length $L = 7$ to the length $L = 13$.

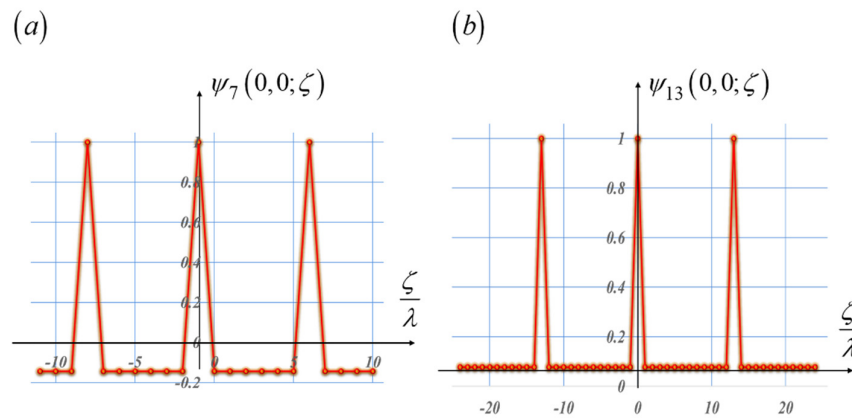


Figure 10. Cyclic autocorrelation associated with the Barker sequences in Figure 9.

As previously indicated, in optics one is dealing with noncyclic autocorrelations. This is due to the finite size of the masks. The autocorrelations do not repeat themselves. To visualize the impact of this feature, we present the noncyclic autocorrelations of the pseudo random sequences, as well as the noncyclic autocorrelations of the Barker sequences.

From Figure 11 and from Figure 12, we note that the noncyclic autocorrelations do not have replicated peaks. Furthermore, associated with the pseudorandom sequences, the noncyclic autocorrelations exhibit sidelobes variable values.

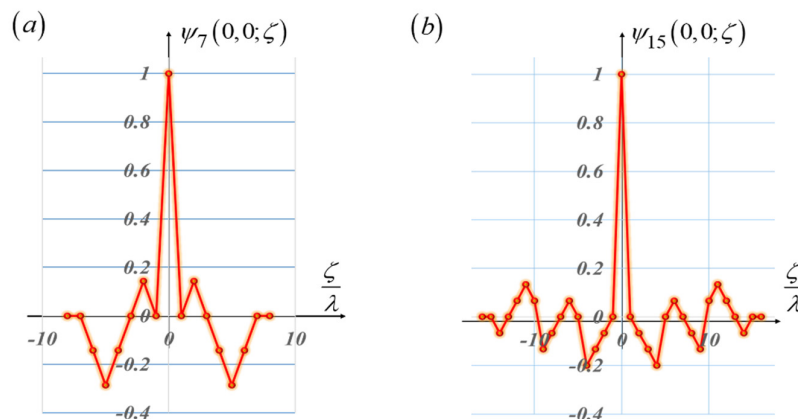


Figure 11. Like Figure 7. However, autocorrelations are noncyclic. Now, the pseudo random sequences have values of unity or minus unity. The noncyclic autocorrelations have a distinctive central peak, with nonuniform sidelobes.

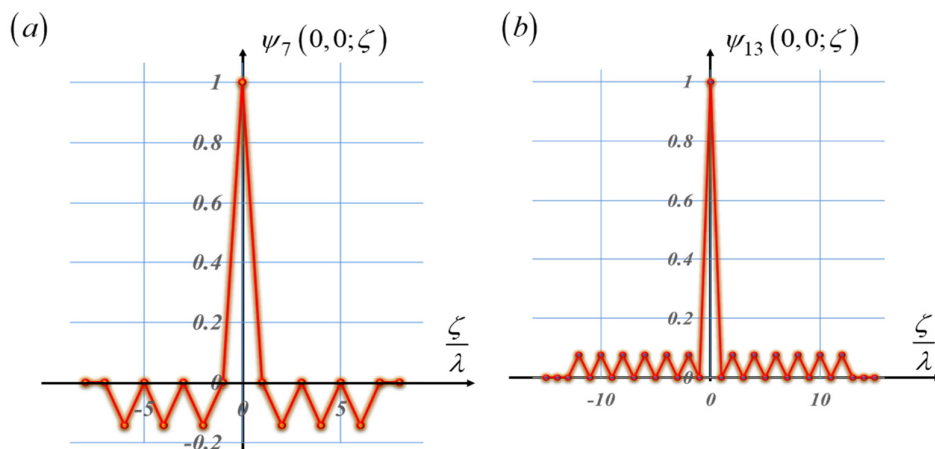


Figure 12. Like Figure 10. However, now the autocorrelations are noncyclic. They exhibit a single central peak, with sidelobes that have constant value.

From Figure 12, we observe that the Barker sequences generate noncyclic autocorrelations, which exhibit a distinctive central peak, with replicated sidelobes. Hence, Barker codes can be a desirable choice implementing an automatic sensor, based on optical autocorrelations. In the absence of phase gradients, the noncyclic autocorrelations have a central distinctive peak. On the other hand, if there are phase gradients, then the noncyclic autocorrelations have reduced values.

For the implementation of an automatic sensor, one has two choices. In Figure 13a, we show an unfolded version of the sensor. optical setup is shown in Figure 13a. And in Figure 13b, we depict folded optical setup.

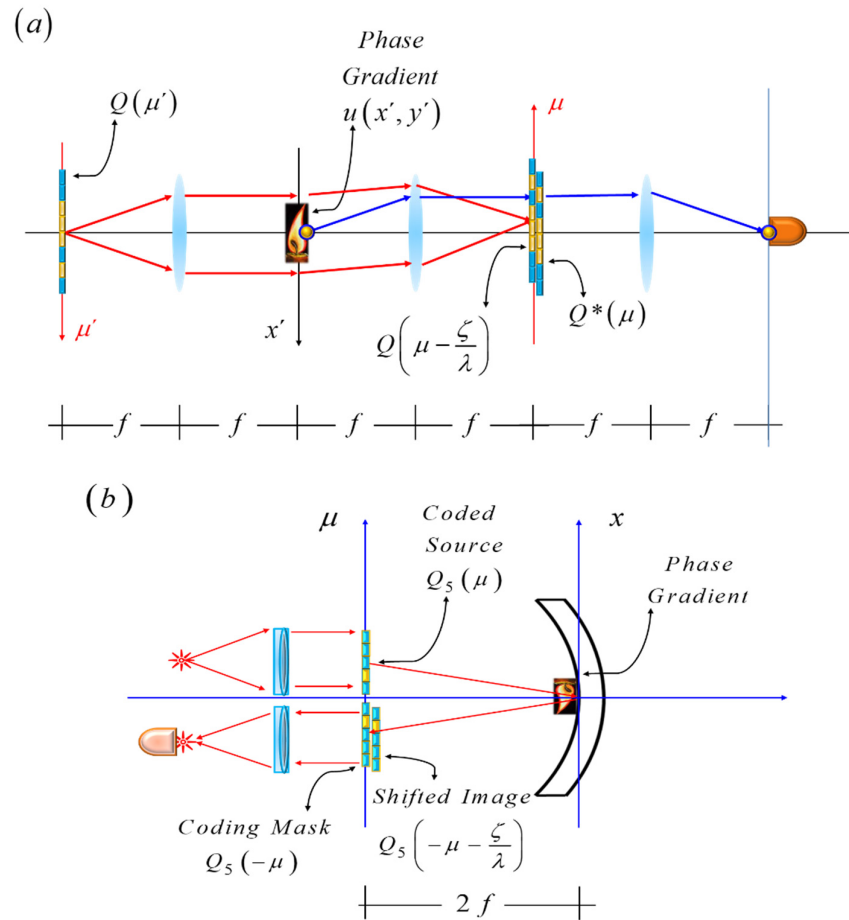


Figure 13. Schematics of a Schlieren technique that acts as an automatic sensor of phase gradients. In (a) an unfolded version. In (b) a folded version.

Hence, Barker codes can be a desirable choice for coding the source and the spatial filter. And in this manner one can have a correlation method for automatically detecting the presence of phase gradients. For this purpose, the optical setup is shown in Figure 13. An unfolded version of the optical setup is shown in Figure 13a. And in Figure 13b, we use a spherical mirror for depicting a folding version of the optical setup.

6. Conclusions

We have described a theoretical framework for designing nonconventional Schlieren techniques. These techniques employ optical masks for rendering visible phase gradients, like the Foucault knife-edge.

We have revisited the use of an effective transfer function. This discussion clarifies the relevance of employing arguments of symmetry, when designing Schlieren techniques.

We have disclosed a statistical model that describes phase gradients as randomly located prisms.

We have unveiled a link between the generation of optical autocorrelations and the use of Schlieren techniques. We employ two masks that at geometrical optics conjugates planes. The first mask is for coding the source. And a second mask is for coding the spatial filter. In this manner, we obtain a Schlieren technique with high light throughput.

We have indicated that the use of two masks is linked to the implementation of an optical correlator. This link is useful for implementing an automatic method that detects phase gradients.

The optical correlator exploits the use of Barker sequences for generating highly peaked, noncyclic irradiance distributions.

Author Contributions: Conceptualization, JOC; methodology, JOC; software, CMGS.; validation, JOC., and CMGS; writing—original draft preparation, JOC; writing—review and editing, CMGS. All authors have read and agreed to the published version of the manuscript.

Data Availability Statement: Currently, we do not include the numerical procedures for evaluating the graphs and the figures in this paper.

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Conflicts of Interest: The authors declare no conflicts of interest.

Appendix A: Symmetry Considerations

As pointed out in Equation (), we write the amplitude transmittance as the sum of an even function, $E(\mu)$, and of an odd function $\Theta(\mu)$.

$$Q(\mu, 0) = E(\mu) + \Theta(\mu). \quad (\text{A1})$$

One finds the even function by using the formula

$$E(\mu) \triangleq \left(\frac{1}{2}\right) [Q(\mu, 0) + Q(-\mu, 0)]. \quad (\text{A2})$$

$$\Theta(\mu) \triangleq \left(\frac{1}{2}\right) [Q(\mu, 0) - Q(-\mu, 0)]. \quad (\text{A3})$$

By substituting Equation (A1) in Equation (5) we have that

$$u(x, y) = Q(0, 0) + \pi \left(\frac{a}{\lambda}\right) \left[\begin{aligned} &\left[E\left(\frac{1}{d}\right) + \Theta\left(\frac{1}{d}\right) \right] \exp\left(i 2 \pi \frac{x}{d}\right) \\ &- \left[E\left(\frac{1}{d}\right) - \Theta\left(\frac{1}{d}\right) \right] \exp\left(-i 2 \pi \frac{x}{d}\right) \end{aligned} \right]. \quad (\text{A4})$$

Equation (A4) can also be written as

$$\begin{aligned} u(x, y) = & Q(0, 0) + \left(2 \pi \frac{a}{\lambda}\right) \Theta\left(\frac{1}{d}\right) \cos\left(2 \pi \frac{x}{d}\right) + \\ & + i \left(2 \pi \frac{a}{\lambda}\right) E\left(\frac{1}{d}\right) \sin\left(i 2 \pi \frac{x}{d}\right). \end{aligned} \quad (\text{A5})$$

Equation (A5) appears as Equation (7) in the main text.

Appendix B: Statistical Considerations

We use a Fourier transform for expressing the even component in Equation (19) as follows

$$e_H(x) = \int_{-\infty}^{\infty} E_H(\mu) \exp(i 2 \pi x \mu) d\mu. \quad (\text{B1})$$

We note that the function $e_H(x)$ is a real, even function of the variable x . We note that the odd component, in Equation (20), is a real function. Its Fourier transform is also an odd function. However, its Fourier transform is a purely imaginary function. Hence, it is convenient to use the following expression

$$i \mathcal{G}_H(x) = \int_{-\infty}^{\infty} \Theta_H(\mu) \exp(i 2 \pi x \mu) d\mu. \quad (\text{B2})$$

Thus, at the image plane, the average amplitude distribution is

$$\langle u(x, y) \rangle = Q(0, 0) + i \left(2 \pi \frac{a}{\lambda} \sin(\theta) \right) [e_H(x) + i \mathcal{G}_H(x)]. \quad (\text{B3})$$

Or equivalently, the average amplitude distribution is

$$\langle u(x, y) \rangle = Q(0, 0) - \left(2 \pi \frac{a}{\lambda} \sin(\theta) \right) \mathcal{G}_H(x) + i \left(2 \pi \frac{a}{\lambda} \sin(\theta) \right) e_H(x). \quad (\text{B4})$$

Next, we use the following definition for the normalized, average irradiance distribution

$$\langle I(x, y) \rangle \triangleq \frac{|\langle u(x, y) \rangle|^2}{Q(0, 0)}. \quad (\text{B5})$$

By substituting Equation (B4) in Equation (B5) we have that

$$\langle I(x, y) \rangle = 1 - \left(4 \pi \frac{a}{\lambda} \sin(\theta) \right) \mathcal{G}_H(x). \quad (\text{B6})$$

Equation(B6) appears as Equation (9) in the main text.

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