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Posted Date: 9 September 2025

doi: 10.20944/preprints202502.0683.v10

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Article

Seismic Activation Modeling with Statistical Physics

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Abstract

A correspondence between fault damage mechanics and critical point models of seismic activation is presented, and a method of testing the correspondence against seismic measurements is outlined.

Keywords: seismic activation; fault dynamics; statistical physics; signal processing

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Introduction

An increase in the number of intermediate sized earthquakes of moment magnitude $\mathrm{M} > 3.5$ in a seismic region preceding the occurrence of a mainshock event, referred to as seismic activation, has been documented by various researchers [7]. For example, seismic activation was observed in a geographic region spanning $21°N - 26°N \times 119°E - 123°E$ for a period of time between 1991 and 1999 preceding the magnitude 7.6 Chi-Chi earthquake [10]. Figure 1 shows a schematic plot of the cumulative distribution of earthquakes of different magnitudes in a seismic activation region in two different time intervals of equal duration preceding occurrence of a major ($7 < \mathrm{M} < 8$) earthquake at time $\tau = \tau_0$. In this figure, τ is a real time parameter, and τ_0 is the characteristic time of major earthquake recurrence assuming an earthquake of similar magnitude occurred in the same region at $\tau = 0$ [27,36]. Importantly, the cumulative distribution of earthquake magnitudes in a time interval of fixed width increasingly deviates away from a Gutenberg-Richter distribution as the end of the time interval approaches τ_0.

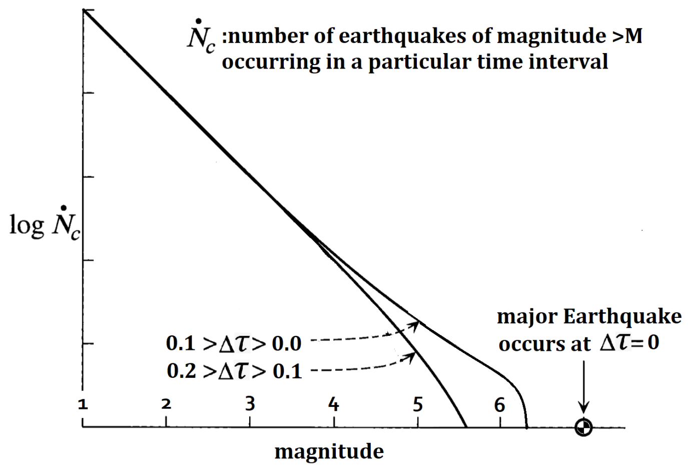

Figure 1. Plot of the cumulative distribution of earthquakes of different moment magnitudes in a seismic activation region in two different time intervals of equal width preceding occurrence of a major earthquake at $\Delta\tau = \tau_0 - \tau = 0$ [27,36].

As a means of predicting the time $\tau = \tau_0$ at which a mainshock event preceded by seismic activation occurs, it has been hypothesized that the average seismic moment $\overline{\mathcal{M}}_\tau$ of earthquakes occuring in time intervals $(\tau, \tau + \Delta\tau)$ preceding a mainshock event obeys an inverse power of time to failure law:

$$\overline{\mathcal{M}}_\tau \propto \frac{1}{(\tau_0 - \tau)^{\gamma_1}} \quad (1)$$

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and that the cumulative Benioff strain C(τ), defined as:

$$\mathcal{C}(\tau) = \sum_{i=1}^{n(\tau)} \mathcal{M}_{0,\mathrm{i}}^{1/2}, \tag{2}$$

where $\mathcal{M}_{0,i}$ is the seismic moment of the i^{th} earthquake in the region starting from a time $\tau = 0$ preceding the mainshock event, and $n(\tau)$ is the number of earthquakes occurring in the region up to time τ, satisfies [34]:

$$\mathcal{C}(\tau) = c_0 - c_1(\tau_0 - \tau)^{\gamma_2},\ \gamma_2 = 1 - \gamma_1/2. \tag{3}$$

The exponent selection of $1/2$ in equation (2) is not necessary to derive formula (3) with a different arithmetic relation between γ_1 and γ_2, but appears to have been selected by previous researchers based on Benioff's finding that the elastic rebound of an earthquake is proportional to the square root of its seismic moment. When formula (3) is fit to real seismic data, a typical value of γ_2 is 0.3 [7,35]. Notably, validity of equation (3) as a quantitative description of seismic activation earthquake occurrence has been refuted by some researchers [17,39].

A model of seismic activation based on fault damage mechanics (FDM) has been used to derive equation (3) with a value $\gamma_2 = 1/3$ [5]. In this model, the seismic activation region is defined as a 1D fault consisting of perfectly plastic material, and a time evolution equation for a fault damage parameter, quantifying fault material microcrack density, is derived based on non-equilibrium thermodynamic considerations [4]. In terms of fault material shear strain $\epsilon(\tau)$, this damage evolution equation is:

$$\frac{d\epsilon}{d\tau} \propto \epsilon^4, \tag{4}$$

implying finite time divergence of the strain:

$$\epsilon(\tau) \propto \frac{1}{(\tau_0 - \tau)^{1/3}} \tag{5}$$

and power law scaling (1) of the average earthquake seismic moment time with scaling exponent $\gamma_1 = 4/3$ in the limit $\tau \to \tau_0$.

In addition to the FDM model of seismic activation, a statistical mechanics model of seismic activation known as the Critical Point (CP) model has been put forth to derive equation (3) with a value $\gamma_2 = 1/4$ [27]. In this derivation, time scaling:

$$\overline{\mathcal{M}}_\tau \propto \frac{1}{(\tau_0 - \tau)^{3/2}} \tag{6}$$

of the average earthquake seismic moment at time τ is derived by identifying the mean rupture length $\overline{\mathcal{R}}_\tau$ of seismic activation earthquakes occuring at time τ with the correlation length of a τ dependent statistical mechanical system. The correlation length dependence on τ is specified by asserting that the statistical mechanical system order parameter satisfies a time dependent Ginzburg-Landau equation, whereby:

$$\overline{\mathcal{R}}_\tau \propto \frac{1}{(\tau_0 - \tau)^{1/2}}, \tag{7}$$

and relation (6) follows assuming $\overline{\mathcal{M}}_\tau \propto \overline{\mathcal{R}}_\tau^3$ [28]. **Table 1** shows typical fault material displacements and rupture lengths for earthquakes of different moment magnitudes.

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Table 1. Approximate relation between earthquake magnitude, fault material displacement, and fault rupture length.

Moment Magnitude	Average Fault Material Displacement (m)	Fault Rupture Length (km)
4	0.05	1
5	0.15	3
6	0.5	10
7	1.5	30
8	5	100

Importantly, previous work has not clearly explained how FDM and CP seismic activation models are related. Therefore, the first objective of this article is to conjecture how FDM and CP models of seismic activation are in correspondence with each other and provide computational evidence for this conjecture. The second objective is to outline how the conjectured correspondence can be used to advance earthquake forecasting technology.

To meet these objectives, two different pre-existing seismicity models are developed: the earthquake cascade model and the renormalization group model of earthquakes [31,33]. The earthquake cascade model provides an explanation for earthquake occurrence statistics in windows of time preceding the mainshock event in terms of the size distribution of metastable clusters of fault material within the activation region that can join together to create larger clusters or undergo slip events that initiate earthquakes. The renormalization group theory of earthquakes explains how the formalism of equilibrium statistical mechanics accounts for time scaling of cumulative Benioff strain during seismic activation in terms of statistical mechanics critical scaling theory, without explicitly identifying the geophysically relevant statistical mechanical system. For the purposes of this article, the earthquake cascade model is developed into a random matrix model of the seismic activation region tangent stiffness matrix, and the renormalization group theory of earthquakes is developed into a description of the time evolution of the random matrix model during mainshock rupture nucleation. In so doing, correspondence between FDM and CP seismic activation models is clarified, because the random matrix model is an FDM description of crack growth in a seismic region, and time evolution of the random matrix model is specified by statistical mechanics critical scaling theory. The conjectured correspondence between FDM and CP seismic activation models can be tested against seismic measurements because it predicts a finite dimensional nonlinear dynamical system quantifies spatial variation of the activation region Gutenberg-Richter b-value occurring with mainshock rupture nucleation [37].

The outline of the article is as follows. Section 2 introduces the earthquake cascade model with reference to laboratory studies of rock fracture, and explains how this model is related to a random matrix model of the seismic activation region tangent stiffness matrix. Section 3 provides a computation of the eigenvalue distribution of the random stiffness matrix model, conjectures how time evolution of the random stiffness matrix ensemble is described by 2D statistical mechanics critical scaling theory, and outlines how this conjecture can be tested with seismic signal processing. Section 4 comments on application of seismic activation modeling with statistical physics to advancing earthquake forecasting technology.

Materials and Methods

It has been observed that fracture processes occurring within the Earth preceding unstable slip along a mainshock fault are analogous to fracture processes occurring in triaxially loaded rock samples preceding sample failure [24]. Triaxial test results indicate 4 stages of rock deformation leading to sample failure [12]:

- crack closure
- elastic deformation
- stable microcrack growth and extension

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- unstable crack extension

Triaxial loading of sandstone samples with recording of acoustic emissions has further indicated that microcracks nucleate in zones along faults where unstable cracking initiates, in a process called shear localization [13]. This microcrack nucleation process is the laboratory analog of earthquake rupture nucleation, which leads to acceleration of an earthquake rupture front across a rupture area consisting of metastable fault material where shear stress has reached material shear strength. It is the size distribution of metastable clusters of fault material in a seismic region that is quantified by the earthquake cascade model.

Earthquake Cascade Model

Mathematically, the earthquake cascade model divides a 2D earthquake fault into a grid of squares of unit area, and defines $N_k(\tau)$ as the number of metastable clusters of fault material of area 2^k at time τ, where a metastable cluster is defined as the region over which an earthquake rupture propagates after initiation. The model also provides the time evolution equations:

$$\frac{dN_0}{d\tau} = C_0 - C_{0,1}N_0^2 - D_0 N_0, \tag{8}$$
$$\frac{dN_1}{d\tau} = \frac{1}{2}C_{0,1}N_0^2 - C_{1,2}N_1^2 - D_1 N_1, \tag{9}$$
$$\vdots \tag{10}$$
$$\frac{dN_n}{d\tau} = \frac{1}{2}C_{n-1,n}N_{n-1}^2 - C_{n,n+1}N_n^2 - D_n N_n, \tag{11}$$

where C_0 is the rate at which squares of unit area become metastable, $\frac{1}{2}C_{k-1,k}N_{k-1}^2$ is the rate at which clusters of area 2^{k-1} join to form clusters of area 2^k, and $D_k N_k$ is the rate at which clusters are lost due to earthquake slip initiation. It is also asserted that $C_{k,k+1} \propto (2^k)^\epsilon$, and $D_k \propto 2^k$.

If earthquake occurrence rates are negligible in comparison to cluster joining rates, the earthquake cascade model specifies a steady state cluster size distribution by the requirement:

$$\frac{N_{k-1}}{N_k} = 2^{(\epsilon+1)/2}, \tag{12}$$

whereby:

$$N_k = N_0 \cdot 2^{-k(\epsilon+1)/2}. \tag{13}$$

In turn, equation (13) implies:

$$D_k N_k \propto (2^k)^{(1-\epsilon)/2}. \tag{14}$$

which for metastable cluster area $\mathcal{A}_k = 2^k$, specifies an earthquake occurrence rate proportional to:

$$\mathcal{A}_k^{(1-\epsilon)/2} dk \propto \mathcal{A}_k^{-(1+\epsilon)/2} d\mathcal{A}_k. \tag{15}$$

Equation (15) can be interpreted as a Gutenberg-Richter scaling relation between frequency of earthquake occurrence and earthquake rupture area with b-value $b = (\epsilon - 1)/2$. Specifically, if N_{cM} is the number of earthquakes of moment magnitude greater than or equal to M occurring during the time interval $(\tau, \tau + \Delta\tau)$, the Gutenberg-Richter law implies:

$$N_{cM} \propto 10^{-bM}, \tag{16}$$

which using the seismic moment-magnitude relation:

$$\mathrm{M} = \left(\frac{\log_{10}\mathcal{M} - 9}{1.5}\right), \tag{17}$$

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and corner frequency scaling relation:

$$\mathcal{M} \propto \omega^{-3}, \tag{18}$$

implies the number of earthquakes $f(\omega)d\omega$ occurring with corner frequency in the interval $(\omega, \omega + d\omega)$ satisfies:

$$f(\omega)d\omega \propto \omega^{2b-1}d\omega \propto \mathcal{A}^{-b-1}d\mathcal{A}, \tag{19}$$

for $\omega \propto 1/\sqrt{\mathcal{A}}$. For example, if $\epsilon = 1.72$, then $b = 0.36$, in which case a higher than average proportion of intermediate sized earthquakes as compared to small earthquakes are occurring, as occurs during seismic activation.

FDM Random Matrix Model

To relate the earthquake cascade model to statistical mechanics, it is conjectured that the distribution of metastable cluster sizes can be related to the eigenvalue distribution of a random matrix ensemble. As a starting point for explaining this conjecture, **Figure 2** shows an earthquake fault containing cracks of length R separated by metastable clusters of uncracked material of diameter D. The fault material is sheared by applying a shear displacement across a constant height of material. If the crack configuration is static, the shear displacement may be increased until one or more crack tip stress intensity factors reach the Mode II fracture toughness of the uncracked material, at which point crack propagation initiates. In terms of time independent finite element analysis, at this critical shear displacement the tangent stiffness matrix of the fault material region has one or more zero eigenvalues, identifying one or more marginally stable modes of crack growth.

Mathematically, assuming a region of the Earth is contained within a hemisphere $\mathcal{H}$ centered on the Earth's surface, and subsurface material within the hemisphere is meshed with finitely many structural elements, a nodal displacement vector $\mathbf{U}(t) = e^{-i\omega t}\mathbf{u}$ perturbing a static finite strain elastoplastic deformation of material within the hemisphere at time satisfies the equation:

$$\left(\tilde{K}(\omega) - i\omega\tilde{D}(\omega) - \omega^2\tilde{M}\right)\mathbf{u} = 0, \tag{20}$$

where $\tilde{K}(\omega)$, $\tilde{D}(\omega)$, and $\tilde{M}$ are the frequency dependent finite element tangent stiffness, damping, and mass matrices [6]. Based on this equation, the condition:

$$\det\left(\tilde{K}(\omega) - i\omega\tilde{D}(\omega) - \omega^2\tilde{M}\right) = 0, \tag{21}$$

has non-zero complex valued solutions ω specifying resonant frequencies of the region and its static crack configuration, and zero solutions whose associated eigenvectors $\mathbf{u}$ characterize marginally stable modes of crack growth. Nonlinear eigenvalue problem (20) can also be written in the form:

$$\tilde{L}(\omega)\begin{bmatrix}\mathbf{u}\\ -i\omega\tilde{M}\mathbf{u}\end{bmatrix} = -i\omega\begin{bmatrix}\mathbf{u}\\ -i\omega\tilde{M}\mathbf{u}\end{bmatrix}, \tag{22}$$

where:

$$\tilde{L}(\omega) = \begin{bmatrix}0 & \tilde{M}^{-1}\\ -\tilde{K}(\omega) & -\tilde{D}(\omega)\tilde{M}^{-1}\end{bmatrix}. \tag{23}$$

and for brevity, solutions to equation (22) will be referred to as eigenvalues of the normalized tangent stiffness matrix $\tilde{L}(\omega)$. At low frequencies, where $\tilde{K}(\omega)$ and $\tilde{D}(\omega)$ can be approximated as being independent of ω, nonlinear eigenvalue problem (21) is equivalent to the linear eigenvalue problem:

$$\tilde{L}(0)\begin{bmatrix}\mathbf{u}\\ -i\omega\tilde{M}\mathbf{u}\end{bmatrix} \equiv \begin{bmatrix}0 & \tilde{M}^{-1}\\ -\tilde{K}(0) & -\tilde{D}(0)\tilde{M}^{-1}\end{bmatrix}\begin{bmatrix}\mathbf{u}\\ -i\omega\tilde{M}\mathbf{u}\end{bmatrix} = -i\omega\begin{bmatrix}\mathbf{u}\\ -i\omega\tilde{M}\mathbf{u}\end{bmatrix}, \tag{24}$$

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which determines oscillatory solutions of the differential equation:

$$\frac{d}{d\tau}\begin{bmatrix}\mathbf{u}(\tau)\\ \tilde{M}\frac{d\mathbf{u}(\tau)}{d\tau}\end{bmatrix} = \tilde{L}(0)\begin{bmatrix}\mathbf{u}(\tau)\\ \tilde{M}\frac{d\mathbf{u}(\tau)}{d\tau}\end{bmatrix}, \tag{25}$$

characterizing time variation of nodal displacement and momentum.

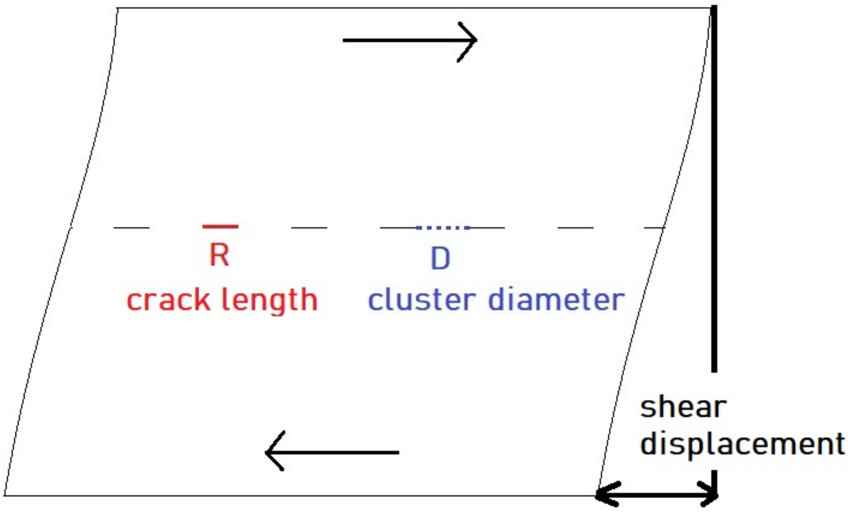

Figure 2. Cracks of length R separated by uncracked material of diameter D.

Physically, earthquakes occur with propagation of cracks within the Earth, so the assumption of static elastoplastic material deformation is not valid for a seismic region in which cumulative Benioff strain is increasing at a constant rate or accelerating [26]. For this reason, it is not logical to associate metastable clusters of the earthquake cascade model with zero eigenvalues of an elastostatic tangent stiffness matrix. However, it is known that stressed regions within the Earth can migrate at the fronts of slow strain waves propagating within the Earth [9]. Therefore, to associate metastable clusters of the earthquake cascade model with tangent stiffness matrix eigenvalues, it is conjectured that rather than being defined as static material configurations, metastable clusters should be defined as dynamic material configurations existing at the tips of cracks growing within the Earth. In principle, this allows for association of metastable clusters with eigenvalues of a normalized tangent stiffness matrix $\tilde{L}(\tau;\omega)$ that depends on τ, and is defined in relation to a crack phase field.

To elaborate further on the previous statements, it is recalled that in 1 spatial dimension, slow strain waves propagating along crustal faults have been analyzed as kink solutions to a sine-Gordon equation with solitary wave velocity profiles:

$$v(x,\tau) \propto V\,\mathrm{sech}\left(\frac{x - V\tau}{\delta}\right), \tag{26}$$

where δ is the thickness of fault material, and the wave velocity V can range from a velocity of seismicity migration (10^{-3} to 10^{-2} m/s) to an earthquake rupture velocity (10^2 to 10^3 m/s) depending on the frictional resistance of the fault material [9]. This analysis also suggests that if fault material frictional resistance decreases with increasing velocity V, there is a critical velocity at which acceleration of the strain wave to an earthquake rupture velocity is triggered. For this reason, it appears qualitatively reasonable to identify δ with the diameter of a fault material cluster that is metastable at a velocity depending on δ.

Mathematically, the quantity δ appearing in equation (26), while related to a bound state eigenvalue of the sine-Gordon inverse scattering problem in 1 spatial dimension, cannot be directly related to an eigenvalue of a normalized tangent stiffness matrix $\tilde{L}(\tau;\omega)$ for a 3D seismic region [2,18]. Therefore,

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it is conjectured that each metastable cluster diameter D at time τ is proportional to $1/|\omega|$, where ω is an eigenvalue of $\tilde{L}(\tau;\omega)$ for which ω^2 is negative, based on the following qualitative observations:

- modes of localized plastic deformation of a geomechanical system can be associated with eigenvalues of the system tangent stiffness matrix, with nodal velocity eigenvectors characterizing the spatial extent of deformation [11]
- localized plastic deformation at a propagating crack tip is accompanied by strain energy release [14]

Furthermore, to facilitate relation with statistical mechanics, it is conjectured that a stochastic process model of a seismic region stiffness matrix exists whereby the eigenvalues of $\tilde{L}(\tau;\omega)$, viewed as being selected from a random matrix ensemble, describes the distribution of metastable cluster diameters and velocities in the seismic region at time τ.

Results

FDM Random Matrix Eigenvalue Distribution

Suppose the distribution of eigenvalues of the matrix $\tilde{L}(\tau,\omega)$ is time invariant in a seismic region for which cumulative Benioff strain increases at a constant rate. In this case, if $F(|\omega|)d|\omega|$ is the density of eigenvalues for which ω^2 is negative, and each metastable cluster undergoes earthquake rupture in the time interval $(\tau,\tau+\Delta\tau)$ with equal probability, the Gutenberg-Richter law requires:

$$F(|\omega|) \propto |\omega|^{2b-1}. \tag{27}$$

Equation (27) has the mathematical form of a density of vibrational modes localized on a fractal of dimension $2b$ [3]. This is geophysically reasonable if $2b$ approximates the seismic region fault system fractal dimension, and vibrational modes of the fault system identify strain energy release modes of the seismic region.

2D Critical Scaling Theory

Previous research documents decrease in the Gutenberg-Richter b-value of a seismic region preceding a mainshock event [23]. If such decrease is a general feature of seismic activation, the renormalization group theory of earthquakes may provide a systematic means of quantifying this decrease, in which case statistical mechanical critical scaling theory should apply to describe time evolution of the spatial distribution of Gutenberg-Richter b-values in the region. From this point of view, characterizing the metastable cluster distribution of a seismic region with a fixed value of b in terms of the eigenvalue distribution of the region's normalized tangent stiffness matrix is of value if it enables identification of a geophysically relevant statistical mechanical system and renormalization group fixed points. For the purposes of this article, it is conjectured that the relevant system is a 2 dimensional Coulomb gas, without theoretical derivation, and conclusions are drawn from this conjecture to enable testing its validity [40].

The energy of a 2D Coulomb gas with m positive and m negative charges is:

$$H_{2m} = -2\pi \sum_{i<j}^{2m} e_i e_j \ln \sqrt{\frac{r_0^2 + r_{ij}^2}{r_0^2}} \tag{28}$$

where $e_i = \pm|e|$ is the particle charge [16]. At thermal equilibrium, this system behaves like a neutral gas of bound pairs (i.e. dipoles) of charges of opposite signs below the KT transition temperature:

$$T_c = \frac{\pi e^2}{2k_B}, \tag{29}$$

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and the density$\Gamma(l)dl$ of dipoles of length l satisfies [32]:

$$\Gamma(l)dl \propto l^{-2\pi e^2/k_B T} dl. \tag{30}$$

Correspondence between earthquake occurrence statistics in a seismic region with constant Gutenberg-Richter b-value and statistical mechanics is conjectured to occur with equality of metastable cluster diameter and dipole length distributions, which requires:

$$-2b - 1 = -\frac{2\pi e^2}{k_B T}. \tag{31}$$

or:

$$1 + 2b = 2\pi\beta, \tag{32}$$

for $\beta = e^2/k_B T$.

In a seismic activation region, the Gutenberg-Richter b-value is anticipated to depend on τ, and vary spatially. More specifically, the fractal dimension of of the seismic activation region fault system where mainshock rupture nucleation occurs may differ from the fractal dimension of the fault system elsewhere [15]. To avoid this difficulty in defining correspondence between seismic activation region earthquake occurence statistics and dipole length statistics of an ordered 2D Coulomb gas, it is suggested that the Gutenberg-Richter b-value be measured in a spherical subregion of the seismic activation region centered on the mainshock hypocenter with diameter equal to the mainshock rupture length. From this point of view, time dependence of the Gutenberg-Richter b-value correspondence to renormalization group flow of the parameter β.

Geophysical Test

Previous research has demonstrated that fractal dimension of a fault system in a seismic region can be extracted from the power spectral densities of coda wave displacement seismograms of local earthquakes [1]. Moreover, based on the observed correlation between coda Q and Gutenberg-Richter b-values, coda Q monitoring has been identified as a possible earthquake prediction technology. If valid, the FDM random matrix model may contribute to these previous efforts by specifying time evolution of statistical velocity models of seismic activation regions in terms of nonlinear dynamical systems. As an initial test of validity, the seismic record of an activation region may be considered for a set of stations within a rupture length of the mainshock epicenter, and local earthquake coda Q values of local earthquake in different channels at different stations recorded at regular time intervals. Alternatively, using ambient seismic noise, coda Q values may be computed between station pairs at regular time intervals [22]. In turn, these coda Q values may be input to a time domain multichannel singular spectrum analysis algorithm, where the number of channels equals the number of stations or station pairs, and the number of singular values output by the algorithm in time windows preceding a mainshock event counts the number of unstable cracks contributing to mainshock rupture nucleation [8]. With reference to previous geophysical application of singular spectrum analysis, performed in the frequency domain, the signal processing algorithm suggested here is different in that it should be carried out in the time domain τ rather than the frequency domain [30].

Discussion

Previous work has identified predicting the time of occurrence of mainshock events as an application of statistical physics models of seismic activation, but this application has not yet been realized [7]. In more recent times, the artificial intelligence algorithm QuakeGPT has been developed for the purpose of forecasting earthquake occurrence, using seismic event record training data created with a stochastic simulator [29]. Therefore, a practical application of statistical mechanics models of seismic activation may be to be improve stochastic simulation of seismic event records for use in

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earthquake forecasting technology, acknowledging that rigorous tests of model validity against real seismic data must be passed before achieving this objective can be considered a realistic possibility.

In conclusion, work towards improving current earthquake early warning systems can proceed in two directions. Firstly, work can be done to determine whether or not observed changes of the Earth's elastic velocity model preceding mainshock events can be processed to extract an integer identifiable as the phase space dimension of a nonlinear dynamical system. Secondly, theoretical work can be done to elaborate upon the statistical mechanics model of seismic activation presented in this article to determine other tests of its scientific validity and potential for practical application.

Funding: This research received no external funding.

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Acknowledgments: Thanks to my family for their support throughout completion of this research. Thanks to Dr. Girish Nivarti, Dr. Evans Boney, and Professor Richard Froese for their willingness to entertain communication regarding the content of the article.

Conflicts of Interest: The authors declare no conflicts of interest.

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