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The Symmetry Number Structure about Line-1/2

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Keywords: P/2n; prime numbers conjectures; riemann hypothesis

The Symmetry Number Structure about Line-1/2

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Abstract In this paper, we discuss the symmetry number structure about line-1/2**.** We find that using the symmetry characters of those structures we can give proofs of the number Conjectures: Goldbach Conjectur, eTwins Prime Conjecture and Polignac's conjecture and the Riemann Hypothesis. **In this paper, we also gave concise proofs of the Fermat' Last Theorem and the 3n+1 conjecture.**

Keywords: P/2n; Prime numbers Conjectures; Riemann Hypothesis

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1. The Symmetry of P/2n and Prime Numbers Conjectures

We have

$$\frac{P}{2n} = \begin{cases} \frac{1}{2^{N+1}} \; n = 2^N P \\ \frac{3}{4} \; n = 2 \; P = 3 \\ 1 \; n = 1 \; P = 2 \end{cases}$$

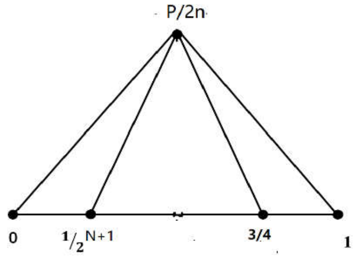

Figure 1. P/2n number structure with points [0 $1/2^{N+1}$ 3/4 1].

N~(0, 1 , 2 , 3 , 4 , ...) All natural numbers
n~(1 , 2 , 3 , 4 , ...) All natural numbers excepted 0
P~(2 , 3 , 5 , 7 , ...) All prime numbers
And

$$\frac{P}{2n} = \begin{cases} \frac{1}{2} - \frac{1}{2n} \; P = n - 1 \; (n \geq 3) \\ \frac{1}{2} \; P = n \\ \frac{1}{2} + \frac{1}{2n} \; P = n + 1 \end{cases}$$

And We have
p0∈P ~(0, n] ($n \geq 2$)
And based on Bertrand -Chebyshev Theorem: when $n \geq 2$, there are at least a prime number between n and 2n.
pn∈P ~[n, 2n) ($n \geq 2$)
So we have:

$$0 < \frac{p0}{2n} \leq \frac{1}{2}$$

$$\frac{1}{2} \leq \frac{pn}{2n} < 1$$

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So we have

$$\frac{p0}{2n} = \frac{1}{2} - \frac{1}{2n} \ (n \geq 3)$$

$$\frac{pn}{2n} = \frac{1}{2} + \frac{1}{2n}$$

So

$$(\frac{1}{2} - \frac{1}{2n}) + (\frac{1}{2} + \frac{1}{2n}) = \frac{p0}{2n} + \frac{pn}{2n}$$

$$2n = p0 + pn \ \ (n \geq 3)$$

This is the proof of Goldbach conjecture.
And

$$\frac{pn}{2n} - \frac{p0}{2n} = \left(\frac{1}{2} + \frac{1}{2n}\right) - (\frac{1}{2} - \frac{1}{2n})$$

$$pn - p0 = 2$$

This is the proof of Twin Primes Conjecture
And we also have

$$0 < \frac{p0}{2n} = \frac{2k_1+1}{2n} \ll 1/2 \ \ (n \geq 3)$$

$$0 < 2k_1 + 1 \ll n$$

$$0 < k_1 \ll \frac{n-1}{2}$$

$$k_1 \ is \ a \ positive \ integer$$

$$so \ k1 \sim 1,2,3 \cdots\cdots \left[\frac{n-1}{2}\right] \ \ (n \geq 3)$$

And

$$\frac{1}{2} \ll \frac{pn}{2n} = \frac{2k_2 + 1}{2n} < 1$$

$$n \ll 2k_2 + 1 < 2n$$

$$\frac{n-1}{2} \ll k_2 < \frac{2n-1}{2} (n \geq 3)$$

$$k_2 \ is \ a \ positive \ integer$$

$$so \ k_2 \sim \left[\frac{n-1}{2}\right], \left[\frac{n-1}{2}\right] + 1, \cdots\cdots \left[\frac{2n-1}{2}\right] \ \ (n \geq 3)$$

we have

$$\frac{pn}{2n} - \frac{p0}{2n} = \frac{2k_2 + 1}{2n} - \frac{2k_1 + 1}{2n}$$

$$pn - p0 = 2(k2 - k1)$$

$$(pn - p0)_{max} = 2(k2_{max} - k1_{min}) = 2\left(\left[\frac{2n-1}{2}\right] - 1\right) = 2(n-2) \ (n \geq 3)$$

This is the proof of Polignac's conjecture.
So we get a symmetry structure of P/2n as Figure 2

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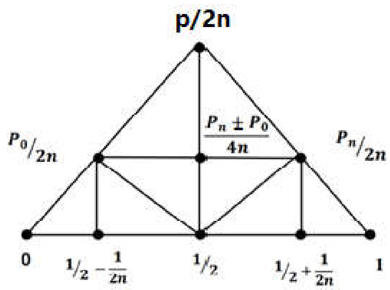

Figure 2. a symmetry structure of P/2n about line-1/2.

$$\frac{p0}{2n} = \frac{1}{2} - \frac{1}{2n} \ (n \geq 3)$$

$$\frac{pn}{2n} = \frac{1}{2} + \frac{1}{2n}$$

$$\frac{pn \pm p0}{4n} = \frac{1}{2}$$

2. A Concise Proof of The Fermat' Last Theorem

The Fermat' Last Theorem:

$\boldsymbol{x^n + y^n = z^n}$ $(x, y, z \in n, xyz \neq 0\ n > 2)$ **has no solution.**

$\boldsymbol{n}$**~(1, 2, 3, 4, 5, 6.........)** all the natural numbers excepted 0

The equivalent proposition of this conjecture is

$$\boldsymbol{(\frac{x}{z})^n + (\frac{y}{z})^n = 1}$$

$(x, y, z \in n, xyz \neq 0\ n > 2)$ **has no solution.**

We have

$$(\frac{x}{z})^n + (\frac{y}{z})^n = 1 = \sum \frac{1}{2^n}$$

n~(1,2,3,4,5, 6.........) all the natural numbers excepted 0

$$(\frac{x}{z})^n + (\frac{y}{z})^n = 1$$
$$= (\frac{1}{2^n} - \frac{1}{2^n}) + 1$$
$$= (\frac{1}{2} - \frac{1}{2^n}) + (\frac{1}{2} + \frac{1}{2^n})$$
$$= \frac{1}{2^n} + (1 - \frac{1}{2^n})$$

Only When $n = 1$ we have

$$\left(\frac{1}{2^n} - \frac{1}{2^n}\right) = \left(\frac{1}{2} - \frac{1}{2^n}\right) = 0$$

$$1 = (\frac{1}{2} + \frac{1}{2^n}) = (\frac{1}{2} + \frac{1}{2})$$

And Only When $n = 2$

$$\left(\frac{1}{2^n}\right) = \left(\frac{1}{2} - \frac{1}{2^n}\right) = \frac{1}{4}$$

$$\left(1 - \frac{1}{2^n}\right) = \left(\frac{1}{2} + \frac{1}{2^n}\right) = \frac{3}{4}$$

And We can get the figures as Figure 3.

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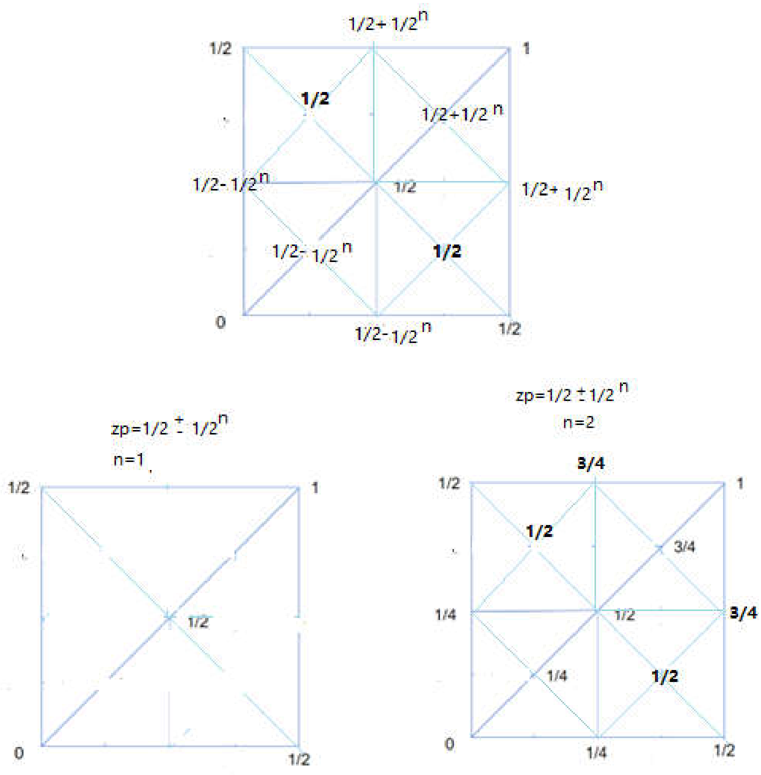

Figure 3. D$1/_{2}$+$1/_{2}$ with points $1/2\text{-}1/2^n$ and $1/2\text{-}1/2^n$ n~(1 , 2 , 3 , 4 , ……….).

$$\boldsymbol{n = 1 \text{ and } n = 2}$$

In fact we have

$$1 = \frac{1}{2^1} + \frac{1}{2^1} = \frac{1}{2^2} + \frac{3}{2^2} = \frac{1}{2^3} + \frac{7}{2^3} \, \boldsymbol{or} \, \frac{3}{2^3} + \frac{5}{2^3}$$

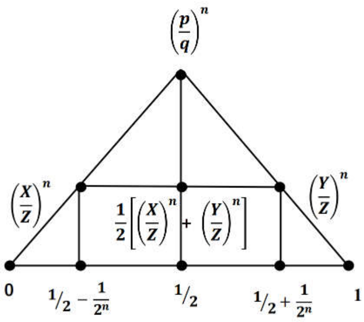

Figure 4. a symmetry structure of $(\frac{p}{q})^{\boldsymbol{n}}$ about line-1/2.

$(\frac{p}{q})^{\boldsymbol{n}}$ p, q is relatively prime and n~(1 , 2 , 3 , 4 , ……….)

$$\mathbf{1/2}[(\frac{x}{z})^{\boldsymbol{n}} + \left(\frac{y}{z}\right)^{\boldsymbol{n}}] = \mathbf{1/2} \leftrightarrow (\frac{x}{z})^{\boldsymbol{n}} + \left(\frac{y}{z}\right)^{\boldsymbol{n}} = \mathbf{1}$$

$$(\frac{x}{z})^n = \frac{1}{2} - \frac{1}{2^n}$$

$$(\frac{y}{z})^n = \frac{1}{2} + \frac{1}{2^n}$$

3. A Concise Proof of Collatz Conjecture

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Collatz Conjecture:

$$f(n) = \begin{cases} \frac{n}{2} \; if n \equiv 0 \; (mod2) \\ 3n + 1 \; if n \equiv 1 \; (mod2) \end{cases}$$

$k \in N \; \rightarrow f^k(n) = 1$

n~(1 , 2 , 3 , 4 , ……….)all the natural numbers excepted 0

$$\frac{[\frac{n+1}{2}]+[\frac{n-1}{2}]}{[\frac{n}{2}]} = \frac{2n+2}{n+1} = \frac{(n-1)+(3n+1)}{2n} = \frac{4n}{2n} = \frac{4}{2} = \frac{2}{1} = \frac{1}{\frac{1}{2}}$$

$$= \sum \frac{1}{2^N}$$

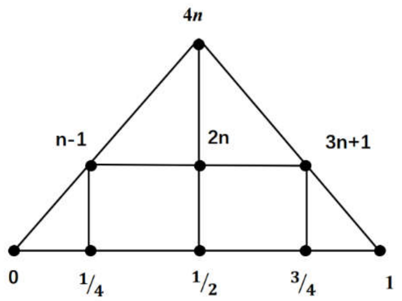

Figure 5. a symmetry structure of $\mathbf{4n}$ about line-1/2.

$$\mathbf{2n = 1/2[(n - 1) + (3n + 1)]}$$

$$\lim_{n\to\infty} \left(\frac{\mathbf{n - 1}}{\mathbf{4n}}\right) = \mathbf{1/4}$$

$$\lim_{n\to\infty} \left(\frac{\mathbf{3n + 1}}{\mathbf{4n}}\right) = \mathbf{3/4}$$

4. The Proof of Riemann Hypothesis

Riemann Zeta-Function

$$\xi\,(s) = \sum_{n=1} \frac{1}{n^s} = \prod \frac{1}{1 - p^s} \qquad (s = a + bi)$$

$$\boldsymbol{S > 1\; \xi\,(s) \rightarrow const}$$

The trivial zero-points of Riemann Zeta-Function is -2n (n~1,2,3,…….)

Riemann Hypothesis: all the Non-trivial zero-point of Zeta-Function $Re(s) = {}^1/_2$.

We can get a symmetrical structure including all numbers about the line-1/2 as Figure 6

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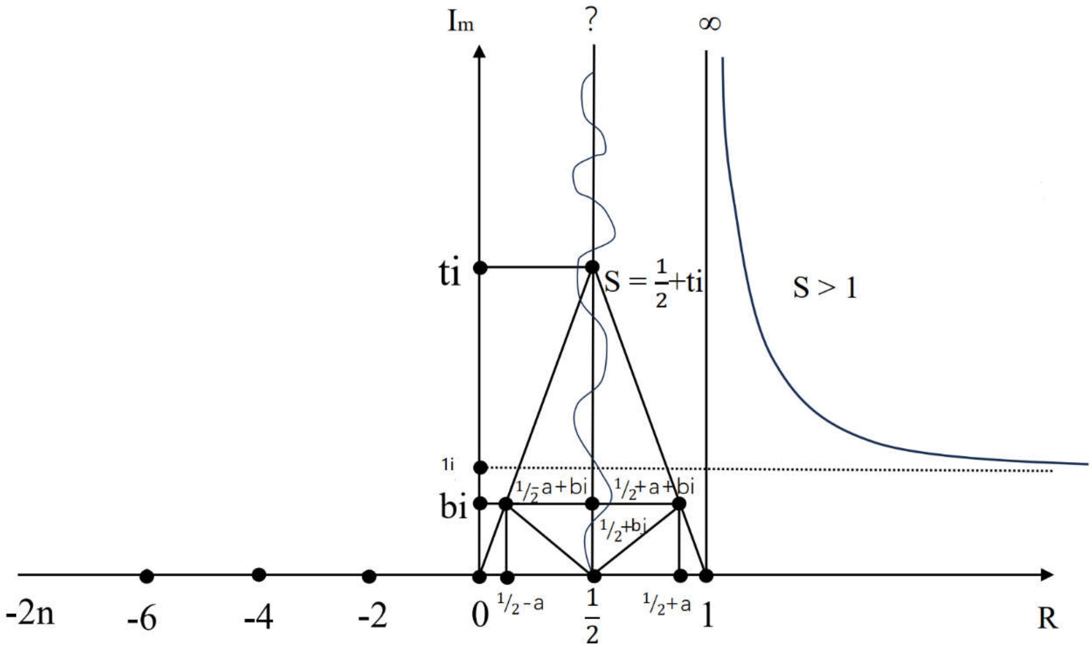

Figure 6. Riemann Hypothesis: all the non-trivial Zero points of Riemann zeta-function are on the 1/2 axis.

$$s = \frac{1}{2} + ti \quad t \in R$$

$$zp1 = \frac{1}{2} - a + bi \; zp0 = \frac{1}{2} + bi \; zp2 = \frac{1}{2} + a + bi$$

$$zp1 + zp2 = \left(\frac{1}{2} - a + bi\right) + \left(\frac{1}{2} + a + bi\right) = 1+2bi$$

$$zp2 - zp1 = \left(\frac{1}{2} + a + bi\right) - \left(\frac{1}{2} - a + bi\right) = 2a$$

$$a, b \in R \; 0 \le a \le \frac{1}{2}$$

As the Figure 7. If we have zero points of $\xi\ (s)$ on line-1/2±a as

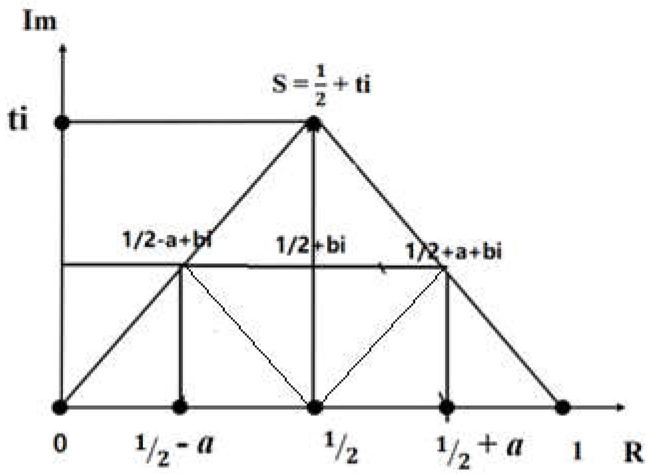

Figure 7. a symmetry structure about line1/2+/-a at the zero piont s=1/2+ti.

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$$zp1 = \frac{1}{2} - a + bi \; zp2 = \frac{1}{2} + a + bi$$

And $s = \frac{1}{2} + ti \;\; t \in R$ is the first zero point on line-1/2
We can get a zero point as

$$zp0 = \frac{1}{2} + bi \;\; b < t \; b, t \;\in R$$

It is contrary to that $s = \frac{1}{2} + ti \;\; t \in R$ is the first zero point on line-1/2
As the Figure 8. If we have zero points of $\xi\ (s)$ on line 1/2±a as

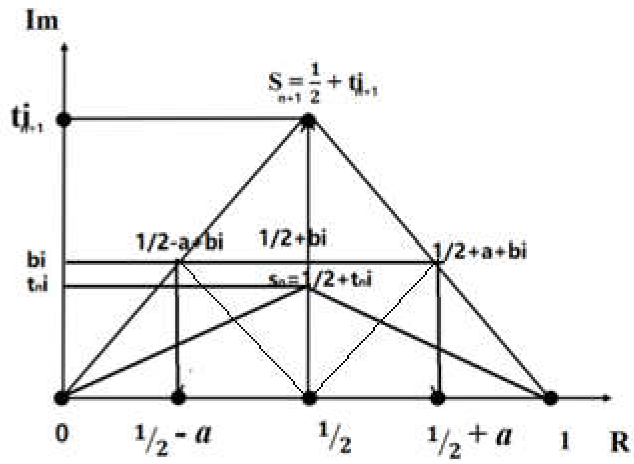

Figure 8. a symmetry structure about line1/2+/-a at the zero point s_n=1/2+t_ni and s_{n+1}=1/2+t_{n+1}i.

$$zp1 = \frac{1}{2} - a + bi \; zp2 = \frac{1}{2} + a + bi$$

And $s_n = \frac{1}{2} + t_n i \;\; t \in R$ is the No. n zero point on line-1/2
$s_{n+1} = \frac{1}{2} + t_{n+1} i \;\; t \in R$ is the No. n+1 zero point on line-1/2
We can get a zero point between s_n ***and*** s_{n+1} ***on line*** $-\ 1/2$ as

$$zp0 = \frac{1}{2} + bi \;\; t_n < b < t_{n+1} \; b, t \;\in R$$

It is contrary to that s_n ***and*** s_{n+1} are the adjacent zero points on line-1/2
So on complex plane, We can have the symmetry structure about the line-1/2 with zp=1/2±a ($0 \le a \le \frac{1}{2} \; a \;\in R$)show as on Figure 9.

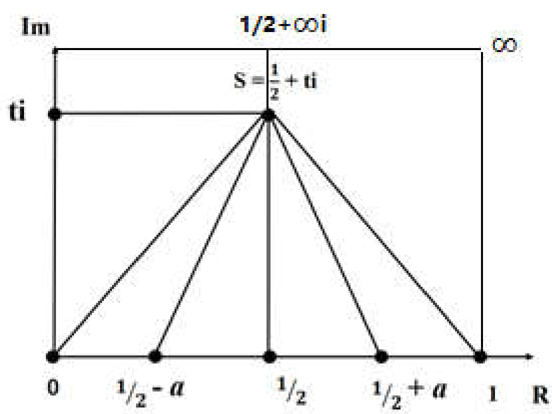

Figure 9. symmetry structure about the line-1/2 with zp=1/2±a.

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$$S = \frac{1}{2} + ti\ (t \in R)$$

$$zp = \frac{1}{2} \pm a\ \ (0 \leq a \leq \frac{1}{2}\ a \in R)$$

$$zp1 = \frac{1}{2} - a\ zp0 = \frac{1}{2}\ zp2 = \frac{1}{2} + a$$

$$zp1 + zp2 = \left(\frac{1}{2} - a\right) + \left(\frac{1}{2} + a\right) = 1$$

$$zp2 - zp1 = \left(\frac{1}{2} + a\right) - \left(\frac{1}{2} - a\right) = 2a$$

This is mean that there are no zero points on line-1/2±a $(0 \leq a \leq \frac{1}{2}\ a \in R)$.

Hardy and Littlewood give a proof that there are infinite zero points on line-1/2 (Hardy and Littlewood. 1914)

So we give a proof that all the non-trivial Zero points of Riemann zeta-function are on the Line-1/2. This is the proof of Riemann Hypothesis.

5. The Symmetry Number Structure About Line-1/2 Including All Numbers

In fact, we have a symmetrical number structure about line-1/2 as figure.10.

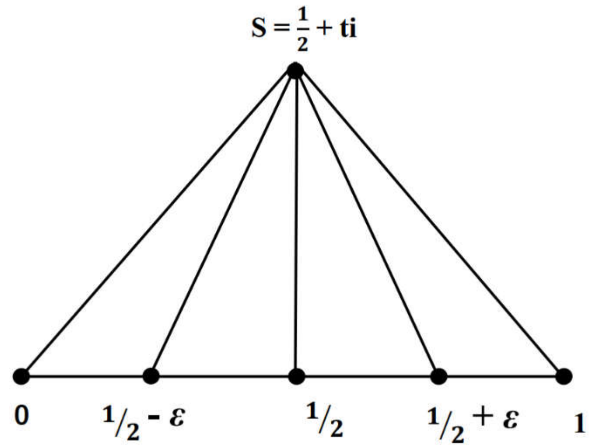

Figure 10. symmetry structure about the line-1/2 with zp=1/2±ε.

$$S = \frac{1}{2} + ti\ (t \in R)$$

$$zp = \frac{1}{2} \pm \varepsilon\ (\varepsilon = a + bi\ a, b\ \in R\ 0 \leq a \leq \frac{1}{2})$$

And we can get a symmetry number structure about line-1/2 as Figure 11. We should call it **Reimann dynamic space.**

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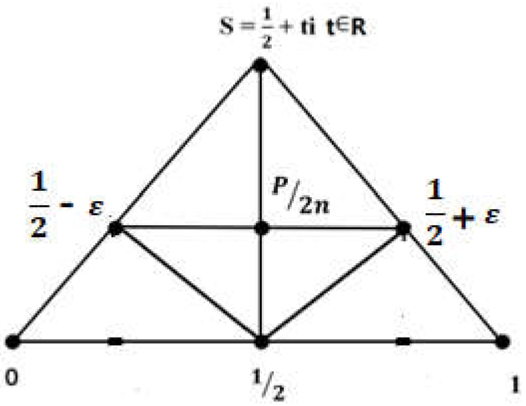

Figure 11. Reimann dynamic space.

$$1+\ i^2 = 0$$

$$1 + 1/2\ (i+1)\ (i-1) = 0$$

$$\boldsymbol{S} = \frac{1}{2} + \boldsymbol{ti}\ (\boldsymbol{t}\ \in \boldsymbol{R})$$

$$\boldsymbol{zp} = \frac{1}{2} \pm \boldsymbol{\varepsilon}\ (\boldsymbol{\varepsilon} = \boldsymbol{a} + \boldsymbol{bi}\ \boldsymbol{a},\boldsymbol{b}\ \in \boldsymbol{R}\ \boldsymbol{0} \le \boldsymbol{a} \le \frac{1}{2})$$

$$\frac{P}{2n} = \begin{cases} \frac{1}{2^{N+1}}\ n = 2^N P \\ \frac{3}{4}\ n = 2\ P = 3 \\ 1\ n = 1\ P = 2 \end{cases}$$

N~(0, 1 , 2 , 3 , 4 , ……….) All natural numbers
n~(1 , 2 , 3 , 4 , ……….) All natural numbers excepted 0
P~(2 , 3 , 5 , 7 , ……….) All prime numbers

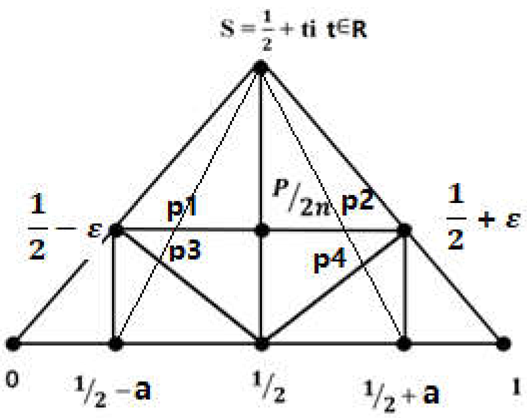

Figure 12. Reimann dynamic space with p1 p2 p3 p4.

We can have point p1 p2 p3 p4 and

$$\boldsymbol{p1} \in \left(\frac{\boldsymbol{1}}{\boldsymbol{2}} - \boldsymbol{\varepsilon}, \frac{\boldsymbol{p}}{\boldsymbol{2n}}\right)$$

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$$p2 \in \left(\frac{p}{2n}, \frac{1}{2} + \varepsilon\right)$$

$$p3 \in \left(\frac{1}{2} - \varepsilon, \frac{1}{2}\right)$$

$$p4 \in \left(\frac{1}{2}, \frac{1}{2} + \varepsilon\right)$$

And we can get Figure 13.

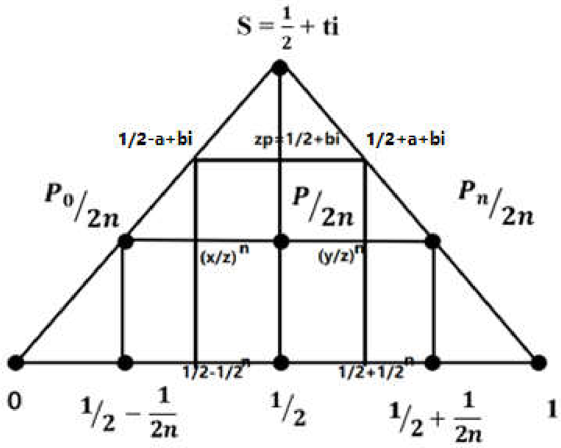

Figure 13. Reimann dynamic space and number conjectures.

1. $zp = \frac{1}{2} + bi \;\; 0 < b < t \; b, t \;\in R$ (the proof of RH)

2. $(\frac{x}{z})^n + \left(\frac{y}{z}\right)^n = 1$

$$(\frac{x}{z})^n \leftrightarrow \frac{1}{2} - \frac{1}{2^n}$$

$(\frac{y}{z})^n \leftrightarrow \frac{1}{2} + \frac{1}{2^n}$ **(the proof of F.L.T)**

3. $\frac{p0}{2n} \leftrightarrow \frac{1}{2} - \frac{1}{2n}$

$\frac{pn}{2n} \leftrightarrow \frac{1}{2} + \frac{1}{2n}$ **(the proof of GC/BC/TPC)**

$$\frac{p}{2n} \leftrightarrow \frac{1}{2}$$

And we have

$$1/2 = 1/2 \;\; 0 = 1/2\text{-}1/2 \;\; 1 = 1/2 + 1/2$$

$$1 + (\pm\ i)^2 = 0$$

$$1 + 1/2(i+1)\ (i-1) = 0$$

$$\infty = 1 + 1 + 1 + 1 + \cdots$$

We called it $L^{1/2\pm\varepsilon}$ [0 1/2 1] and analytic continuation to $\begin{bmatrix} +\infty & -\infty \\ -\infty & +\infty \end{bmatrix}$ we can get Figure 14.

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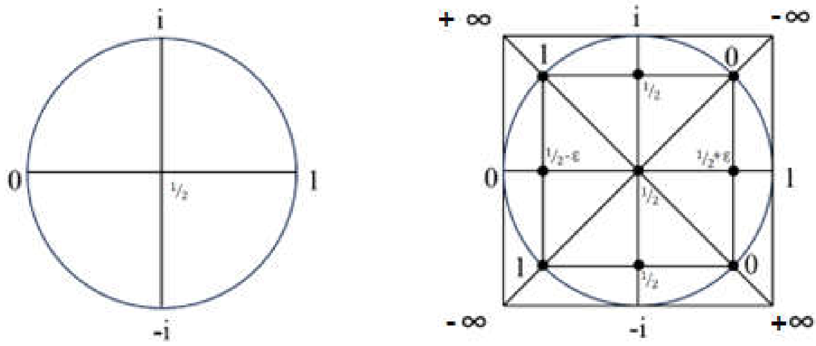

Figure 14. The Symmetry of $L^{1/2\pm\varepsilon}$ $[0\ 1/2\ 1]$ with $\boldsymbol{zp} = \frac{1}{2} \pm \boldsymbol{\varepsilon}$.

So we have:

$$1+\begin{bmatrix} +\infty & i & -\infty \\ 0 & 1/2 & 1 \\ -\infty & -i & +\infty \end{bmatrix} \begin{bmatrix} 1 & 1/2 & 0 \\ \frac{1}{2}-\varepsilon & 1/2 & \frac{1}{2}+\varepsilon \\ 1 & 1/2 & 0 \end{bmatrix}^{-1} = 0$$

$$\boldsymbol{zp} = \frac{\mathbf{1}}{\mathbf{2}} \pm \boldsymbol{\varepsilon}$$

$$\boldsymbol{\varepsilon} = \boldsymbol{a} + \boldsymbol{bi}\ (\boldsymbol{a}, \boldsymbol{b}\ \in \boldsymbol{R}\ \mathbf{0} \le \boldsymbol{a} \le \tfrac{\mathbf{1}}{\mathbf{2}})$$

$$\boldsymbol{zp1} = \frac{\mathbf{1}}{\mathbf{2}} - \boldsymbol{\varepsilon}\ \boldsymbol{zp2} = \frac{\mathbf{1}}{\mathbf{2}} + \boldsymbol{\varepsilon}$$

We have

$$\boldsymbol{zp1} + \boldsymbol{zp2} = \left(\frac{\mathbf{1}}{\mathbf{2}} - \boldsymbol{\varepsilon}\right) + \left(\frac{\mathbf{1}}{\mathbf{2}} + \boldsymbol{\varepsilon}\right) = \mathbf{1}$$

$$\boldsymbol{zp2} - \boldsymbol{zp1} = \left(\frac{\mathbf{1}}{\mathbf{2}} + \boldsymbol{\varepsilon}\right) - \left(\frac{\mathbf{1}}{\mathbf{2}} - \boldsymbol{\varepsilon}\right) = \mathbf{2}\boldsymbol{\varepsilon} = \mathbf{2}(\boldsymbol{a} + \boldsymbol{bi})$$

And we have

$$n^2 = \frac{1}{2} \cdot n \cdot 2n = \sum^{N} \frac{1}{2} \sum^{N} \frac{1}{2^N} [\left(\frac{\mathbf{1}}{\mathbf{2}} - \boldsymbol{\varepsilon}\right) + \left(\frac{\mathbf{1}}{\mathbf{2}} + \boldsymbol{\varepsilon}\right)]$$

N~(0, 1 , 2 , 3 , 4 , ……….) All natural numbers
n~(1 , 2 , 3 , 4 , ……….) All natural numbers excepted 0
We can get a matrix $(n \times n)$

$$\begin{bmatrix} 1/2 \ \dots\dots\ \frac{1}{2^n}(1/2+\boldsymbol{\varepsilon}) \\ \dots\dots\ 1/2\ \dots\dots\dots \\ \frac{1}{2^n}(1/2-\boldsymbol{\varepsilon})\ \dots\dots\ 1/2 \end{bmatrix} (n \times n)$$

The tr(A)=1/2*n
We have

$$0 = \frac{1}{2} - \frac{1}{2}\ 1 = \frac{1}{2} + \frac{1}{2}\ 2 = 1 + 1$$

$$1 + i^2 = 0\ \ \mathbf{1/2} + \boldsymbol{i}^{\mathbf{4N+1}} = \mathbf{1/2} + \boldsymbol{i}$$

$$\infty = 1 + 1 + 1 + 1 + \cdots$$

$$\mathbf{p0 \in P\ \le 2n\ pn \in P \ge 2n}$$

N~(0 , 1 , 2 , 3 , 4 , ……….) all the natural numbers.

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n~(1 , 2 , 3 , 4 , ……….) All natural numbers excepted 0
P~(2 , 3 , 5 , 7 , ……….) All odd prime number

$$\boldsymbol{S = \frac{1}{2} + t\ (t\ \in R)}$$

$$\boldsymbol{zp = \frac{1}{2} \pm \varepsilon\ (\varepsilon = a + bi\ a, b\ \in R\ 0 \leq a \leq \frac{1}{2})}$$

And we find that
1. $\boldsymbol{1 + e^{\pi i}}$ =0 (Euler's Formula)

$$1+i^2=0\quad 1 + \frac{1}{2}(i + 1)(i - 1) = 0\ (1+i)(1-i)\ = \sum \frac{1}{2^N}$$

$$\boldsymbol{1 + e^{\pi i} = 0\ 1 + \frac{1}{2}(e^{ip\pi} - e^{i2N\pi}) = 0}$$

N~(0 , 1 , 2 , 3 , 4 , ……….) all the natural numbers.
p~(3 , 5 , 7 , ……….) All odd prime number

$$\boldsymbol{2(n \pm 1) = pn \pm p0}$$

2. $\boldsymbol{pn - 2n + p0 = 2}$

And

$$\boldsymbol{2n - pn + p0=2}$$

It is like the Euler's Polyhedron Formula
We can get Figure 15. This is a symmetry number structure about line-1/2 including all numbers.

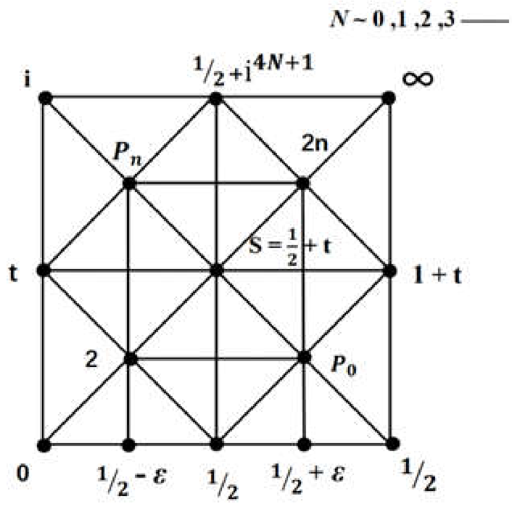

Figure 15. The Symmetry of S∞+i.

Data Availability Statement: No datasets were generated or analyzed during the current study.

Conflicts of Interest: The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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