

Relative-Entropy Variational Principle for Semiclassical Gravity with Finite-Resolution Boundaries

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Article

Relative-Entropy Variational Principle for Semiclassical Gravity with Finite-Resolution Boundaries

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Abstract

We propose a causal-diamond formulation of semiclassical gravity in which a finite-resolution boundary regulator supplies the edge structure for a local Wheeler–DeWitt description. Because the diffeomorphism-invariant Hilbert space does not factorize, each diamond is equipped with a boundary-completed algebra A_O , ensuring that the operational state ρ_O and the semiclassical reference family $\sigma_O[\Lambda]$ are defined on identical operator content. Dynamics are posed as local statistical inference: the relative-entropy functional $S_{\text{rel}}(\rho_O \parallel \sigma_O[\Lambda])$ quantifies the mismatch between data and reference on the fixed algebra. Discretizing gravitational stiffness as a finite response capacity of the boundary completion renders the variational problem well-defined at fixed resolution. The framework synthesizes three previously disconnected foundations: causal diamonds for covariant locality, finite-resolution boundaries for capacity regulation, and relative entropy as the governing variational principle. This synthesis determines the minimal operational axioms required to define subsystems, intrinsic clocks and regulated observables in a background-independent setting. The axioms force the relative-entropy Hessian (the Kubo–Mori metric) to block-diagonalize into orthogonal tensor, vector and scalar sectors in the modular/KMS regime. A quasi-local heat-kernel expansion maps the near-equilibrium response to a matching-scale effective field theory, recovering Einstein stiffness, Yang–Mills susceptibilities, and mass gaps. Topological edge-mode counting fixes an effective internal degeneracy $N \approx 1.23 \times 10^{11}$; combined with Newton’s constant G , this calibrates the fundamental matching scale $M_s \approx 3.02 \times 10^{13}$ GeV. The tensor sector, scaling extensively with N , yields a plateau universality class with tensor-to-scalar ratio $r \sim 10^{-3}$, while scalar and vector responses remain strictly intensive. Translations between nested diamonds act as completely positive trace-preserving updates that drive non-abelian symmetry data toward commuting Cartan phases. This induces the canonical Lie-theoretical filtration $G \rightarrow g \rightarrow T$. Treating couplings and mass ratios as discrete topological invariants on the fixed history manifold $S^3 \times S^1$, the architecture yields correlated, falsifiable relations for cosmological parameters, electroweak scales, gauge couplings and the lepton mass spectrum, all governed by the single fundamental scale M_s .

Keywords: quantum gravity; general relativity; semiclassical gravity; Wheeler-DeWitt; causal diamond; coherency screen; boundary completion; Hilbert space; non-factorization; relative entropy; relative-entropic action; mismatch functional; variational principle; Kubo-Mori Metric; Hessian decoupling; block-Diagonal Hessian; tensor sector; vector sector; scalar sector; topological budget; modular flow; KMS condition; modular locality; CPTP; GKSL; Lindblad; Lie filtration; Cartan subalgebra; Cartan torus; Heat-Kernel expansion; spectral trace; Gauss-Bonnet theorem; Euler characteristic; noncommutative geometry; spectral action; thermodynamic gravity; entropic gravity; holographic principle; emergent gravity; quantum information; quantum Fisher; Schur lemma; gauge topology; coherency units; Bekenstein-Hawking entropy; Yang-Mills; fine-structure constant; spectral tilt; tensor ratio; dark energy density; MOND scale; Higgs Mass; Top Quark Mass; Top Yukawa; background-independence; finite-resolution

1. Conceptual Foundations

We propose a finite-resolution framework for defining local quantum subsystems in semiclassical gravity, organized around the Wheeler–DeWitt (WDW) constraint [1]:

$$\hat{H}\Psi = 0$$

where the global state Ψ is constrained rather than time-evolved, and the operator $\hat{H}$ encodes the gravitational constraints and diffeomorphism invariance.

To ensure background independence, we treat dynamics as local statistical inference on causal diamonds $\mathcal{O}(p, q)$ [2], the fundamental units of accessible correlations [3]. In the small-diamond regime, modular flow supplies the intrinsic local clock [4,5]. Geometry is inferred from an entropic mismatch functional between the actual boundary-completed state ρ_O and a semiclassical reference family $\sigma_O[\Lambda]$ defined by the background fields Λ [6–9].

The minimal architecture consists of a causal diamond equipped with boundary completion that stores the edge data required for a subregion algebra [10]. Topological quantization then yields an effective internal degeneracy N and, upon calibration to Newton’s constant G , a matching-scale resolution M_s .

1.1. Axiomatic Basis: Minimal Architecture

We do not start from a preferred lattice or field content; instead, we approach quantum gravity as a problem of local inference. We select the minimal ingredients (axioms P1 to P7) required to define subsystems, clocks and regulated observables in a background-independent finite-resolution setting, organized under three foundational principles:

Principle A: Operational Regions

Locality is defined operationally by the minimal covariant unit: a finite causal diamond [2]. This geometry fixes the boundary interface where subsystem-defining information must reside [10].

P1 (Non-factorization). The diffeomorphism-invariant physical Hilbert space cannot factorize across spatial subregions ($\mathcal{H}_{\text{phys}} \neq \mathcal{H}_A \otimes \mathcal{H}_B$) [10,11]. Constraint relations tie interior data to boundary charges and fluxes, forcing explicit boundary data (edge modes) to define a unique subregion algebra.

P2 (Causal Diamonds). A finite experiment is a closed query–response loop between two timelike-separated events p and q . The causal diamond $\mathcal{O}(p, q) \equiv J^+(p) \cap J^-(q)$ [2] is the observer causal workspace: it contains all degrees of freedom that can be influenced by the query at p and still influence the response at q . This fixes the operational region covariantly without choosing a foliation. Algebraic gluing (P3) then requires the maximal spatial slice to be a simply connected 3-ball B^3 with boundary cut S^2 ; any nontrivial topology introduces extra discrete labels and is therefore excluded.

P3 (Boundary Completion). Since the physical Hilbert space does not factorize (P1), a well-defined subregion algebra $\mathfrak{A}_O$ requires boundary completion. States are positive, normalised functionals on $\mathfrak{A}_O$. This construction can be visualised as a coherency screen at the diamond waist S^2 that reconciles overlapping past and future histories into a consistent record. Boundary completion adjoins a centre $\mathcal{Z}(\mathfrak{A}_O)$ of gluing and charge labels, ensuring ρ_O and $\sigma_O[\Lambda]$ live on identical operator content and define a variational domain (Chapter 2) [10,12].

Principle B: Finite-Resolution Modular Information

A boundary-defined subsystem requires finite capacity to avoid unbounded information density [13]. In the small-diamond/KMS regime [14,15] modular flow supplies the intrinsic local clock [4,5].

P4 (Modular Locality). In the local Rindler regime the vacuum restricted to $\mathfrak{A}_O$ is approximately KMS with respect to a geometric modular flow. The modular Hamiltonian is well-approximated by the local boost generator, fixing KMS periodicity 2π [4,5].

Corollary (Canonical History Manifold). Operational locality (P2) dictates that the causal diamond consists of two spatial 3-balls (B^3) glued along the waist S^2 . Modular flow imposes a fixed period of 2π on the temporal direction (P4), and boundary completion requires this gluing to be closed without additional discrete labels (P3). Together, these strict physical and algebraic conditions uniquely determine the compact history manifold on which all local response is evaluated: $S^3 \times S^1$. Any alternative topology would inherently violate the closed gluing or introduce extrinsic labels explicitly forbidden by the axioms.

P5 (Finite Resolution). Defining a local subsystem requires finite resolution to ensure a stable restriction to $\mathfrak{A}_O$ and a bounded mode density at the interface. We introduce a proper-distance resolution length $L_s \equiv M_s^{-1} = \delta$, realized as a stretched screen placed a fixed proper distance δ from the diamond waist S^2 . This choice is a consistency requirement: it renders the boundary completion finite-capacity and guarantees that ρ_O and $\sigma_O[\Lambda]$ admit a well-defined relative-entropy comparison, consistent with holographic entropy bounds [13,16,17].

Corollary (Resolved Modular Interval). Because local proper time at the stretched screen scales as $t \simeq \delta\tau$, the physical resolution limit δ imposes a minimal resolvable modular step $\tau_{\min} = 1$. The fully resolved modular interval is therefore strictly bounded to $\tau \in [1, 2\pi - 1]$.

Corollary (Monotone-Penalty Spectral Selection). Relative entropy is monotone under the restriction of the algebra to a subregion (P3), while finite resolution imposes a strict limit on the total information capacity of the boundary (P5). Therefore, in any spectral expansion of the response, an admissible correction must strictly decrease as the history volume or boundary capacity increases. Terms that would grow with better resolution violate this monotonicity and are explicitly excluded.

Principle C: Topological Quantization and Minimal Transport

Consistency on a bounded region quantizes boundary charge sectors: invariance under large gauge transformations and Gauss-law completion of the subregion algebra (P3) enforce discrete (integer-level) boundary data rather than continuously tunable normalizations [18]. Given finite resolution, an isotropic transport layer then fixes the local discrete routing rule [19].

P6 (Gauge Topology). Boundary charge sectors are modelled by a bulk Chern–Simons theory [18], whose boundary reduction is a Wess–Zumino–Witten current algebra [20]. Invariance under large gauge transformations quantizes the level $k \in \mathbb{Z}$, which fixes the edge-current normalisation and the matching-scale inverse coupling $\alpha^{-1}(M_s)$ in linear response.

P7 (Discrete Isotropic Transport). A finite-resolution boundary architecture requires (i) a local transport layer that is isotropic in three spatial dimensions and inversion-symmetric and (ii) an internal lift of local rotations to support spinorial matter. The transport layer must admit a faithful discrete realization of the modular automorphism group such that every local excitation returns exactly to its initial state after one full 2π period, without directional bias or spurious phase accumulation. In three spatial dimensions the only connectivity satisfying isotropy, inversion symmetry and this exact closure condition is the signed spanning set $\{\pm x, \pm y, \pm z\}$ (three antipodal axis pairs). This uniquely fixes the coordination $z = 6$ and the L_1 (octahedral) step rule. For matter, admitting local spinors requires lifting $SO(3)$ to its double cover $SU(2)$ (topologically S^3) so that projective representations become available; this introduces a minimal $\mathbb{Z}_2$ twist sector.

Corollary (Transport-Axis Redundancy $|\Gamma|$). The strict isotropic, inversion-symmetric router dictated by discrete transport (P7) explicitly requires an octahedral geometry $\{\pm x, \pm y, \pm z\}$ consisting of exactly three independent antipodal axes. Because flipping the sign of any axis leaves the overall transport isotropy undisturbed, this structure generates an 8-fold labeling redundancy $|\Gamma| = 2^3 = 8$ on the commuting phases. As repeated modular updates drive the data toward a stable fixed-point structure, these specific sign labels lose their distinct identity. After coarse-graining, all eight combinations become physically indistinguishable and must be treated as a single redundant layer.

Corollary (Tip Defect Parameter κ). Near the diamond tips (p, q) , the transverse cross-section becomes sub-resolution at fixed L_s (P5). Representing the geometric modular flow with the discrete transport layer incurs an irreducible overhead; we parameterize this tip-induced coordination cost

by a dimensionless scalar κ . A closed modular cycle traverses both diamond tips (P2), so the total operational overhead per transport axis is 2κ .

Corollary (Pixel). At finite resolution L_s , a pixel is one distinguishable boundary patch on the diamond waist, defined by the triple $(\mathfrak{A}, \mathcal{H}, D)$: the local gauge algebra $\mathfrak{A}$ (P6) acts on internal degrees of freedom $\mathcal{H}$ (P7) and the transport operator D connects adjacent pixels.

1.2. Boundary Architecture, Resolution and Connectivity

A causal diamond $O(p, q)$ (P2) is bounded by two null hypersurfaces (future- and past-directed light-sheets) generated by null rays from p and q [2]. Their intersection defines a distinguished spacelike waist 2-sphere S^2 , the maximal-area cross-section of the diamond. This waist serves as the local Rindler cut for the small-diamond description: it is the canonical interface on which a local subregion algebra can be completed when $\mathcal{H}_{\text{phys}}$ does not factorize (P1–P3) [10,12]. This construction is purely local and does not rely on AdS asymptotics or an AdS/CFT embedding.

Finite resolution (P5) pixelizes the diamond waist: S^2 operates as a finite set of distinguishable boundary patches (pixels) at resolution δ ; sub- δ structures are operationally aliased [16]. Finite resolution is implemented by replacing the idealized null boundary with a stretched screen placed a fixed proper distance δ from the waist S^2 [21]. The screen is part of the definition of $\mathfrak{A}_O$: it fixes which edge observables are operationally available and hence fixes the common operator content on which both ρ_O and the reference family $\sigma_O[\Lambda]$ are defined (P3). Operationally, the null generators of the diamond define the update channels: when excitations cross the boundary, the screen registers their passage as discrete arrival events along these generators, providing the raw record structure that is compared against $\sigma_O[\Lambda]$.

In the local Rindler regime (P4), the local proper time at the screen scales as $t \simeq \delta\tau$ [5]. Identifying the minimal resolvable proper time with the screen resolution ($t_{\min} \sim \delta$) therefore forces $\varepsilon \equiv \tau_{\min} = 1$. Any deviation would physically reposition the screen and redefine the matching-scale M_s ; ε is thus the direct operational consequence of working at the resolution cutoff rather than a free parameter. While the absolute value of the coupling depends on this placement, small variations around $\varepsilon = 1$ rescale the kinematic integration window uniformly across all gauge sectors. The resulting integer-level ratios of inverse couplings remain topologically protected, depending only on the levels k_i . In short, the gauge structure at the matching-scale is fixed by the boundary architecture itself.

Transport across pixels follows the minimal router of P7, implemented in local orthonormal frames on S^2 (a global smooth tangent frame is obstructed by the Hairy Ball Theorem) [22,23]. The induced L_1 step metric must approximate the smooth L_2 isotropic limit; the mismatch becomes significant near the diamond tips where the cross-section becomes sub-resolution, producing an irreducible digitization overhead, parameterized by κ (P7) [24].

The interface is the waist S^2 . Because boundary completion represents interior conserved charge sectors as boundary flux labels (P1, P3, P6) (the Gauss-law requirement that charges in a region be encoded by flux through its boundary), the normalization of the local vector response is tied to the surface geometry. We therefore use the standard infrared convention $\alpha \equiv g^2/4\pi$, associating the 4π factor with the total solid angle of the S^2 interface. This maps discrete boundary response coefficients to the usual flux normalizations of continuum gauge theory (Chapter 3).

1.3. Topological Budget: Effective Internal Degeneracy

With the boundary architecture set, we determine the effective internal degeneracy (channel multiplicity per pixel) N from the vacuum capacity budget $S_{\text{vac}} \equiv \sum_i S_i$ implied by the minimal construction (P2–P5, P7).

The diamond waist S^2 serves as the observer's instantaneous coherency screen: it is the unique spatial intersection where the past light cone (incoming data from initiation p) meets the future light cone (outgoing responses registered at q). This intersection forms the observer's actual experimental horizon, the stage on which every local experiment takes place.

Its composite informational budget S_{vac} determines the effective internal degeneracy via the standard relation $N \equiv e^{S_{\text{vac}}}$, which is thermodynamically consistent with the local Unruh temperature and the 2π periodicity of the modular flow. This converts the additive entropy cost into a multiplicative count of response channels and defines an effective capacity per pixel (number of independent informational channels the coherency screen can support) rather than a literal Hilbert-space dimension, rendering non-integer values for N physically admissible.

Bulk Topology (S_{bulk})

The minimal entropy budget from boundary completion is the unique additive, diffeomorphism-invariant, scale-free functional on the closed cut S^2 required by algebraic gluing (P3). Because the resolution scale M_s is a derived outcome calibrated only later, the initial vacuum budget cannot depend on local metric data such as length or area. At this pre-geometric stage, the only available invariant is the Euler characteristic density given by the Gauss–Bonnet theorem:

$$S_{\text{bulk}} \equiv \oint_{S^2} R^{(2)} dA = 4\pi\chi(S^2) = 8\pi$$

This quantifies the total integrated curvature of the complete closed intersection of past and future light cones at the diamond waist (P2). The same 8π also supplies the discrete labels for the center $\mathcal{Z}(\mathfrak{A}_O)$, topologically locking the algebraic gluing demanded by non-factorization (P1).

A hemisphere (4π) is excluded because the screen must form a topologically closed surface. Any obstructing boundary circle would prevent the completion of a well-defined subregion algebra and introduce a leak that violates the Gauss-law for interior charge encoding (P1, P3). Any other functional would require extra structure (metric dependence, preferred scale, additional discrete labels), violating the minimal-architecture premise.

The octahedral router with coordination $z = 6$ is likewise forced by P7. In a 2π modular cycle, a field excitation travels along a closed great-circle path through the diamond. To complete this cycle without phase or directional bias, every transport axis must possess its exact negative counterpart (inversion-symmetric pairing). The minimal signed spanning set $\{\pm x, \pm y, \pm z\}$ uniquely satisfies this requirement while remaining isotropic and compatible with antipodal partitioning under fixed modular period (P4). Tetrahedral coordination ($z = 4$) lacks such antipodal mapping and would therefore break inversion symmetry; icosahedral coordination ($z = 12$) adds redundant degrees of freedom that violate minimality. The $z = 6$ router is thus uniquely forced; no free parameters or alternative topologies remain.

Tip Defect Parameter (κ)

The tips (p, q) constitute geometric bottlenecks where the null generators converge and the transverse cross-section drops below the resolution length $L_s \equiv \delta$ (P5). In this sub-resolution regime, the continuous L_2 -isotropic modular flow (P4) can only be realized using the discrete L_1 transport structure of the screen (P7).

A full 2π modular cycle requires an excitation to traverse the transport network and return to its starting orientation without directional bias. For the unique minimal router $\{\pm x, \pm y, \pm z\}$, which consists of three independent antipodal axes, this traversal necessitates exactly one unavoidable coordination junction per cycle to maintain isotropy. Normalizing this single irreducible event across the three transport channels over the fixed modular period yields:

$$\kappa = \frac{1}{3 \times 2\pi} = \frac{1}{6\pi} \approx 0.053$$

The same geometric mismatch between the discrete router and the continuous boost generator simultaneously limits the fraction of internal channels that can participate phase-coherently over a full modular period. Bookkeeping overhead and the coherent-participation (duty-cycle) factor therefore

share the same origin and the same numerical value κ . This factor is fixed uniquely by the constraints of modular periodicity, resolution and lattice connectivity (P4, P5, P7).

Twist Contribution (S_{twist})

The discrete transport structure demanded by P7 must support local spinors. Because ordinary rotations $SO(3)$ admit only integer-spin representations, the inclusion of spin-1/2 matter necessitates the double cover $SU(2)$. This topological lift introduces a $\mathbb{Z}_2$ grading on the boundary algebra: a twist sector that distinguishes fermionic from bosonic paths around closed transport loops.

Among all realizations of such a $\mathbb{Z}_2$ twist, the minimal quantum dimension is $d_\sigma = \sqrt{2}$. Any larger non-Abelian extension would increase the effective degeneracy N and is therefore excluded by the minimality premise. The additive contribution to the vacuum entropy budget is the half-bit cost:

$$S_{\text{twist}} = \ln d_\sigma = \ln \sqrt{2} \approx 0.347$$

This value is forced uniquely by P7 and the requirement that projective representations exist for fermions; no free parameter or alternative grading survives the axioms. A trivial grading ($d_\sigma = 1$) is excluded as it would fail to distinguish the 4π rotation required by the $SU(2)$ lift, precluding the existence of fermions.

Resulting Effective Internal Degeneracy (N)

The composite quantity S_{vac} quantifies the total irreducible entropy cost per pixel imposed by the architecture. Summing the bulk, tip and twist contributions identifies the total budget as:

$$S_{\text{vac}} = 8\pi + \frac{1}{6\pi} + \ln \sqrt{2}$$

In statistical mechanics and holographic treatments, entropy measures the logarithm of the effective number of microstates or response channels ($S = \ln \Omega$) [13,25,26]. Exponentiation therefore converts this additive entropy budget into a multiplicative degeneracy $N \equiv e^{S_{\text{vac}}}$. This yields the topology-locked effective internal degeneracy (channel multiplicity per pixel), rather than a literal Hilbert-space dimension (rendering non-integer values physically admissible):

$$N = \sqrt{2} \exp\left(8\pi + \frac{1}{6\pi}\right) \approx 1.23 \times 10^{11}$$

This multiplicity quantifies the total capacity per pixel for internal response sectors (spin, charge, edge modes) on the boundary algebra. These degrees of freedom remain localized to the boundary completion, so the high channel count introduces no additional propagating fields.

While N represents the total available capacity, gravitational stiffness depends only on the fraction of channels that remain phase-coherent during one modular cycle. The same tip defect that contributes to the entropy budget also limits coherent participation, leading to the constitutive relation:

$$M_p^2 = N_{\text{eff}} M_s^2, \quad N_{\text{eff}} \equiv \kappa N$$

Gravitational rigidity is therefore determined solely by the fraction of internal capacity that survives the architectural defect at the diamond tips. The result follows rigidly from the axioms, with no free parameters or alternative topological choices.

Structural Rigidity and Consistency

Structural rigidity follows directly from the topological invariants. The bulk contribution $S_{\text{bulk}} = \oint_{S^2} R^{(2)} dA = 8\pi$ is fixed by the Gauss–Bonnet theorem [27] as the Euler characteristic of the closed S^2 waist, mandated by algebraic gluing (P1–P3). Alternative normalizations (e.g., a 4π hemisphere or $z \neq 6$) would inherently violate this gluing requirement or the minimal parity-symmetric transport

(P7). Similarly, the tip defect $\kappa = 1/(6\pi)$ is locked by the exact 2π closure of the modular cycle on the discrete L_1 router, matching the normalized Regge deficit $\delta_{\text{def}}/(2\pi)^2$ for a minimal curvature bottleneck [24]. Finally, the $\mathbb{Z}_2$ twist $\ln \sqrt{2}$ is the minimal quantum dimension admitting spinors. Even a small shift in κ or S_{twist} would break modular periodicity, violate integer-level gauge quantization (P6) or destroy the lock between coherent participation N_{eff} and GKSL leakage. The derived fundamental scale M_s is therefore shielded from arbitrary modulation.

1.4. Constitutive Relation and Calibration

With the effective internal degeneracy N determined by the architectural axioms (P1–P7), the remaining step is to calibrate the resolution scale M_s using Newton's constant. We use the reduced Planck mass $M_P \equiv (8\pi G)^{-1/2}$ throughout.

In the Einstein–Hilbert action [28], the coefficient of curvature is the inverse gravitational constant. In the linear-response regime, the stiffness of the boundary scales with the number of effective channels [29,30]. The macroscopic stiffness is additive over channels that remain phase-coherent under modular flow on the resolved algebra.

Here κ enters as a dynamical participation factor: the fraction of the available sector capacity that can contribute coherently over one modular cycle in linear response, limited by sub-resolution coordination at the tips. The same tip bottleneck (κ) already contributed a fixed overhead term to the capacity budget (capacity bookkeeping vs. coherent participation). In both roles it is fixed by the clock and transport normalizations rather than tuned to match G . We therefore define:

$$M_P^2 = \sum_{a=1}^{N_{\text{eff}}} M_s^2 = N_{\text{eff}} M_s^2, \quad N_{\text{eff}} \equiv \kappa N.$$

This identifies M_P as the extensive stiffness obtained by summing over finite channels (rather than discretizing a manifold or quantizing the graviton). This stiffness additivity is conceptually aligned with spectral and induced-gravity viewpoints, where effective couplings are controlled by mode counting and mirrors the species-bound logic in which gravitational strength is diluted by the number of active degrees of freedom [30,31].

Numerical Calibration

We now calibrate the resolution scale M_s against Newton constant G using the reduced Planck mass $M_P = (8\pi G)^{-1/2} \approx 2.435 \times 10^{18}$ GeV to align with the standard Einstein–Hilbert action. By substituting the constitutive law $M_P^2 = N_{\text{eff}} M_s^2$ and solving for the resolution scale, we obtain:

$$M_s = \frac{M_P}{\sqrt{N_{\text{eff}}}} \approx 3.02 \times 10^{13} \text{ GeV}.$$

M_s is the fundamental resolution scale implied by δ . It is the physical resolution limit of the algebra (sharp focus) where the EFT problem is well-posed: gravitational stiffness, gauge couplings and mass gaps enter as matching data, and in quantized sectors some conditions reduce to closed-form constraints (Chapter 3). Note that this derived scale coincides with the phenomenological intermediate scale required by neutrino seesaw mechanisms [32] and plateau inflation [33].

In these units, the derived Planck length is simply $L_P = L_s / \sqrt{N_{\text{eff}}}$, so M_s remains the fundamental resolution limit while the Planck scale emerges only as the macroscopic stiffness of the boundary, successfully removing the singular limit $L_P \rightarrow 0$ from the physics.

Pixel Saturation Limit

The finite resolution (P5) of the boundary algebra implies a fundamental limit on the mass of localized excitations. With M_s fixed, the per-pixel saturation unit is $E_{\text{pix}} \equiv M_s / n_{\text{ch}}$, where $n_{\text{ch}} \equiv N / \sqrt{2}$

is the addressable channel count after factoring out the $\mathbb{Z}_2$ twist quantum dimension $d_\sigma = \sqrt{2}$ (P7), treated as structural overhead rather than a data-carrying response channel. This yields:

$$E_{\text{pix}} \equiv M_s/n_{\text{ch}} \approx 348.19 \text{ GeV}.$$

As established in Axioms P1 and P3, localized charged excitations on $\mathfrak{A}_O$ are gauge-invariant only as dressed dipoles (pair excitations). Consequently, a single pixel update must accommodate the pair energy $2m_f$, yielding the representability bound $2m_f \lesssim E_{\text{pix}}$, hence:

$$m_{\text{max}} \approx E_{\text{pix}}/2 \approx 174.1 \text{ GeV}.$$

At the resolution limit M_s , the maximum energy density localizable within a single boundary pixel is bounded by the saturation limit. This value is close to the top-quark mass ($\approx 173 \text{ GeV}$), providing a simple consistency check of the per-pixel energy budget.

Consistency Check: Area Law

We verify that this calibration preserves the Bekenstein–Hawking entropy structure [25,34]. For a horizon of area A , the standard entropy (in natural units) is $S_{\text{BH}} = A/4G$. Substituting $1/G = 8\pi M_p^2$ we obtain:

$$S_{\text{BH}} = 2\pi M_p^2 A = 2\pi(N_{\text{eff}}M_s^2)A = 2\pi N_{\text{surf}}N_{\text{eff}},$$

where $N_{\text{surf}} \equiv A/L_s^2 = AM_s^2$ is the number of resolvable boundary pixels on a waist of area A (set purely by geometry); the entropy factors as pixels $\times$ effective depth.

Converting to bits ($S/\ln 2$) and taking $A_{\text{cosm}} \sim 4\pi H_0^{-2}$ gives $\sim 10^{122}$ bits. This reproduces the standard Bekenstein–Hawking area law and the familiar cosmic entropy scale, providing a consistency check of the calibration.

Although the result follows algebraically from the constitutive lock $M_p^2 = N_{\text{eff}}M_s^2$, it implies that the standard horizon entropy is realized as pixel count (N_{surf}) times effective depth (N_{eff}). The required information density is achieved via channel multiplicity, rather than by shrinking the pixel size to L_p . This recovers the Bekenstein–Hawking scaling using the finite resolution scale L_s , demonstrating that M_s is the true physical regulator while L_p emerges only as a derived, effective stiffness scale.

1.5. Emergent Time

In a Wheeler–DeWitt setting ($\hat{H}\Psi = 0$), time is relational and defined by conditioning on an internal clock (Page–Wootters) [3]. For a partition $\mathcal{H}_{\text{tot}} = \mathcal{H}_C \otimes \mathcal{H}_R$, the conditional state is $|\psi(t)\rangle_R = \langle t|_C|\Psi\rangle$. In the small-diamond regime (P4), the restricted vacuum is approximately KMS, while the associated modular flow provides a canonical local ordering of correlations [4,5]. We take the modular parameter τ as this intrinsic ordering variable. Proper time t arises only after coarse-graining many resolved modular updates; at the stretched screen, one has $t \simeq \delta\tau$ [5]. Time is therefore diamond-local: the operational clock is the sequence of distinguishable boundary updates at the fundamental resolution scale M_s .

With the container $(O, \mathfrak{A}_O, \delta)$ and the local clock fixed, dynamics is posed as inference on the fixed operator content. In the operator-algebraic viewpoint, the modular automorphism group defines intrinsic evolution on the algebra. Semiclassical Wheeler–DeWitt solutions admit a WKB form $\Psi[g] \approx \mathcal{A}[g]e^{iS_{\text{eff}}[g]}$; when modular flow is geometric, the phase functional S_{eff} thus motivates the relative-entropy variational principle introduced in Chapter 2.

We vary the semiclassical reference family $\sigma_O[\Lambda]$ and minimize the mismatch functional $S_{\text{rel}}(\rho_O\|\sigma_O[\Lambda])$ [35,36]. Relative entropy is the natural choice because it vanishes when reference and data coincide and is monotone under algebraic restriction (P3).

The fixed resolution scale δ converts the near-equilibrium response into a controlled expansion. Chapter 2 shows that the second variation (Hessian) of the relative-entropy functional (the Kubo–Mori metric) block-diagonalizes into orthogonal tensor, vector and scalar sectors. This structure admits

a quasi-local derivative expansion organized by standard heat-kernel methods [37], providing the mechanical link between the fixed inputs (N, κ, M_s) and the leading field equations.

2. Relative-Entropy Functional and Dynamics

Dynamics on a causal diamond emerge not as the time-evolution of a global Wheeler–DeWitt state [1], but as local statistical inference. The reduced boundary state ρ_O is the operational data. Geometry and other background fields enter only through a semiclassical reference family $\sigma_O[\Lambda]$ defined on the same boundary-completed algebra $\mathfrak{A}_O$.

Related entanglement-equilibrium approaches recover the tensor sector under locality assumptions [6–8,36,38–42]. Here, the subsystem is fixed at finite resolution on a boundary-completed algebra, so the variational problem becomes a matching-scale response theory at $M_s = \delta^{-1}$, rather than relying on a continuum limit, making the recovery of tensor, vector and scalar sectors possible.

The dynamics are defined by minimizing the informational mismatch $I[\Lambda; O] = S_{\text{rel}}(\rho_O \| \sigma_O[\Lambda])$ at fixed $(O, \mathfrak{A}_O, \delta)$ [35,43,44], wherein its relative-entropy Hessian simultaneously decouples the macroscopic tensor stiffness, the vector (gauge) and scalar (mass) susceptibilities.

We use N for total internal channel multiplicity per pixel, $N_{\text{eff}} = \kappa N$ for effective coherent participation, κ for the tip impedance factor, $k \in \mathbb{Z}$ for the WZW level, α for the gauge coupling and M_s for the resolution scale.

2.1. Entropic Variational Principle and Operational Domain

We fix the container $(O, \mathfrak{A}_O, \delta)$ with proper-distance regulator $\delta = M_s^{-1}$ ($\delta \ll \ell_O \ll R_{\text{curv}}$), varying only the reference family $\sigma_O[\Lambda]$ on the fixed algebra $\mathfrak{A}_O$. We define the variational functional [35]:

$$I[\Lambda; O] := S_{\text{rel}}(\rho_O \| \sigma_O[\Lambda]).$$

$S_{\text{rel}} \geq 0$ is monotone under restriction, enforcing the overlap consistency required by boundary completion (Axiom P3) [15,35,44,45]. On the fixed algebra $\mathfrak{A}_O$, Jaynes' principle selects the maximum-entropy exponential reference family compatible with the chosen sources Λ [43,44]:

$$\sigma_O[\Lambda] = \frac{e^{-K_O[\Lambda]}}{\text{Tr}(e^{-K_O[\Lambda]})}.$$

The modular generator $K_O[\Lambda]$ couples the local background multiplet Λ to the boundary operator multiplet $\mathcal{O}$, defining deformations via the inner product:

$$\delta K = \int d^4x \sqrt{-g} \delta \Lambda(x) \cdot \mathcal{O}(x).$$

In the KMS regime, relative-entropy takes the modular free-energy form [35]. Because the vacuum σ_0 is an equilibrium state, the first law of entanglement ($\delta S_{\text{ent}} = \delta \langle K \rangle$) ensures the variational functional I is stationary at the reference background Λ_0 :

$$S_{\text{rel}}(\rho \| \sigma) = \Delta \langle K_\sigma \rangle - \Delta S \quad (K_\sigma = -\log \sigma), \quad \delta I|_{\Lambda_0} = 0.$$

With linear driving forces strictly vanishing, the leading restoring dynamics arise from the second variation (the Hessian), mirroring a harmonic elastic response.

Variational Domain

We vary the reference background $g \mapsto \sigma_O[g]$ at fixed container $(O, \mathfrak{A}_O, \delta)$. In the modular KMS regime, the reference state takes the form $\sigma_O[g] = e^{-K_O[g]} / \text{Tr}(e^{-K_O[g]})$ on the same boundary-completed algebra. Metric variations $\delta g_{\mu\nu}$ therefore deform the modular generator $K_O[g]$ and correlators on $\mathfrak{A}_O$ without altering the underlying degrees of freedom. The causal diamond O is fixed

relationally by the endpoints (p, q) rather than a coordinate box. Admissible variations preserve the regulator placement at a fixed proper distance $\delta = M_s^{-1}$ from the waist.

Fixing δ pushes the diamond tips below resolution under modular flow, recovering the tip defect parameter κ and locking gravitational stiffness to the same substrate scale as leakage strength.

2.2. Hessian Bridge, Spectral Extraction and EFT Coefficients

Local background fields (metric $g_{\mu\nu}$, gauge field A_μ , scalar φ) couple on the boundary algebra to three composite operators: the stress tensor $T^{\mu\nu}$, the conserved current J^μ , and the scalar density M . Within the finite-resolution boundary-completed algebra $\mathfrak{A}_O$, the Hessian kernel $G_{IJ}(x, y)$ is strictly block-diagonal across these sectors. This separation is forced by the architecture itself.

Modular flow, generated by the geometric boost $K_{\text{mod}}(P4)$, is fixed geometrically by the causal diamond and its waist. Any symmetry compatible with the boundary-completion architecture commutes with this flow: $[K_{\text{mod}}, Q] = 0$ [10,46]. As a result, the metric, gauge-connection and scalar deformations transform independently and cannot mix in the quadratic response functional. Because the Kubo–Mori inner product is the unique invariant bilinear form on the space of deformations, Schur’s lemma forces all cross terms to vanish identically: $G_{IJ}(x, y) = 0$ for $I \neq J$.

The discrete L_1 router (P7) contributes only $O(M_s^{-2})$ anisotropy that averages to zero over the modular period 2π (as the geometric boost generator is invariant under the full octahedral group), while the $\mathbb{Z}_2$ twist acts as a topological superselection rule that partitions the algebra into bosonic and fermionic sectors whose center is preserved by modular flow. In practice, no mixed correlators survive the KMS weighting. To fix normalizations unambiguously, the source–operator pairing is:

$$\delta K = \int d^4x \sqrt{-g} \left(\frac{1}{2} \delta g_{\mu\nu} T^{\mu\nu} + \delta A_\mu J^\mu + \delta \varphi M \right),$$

where the factor $1/2$ is the usual metric-source convention. Expanding the variational functional around the KMS state Λ_0 gives:

$$I[\Lambda_0 + \delta\Lambda] = I[\Lambda_0] + \int d^4x \delta\Lambda(x) \cdot \frac{\delta I}{\delta\Lambda(x)} \Big|_{\Lambda_0} + \frac{1}{2} \iint d^4x d^4y \delta\Lambda(x) \cdot G(x, y) \cdot \delta\Lambda(y) + R_{\geq 3}[\delta\Lambda].$$

At equilibrium the state is $\rho_O = \sigma_0$ on the resolved algebra, so $I[\Lambda_0] = 0$. The entanglement first law then guarantees stationarity at Λ_0 . For small deformations the leading response is purely quadratic and governed by the Hessian

$$G(x, y) := \frac{\delta^2 I}{\delta\Lambda(x)\delta\Lambda(y)} \Big|_{\Lambda_0}.$$

This object is precisely the Kubo–Mori metric, the KMS-weighted connected two-point response in the vacuum (Minimal History Manifold corollary to $S^3 \times S^1$) [14,35]. It behaves like an elastic modulus: small changes in the reference geometry or fields cost a quadratic penalty, and its eigendirections label the independent stiffness modes at long wavelengths. As the second variation of the relative-entropy functional $I[\Lambda; O]$, near KMS, the Hessian reduces to the quadratic form:

$$\delta^2 I = \frac{1}{2} \langle \delta K | \delta K \rangle_{\text{KM}}.$$

When evaluated on the resolved algebra at finite bandwidth, it takes the form $\delta^2 I \approx \text{Tr } f(D_{\mathcal{A}}^2 / M_s^2)$, where $D_{\mathcal{A}}$ is the transport operator fixed by the axioms and the Monotone-Penalty Spectral Selection corollary (P5). The Einstein–Hilbert term thus emerges as the leading quadratic coefficient through symmetry-induced block-diagonalization, the finite-resolution bandwidth limit and heat-kernel expansion on the fixed modular manifold $S^3 \times S^1$.

All relations are fixed uniquely by the axioms and the monotonicity of relative entropy. The resulting coefficients are locked and stable under perturbations because they are fully determined by four rigid structures forced by the axioms: the modular period 2π , the unique compact manifold $S^3 \times S^1$, the $\mathbb{Z}_2$ grading for spinors and the resolution cutoff δ . No free continuous parameters remain.

The same finite resolution $M_s \equiv \delta^{-1}$ that defines the stretched screen also sets the Nyquist frequency of the modular clock. Any deformation with wave-number $k > M_s$ is simply aliased away by the coherency architecture. At this natural bandwidth limit, the quadratic response is best captured by a spectral trace over the resolved modes of the transport operator $D_{\mathcal{A}}$ (the minimal covariant Dirac/Laplace operator fixed by P7 that couples to all three sectors):

$$\delta^2 I \approx \text{Tr} f\left(\frac{D_{\mathcal{A}}^2}{M_s^2}\right).$$

The modular generator $K_O[\Lambda]$ is the unique operator compatible with the KMS condition on the boundary-completed algebra. The entanglement first law forces stationarity at the vacuum, equating the second variation to the Kubo–Mori metric $\frac{1}{2}\langle\delta K|\delta K\rangle_{\text{KM}}$. On the fixed $S^3 \times S^1$ manifold, Tomita–Takesaki flow evaluates this quadratic form directly as a local spectral trace at the resolution bandwidth M_s . This relational clock recovers the global Wheeler–DeWitt constraint $\hat{H}\Psi = 0$ locally. The Einstein–Hilbert term and its leading R^2 saturation therefore follow as the unique low-curvature response of the architecture.

We express the window f via Laplace transform and insert the short-time heat-kernel expansion $\text{Tr} e^{-tD^2} \sim \sum_k a_k t^{(k-4)/2}$. With the minimal history manifold fixed by P2–P4 and the correction terms constrained by the monotone-penalty rule from P3 and P5, the spectral expansion follows directly, yielding finite EFT coefficients c_k at M_s :

$$\text{Tr} f(D^2/M_s^2) \sim \sum_k a_k \left(\int_0^\infty dt t^{(k-4)/2} \tilde{f}(t) \right) M_s^{4-k}.$$

At low derivative order, locality and symmetry fix the operator basis, giving:

$$\delta^2 I \simeq \underbrace{\frac{1}{2} \iint \delta g_{\mu\nu} G_{TT} \delta g_{\rho\sigma}}_{\Rightarrow \int d^4x \sqrt{-g} \frac{M_P^2}{2} R} + \underbrace{\frac{1}{2} \iint \delta A_\mu G_{JJ} \delta A_\nu}_{\Rightarrow \int d^4x \sqrt{-g} \frac{1}{4g^2} F^2} + \underbrace{\frac{1}{2} \iint \delta \varphi G_{MM} \delta \varphi}_{\Rightarrow \int d^4x \sqrt{-g} \mathcal{L}_{\text{mass}}}.$$

These are the response properties defined at M_s . Below that scale, standard renormalization group running takes over. The heat-kernel machinery is therefore not an extra assumption but an organisational tool that enforces locality and symmetry while packaging all the boundary, interface and defect contributions from the fixed architecture. The cutoff itself is the physical operational scale M_s set by the regulated boundary [47,48].

2.3. Coupling Factorization and Spectral–Volume Correspondence

Quasi-locality enforces a strict volume–coupling factorization. For an internal gauge field on an operational domain $\mathcal{D}$, the integrated quadratic cost factors cleanly:

$$\int_{\mathcal{D}} d^4x \sqrt{-g} \frac{1}{4g_{\text{YM}}^2} F^2 \quad \Rightarrow \quad \text{Vol}_4(\mathcal{D}) \cdot (1/g_{\text{YM}}^2) \cdot \langle F^2 \rangle_{\mathcal{D}}.$$

This makes explicit the extensive vs. intensive separation: the Einstein term scales with the number of participating sectors, while $1/g^2$ and mass parameters are intrinsic susceptibilities at M_s .

The spectral trace over the minimal compact operational history then establishes a direct spectral–volume correspondence. In the small-diamond/KMS regime modular time forms a thermal circle S^1 [4,15]. Identifying the spatial part as two B^3 halves glued across the waist ($B^3 \cup_{S^2} B^3 \simeq S^3$),

the minimal manifold for the trace is $S^3 \times S^1$ [37]. On this manifold (fixed by the Minimal History Manifold corollary, P4) the leading heat-kernel coefficient directly counts the number of modes at resolution M_s , so the effective four-volume reads:

$$\text{Vol}_4(\mathcal{D}) = \frac{1}{M_s^4} \frac{\text{Tr}_{\mathcal{D}} \mathbf{1}_{\mathfrak{A}}}{v_{\text{int}}}.$$

Here, v_{int} is the rigid internal degeneracy of the algebra. This mode-counting expression anchors the extensive response of the tensor sector and sets the informational limit for accessible symmetry data via the canonical Lie-theoretical filtration $G \rightarrow \mathfrak{g} \rightarrow T$ (detailed later).

This non-trivial correspondence converts the abstract spectral trace into a physical volume at the resolution scale M_s . Volume emerges not as a primitive coordinate measure, but as the total number of distinguishable response channels (normalized by the resolution scale). The 4-volume becomes an informational quantity derived from the boundary algebra, cleanly separating extensive gravitational stiffness from intensive gauge and mass susceptibilities.

2.4. Tensor Sector: Gravity as Extensive Stiffness

Applying the Hessian mapping to $\delta g_{\mu\nu}$, the tensor block G_{TT} measures the stiffness of the geometric pixel under modular flow. In the low-curvature regime $|R| \ll M_s^2$, linear response around the modular vacuum governs the dynamics [6]. In the small-diamond KMS regime, the modular generator couples $\delta g_{\mu\nu}$ to the stress tensor $T^{\mu\nu}$ (via the conformal Killing vector), supplemented by the boundary completion term required on bounded regions. Its variation reproduces the geometric entropy functional of the cut (area/Wald-type) [4,10,13,25].

With first-order variations vanishing at equilibrium, this entropy functional drives the second-order response [7,49], selecting the Einstein universality class at leading order. The leading two-derivative diffeomorphism invariant at the matching-scale M_s is $\int d^4x \sqrt{-g} R$, with coefficient $M_p^2/2$ [28]. The tensor entry therefore isolates the Einstein stiffness coefficient M_p^2 , which is strictly extensive in the number of coherently participating channels: $M_p^2 = N_{\text{eff}} M_s^2$.

2.5. Vector Sector: Gauge as Intrinsic Susceptibility

Applying the Hessian mapping to δA_μ , the vector block G_{JJ} measures the intrinsic susceptibility of the boundary current, specifically the linear response of the boundary state to an external connection. A weak background connection couples to the conserved edge current J^μ , required by the CS/WZW structure [18,20], through the modular generator:

$$\delta K[A] = \int d^4x \sqrt{-g} A_\mu(x) J^\mu(x).$$

Ward identities enforce gauge invariance, isolating $F_{\mu\nu} F^{\mu\nu}$ as the leading two-derivative operator [37]. In this framework, susceptibility is not a tunable material property but a geometric one, determined by the fluctuation-dissipation relation of the boundary algebra.

By P5 corollary, finite resolution truncates the operational modular interval to $\tau \in [1, 2\pi - 1]$. In the KMS regime, the edge-current correlator, which quantifies the vacuum fluctuations, takes the universal form $\langle J(\tau) J(0) \rangle = \frac{k}{4} \sin^{-2}(\tau/2)$ [50], where $k \in \mathbb{Z}$ is the WZW level. Integrating this correlator over the resolved window yields the total pixel susceptibility:

$$\chi_{\text{pix}} = \frac{k}{4} \int_1^{2\pi-1} \frac{d\tau}{\sin^2(\tau/2)} = k \cot(1/2).$$

Factoring out the kinematic integral isolates the normalized susceptibility $\tilde{\chi}_{\text{pix}} = k/4$. Applying the volume-coupling factorization and the standard flux convention $\alpha \equiv g^2/4\pi$ yields the matching-scale inverse coupling:

$$\alpha^{-1}(M_s) = 4\pi k.$$

This intensive coefficient depends only on the integer level k and the fixed modular protocol ($\varepsilon = 1$), unlike the extensive tensor sector. It represents the stiffness of the gauge sector, locked by the topology of the boundary completion.

2.6. Scalar Sector: Mass as Occupancy Stiffness

Applying the Hessian mapping to $\delta\varphi$, the scalar block G_{MM} measures the occupancy stiffness of localized matter. We couple a scalar deformation to the gauge-invariant density operator $M = \bar{\psi}\psi$ via $\delta K_M = \int d^4x \sqrt{-g} \delta\varphi(x) \bar{\psi}(x)\psi(x)$. In the symmetry decomposition, $\bar{\psi}\psi$ is a Lorentz scalar, so mass resides in this block. The Hessian identifies the leading contribution as a mass term:

$$S_{\text{mass}} \approx - \int d^4x \sqrt{-g} m \bar{\psi}\psi.$$

The $\mathbb{Z}_2$ grading (P7) admits spinor representations [51], and locality selects Dirac-type transport with the operator $D_\mu = \partial_\mu + \frac{1}{4}\omega_\mu^{ab}\gamma_{ab} - iA_\mu^a T^a$. Finite capacity bounds localized defect modes, yielding graded fermionic occupancy $n \in \{0, 1\}$. In discrete modular time, a local mass gap acts as the on-site phase accumulated per update.

Parameterizing $\theta_f \simeq m_f / E_{\text{pix}}$ (subject to $|\theta_f| \leq \pi$) and using the per-pixel energy budget E_{pix} (Chapter 1) gives the scaling $m \propto m_{\text{anchor}} / \Omega$, where Ω is the effective accessible volume counted by the number of resolvable modes of the internal Laplacian at the fixed cutoff M_s [37,52].

2.7. Unified Matching-Scale Action at M_s

The operators derived above for tensor, vector and scalar sectors govern the low-curvature regime. However, as the Einstein term contains no intrinsic saturation scale and extrapolation to $|R| \gtrsim M_s^2$ lies outside the finite-resolution regime, the quasi-local derivative expansion must be completed.

In the spectral/heat-kernel expansion, four-derivative invariants appear after the Einstein term. Because the discrete transport router (P7) is isotropic and the response is averaged over the full modular period, deformations project exclusively onto the scalar-curvature channel. Anisotropic four-derivative operators such as the Weyl tensor squared are orthogonal under the octahedral symmetry of the router and average to zero or become total derivatives in the quasi-local limit.

The Gauss–Bonnet combination is likewise a topological invariant on the compact modular manifold and does not contribute to local dynamics. The leading saturation completion is therefore the scalar-curvature term R^2 .

The coefficient λ in the saturated gravity sector $\frac{M_p^2}{2}R + \lambda R^2$ follows directly from the ratio of heat-kernel coefficients a_2 and a_4 evaluated under the Laplace-transformed window function f . The resulting structural lock is

$$\lambda = \frac{M_p^2}{12M_s^2}.$$

This ratio requires no additional tuning; it emerges naturally from the modular weighting that defines every matching-scale coefficient. Identifying M_s with this stiffness saturation places the high-curvature regime in a plateau universality class with r at the 10^{-3} level, consistent with Starobinsky-type inflation. The R^2 saturation coefficient follows directly from the ratio of heat-kernel coefficients a_4/a_2 evaluated on the axiomatically fixed $S^3 \times S^1$ manifold. No additional entropy modulation is required.

Assembling the continuum action requires mapping the extracted stiffness coefficients to their canonical local operators. Summing the three blocks gives:

$$S_{\text{eff}} \approx \int d^4x \sqrt{-g} \left[\underbrace{\frac{M_p^2}{2} \left(R + \frac{1}{6M_s^2} R^2 \right)}_{\text{Tensor}} - \underbrace{\frac{1}{4g(M_s)^2} F_{\mu\nu} F^{\mu\nu}}_{\text{Vector}} + \underbrace{\bar{\psi}(i\gamma^\mu D_\mu - m)\psi}_{\text{Scalar}} \right].$$

This matching-scale serves as the transition point where discrete dynamics freeze out and continuous evolution takes over. The coefficients represent response properties defined at M_s , treating couplings as a property of the observation layer (the lens) rather than the underlying dynamics (the object). Quantum fluctuations are not ignored; they are effectively resummed into the response coefficients at the matching-scale. Below M_s , standard EFT renormalization proceeds normally.

2.8. Relation to Continuum Approaches

Our formulation complements continuum entropic programmes [6,8,36] by recasting gravitational dynamics as local inference on a regulated subsystem. We begin from a fixed operational container $(\mathcal{O}, \mathfrak{A}_O, \delta)$. Non-factorization (P1) forces boundary completion with finite capacity, and finite resolution (P5) supplies a physical bandwidth $M_s = \delta^{-1}$. These two inputs render the inference problem well-posed: ρ_O and $\sigma_O[\Lambda]$ live on identical operator content, and all response is defined at a fixed physical scale rather than in a formal continuum limit. The Hessian of the relative-entropy functional, identified as the Kubo–Mori metric [14], then governs the dynamics. In the KMS/quasi-local regime, the tensor block admits a derivative expansion whose unique leading two-derivative invariant is $\int d^4x \sqrt{-g} R$, recovering the Einstein universality class at leading order.

The construction also connects to spectral-action principles in noncommutative geometry [47,48] by using a spectral trace to represent the mode-summed Kubo–Mori response. Here, the cutoff is not an auxiliary Λ but the operational threshold M_s , calibrated by the boundary architecture through G . The resulting EFT coefficients are matching-scale data; standard renormalisation applies only below this physical bandwidth, where the continuum description remains valid.

2.9. Time Evolution as Open Modular Dynamics and Canonical Lie Filtration

The Hessian governs restoring forces inside a fixed diamond. Physical time evolution requires shifting the operational window to the next causal diamond. Finite resolution forces the diamond tips below the regulator scale δ , splitting the algebra as $\mathfrak{A}_{\text{full}} \simeq \mathfrak{A}_{\text{res}} \otimes \mathfrak{A}_{\text{sub}}$. Each shift therefore involves a lossy partial trace $\rho_{\text{res}} = \text{Tr}_{\text{sub}} \rho_{\text{full}}$ over exiting degrees of freedom, rendering observable evolution intrinsically non-unitary.

Probability and positivity are preserved if the update is a completely positive trace-preserving (CPTP) map. In modular time τ these repeated steps follow the Gorini–Kossakowski–Sudarshan–Lindblad (GKSL) equation:

$$\frac{d\rho}{d\tau} = -i[K_{\text{mod}}(\tau), \rho] + \sum_{a,\omega} \gamma_a(\omega; \tau) \left(L_{a,\omega} \rho L_{a,\omega}^\dagger - \frac{1}{2} \{L_{a,\omega}^\dagger L_{a,\omega}, \rho\} \right),$$

where the dissipator strength is fixed by the same tip defect κ and modular window $\varepsilon = 1$ that determine gravitational stiffness [53,54]. Proper time for an external observer is simply $t \simeq \delta\tau$.

Finite resolution continuously leaks information through the tip bottlenecks, so relative entropy contracts at every step. This supplies a thermodynamic arrow of time that emerges naturally from the same axioms (P1, P5) that fix spacetime rigidity [35]. This local dissipation engine scales directly to macroscopic horizons. Locking the leakage strictly to κ yields the emission scaling $P_\infty \propto \kappa_H^2$, reproducing the semiclassical Hawking prefactor up to greybody factors [34,55].

The long-time fixed point is a pointer algebra $\mathfrak{A}_{\text{eff}} = \text{Fix}(\mathcal{E}_\tau)$ of stable, commuting observables [56]. For any compact non-abelian symmetry, repeated dephasing drives accessible data toward commutativity along the canonical filtration:

$$G \rightarrow \mathfrak{g} \rightarrow T,$$

where G is the full global group (maximal phase coherence), $\mathfrak{g}$ the local tangent dynamics (directionality without global connectivity) and T the maximal torus of mutually commuting Cartan phases. The inevitability of this distillation is anchored in the following axiomatic chain:

$$(P1, P3, P5) \Rightarrow \rho_{\text{res}} \xrightarrow{\text{CPTP}} \mathcal{E}_\tau = e^{\tau\mathcal{L}} \xrightarrow{\text{GKSL (P4)}} \mathfrak{A}_{\text{eff}} = \text{Fix}(\mathcal{E}_\tau) \xrightarrow{\text{dephase}} G \rightarrow \mathfrak{g} \rightarrow T$$

The split algebra forces CPTP updates, the modular clock drives the GKSL structure and representation theory restricts the stable path to these three irreducible layers.

This finite-resolution, open-modular architecture therefore supplies the natural origin of the three-generation hierarchy explored in [Chapter 3](#). In this picture, "now" is the latest stable record after a CPTP update, and "time passing" is the irreversible sequence of such updates.

2.10. Recovery of Standard Limits and Entropic Arrow of Time

Einstein Limit

Standard limits follow directly by varying the unified matching-scale action obtained from the relative-entropy Hessian. For the gravitational sector the R^2 saturation term yields the corrected Einstein equations:

$$M_P^2 G_{\mu\nu} + \frac{M_P^2}{6M_s^2} H_{\mu\nu}^{(R^2)} = T_{\mu\nu}^{\text{eff}}, \quad H_{\mu\nu}^{(R^2)} = 2RR_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R^2 - 2\nabla_\mu \nabla_\nu R + 2g_{\mu\nu}\square R.$$

In the low-curvature regime $|R| \ll M_s^2$ the higher-order correction is heavily suppressed. Stationarity of the variational functional constrains the background to the Einstein universality class [\[6,49\]](#), recovering $M_P^2 G_{\mu\nu} \approx T_{\mu\nu}^{\text{eff}}$ with stiffness set by the extensive channel capacity N_{eff} [\[30\]](#).

Yang–Mills and Dirac Limits

The same Hessian blocks recover Yang–Mills and Dirac kinematics as the minimal covariant structures compatible with the boundary architecture. The vector block is fixed by the quantized WZW current algebra (P6) together with the universal modular correlator $\langle J(\tau)J(0) \rangle \propto \sin^{-2}(\tau/2)$ evaluated on the $S^3 \times S^1$ history manifold; its quadratic response directly yields the gauge kinetic term and the continuity equation $\nabla_\mu F^{\mu\nu} = J^\nu$. The scalar block, shaped by the $\mathbb{Z}_2$ spinor-lift grading required by discrete isotropic transport (P7) together with per-pixel saturation E_{pix} , produces the Dirac operator $(i\gamma^\mu D_\mu - m)\psi = 0$.

These standard field equations emerge as the low-density limit of the boundary response. While deriving the Einstein sector from entropic logic is an established method [\[6,38\]](#), the framework yields Dirac and Yang–Mills kinematics as the minimal covariant structures required by the assumed boundary completion, discrete transport and sector decomposition. This unique combination of axioms forces the three sectors to decouple in this way, making the simultaneous recovery of the Einstein, Yang–Mills and Dirac equations a single, rigid consequence of the same finite-resolution architecture.

Entropic Arrow of Time

Finite resolution supplies a structural basis for the arrow of time. Tracing out the sub-resolution sector $\mathfrak{A}_{\text{sub}}$ induces the CPTP map Φ on the resolved state ρ_{res} . Relative entropy is strictly contractive under any CPTP map [\[54,57\]](#), so state distinguishability decreases monotonically in modular time:

$$S_{\text{rel}}(\Phi(\rho) \parallel \Phi(\sigma)) \leq S_{\text{rel}}(\rho \parallel \sigma) \quad \Rightarrow \quad \frac{dS_{\text{rel}}}{d\tau} \leq 0.$$

Thus, the thermodynamic arrow of time (the irreversible loss of distinguishability) arises from the exact same architecture that fixes gravitational stiffness, gauge couplings and matter generations. While most standard approaches must postulate the arrow of time, here it emerges naturally as a mandatory algebraic consequence of the entropic contractivity of the finite-capacity boundary.

3. Numerical Predictions Across Scales

This chapter summarizes the quantitative predictions derived in the [Appendices](#) from the core ingredients of [Chapters 1](#) and [2](#). Specifically, all entries arise from a fixed container $(\mathcal{O}, \mathfrak{A}_\mathcal{O}, \delta)$ defined by the relative-entropic variational principle and the Hessian mapping, where the tensor, vector and scalar sectors decouple into a matching-scale response problem.

The objective is to demonstrate the numerical implications of these constraints in a central location. In this framework, the fundamental parameters of nature are not arbitrary coefficients to be fitted; rather, once the effective internal degeneracy $N \approx 1.23 \times 10^{11}$ is fixed by the entropic geometry and the fundamental resolution scale $M_s \approx 3.02 \times 10^{13}$ GeV is anchored to Newton’s constant G , these values emerge as the necessary operating settings of the regulated boundary itself.

While these predictions span disparate domains, we present them as a single summary to emphasize their shared origin. The same architecture constrains inflation, vacuum leakage and the Higgs energy budget. Splitting these results would hide the shared mechanism and make the correlations look coincidental.

3.1. Predictions vs. Observations

Table 1 synthesizes these results, mapping each prediction back to the foundational axioms (P1–P7) and the specific internal variables $(N, \mathcal{N}_*, m_p)$ that drive the numerical output. To ensure an auditable derivation, we distinguish between primary geometric consequences and dependent algebraic relations.

From a single dimensional anchor (G) and seven natural axioms ([Chapter 1](#)), evaluated through the relative-entropy Hessian and local spectral trace ([Chapter 2](#)), the appendices predict 25 benchmark quantities spanning many orders of magnitude. The intent is to show how rigidly this open-modular architecture correlates sectors that are usually treated independently.

Classification [X, Y]: X specifies the comparator class (O=Observed/PDG, I=Inferred, S=SM/RGE, B=Bound, H=Heuristic); Y the derivation class (P=Primary, D=Dependent/Algebraic, E=Exploratory).

Table 1. Summary of Predictions vs. Observations.

Quantity	Prediction	Observation / Reference	Deviation	Class	Derivation Basis
Cosmology: Capacity Saturation and Entropic Response (Appendix A)					
Coherence $\mathcal{N}_*$	$18\pi \approx 56.55$	56.98 [58]	0.76%	[I,P]	[P4, P5, P7]
Scalar amplitude A_s	2.08×10^{-9}	2.10×10^{-9} [58]	1.0%	[O,P]	[P4, P5, P7]
Spectral tilt n_s	0.9646	0.9649 ± 0.0042 [58]	0.03%	[O,P]	$\mathcal{N}_*$; [P4, P5, P7]
Tensor ratio r	0.0038	< 0.036 [59]	Consistent	[B,P]	$\mathcal{N}_*$; [P4, P5, P7]
Running α_{run}	-6.3×10^{-4}	-0.0045 ± 0.0067 [58]	Consistent	[O,P]	$\mathcal{N}_*$; [P4, P5, P7]
Scalaron mass M_R	3.02×10^{13} GeV	3.00×10^{13} GeV [58]	0.7%	[I,D]	[P5]
Stiffness λ_{R^2}	5.42×10^8	5.44×10^8 [58]	0.4%	[I,D]	[P5, P7]
Vacuum Ω_Λ	0.667	0.689 ± 0.006 [60]	3.2%	[O,P]	[P3, P5, P7]
Struct. growth S_8	0.816	$0.76\text{--}0.84$ [58,61]	Consistent	[O,E]	[P4, P5, P7]
Accel. floor a_0	1.04×10^{-10}	$\sim 1.2 \times 10^{-10}$ [62]	13%	[H,E]	[P4]
Electroweak Saturation and Mass Generation (Appendix B)					
Higgs mass m_H	125.7 GeV	125.25 ± 0.17 [60]	0.36%	[O,P]	[P3, P5–P7]
VEV v	246.3 GeV	246.22 [60]	0.03%	[I,P]	[P4, P5, P7]
Top mass m_t	174.1 GeV	172.69 ± 0.30 [60]	0.82%	[O,P]	[P3, P5–P7]
Top Yukawa y_t	1.00	0.99 ± 0.01 [60]	1.0%	[I,D]	m_t, v ; [P3–P7]
Higgs quartic λ_H	0.130	0.126 ± 0.001 [60]	3.2%	[I,D]	m_H, v ; [P3–P7]
Gauge Couplings as Entropic Stiffness (Appendix C)					
Fine-structure α_0^{-1}	137.035999216	137.035999206 [63]	$< 10^{-10}$	[O,P]	[P2, P4, P5, P7]
UV EM $\alpha_{\text{em}}^{-1}(M_s)$	113.10	113.4 [64]	0.2%	[I,P]	[P5–P7]
UV Strong $\alpha_s^{-1}(M_s)$	37.7	38 ± 2 [64]	1.0%	[S,P]	[P5–P7]
Coupling Ratio $\mathcal{R}$	3.00	3.03 [64]	1.0%	[S,D]	[P6, P7]
Weak mix $\sin^2 \theta_W$	0.375	0.375 ± 0.005 [60]	Consistent	[S,P]	[P6, P7]
Lepton Mass Spectrum via Spectral Filtration (Appendix D)					
Proton mass m_p	942 MeV	938 MeV [60]	0.4%	[O,P]	[P5, P6]
Ratio m_p/m_e	1836.12	1836.15 [60]	10^{-5}	[O,P]	[P5, P7]
Electron mass m_e	0.511009 MeV	0.510999 MeV [60]	10^{-5}	[O,D]	m_p ; [P5, P7]
Muon mass m_μ	104.25 MeV	105.66 MeV [60]	1.3%	[O,P]	m_p ; [P5, P7]
Tau mass m_τ	1.788 GeV	1.777 GeV [60]	0.6%	[O,P]	[P5–P7]

3.2. Derivation Summary and Insights

The architectural foundation begins with a single operational object defined in [Chapter 1](#): a causal diamond equipped with a boundary-completed algebra, a fixed resolution scale M_s , and an effective

internal degeneracy N . Rather than tuning independent phenomenological sectors or attempting to quantize the graviton from the bottom up, gravitational stiffness is discretized directly at the boundary. The constants N and M_s are rigidly fixed by the entropic geometry of the regulated boundary. These two constants then govern every sector: M_s sets saturation and activation thresholds, while N sets collective averaging and stiffness.

[Chapter 2](#) supplies the dynamical engine. The physical action emerges not as a postulate, but as the relative-entropy mismatch functional between the actual boundary state and a semiclassical reference family. The Hessian of this functional (the Kubo–Mori metric) strictly decouples into orthogonal tensor, vector, and scalar blocks. The tensor block fixes the Planck scale M_P , the vector block fixes gauge couplings, and the scalar block fixes mass gaps. Finite resolution turns every translation between nested diamonds into an open modular update governed by a CPTP semigroup. The resulting coarse-graining drives the algebra to a stable fixed-point pointer structure whose spectral counting becomes a physical observable.

Cosmology: Capacity Saturation and Entropic Response ([Appendix A](#))

Ultraviolet saturation produces the Starobinsky plateau with a scalaron mass $M_R = M_s$. The scalar amplitude A_s becomes a pure capacity diagnostic following from channel averaging over N , and the coherence duration $\mathcal{N}_*$ follows from the discrete transport overhead. In the deep infrared, open-system leakage balances against the Hubble volume, mathematically forcing the vacuum fraction to the exact topological ratio $4/6$ dictated by the six spatial transport axes.

Electroweak Saturation and Mass Generation ([Appendix B](#))

The split between extensive and intensive boundary responses resolves the geometric origin of the hierarchy problem. The Planck mass is extensive, reflecting the collective stiffness of N_{eff} channels acting in parallel. Conversely, the electroweak scale is intensive, set by the bare per-pixel energy budget E_{pix} . The Higgs and top quark emerge as the necessary geometric structures required to normalize the $SU(2)$ charge basis and saturate a single-pixel unitary step, yielding the observed hierarchy without introducing any new fundamental parameters.

Gauge Couplings as Entropic Stiffness ([Appendix C](#))

Gauge couplings are read as vector-sector stiffness. Finite modular history strictly fixes the integration window ($\varepsilon = 1$), while topological quantization (Axiom P6) replaces continuous normalization freedom with integer levels k . The resulting ultraviolet couplings and the static fine-structure constant $\alpha^{-1}(0)$ become rigid spectral sums on the fixed history manifold $S^3 \times S^1$, transforming coupling ratios from adjustable parameters into discrete topological invariants.

Lepton Mass Spectrum via Spectral Filtration ([Appendix D](#))

Generations arise as stable accessibility limits under open modular coarse-graining. The Canonical Lie Filtration corollary forces any compact non-abelian symmetry to collapse into exactly three irreducible layers: $G \rightarrow \mathfrak{g} \rightarrow T$. The Transport-Axis Redundancy corollary supplies the strict integer factors 3 and 8 on every accessibility count, locking the mass ratios $m_e : m_\mu : m_\tau$ to the exact geometric constants $6\pi^5$, 9 and $2\pi^4$. Every empirical anchor is explicitly isolated to precision-test these dimensionless ratios, never to tune the architecture.

4. Conclusions and Outlook

A finite-resolution boundary architecture is sufficient to define local physics in a diffeomorphism-invariant quantum theory. Working in a Wheeler–DeWitt setting where the global state is constrained rather than evolved, the central operational problem is not to quantize the graviton, but to make a local subsystem well-defined when Hilbert spaces do not factorize cleanly across subregions.

Solving this non-factorization problem requires synthesizing three previously disconnected foundations. Causal diamonds supply covariant locality, finite-resolution screens regulate boundary

capacity and relative entropy provides the governing variational principle. This integration strictly determines the minimal operational axioms required to define subsystems, intrinsic clocks and regulated observables in a background-independent setting. By fixing the boundary structure, this synthesis eliminates the need for spacetime background or bottom-up graviton quantization.

4.1. Discretizing Gravity and Hierarchy of Scales

Gravitational stiffness discretizes directly at the regulated boundary. From the entropic geometry of the coherency screen, two fundamental constants emerge to govern the entire framework:

$$N = \sqrt{2} \exp\left(8\pi + \frac{1}{6\pi}\right) \approx 1.23 \times 10^{11}$$

$$M_s = \frac{M_P}{\sqrt{\kappa N}} \approx 3.02 \times 10^{13} \text{ GeV}$$

These constants carry the explanatory load. M_s sets the absolute resolution cutoff and saturation thresholds, while N determines collective averaging and stiffness. This geometric separation provides a structural resolution to the hierarchy problem: gravity emerges as an extensive response shared across N_{eff} coherent channels, while gauge couplings and electroweak mass gaps remain intensive, per-pixel quantities set directly by the fundamental scale M_s .

4.2. The Dynamical Engine

The dynamical engine operates without new physical postulates. The physical action is identical to the relative-entropy mismatch functional between the actual boundary state and a semiclassical reference family. The Hessian of this functional (the Kubo–Mori metric) decouples naturally into orthogonal tensor, vector and scalar blocks, yielding Einstein stiffness, gauge couplings (Yang–Mills) and mass terms (Dirac) directly from boundary response. Finite resolution turns translations between nested diamonds into open modular updates governed by a completely positive trace-preserving (CPTP) semigroup. This coarse-graining supplies both the relational clock and the thermodynamic arrow of time.

4.3. Coherency Units and Geometric Naturalness

In natural coherency units where the matching scale is set to $M_s = 1$, every derived constant of the theory reduces to a pure geometric coefficient fixed by the topology of the causal diamond. The Planck mass, the R^2 saturation coefficient, the tensor-to-scalar ratio and the gauge-level ratios become dimensionless values. This geometric unit system mathematically clarifies that the observed parameters of nature are not arbitrary constants, but structural necessities of the finite-resolution architecture.

4.4. Predictive Rigidity Across Sectors

The predictive power of this architecture manifests by systematically replacing continuous parametric freedom with discrete topological invariants. The deep-infrared vacuum fraction is fixed to the topological ratio $4/6$, yielding the Starobinsky plateau and scalar amplitude as pure capacity-counting statistics. Electroweak scales extract strictly from the intensive single-pixel energy budget. Gauge couplings emerge as rigid vector-sector stiffnesses, culminating in the fine-structure constant as a spectral sum on the fixed history manifold. Finally, the canonical Lie-theoretical filtration dictates the lepton mass spectrum, locking the generation ratios $m_e : m_\mu : m_\tau$ via the Transport-Axis Redundancy corollary to the exact geometric invariants $6\pi^5$, 9 and $2\pi^4$. Every prediction is falsifiable, and every coefficient traces transparently back to the foundational axioms.

4.5. Outlook

The framework unifies scales by tying gravity, gauge response and mass gaps to a single finite-resolution boundary on a causal diamond. With N fixed by boundary topology and M_s anchored to Newton's constant, these sectors cease to be independent phenomenological domains.

The architectural core of the Standard Model spectrum emerges as a geometric necessity of this boundary structure. The resulting physics is a single, tightly constrained architecture in which gravity, gauge response and mass are inextricably linked at the fundamental resolution scale M_s .

In this sense, the observed scales of nature appear less as unrelated constants and more as stable, self-consistent limits of what a finite-resolution quantum theory can represent.

Appendix A. Cosmology: Capacity Saturation and Entropic Response

This appendix derives UV and IR cosmological constraints from the finite-resolution boundary response set by (N, κ, M_s) and the fixed architecture.

In the UV, geometric stiffness saturates the information capacity (inflationary plateau); in the deep IR, the finite horizon surface enforces maintenance costs (vacuum fraction) and coherence limits (growth suppression and acceleration floor). Inputs such as H_0 and T_{CMB} are treated as boundary conditions of the local patch, while the functional forms and dimensionless locks are traced to the fixed boundary architecture (N, κ, M_s) and the dynamical engine of [Chapter 2](#).

In the cosmological context, the operational causal diamond $\mathcal{O}$ is identified with the observer (instantaneous) Hubble patch, ensuring the waist S^2 coincides with the apparent horizon where entropic matching conditions apply.

Appendix A.1. Geometric Stiffness and Activation (UV Saturation)

In [Chapter 2](#), we showed that the regulated boundary remains weakly coupled at intermediate scales ($|R| \ll M_s^2$), naturally reproducing General Relativity at low curvature where discrete transport overhead is negligible. However, finite resolution (Axiom P5) imposes a hard limit on both geometry and the effective field theory. As the curvature scale approaches the pixel resolution ($|R| \rightarrow M_s^2$), finite resolution bounds the growth of the response; the leading curvature-squared truncation is captured by an $R + R^2$ effective action, which places the dynamics in the Starobinsky plateau universality class [\[33\]](#). The effective action in the saturation regime is:

$$S_{\text{eff}} \approx \int d^4x \sqrt{-g} \left[\frac{M_P^2}{2} R + \lambda_{R^2} R^2 \right].$$

The standard plateau dictionary defines the mass scale M_R via $\lambda_{R^2} \equiv M_P^2 / (12M_R^2)$. The ratio λ_{R^2} / M_P^2 is fixed by the ratio of heat-kernel coefficients a_4 / a_2 evaluated on the modular manifold $S^3 \times S^1$ ([Chapter 2](#)). Because the spectral trace is cut off at the physical resolution $\delta = M_s^{-1}$ (Axiom P5), the curvature scale at which the R^2 term becomes order-1 relative to the Einstein term is forced to be exactly $M_R = M_s$ (no additional prefactors):

$$M_R = M_s \simeq 3.02 \times 10^{13} \text{ GeV}$$

(CMB-inferred (Starobinsky): $3.00 \times 10^{13} \text{ GeV}$ [\[58\]](#); Relative Deviation: 0.7%)

Using $M_P^2 = N_{\text{eff}} M_s^2 = \kappa N M_s^2$, the R^2 stiffness coefficient becomes:

$$\lambda_{R^2} = M_P^2 / (12M_s^2) = \kappa N / 12 \approx 5.42 \times 10^8$$

(CMB-inferred (Starobinsky): 5.44×10^8 [\[58\]](#); Relative Deviation: 0.4%)

Appendix A.2. Coherence Duration ($\mathcal{N}_*$)

We determine the duration of the inflationary phase by analyzing the stability of the resolved sector against discrete transport overhead. The minimal discrete transport structure (P7) induces an irreducible mismatch κ per resolved modular update $\Delta\tau = 1$ (P5).

The distinguishability threshold is the minimal resolvable phase shift that the coherency screen can register. Each causal diamond has two tip bottlenecks (P2). The discrete L_1 router incurs an irreducible mismatch κ per axis at each tip (P7 corollary). Thus, a full modular update across the diamond costs 2κ per axis. This mismatch accumulates independently along each of the $D = 3$ spatial transport axes. The distinguishability threshold is reached when the accumulated relative-entropy mismatch per axis reaches $\mathcal{O}(1)$ under successive CPTP updates (contractivity and pointer-algebra stabilization in Chapter 2). Summing over the $D = 3$ axes therefore yields $m_* = 2D/\kappa$ resolved steps, leaving no free parameters. On the plateau, $H \approx M_R/2$ and the Unruh relation $t \simeq \delta\tau$ (Chapter 1) imply $\Delta\mathcal{N} = 1/2$ per modular step, hence:

$$\mathcal{N}_* \approx m_* \cdot \Delta\mathcal{N} = \frac{D}{\kappa} = 18\pi \approx 56.55.$$

(Observed: ≈ 56.98 [58] (n_s -inferred); Relative Deviation: 0.8%)

Appendix A.3. Primordial Observables (A_s, n_s, r, α_s)

The primordial amplitude is a capacity diagnostic reflecting the thermal KMS fluctuations of the local de Sitter patch rather than standard zero-point field quantization; the variance scales exactly as $1/N$, because the coarse-grained curvature readout averages over the N independent channels of the vacuum budget. The standard plateau amplitude then gives:

$$A_s = \frac{M_R^2 \mathcal{N}_*^2}{24\pi^2 M_P^2} = \frac{81\pi}{N} \simeq 2.08 \times 10^{-9}$$

(CMB-inferred: $(2.10 \pm 0.03) \times 10^{-9}$ [58]; Relative Deviation: 1.0%)

Substituting the stiffness relation $M_P^2 = \kappa N M_s^2$ (Chapter 1) and $\mathcal{N}_* = 18\pi$ collapses the standard plateau expression algebraically to $A_s = 81\pi/N$.

With $\mathcal{N}_* \approx 56.55$, the standard plateau consistency relations yield the following spectral parameters (spectral index n_s , tensor-to-scalar ratio r , running of spectral index α_s):

$$n_s \approx 1 - \frac{2}{\mathcal{N}_*} = 1 - \frac{1}{9\pi} \approx 0.9646$$

(Observed: 0.9649 ± 0.0042 [58]; Relative Deviation: 0.03%)

$$r \approx \frac{12}{\mathcal{N}_*^2} \approx 0.0038$$

(Constraint: $r < 0.036$ [65]; Consistent)

$$\alpha_s \approx -\frac{2}{\mathcal{N}_*^2} \approx -6.3 \times 10^{-4}$$

(Constraint: $\alpha_s = -0.0045 \pm 0.0067$ [58]; Consistent)

Appendix A.4. Cosmological Constant as Entropic Horizon Leakage

In the deep IR, the boundary does not saturate; instead, the horizon surface becomes informationally sparse [2]. We derive the cosmological constant not as a bulk vacuum energy summation (which diverges), but as the steady-state horizon leakage cost required to balance the open modular update (Chapter 2) [66,67].

For a flat universe, the critical density is $\rho_{\text{crit}} = 3M_P^2 H^2$. As derived in Chapter 2, the local GKSL dissipation rate [54,68] redshifts to a macroscopic power $P_\infty \propto \kappa C H^2$, and M_s cancels at the level of

the scaling estimate. Balancing this continuous leakage over a Hubble 4-volume implies an effective vacuum density $\rho_\Lambda \propto H^2$. Since $\rho_{\text{crit}} \propto H^2$, the vacuum fraction Ω_Λ reduces to a dimensionless coefficient fixed by the minimal discrete-transport topology:

$$\Omega_\Lambda \equiv \frac{\rho_\Lambda}{\rho_{\text{crit}}} = L \times \left(\frac{\mathcal{C}_0}{z} \right)$$

The leakage coefficient is unity by the definition of the GKSL rate (Chapter 2); the scaling $\rho_\Lambda \propto \kappa CH^2$ is exact once M_s cancels, ensuring that macroscopic IR dynamics decouple cleanly from the discrete UV grid to preserve Lorentz invariance at large distances.

The six orthogonal directions $\{\pm x, \pm y, \pm z\}$ of the minimal discrete transport structure (P7) set the number of independent leakage channels $z = 6$. In the deep IR, only massless long-range fields remain actively supported: photon (2 polarizations) and graviton (2 polarizations), yielding an active-channel factor $\mathcal{C}_0 = 4$.

Equating the volumetric vacuum energy density required to sustain the Hubble volume with the surface dissipation rate through $z = 6$ transport channels (GKSL leakage strength $\propto \kappa \mathcal{C}_0 H^2$ from Chapter 2) produces the purely topological ratio in the late universe:

$$\Omega_{\Lambda,0}^{\text{pred}} = \mathcal{C}_0/z = \frac{4}{6} \approx 0.667$$

(Observed: 0.689 ± 0.006 [60]; Relative Deviation: 3.2%)

Because additional massless long-range species would strictly increase $\mathcal{C}_0$ and shift the predicted band upward, this is strictly falsifiable.

At earlier epochs, the active-channel factor becomes horizon-scale dependent, $\mathcal{C}(T_H)$, because species with $m \lesssim T_H$ (equivalently $m \lesssim H/2\pi$) can contribute to the dissipative inventory, while heavier species are suppressed. A minimal interpolation replaces $\mathcal{C}_0$ with a thresholded sum:

$$\Omega_\Lambda = L \frac{\mathcal{C}(T_H)}{z} \quad (z = 6) \quad \mathcal{C}(T_H) = \sum_i g_i w\left(\frac{m_i}{T_H}\right),$$

where g_i counts helicity/polarization states and $w(x)$ is a smooth threshold with $w \rightarrow 1$ for $x \ll 1$ and $w \rightarrow 0$ for $x \gg 1$.

The vacuum fraction Ω_Λ departs from $4/6$ only if additional species become effectively massless on the horizon scale. Since $T_H \sim H/2\pi$ remains negligible compared to SM masses throughout the post-recombination epoch, $\mathcal{C}(T_H) \approx \mathcal{C}_0$ strictly holds in the late universe. Any detectable deviation in this regime would point to new ultra-light degrees of freedom.

Appendix A.5. IR Regime: Horizon Coherence and S_8 (Exploratory)

The following late-time dynamic effects (S_8, a_0) are classified as exploratory. We model late-time growth phenomenology as a coherence impedance effect to test if the channel multiplicity (N) remains the correct measure of degrees of freedom in the IR.

As the causal patch grows toward the deep IR, the discrete transport on the boundary becomes susceptible to entropic drift. We define the effective clustering strength via the tensor-sector Kubo–Mori stiffness $\mathcal{K}(O_a)$ of a horizon-sized diamond O_a :

$$f(a) \equiv \frac{G_{\text{eff}}(a)}{G_N} = \frac{\mathcal{K}(O_0)}{\mathcal{K}(O_a)}.$$

The system maintains GR-like stiffness ($f \approx 1$) only while the boundary transport remains coherent. The onset of decoherence a_* occurs when the accumulated relative-entropy drift over one Hubble time reaches the distinguishability threshold fixed by the tip defect parameter corollary. Beyond this

threshold, the update map becomes lossy ($f(a) < 1$). This suppression satisfies relative-entropic contractivity, consistent with monotonic information loss along RG flow.

Because Kubo–Mori stiffness is extensive over the four-volume counted by the spectral trace on the $S^3 \times S^1$ minimal history manifold (Chapter 2), large-scale coherence dilutes isotropically as the physical spacetime volume grows. The suppression therefore scales with the inverse spacetime dimension $d = 4$, yielding an effective exponent $\nu_{\text{eff}} = 1/d = 1/4$. We obtain:

$$N_{\text{part}}(a) \propto \left(\frac{H(a)}{H(a_*)} \right)^{\nu_{\text{eff}}}.$$

This suppression appears as a finite-resolution effect in the S_8 parameter. Using the standard Λ CDM growth index approximation Ω_m^γ with $\gamma \approx 0.55$:

$$S_8^{\text{pred}} \approx S_8^{\text{Planck}} \times f(1)^{0.55} \approx 0.816$$

(Observed: 0.76–0.84 [58,61]; Consistent)

Appendix A.6. Galactic Regime: Acceleration Floor (a_0) (Exploratory)

The acceleration floor inherits the same relative-entropy contractivity and horizon KMS background (P4, Chapter 2) that generates the thermodynamic arrow of time. This implies a thermal horizon background $T_H \simeq \hbar H / 2\pi k_B$ [6,69,70]. An inertial acceleration signal a becomes indistinguishable from the horizon noise when its Unruh temperature T_U [5] drops below T_H . The crossover $T_U \sim T_H$ fixes an acceleration floor: $a_0(z) \propto cH(z)$. Unlike static modifications of gravity, this floor is dynamic and evolves with redshift. Using the shared 2π KMS normalization:

$$a_0 = \frac{1}{2\pi} cH_0 \approx 1.04 \times 10^{-10} \text{ m s}^{-2}$$

(Observed: $\approx 1.2 \times 10^{-10} \text{ m s}^{-2}$ [62,71]; Consistent)

Appendix A.7. Summary of Predictions

Inputs: (N, κ, M_s) with $N \approx 1.23 \times 10^{11}$; $\kappa = 1/(6\pi)$; $M_s \approx 3 \times 10^{13} \text{ GeV}$.

Outputs: Inflationary plateau ($n_s \simeq 0.965$, $r \simeq 0.004$, $\alpha_s \simeq -6 \times 10^{-4}$, $A_s \simeq 2.1 \times 10^{-9}$, $\mathcal{N}_* \simeq 56.6$); stiffness ($\lambda_{R^2} \simeq 5.4 \times 10^8$); vacuum fraction ($\Omega_{\Lambda,0} \simeq 0.67$); growth suppression ($S_8 \simeq 0.82$); acceleration floor ($a_0 \simeq 1.0 \times 10^{-10}$).

Falsifiers: Violation of plateau consistency (e.g. r well below 10^{-3}); vacuum fraction significantly outside the 4/6 topological band; absence of late-time growth suppression ($S_8 \approx S_8^{\text{Planck}}$); redshift-independent acceleration floor ($a_0 \not\propto H(z)$).

All functional forms and numerical prefactors follow from the axioms, corollaries and spectral extraction in Chapter 2. The active-channel count $\mathcal{C}_0 = 4$ is the natural infrared count of massless long-range bosons on the horizon scale: two photon polarizations plus two graviton helicities (Axioms P4 and P7).

Appendix B. Electroweak Saturation and Mass Generation

This appendix demonstrates that the mass scales of the electroweak sector (m_H, v, m_t) are constrained by finite-resolution boundary physics: structural entropic cost (Higgs), phase-noise floor (VEV) and principal-branch phase saturation (top quark), defined at the boundary matching scale E_{pix} . To compare our bare parameters with experimental data, we use standard SM scheme conversions [60].

Appendix B.1. Per-Pixel Energy Budget (E_{pix})

The electroweak sector emerges as the saturation of the finite per-pixel capacity E_{pix} at the boundary algebra (Chapter 1, Axiom P5), read out through the scalar block of the relative-entropy Hessian (Chapter 2).

In the modular/KMS regime (Axiom P4), local dynamics are controlled by linear response via the Kubo–Mori metric [14]. The physical resolution $\delta = M_s^{-1}$ (Axiom P5) sets the shortest modular update $\varepsilon = 1$, fixing the resolution scale M_s of the boundary algebra (Axiom P3).

A pixel carries an effective internal degeneracy N (Chapter 1). Axiom P7 adds a $\mathbb{Z}_2$ spinor-lift overhead with quantum dimension $d_\sigma = \sqrt{2}$. This supplies spin structure but is not an independent dynamical channel; the addressable internal degrees of freedom count is $n_{\text{ch}} = N/\sqrt{2}$. In linear response, stiffness is additive. The pixel-saturation energy stated in Chapter 1 is then:

$$E_{\text{pix}} \equiv \frac{M_s}{n_{\text{ch}}} = \frac{\sqrt{2}M_s}{N} \approx 348.2 \text{ GeV}$$

Heavier localized excitations would require multi-pixel encoding. E_{pix} sets the single-pixel activation scale used below.

Appendix B.2. Higgs Boson: Structural Entropic Cost

The Higgs field sets the scale (metric) of the internal charge space. While gauge freedom removes three Goldstone modes at long distances, the regulated boundary algebra must represent the full $SU(2)$ doublet (four real components) to define the vacuum state and normalize gauge charges (Axioms P3 and P6). The regulated boundary algebra (P3) must faithfully represent the full $SU(2)$ doublet (four real components) to normalize gauge charges under Gauss-law completion (P6). Maintaining operational distinguishability of these four vacuum sectors against modular noise (P4) incurs the minimal entropic cost per component already fixed by the $\mathbb{Z}_2$ grading structure required for spinors (P7): exactly one bit ($\ln 2$) each, yielding a total $\Delta S_H = 4 \ln 2$.

By the equipartition of pixel saturation energy across these informational constraints, the scalar Hessian of Chapter 2 converts this distinguishability cost directly into the mass scale:

$$m_H = \frac{E_{\text{pix}}}{\Delta S_H} = \frac{E_{\text{pix}}}{4 \ln 2} \approx 125.7 \text{ GeV}$$

(Measured: $125.25 \pm 0.17 \text{ GeV}$ [60]; Relative Deviation: 0.36%).

Appendix B.3. Vacuum Expectation Value: Noise Floor

The Vacuum Expectation Value (VEV) v represents the magnitude of the Higgs condensate (the radial order parameter). At resolution δ , the system cannot stably lock the absolute phase against an external reference (P4, P5), making the phase effectively random (maximum entropy) [43].

The VEV is the radial order parameter of the same scalar-density operator. Modular flow (P4) on finite-resolution algebra (P5) ergodically samples the full phase circle uniformly, the maximum-entropy state selected by the same Jaynes principle used to construct the reference family in Chapter 2. Because the linear expectation vanishes, the quadratic fluctuation survives as the stable radial order parameter. With its maximum amplitude bounded by the pixel capacity E_{pix} , the VEV is:

$$v^2 = \langle (E_{\text{pix}} \cos \phi)^2 \rangle_\phi = \frac{1}{2} E_{\text{pix}}^2 \quad \rightarrow \quad v = \frac{E_{\text{pix}}}{\sqrt{2}} \approx 246.3 \text{ GeV}$$

(Inferred from G_F : 246.22 GeV [60]; Relative Deviation: 0.03%).

Appendix B.4. Top Quark: Phase Saturation Bound

Fermions cannot exist as isolated local observables (P1, P3); with edge completion the minimal local excitation is therefore a gauge-invariant dressed dipole. Finite resolution (P5) sets the modular update scale $\varepsilon = 1$, so local evolution on the pixel algebra is realized by a discrete unitary step operator U_{pix} whose eigenvalues are pure phases $e^{i\theta}$. To avoid principal-branch ambiguity the resolved single-step phase lies in the domain $\theta \in (-\pi, \pi]$.

The scalar-sector Hessian (Chapter 2) identifies the fermion mass gap with the phase rotation per modular step: $\theta \approx m_f / E_{\text{pix}}$. Representability of the full dipole (pair energy $2m_f$) therefore forces the total rotation angle to fit exactly within the principal domain, yielding:

$$m_t \approx \frac{1}{2} E_{\text{pix}} \approx 174.1 \text{ GeV}$$

(PDG reference value: $172.69 \pm 0.30 \text{ GeV}$ [60]; Relative Deviation: 0.8%).

Because the modular clock ticks at finite discrete intervals (P4, P5), the dipole pair energy $2m_f$ must fit entirely within the principal branch of the discrete modular step operator to avoid phase aliasing. A sub-percent dressing overhead is expected for a gauge-invariant excitation that cannot sit exactly on the principal-branch boundary.

Appendix B.5. Structural Locks and Vacuum Stability

The predictions for m_t and m_H carry intrinsic uncertainties from scheme conversion and finite width ($\sim 1 \text{ GeV}$ for top, $\sim 0.2 \text{ GeV}$ for Higgs) [72]. Physically, the vacuum does not select these masses; it simply bounds any excitation that attempts to exceed the per-pixel capacity of the regulated boundary (E_{pix}). This saturation condition constrains the Standard Model flavor parameters by fixing the dimensionless couplings (specifically the top Yukawa coupling y_t and the Higgs self-coupling λ) as rigid geometric ratios:

$$y_t \equiv \frac{\sqrt{2} m_t}{v} \approx 1$$

(Inferred: 0.99 ± 0.01 [60]; Relative Deviation: $\sim 1\%$)

$$\lambda \equiv \frac{m_H^2}{2v^2} = \frac{1}{16(\ln 2)^2} \approx 0.130$$

(Inferred: 0.126 ± 0.001 [60]; Relative Deviation: 3.2%)

This value places the vacuum right on the edge of stability, consistent with standard RG analyses where λ runs toward a near-critical small value in the 10^{11} – 10^{13} GeV range [73] (within current parameter uncertainties). This provides the matching-scale boundary conditions for the Standard Model inputs, which then evolve according to standard renormalization flows.

Appendix B.6. Summary of Predictions

Inputs: Resolution scale M_s ; effective internal degeneracy N ; spinor overhead $d_\sigma = \sqrt{2}$ (Axiom P7) $\rightarrow E_{\text{pix}} = \sqrt{2} M_s / N$.

Outputs: $m_H \approx 125.7 \text{ GeV}$; $v \approx 246.3 \text{ GeV}$; $m_t \approx 174.1 \text{ GeV}$; derived locks $y_t \approx 1$; $\lambda \approx 0.130$ at the matching scale (vacuum metastability).

Falsifiers: Persistent violation of these relations after standard scheme conversion/RG evolution; discovery of a fermion heavier than the top; measured Higgs quartic incompatible with the predicted boundary condition.

All functional forms and numerical prefactors follow from the pixel-saturation bound (Chapter 1), the $\mathbb{Z}_2$ grading (Axiom P7) and the scalar-sector Hessian (Chapter 2). The only modeling choice is the identification of the Higgs radial mode with the scalar-density operator, already present in the Hessian.

Appendix C. Gauge Couplings as Entropic Stiffness

This appendix derives SM gauge couplings as response properties of the boundary state ρ_O , not as free parameters. We define a gauge coupling by the entropic stiffness of the variational functional: the Kubo–Mori (quantum Fisher) response of the vacuum reference state [14] to a background gauge deformation in the modular/KMS regime (Axiom P4) [4,15], evaluated on the fixed boundary algebra (Axiom P3) at finite resolution (Axiom P5) [10,13]. Topological quantization of the edge current algebra (Axiom P6) [18] fixes an integer level k , while the conventional 4π factor enters through flux normalization on the diamond waist S^2 (Axiom P2).

These couplings emerge strictly from the vector (current) sector G_{JJ} of the relative-entropy Hessian (Chapter 2), locked by non-factorization (P1) and boundary completion (P3); masses, by contrast, follow from the scalar density sector (Appendix D).

Appendix C.1. Entropic Stiffness Quantization

We first establish the quantization rule that turns the vector-sector response into a definite gauge coupling at the matching scale. In Chapter 2, we fixed the measurement protocol: the coupling is read off from the quadratic relative-entropy cost of a weak background gauge deformation in the KMS regime matched to the canonical normalization of $-\frac{1}{4}F^2$.

As explained in Chapter 2, microscopically, the stiffness arises from integrating the universal modular noise kernel $\langle \mathcal{J}(\tau)\mathcal{J}(0) \rangle \propto \sin^{-2}(\tau/2)$ over the resolved modular history $[\varepsilon, 2\pi - \varepsilon]$, yielding the susceptibility factor $I_{\text{mod}} = 4 \cot(\varepsilon/2)$ (obtained in Chapter 2 by integrating the universal WZW kernel over the resolved interval set by the Monotone-Penalty Spectral Selection corollary). Using this protocol with the fixed modular window $\varepsilon = 1$ (Axiom P5) and integer edge level k (Axiom P6), we obtained the master relation:

$$\alpha^{-1}(M_s) = 4\pi k.$$

The factor 4π converts surface flux through the waist S^2 (P2) into the conventional $\alpha \equiv g^2/4\pi$; the integer k is locked by the WZW current algebra (P6) once the modular window is fixed at $\varepsilon = 1$ (P5 corollary).

This relation acts as a dictionary between the discrete edge currents and the continuous field. The factor 4π is not a prediction of the boundary geometry, but simply the standard geometric conversion factor for flux through a sphere ($\int_{S^2} d\Omega = 4\pi$). By fixing the continuum definition to the standard form $\alpha \equiv g^2/4\pi$ and the canonical normalization $-\frac{1}{4}F^2$, we absorb the remaining normalization into the integer level k of the matching protocol. Thus 4π fixes the continuum flux convention at the S^2 interface, while k encodes the matched normalization of the edge current algebra at M_s . Any additional order-one geometric factor from discretization would simply rescale the current and is absorbed into k by definition.

This holds at leading derivative order in the regime $\delta \ll \ell_O \ll R_{\text{curv}}$ and $|p| \ll M_s$. For an Abelian factor, independent charged sectors contribute additively to the quadratic cost, so the effective level k is an additive inventory over the charged sectors active at M_s .

Appendix C.2. Strong Sector Prediction ($k = 3$)

For the strong interaction with gauge group $SU(3)_c$, the problem is reduced to fixing one integer:

$$\alpha_s^{-1}(M_s) = 4\pi k_s$$

Non-Abelian gauge fields carry charge. Physically, the vacuum stiffness is dominated by self-interaction (anti-screening) [74], rather than the sum over matter charges characteristic of the Abelian sector. The edge current algebra must therefore be normalized by an intrinsic property of the $SU(3)$ current algebra itself.

The stiffness level for non-Abelian groups is the adjoint self-coupling normalization, given rigidly by the dual Coxeter number $h^\vee$ of the WZW algebra (P6 and the prepared corollary on current-algebra

normalization). For $SU(3)_c$ we have $k_s = 3$; any other integer would violate the anomaly inflow required by boundary completion (P3). The matching-scale coupling follows immediately:

$$\alpha_s^{-1}(M_s) = 4\pi \cdot 3 = 12\pi \simeq 37.7$$

(SM-inferred at M_s : $\alpha_s^{-1} \simeq 38 \pm 2$ [60,64] + running; Relative Deviation: $\sim 1\%$)

Appendix C.3. Electromagnetic Prediction ($k = 9$) and Consistency Check

We now apply the stiffness quantization $\alpha^{-1}(M_s) = 4\pi k$ to the electromagnetic sector. Stiffness is driven here by matter screening ($\sum Q^2$) of all long-range charged sectors active at M_s . The electromagnetic inventory follows directly from the three-layer stable data produced by the open modular filtration (Chapter 2); at the matching scale this yields three generations plus the Higgs doublet, producing $k_{\text{em}} = 9$ and the integer ratio 3 with the strong sector:

Quarks: 3 generations $\times$ 3 colors (u, d) $\implies 9 \times [(2/3)^2 + (-1/3)^2] = 5$

Leptons: 3 generations (e, ν) $\implies 3 \times [(-1)^2 + 0^2] = 3$

Higgs: One complex charged component in the $SU(2)$ doublet $\implies 1 \times 1^2 = 1$

The total electromagnetic stiffness is the integer sum $k_{\text{em}} = 5 + 3 + 1 = 9$. Substituting this into the master formula:

$$\alpha_{\text{em}}^{-1}(M_s) = 4\pi \times 9 = 36\pi \simeq 113.10$$

We compare this geometric prediction against SM couplings extrapolated to the matching scale $M_s \simeq 3 \times 10^{13}$ GeV. Using the unbroken-basis identity with 1-loop SM values ($\alpha_1^{-1} \approx 42.6, \alpha_2^{-1} \approx 42.4$) [64]:

$$\alpha_{\text{SM}}^{-1}(M_s) = \alpha_2^{-1} + \frac{5}{3}\alpha_1^{-1} \approx 113.4$$

(Prediction: 113.10; Relative Deviation: $\sim 0.2\%$)

This convergence yields an effective mixing angle $\sin^2 \theta_W(M_s) \approx \alpha_{\text{em}}(M_s)/\alpha_2(M_s) \approx 113.1^{-1}/42.4^{-1} \approx 0.375$, recovering the canonical projection factor $3/8$ (standard hypercharge normalization) defined by the ratio $\sum T_3^2 / \sum Q^2$ over a fermion generation [75].

Internal Consistency Check (Integer Lock)

Because both gauge sectors share the same 4π flux normalization, the ratio of inverse couplings depends only on the integers k ($k_s = 3$):

$$\frac{\alpha_{\text{em}}^{-1}(M_s)}{\alpha_s^{-1}(M_s)} = \frac{k_{\text{em}}}{k_s} = \frac{9}{3} = 3$$

(SM-inferred ratio at M_s : ≈ 3.03 ; Relative Deviation: 1%)

This integer lock is a clean falsifiable prediction at the matching scale. Any additional electrically charged sector active below M_s would shift k_{em} and spoil this integer relation.

Appendix C.4. Infrared Static Response: Fine-Structure Constant

In this section, we calculate the macroscopic fine-structure constant $\alpha(0)$ by treating it not as a fundamental given, but as the collective, integrated response of the boundary vacuum to a static electromagnetic field.

We define this observable rigorously using the Euclidean path integral: the static inverse coupling is the coefficient of the quadratic gauge kinetic term ($F_{\mu\nu}^2$) in the effective action defined by the Laplace-type kinetic operator Δ_A on the operational history manifold ($S^3 \times S^1$) [37]. In this interpretation, it is the renormalized $\omega \rightarrow 0$ Kubo–Mori susceptibility [14,35] of the boundary state integrated over the minimal operational history $S^3 \times S^1$ (Minimal History Manifold corollary). To evaluate this functional without explicitly performing RG running, we utilize the standard spectral representation.

Just as hydrodynamics replaces microscopic collisions with effective constitutive laws, this expansion packages microscopic quantum loops into exact geometric invariants [28,37].

The operational axioms completely fix the integration space: the causal diamond with boundary completion on the coherency screen at its waist S^2 (P1–P3). Finite resolution and modular flow then lock the minimal history manifold to $S^3 \times S^1$ (Minimal History Manifold corollary from P2, P4, P7). Because operator-dependent prefactors are factored out by ultraviolet matching at M_s , the static response coefficient reduces to a closed, exhaustive set of dimensionless scale-free invariants. The Monotone-Penalty Spectral Selection corollary (P3, P5) dictates their hierarchy: leading contributions must be extensive volume measures (scaling with history four-volume), while finite-resolution corrections must strictly decrease with both bulk history volume (better averaging) and boundary capacity (better resolution).

Subject to these rigid constraints, exactly four leading invariants survive. The heat-kernel coefficient a_4 on the twisted, glued $S^3 \times S^1$ manifold with discrete transport therefore isolates precisely these four contributions. Each corresponds directly to an irreducible geometric feature of the causal diamond:

Bulk volume ($4\pi^3$): The axioms force the leading extensive term to equal the exact four-volume of the resolved history manifold ($2\pi^2 \times 2\pi = 4\pi^3$). Physically, this counts the total number of independent response channels across the full coherency screen: every modular update registers a local excitation that propagates once around the closed $S^3 \times S^1$ geometry (fixed by P2, P4, P7). No other volume measure is compatible with the diamond construction.

Twist sector ($\pi^2 + \pi$): The spinorial lift required by discrete isotropic transport (P7) imposes a $\mathbb{Z}_2$ grading on closed loops around the screen. This grading forces a twisted sector in the heat-kernel trace; the half-measures $\pi^2 + \pi$ appear directly once the double-cover identification is imposed [37]. Physically, this registers the extra phase cost that fermions acquire when their worldlines wind once around the screen (a direct consequence of the octahedral router and inversion symmetry).

Gluing penalty ($-1/32\pi^4$): Algebraic completion of the subregion algebra (P3) glues past and future light cones across the waist S^2 . The Gauss–Bonnet budget on that closed 2-sphere supplies a fixed capacity $\text{Cap} = 8\pi$ (Chapter 1) [27]. The monotone-penalty rule then isolates the unique leading interface suppression $-1/(\text{Vol} \cdot \text{Cap})$, which first appears at order π^{-4} . This precisely matches standard interface heat-kernel methods [76,77], encoding the irreducible cost of reconciling overlapping histories into a single operational record; any weaker or stronger gluing would violate non-factorization (P1).

Defect correction ($1/64\pi^6$): The Transport-Axis Redundancy corollary fixes six orthogonal directions on the screen ($z = 6$). At the diamond tips, the transverse cross-section drops below resolution, incurring a modular-closure overhead of $(2\pi)^{-6} = 1/(64\pi^6)$ that shares the same origin as the tip impedance $\kappa = 1/(6\pi)$. Physically, this registers the coordination penalty paid every time an excitation traverses a sub-resolution bottleneck on the coherency screen; the six-axis structure makes this correction inevitable and unique.

These four contributions add linearly because each is a dimensionless geometric piece of the same heat-kernel a_4 coefficient evaluated at M_s . The resulting sum is structurally rigid: every coefficient traces directly to a named structural lock on the coherency screen, requiring no intermediate continuum limit or free parameters.

The absence of intermediate powers (such as $\pi^{-1}, \pi^{-2}, \pi^{-3}$) is strictly enforced by the same Monotone-Penalty Spectral Selection corollary. Admissible corrections must decrease with both history four-volume and boundary capacity; terms built solely from the modular period or single-scale factors violate this non-factorization rule and are excluded. This silent band supplies structural stability: even order-unity coefficients in these missing sectors would shift $\alpha^{-1}(0)$ by $\mathcal{O}(10^{-1})$, yet the observed parts-per-billion precision requires no such tuning. The powers 32 and 64 arise directly from the discrete bookkeeping of the coherency screen ($\text{Vol}(S^3 \times S^1) \cdot \text{Cap}(S^2)$ for the gluing interface and

the six-axis closure $(2\pi)^{-6}$ for the tip defect), making the small corrections the irreducible boundary signatures that survive coarse-graining.

Evaluating the vacuum susceptibility over the modular S^1 geometry (Chapter 2) yields $I_{\text{mod}}(\varepsilon) = 4 \cot(\varepsilon/2)$. The window $\varepsilon = 1$ is fixed by Axiom P5 ($\tau_{\min} = 1$) and is therefore not tunable; small shifts $\varepsilon \rightarrow 1 + \delta$ produce order-unity changes in the integral ($I_{\text{mod}} \approx 7.32$ and $dI_{\text{mod}}/d\varepsilon \approx -8.70$ at $\varepsilon = 1$), confirming that the result is strictly regulator-scheme independent rather than fitted. With the operational window fixed, the static response is entirely determined by the dimensionless spectral sum.

Prediction of the Fine-Structure Constant under Spectral Selection

$$\alpha^{-1}(0) = 4\pi^3 + \pi^2 + \pi - \frac{1}{32\pi^4} + \frac{1}{64\pi^6} \approx 137.035\,999\,216$$

(Measured: 137.035 999 206(11) [63]; Relative Deviation $\approx 7 \times 10^{-11}$ ($< 1\sigma$)).

Observation and Falsifiability

The two most precise measurements of α ([63,78]) currently disagree by $> 5\sigma$. Our approach discriminates between them:

LKB 2020 (Rb) [63]: 137.035 999 206(11) $\rightarrow$ Agreement ($< 1\sigma$)

Berkeley 2018 (Cs) [78]: 137.035 999 046(27) $\rightarrow$ Disfavored ($> 5\sigma$)

Our approach supports the LKB 2020 result [63], also acting as a falsifier.

The accuracy of this result comes from rigidity, not tuning. The boundary architecture fixes the subsystem, the clock and the transport rule, so the coupling is read globally as a coarse-grained vacuum stiffness rather than a perturbative loop series. The estimate is highly constrained; changes to the inventory or topology would shift the result at $\mathcal{O}(1)$ in the spectral coefficients.

Appendix C.5. Summary of Predictions

Inputs: Resolution scale M_s .

Outputs: UV relation $\alpha^{-1}(M_s) = 4\pi k$; strong $\alpha_s^{-1} = 12\pi$ ($k_s = 3$); EM $\alpha_{\text{em}}^{-1} = 36\pi$ ($k_{\text{em}} = 9$); Integer lock $\alpha_{\text{em}}^{-1}/\alpha_s^{-1} = 3$; IR static prediction $\alpha^{-1}(0) \approx 137.035999216$ (spectral sum).

Falsifiers: Any new charged sector below M_s (breaks integer lock); confirmation of Berkeley (Cs) measurement [78] over LKB (Rb) [63].

Appendix D. Lepton Mass Spectrum via Spectral Filtration

This appendix derives the charged-lepton mass spectrum (m_e, m_μ, m_τ) from the finite-resolution boundary architecture (Axioms P1–P7), with finite capacity (Axiom P5) providing the central cutoff, and the linear-response framework (Hessian of relative entropy) established in Chapter 2.

The translation between nested causal diamonds acts as an open modular update (GKSL semigroup) (Axioms P3–P5) [53,54]. Because this open time evolution selects a fixed-point pointer algebra that becomes increasingly commutative as coherence decreases, the surviving accessible data for compact non-abelian symmetries is organized by the canonical Lie hierarchy $G \rightarrow \mathfrak{g} \rightarrow T$ (unique up to conjugacy). Imposing a finite capacity implies that accessibility is a physical resource rather than a formal limit [13,16].

Appendix D.1. Dressed Hadronic Anchor m_p

The observable unit of mass for stable matter is the proton (m_p). We require the boundary dynamics to naturally generate this hadronic scale directly through confinement physics [74].

The confinement scale Λ_{QCD} is generated by running the strong coupling from M_s down to the pole, driven by the coupling $\alpha_s^{-1}(M_s) = 12\pi$ (Appendix C) [28]. Using a 1-loop beta function ($\langle b_0 \rangle \approx 7.25$ for effective running):

$$\Lambda_{\text{QCD}} \approx M_s \cdot \exp \left[-\frac{2\pi(12\pi)}{\langle b_0 \rangle} \right] \approx 200 \text{ MeV}$$

Modeling the proton geometrically as the ground state of a spherical confinement cavity of radius $R \sim \Lambda_{\text{QCD}}^{-1}$, the lowest semiclassical orbital mode for a confined massless fermion strictly yields the geometric eigenvalue $x_{1,-1} \approx 3\pi/2$ [79]. Thus, the cavity fundamental sets the mass scale:

$$m_p^{\text{mech}} \approx \frac{3\pi}{2} \Lambda_{\text{QCD}} \approx 942 \text{ MeV}$$

(PDG value: 938 MeV [60]; Relative Deviation $\sim 0.4\%$)

While this confirms the boundary naturally generates the hadronic scale, QCD running and cavity models introduce standard phenomenological uncertainties. To protect the topological precision of the lepton derivations, we use the empirical m_p as geometric anchor.

Appendix D.2. Mass as Spectral Susceptibility

In [Chapter 2](#), we defined local dynamics via the Hessian of relative entropy and showed that, in the linear-response limit (Kubo-Mori formalism [14]), the static susceptibility χ of the vacuum is proportional to the integrated spectral density [80] of the Laplacian Δ_{int} [37,52]:

$$\chi \sim \sum_n \frac{|\langle 0 | \mathcal{O} | n \rangle|^2}{E_n} \implies m \propto \chi^{-1} \propto \frac{m_{\text{ref}}}{\Omega}.$$

The finite-resolution boundary and its open modular updates across nested diamonds ([Chapter 2](#)) induce a symmetry-respecting CPTP coarse-graining semigroup $\mathcal{E}_t$ [54] that drives the observable algebra toward a stable fixed-point pointer structure, $\mathfrak{A}_{\text{eff}} = \text{Fix}(\mathcal{E}_t)$. By the Canonical Lie Filtration corollary, this decoherence reduces any compact non-abelian symmetry into exactly three irreducible layers: the global group G , its local tangent algebra $\mathfrak{g}$, and the maximal abelian Cartan torus T . The surviving phase volume in each layer defines the accessibility Ω (the number of resolvable modes via Weyl's law [80]). Because mass emerges as the inverse spectral susceptibility ($m \propto m_{\text{ref}}/\Omega$), mass and coherence time are strictly co-determined: as coarse-graining irreversibly strips away non-commuting information, the accessible phase volume collapses, simultaneously shortening the lifetime and driving up the mass.

The geometric anchor m_{ref} tracks the surviving algebra. Global and tangent regimes retain non-abelian dressing, fixing the anchor at the hadronic scale ($m_{\text{ref}} = m_p$). The commuting T regime obstructs this dressing, reducing the anchor to the bare pixel energy ($m_{\text{ref}} = E_{\text{pix}}$). This baseline applies strictly to leptons, as quarks lack asymptotic states and require hadronic spectroscopy.

Appendix D.3. Lepton Cascade

The filtration acts on the spinorial sector required by discrete isotropic transport (P7). The geometric correspondence from physical boundary to internal topology naturally generates the target manifold:

$$\text{3D Space (P7)} \xrightarrow{\text{Rotation}} SO(3) \xrightarrow{\text{Quantum}} SU(2) \xrightarrow{\text{Topology}} S^3$$

Because $S^3 \cong SU(2)$ (the group of unit quaternions), this manifold naturally carries the Lie structure required by the isotropic transport (P7), making the filtration $G \rightarrow \mathfrak{g} \rightarrow T$ rigorously applicable. For this $SU(2)$ spinorial sector, this geometric reduction admits no nontrivial intermediate stages: its unique local linearization is $\mathfrak{g}$ and its maximal connected abelian subgroup is $T \cong U(1)$. This topological rigidity mathematically restricts the lepton cascade to exactly three generations.

Furthermore, the full gauge inventory fixed in [Appendix C](#) demands a transitive action for the $SU(3)$ color group; the axioms therefore force the internal manifold to be $\mathcal{M}_{\text{int}} \cong S^3 \times S^5$.

We define the effective spectral volume by $\Omega = \text{Tr}(\Pi_{\text{axes}} \otimes \Pi_{\text{int}})$, where the Transport-Axis Redundancy corollary supplies the spatial trace factor 3. The exact same accessibility observable Ω

underlies all three generations; under the CPTP coarse-graining limits it cleanly reduces to a full phase-volume count (global limit), a local generator count (tangent limit) and a Cartan phase-volume count (commuting limit). The three regimes therefore yield rigid integer accessibility counts.

Generation 1: Electron (stable global G regime; resolves full global symmetry G)

In the stable limit (full non-abelian content remains coherent), the leading Weyl term depends strictly on the total geometric phase volume [80], removing any freedom to reshuffle factors at leading order. The accessibility Ω_e therefore exactly equals the volume of the gauge manifold $S^3 \times S^5$ times the three transport axes (P7 and Transport-Axis Redundancy corollary): $\Omega_e = 3 \times 2\pi^5 = 6\pi^5$. Thus:

$$m_e = \frac{m_p}{6\pi^5} \approx 0.511009 \text{ MeV}$$

(PDG: 0.510999 MeV; relative deviation $\approx 2 \times 10^{-5}$).

Generation 2: Muon (transient tangent $\mathfrak{g}$ regime; resolves local algebra $\mathfrak{g}$)

In the transient limit, only local generator data survive. Operationally, this transient regime resolves local generator directions but does not survive long enough to resolve global topological identifications. The tangent algebra $\mathfrak{g} = \mathfrak{su}(2)$ has dimension 3; multiplied by the three transport axes, it yields $\Omega_\mu = 9$ [81]. Thus:

$$\Omega_\mu = 3 \times \dim(\mathfrak{su}(2)) = 9, \quad m_\mu = \frac{m_p}{9} \approx 104.25 \text{ MeV}$$

(PDG: 105.66 MeV [60]; Relative Deviation $\approx 1.3\%$; Tangent proxy is expected to carry a small systematic offset).

Generation 3: Tau (prompt commuting T regime / Cartan torus; fixed-point algebra)

In the prompt limit, the algebra abelianizes to the maximal Cartan torus T . The Transport-Axis Redundancy corollary fixes the 8-fold labeling redundancy $|\Gamma| = 8$ on the commuting phases (corresponding to the $\mathbb{Z}_2^3$ axis-sign redundancies; axis permutations correspond to valid physical rotations and are therefore not quotiented out). The excitation is pixel-bound, so the anchor is the per-pixel energy E_{pix} (Chapter 1). The phase volume of the torus divided by $|\Gamma|$ gives $\Omega_\tau = (2\pi)^4/8 = 2\pi^4$. Thus:

$$\Omega_\tau = \frac{\text{Vol}(T^4)}{|\Gamma|} = \frac{(2\pi)^4}{8} = 2\pi^4, \quad m_\tau = \frac{E_{\text{pix}}}{2\pi^4} \approx 1.788 \text{ GeV}$$

(PDG value: 1.777 GeV [60]; Relative Deviation $\sim 0.6\%$)

Appendix D.4. Summary of Predictions

Inputs: Resolution scale M_s ; Per-pixel energy budget E_{pix} ; m_p (derived from M_s via confinement, then used as geometric anchor only).

Outputs: $m_e; m_\mu; m_\tau$.

Falsifiers: Failure of geometric integer accessibility ratios ($6\pi^5, 9, 2\pi^4$) fixed by canonical Lie filtration and Transport-Axis Redundancy corollary; Discovery of a 4th charged lepton generation.

All numerical predictions follow directly from the symmetry-respecting CPTP coarse-graining semigroup of Chapter 2 and the Canonical Lie Filtration of the boundary algebra.

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