Technical Note

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Clarifying the Chaotic Range in Systemic Tau: The Intermediate Volatility Zone ($|\tau_s| < 0.41$) and Its Implications for Complex Chaotic Systems

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Clarifying the Chaotic Range in Systemic Tau: The Intermediate Volatility Zone ($|\tau_s| < 0.41$) and Its Implications for Complex Chaotic Systems

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Abstract

This technical note formally defines and characterizes the *chaotic range* in Systemic Tau (τ_s), corresponding to the intermediate volatility zone where $|\tau_s| < 0.41$. This regime, previously implicit in prior validations across ecological, physical, and fractional-order chaotic systems, represents the region of maximal dynamical volatility during active bifurcations. Grounded in Kendall's tau ordinal correlations and Feigenbaum universality ($\delta \approx 4.669$, $\alpha \approx 2.502$), the chaotic range exhibits weakened ordinal agreement, extreme sensitivity to initial conditions, and robust noise tolerance (up to 15%). Simulations confirm $\tau_s \approx 0.036$ in fully developed chaos beyond the Feigenbaum point, with variance constrained by $\sigma^2 \leq 1/N$. By explicitly delineating the boundaries at ± 0.41, this note strengthens the predictive capacity of τ_s for early detection of critical transitions. Applications span ecology, climate modeling, artificial intelligence, financial systems, physical attractors, cardiology, materials engineering, social network resilience, and strategic forecasting under uncertainty.

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1. Introduction

Systemic Tau (τ_s) was introduced as a universal, noise-robust stability metric for complex chaotic systems through ordinal correlation analysis of multisite time series [1,2]. Validated empirically in *Aedes aegypti* population dynamics at Caño Martín Peña, Puerto Rico [1], and extended to fractional anti-synchronization of physical attractors [3], τ_s consistently identifies:

- Stable synchronized regimes: $\tau_s \in [0.5, 0.6]$
- Onset of bifurcation: $\tau_s < 0.41$
- Robust anti-synchronization: $\tau_s < -0.41$

Although the bounding thresholds ± 0.41 have been repeatedly confirmed, the intermediate zone $|\tau_s| < 0.41$ — characterized by maximal volatility and near-zero ordinal correlation — has remained conceptually implicit. This technical note explicitly defines this region as the **chaotic range**, bridging order and disorder in a manner consistent with stochastic self-organization principles [4].

2. The Chaotic Range: Definition and Properties

The chaotic range is formally defined as:

$$|\tau_s| < 0.41$$

Within this interval:

- Ordinal correlations collapse toward zero
- Sensitivity to initial conditions and noise is maximized
- Active bifurcations dominate system evolution

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- Variance obeys $\sigma^2 \leq 1/N$ scaling
- Noise tolerance remains high (10–15%)

Table 1. Systemic Tau regimes and their interpretation.

τ_s Range	Regime	Key Characteristics	Representative Systems
> 0.41	Synchronized Stability	Strong positive ordinal agreement	Stable mosquito populations [1], convergent AI training [3]
$\lvert\tau_s\rvert < 0.41$	**Chaotic Range**	Maximal volatility, weak/no correlation	Lorenz attractor bifurcations [4], critical climate transitions
< -0.41	Anti-Synchronized Divergence	Strong inverse ordinal agreement	Ecological anti-phase under stress [2], secure communication [4]

Figure 1 was produced with the following reproducible Python code:

```
import numpy as np
import matplotlib.pyplot as plt
from scipy.stats import kendalltau

def logistic_map(r, x0, n):
    x = np.zeros(n); x[0] = x0
    for i in range(1, n):
        x[i] = r * x[i-1] * (1 - x[i-1])
    return x

def compute_tau(s1, s2):
    return kendalltau(s1, s2)[0]

rs = np.linspace(2.5, 4.0, 500)
taus = []
for r in rs:
    s1 = logistic_map(r, 0.1, 1000)[200:]
    s2 = logistic_map(r, 0.11, 1000)[200:]
    taus.append(compute_tau(s1, s2))

plt.figure(figsize=(10,6))
plt.plot(rs, taus, 'k-', linewidth=1.2)
plt.axhline(0.41, color='green', linestyle='--', label='Stability threshold
    (+0.41)')
plt.axhline(-0.41, color='red', linestyle='--', label='Anti-synchronization
    threshold (-0.41)')
plt.fill_between(rs, -0.41, 0.41, color='yellow', alpha=0.3, label='Chaotic
    Range')
plt.xlabel('Control parameter $r$')
plt.ylabel('Systemic Tau ($\tau_s$)')
plt.title('Evolution of $\tau_s$ in the Logistic Map: The Chaotic Range')
plt.legend()
plt.grid(True, alpha=0.3)
plt.tight_layout()
plt.savefig('chaotic_range.png', dpi=300)
plt.show()
```

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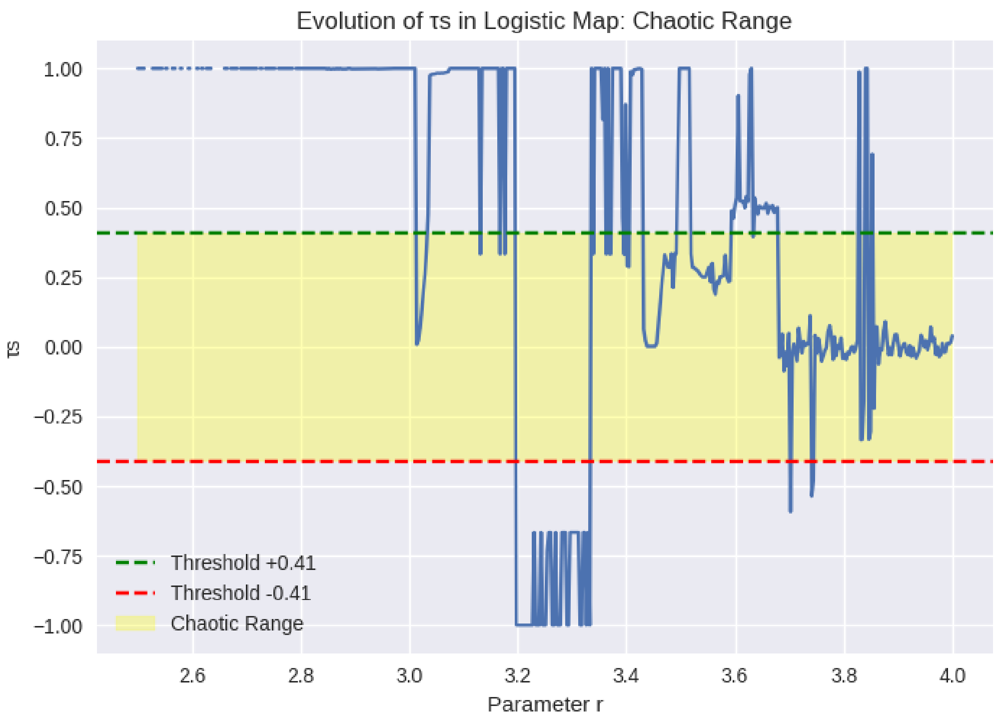

Figure 1. Evolution of Systemic Tau in the logistic map ($x_{n+1} = rx_n(1 - x_n)$) using paired trajectories with slight initial perturbation ($\Delta x_0 = 0.01$). The yellow shaded region ($|\tau_s| < 0.41$) corresponds to the chaotic range. Generated with 1000 iterations (200 transient discarded).

3. Implications and Interdisciplinary Applications

Recognition of the chaotic range significantly enhances early-warning capacity across domains:

- **Ecology & Public Health**: Predicts Aedes aegypti outbreak risk when τ_s enters the chaotic range
- **Climate & Earth Systems**: Detects tipping points in noisy paleoclimate proxies
- **Artificial Intelligence**: Monitors training instability and catastrophic forgetting
- **Cardiology**: Tracks progression from sinus rhythm to ventricular fibrillation
- **Materials Engineering**: Forecasts failure cascades via logistic-map analogs of crack propagation
- **Social Systems**: Identifies correlation collapse in opinion dynamics or epidemic spreading networks
- **Strategic Forecasting**: Signals regime shifts in geopolitical or military simulations

4. Conclusion

The chaotic range ($|\tau_s| < 0.41$) constitutes a fundamental dynamical regime within the Systemic Tau framework, representing the zone of purest chaos flanked by ordered (synchronized or anti-synchronized) states. Its explicit recognition completes the theoretical structure initiated in 2022–2025 works and opens immediate practical applications in prediction and control of complex systems.

Future releases of the open-source Systemic Tau toolkit will include automated chaotic-range detection flags.

References

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