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Posted Date: 18 September 2024

doi: 10.20944/preprints202409.1244.v1

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Article

The Influence of In-Plane Constraints on Fatigue Crack Growth Rate

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Abstract: The geometry of structural members can influence the fatigue crack growth rate. In the paper, the T-stress depending on the element's geometry is used to quantify the effect of geometry on the fatigue crack growth rate. A series of fatigue tests on three-point bending specimens with different initial crack lengths were conducted. As a result, the dependence of the crack growth rate on the T-stress value was obtained.

Keywords: fatigue crack growth; Paris' law; in-plane constraints; brittle fracture; Williams' solution

1. Introduction

In 1957 M.L. Williams [1] developed formulas for stress distribution in front of the crack. The solution has the form of a series of terms. Physical analysis lets to limit the number of elements in a series but still, there is a series of terms. Usually, only the first term, as a dominating, was used to describe the stress field in the neighborhood of the crack tip [2]. In his analysis, Williams assumed that the body is infinite. In reality, a body has a finite size and shape that influence the fracture toughness. This was taken into account in the standard [3] by strict requirements for the shape and dimensions of a specimen for the test to be valid.

In 1973 Larsson and Carlsson [4] showed that the size and shape of the plastic zone ahead of the crack tip depends on the in-plane geometry of the specimen. The influence of the specimen geometry can be considered by the value of the T-stress which is the second term of Williams' expansion.

In the case of elastic-plastic materials, the stress field is explained by the Hutchinson, Rice, and Rosengren equations [5], [6]. They describe so called HRR field. In this case, there is also the quantity that is equivalent to the T-stress. The full solution for HRR field was given by Yang S., Chao Y.J. and Sutton M.A. [7], but the Shih's and O'Dowd's approach turned out to be more popular, mainly due to its simplicity [8], [9], [10], [11]. In this case, the equivalent of the T-stress is the Q-stress, which is the sum of all higher-order elements of the asymptotic expansion. In [9] the simple relationship between fracture toughness and Q-stress value is given.

The influence of in-plane constraints on the fatigue process in the crack initiation phase and in the ultra-low-cycle fatigue range was shown in [12] and [13]. These works showed that the Q/T-stress level affects not only the fatigue crack growth rate and the number of cycles to crack initiation but also the crack initiation site.

There are many laws of fatigue crack growth, but the most popular and comprehensive is Paris' law [14] which can be written as:

$$\frac{da}{dN} C (\Delta K_{eff})^m \quad (1)$$

where a is the crack growth in millimeters per cycle, N is the number of cycles, C and m are material constants,

$$\Delta K_{eff} = \left(\frac{1}{1-R} - \frac{\Delta K_{op}}{\Delta K} \right) \Delta K$$

ΔK is a range of the stress intensity factor (SIF) changes, ΔK_{op} is the SIF value when crack starts to open, R is a stress ratio.

As it results from equation (1), the rate of fatigue crack growth depends primarily on the load, which affects the range of changes in the stress intensity factor, but also on the stress ratio, i.e. on the average load [15], [16], [17]. The last quantity is the threshold value of the stress intensity factor at which the crack opens. Unfortunately, the fatigue cracking process is too complicated to be described by formula (1) alone. Therefore, research at the current stage of scientific development tries to take into account other factors. One of the elements that are difficult to describe mathematically is the effect of the environment. In the work [18], the effect of humidity and temperature was studied simultaneously. This effect was included by describing the changes in the parameters C and m in Paris's law.

Much more difficult to control is the effect of aggressive environment [19]. In such case, the simplest tool to show the effect of the selected factor is the Paris equation plot.

It is equally difficult to describe the effect of the material microstructure on the crack growth rate. For example, in [20] the effect of microstructure on the crack initiation site and the fatigue crack growth path was analyzed in welded joints. In [21] many aspects of fatigue crack growth in railway rails were studied, and one of the elements taken into account was the grain size. It was found that a finer microstructure requires a greater number of cycles to failure and thus reduces the crack growth rate. In [22] the effect of the inhomogeneity of the structure (clusters of small grains near large grains) on the fatigue crack initiation site was studied, as well as how the structure affects the crack growth in its initial phase.

The influence of individual parameters can be enhanced by their mutual influence [23], which is why artificial intelligence is increasingly used to assess the influence of selected parameters on the fatigue process [24], [25].

The influence of parameters that is much easier to describe mathematically is the influence of material properties on fatigue behavior [26]. The influence of the geometry of the tested element seems to be much more important and less intuitive to predict. The importance of the influence of geometry results from the fact that the shapes of elements used in industry differ significantly from the specimens used in laboratories. The mathematical formulation of the influence of geometry will allow for direct transfer of results from the laboratory to engineering practice. Fatigue cracks in structural elements often behave in a way that is difficult to predict. The growth of the crack may differ significantly from the model described by Paris's law, calculated in the laboratory using standard specimens. The main reason for this state of affairs is the geometry of the structural element, because the material used in the tests is usually the same as the one from which the structural element is made. The influence of geometry is divided into two groups due to the constraints that geometry imposes on the development of plastic zones. The influence of thickness on the growth rate of fatigue cracks was tested in [27], and [28]. Thickness creates so-called out-of-plane constraints. The shape and dimensions of the element in turn, constitute the second group of constraints, i.e. in-plane constraints.

The influence of the shape and dimensions of the elements on fatigue behavior are taken into account in different ways. The most popular approach uses the stress concentration factor [29]. The problem in this case is that different formulas are used for each geometry. That is why in [30] the normalized stress gradient is used.

In view of the findings of Larsson and Carlsson [4] combined with the results of O'Dowd [9] in the field of monotonic loading, it seems quite obvious to try to use a two-term approach to describe fatigue crack growth. An additional advantage of such an approach would be a uniform approach to the influence of geometry. Regardless of the shape of the element, the influence of geometry would be described by the same equations.

This paper presents preliminary studies of fatigue crack growth rates for different levels of in-plane constraints.

2. Materials and Methods

The tests were carried out on specimens made of C45 steel (1.0503/1045). The values of material constants achieved in the tensile tests are presented in Tab. 1 and Fig.1a.

Table 1. Material parameters.

Feature	Value
Young's modulus [GPa]	182.9
Yield strength [MPa]	711.45
Tensile strength [MPa]	771.61
Elongation [%]	16
Reduction in area [%]	64.54

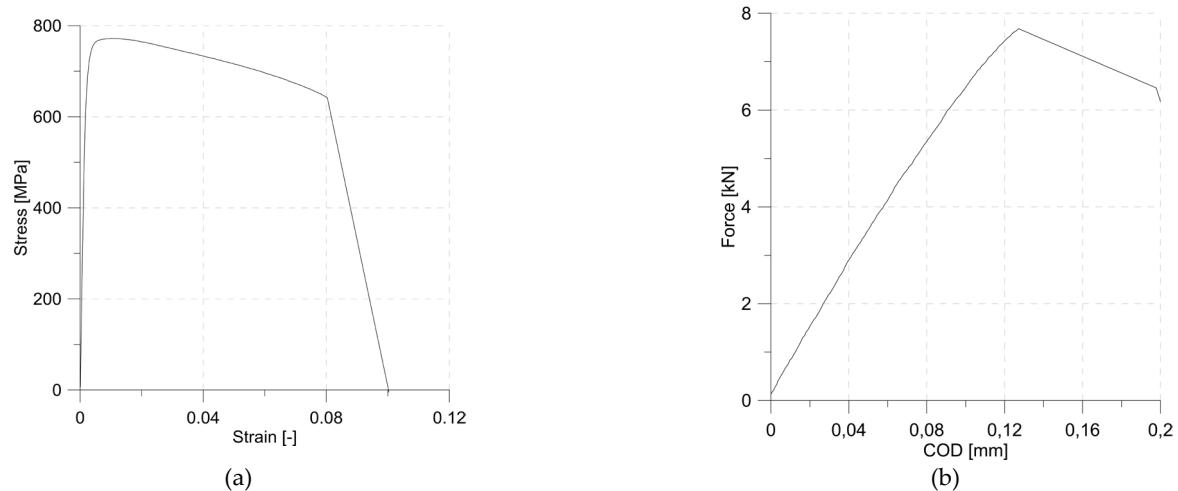


Figure 1. Examples of material test results: stress-strain curve (a), crack opening displacement-force curve (b).

The fatigue load amplitude was made dependent on the critical value of the SIF. To determine it, three three-point bending specimens are prepared with a thickness of $t=10$ mm and other dimensions proportional to this thickness in accordance with the standard [3]. As a result of the experiments, an average value of $K_{IC}=53,5\text{MPa}\sqrt{\text{m}}$ was obtained.

The next step was to make a set of three-point bending specimens with a thickness of $t=10$ mm and other dimensions in accordance with the standard [3] except for the crack length. Three different crack lengths are introduced to the specimens to obtain the relative crack length $a/W=0.25, 0.5, 0.75$, where a is the crack length and W is the specimen width.

The specimens were subjected to fatigue loading with repeating cycles of two maximum values: $0.77K_{IC}$ and $0.5K_{IC}$, while minimum value was always the same and equal 0.1kN . Zero force value was not used due to the possibility of loss of contact between the actuator and the specimen. The force values corresponding to the assumed maximum SIF levels were determined based on the formula (3) [31]:

$$K_I = \frac{3SP}{2tW^2} \sqrt{\pi a} F_I(\alpha) \quad (3)$$

$$\text{where } \alpha=a/W, \text{ and } F_I(\alpha) = \frac{1.99 - \alpha(1-\alpha)(2.15 - 3.93\alpha + 2.7\alpha^2)}{(1+2\alpha)(1-\alpha)^{3/2}}, \text{ and other undefined}$$

parameters are presented in the Fig. 2.

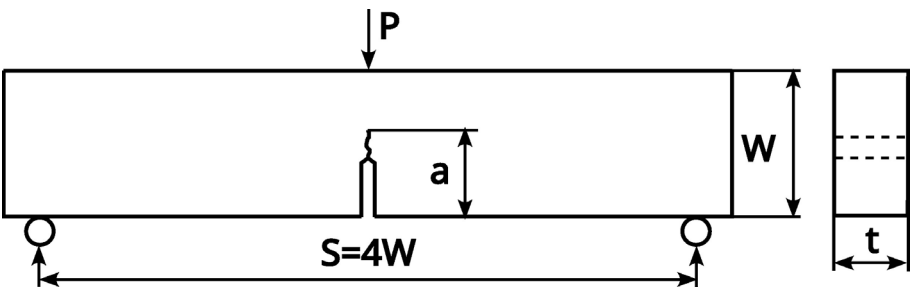


Figure 2. Geometry of the tested specimens.

The number of cycles at which the crack growth reached 1 mm and 2 mm was recorded. Crack growth measurements were made optically on the surface of the specimens.
The maximum forces used to load the samples are shown in Tab. 2.

Table 2. Applied loads [N].

Max. SIF level	a/W=0.25	a/W=0.5	a/W=0.75
0.5 KIC	3985	2004	692
0.77 KIC	6161	3098	1070

To determine the T-stress values, numerical calculations performed in the ABAQUS/CAE 2017 program were used. The specimens were modeled in plane strain using 4-node bilinear elements (CPE4R). Taking advantage of the problem symmetry, only half of the specimen is modeled. In each specimen, the zone in the vicinity of the crack is modeled as a rectangle composed of two squares filled with elements of size 0.003 mm x 0.003 mm (Fig. 3). The size of the elements near the crack tip guarantee stable and accurate results. The model is composed of over 230 thousand elements. The exact number depends on the crack length. For a/W= 0.25 and a/W=0.75, it was 233 thousand elements; for a/W=0.5, it was 236 thousand elements. The loading roller and support roller were modeled as rigid bodies. The displacement was applied to the appropriate roller. Its value is sufficient to obtain the required K-values. The T-stress values was determined automatically by the ABAQUS program.

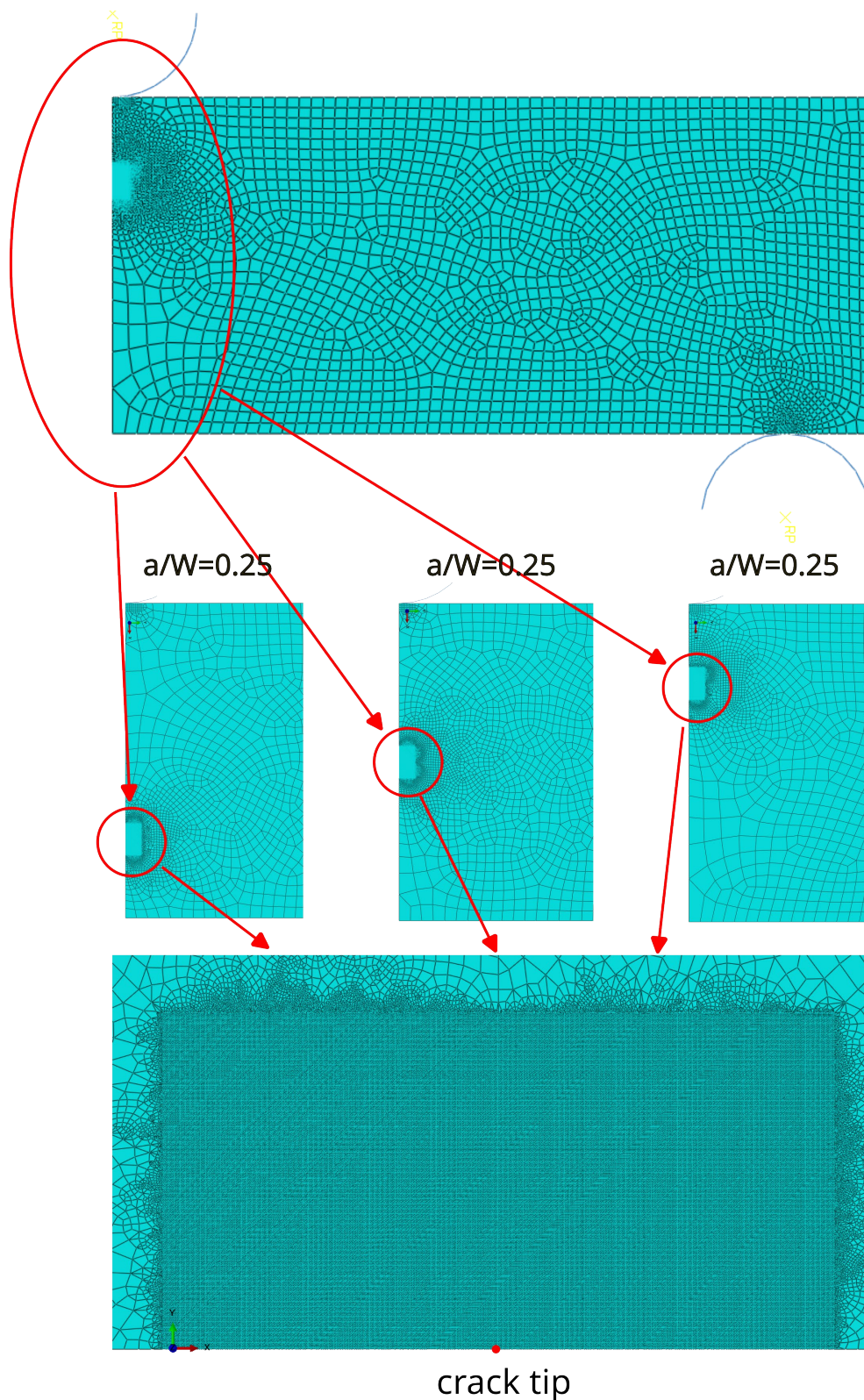


Figure 3. Details of numerical model: general view of the specimen (top row), area of the plane of symmetry (middle row) and crack tip zone (bottom row).

3. Results

During the tests, the number of cycles was recorded after reaching an increase of 1 mm and 2 mm. The measurement results were an average of three samples. In the case of the longest cracks, i.e. $a/W=0.75$, it was not possible to determine the average. In the case of the lower load level, no increase

was observed or it was very small, therefore the tests were stopped after exceeding 1,000,000 cycles, but at a higher load level, one of the specimens broke suddenly before reaching an increase of 2 mm.

Tables 3 and 4 show the average results of measurements of the number of cycles required to achieve the assumed increase.

Table 3. Number of cycles to achieve 1 mm crack growth.

Max. SIF level	a/W=0.25	a/W=0.5	a/W=0.75
0.5 KIC	63793	113274,5	>1000000
0.77 KIC	25855	29477,5	90706

Table 4. Number of cycles to achieve 2 mm crack growth.

Max. SIF level	a/W=0.25	a/W=0.5	a/W=0.75
0.5 KIC	117969	185015	>1000000
0.77 KIC	43713	49085	not conclusive

The T-stress values at the beginning of loading for each load case determined by numerical calculations are presented in Table 5.

Table 5. T-stress values [MPa].

Max. SIF level	a/W=0.25	a/W=0.5	a/W=0.75
0.5 KIC	-38.65	19.09	67.55
0.77 KIC	-59.75	29.49	104.43

4. Discussion

In all cases, the initial load had the same parameters. Both the maximum load value and the stress ratio are identical, nevertheless, the number of cycles to achieve the intended increments is clearly different (Fig. 4).

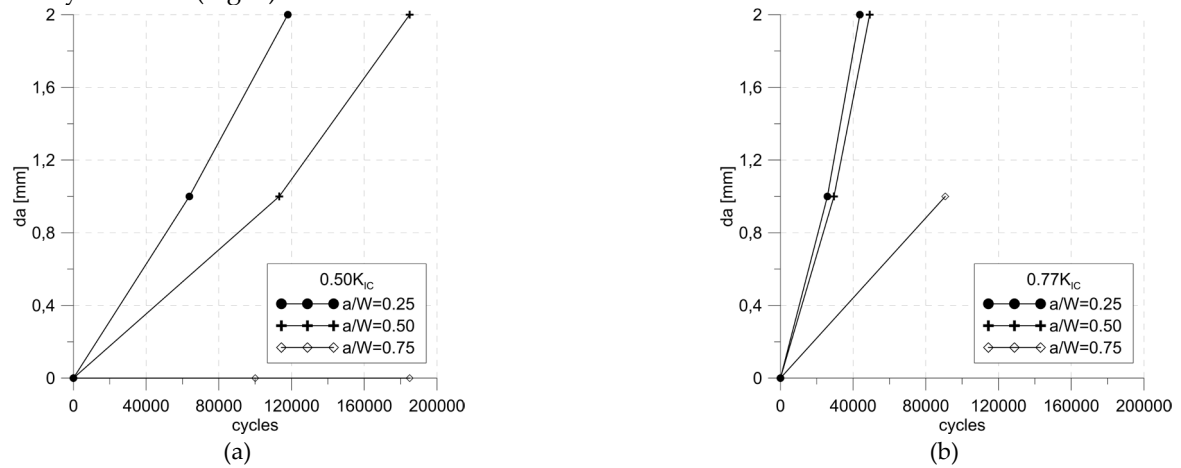


Figure 4. Graphical presentation of the results from Tables 3 and 4.

With the crack length increase at constant force, the maximum value of the stress intensity factor (3) increases, and thus the range of its changes. The maximum values of SIF for assumed increments are shown in Tables 6 and 7.

Table 6. SIF values for crack length increased by 1 mm [MPa√m].

a [mm]	F(0.5K _{IC})	F(0.77K _{IC})
6	30,4	46,0
11	31,6	48,8

16	37,5	58,0
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Table 7. SIF values for crack length increased by 2 mm [MPa√m].

a [mm]	F(0.5K _{IC})	F(0.77K _{IC})
7	34,6	53,5
12	37,9	58,6
17	57,9	89,5

As can be seen from Tables 6 and 7, the SIF value as a result of crack growth increases faster for longer cracks, but the crack growth rate behaves oppositely. The difference in the obtained number of cycles between the specimens increases. For an increase of 2 mm in the case of a higher load, the critical SIF value was reached for all lengths, and despite this, the specimens did not fail rapidly except for the sample with $a/W = 0.75$. However, in this case, the SIF exceeded the critical value by 67%. This geometry should have already failed for an increase of 1 mm, corresponding to an increase in SIF to 58 MPa√m.

Comparing the obtained increases with the stress values T, a close correlation can be seen. Specimens with a higher T-stress value were characterized by lower fatigue crack growth or its arrest, while low values promoted the acceleration of crack growth (Tab. 8 and 9).

Table 8. Crack growth rates in the range 0- 1 mm [mm/cycle].

Max. SIF level	a/W=0.25	a/W=0.5	a/W=0.75
0.5 KIC	1,56757E-05	8,82811E-06	arrested
0.77 KIC	3,86772E-05	3,39242E-05	1,10246E-05

Table 9. Crack growth rates in the range 1- 2 mm [mm/cycle].

Max. SIF level	a/W=0.25	a/W=0.5	a/W=0.75
0.5 KIC	1,84584E-05	1,39391E-05	arrested
0.77 KIC	5,59973E-05	5,10009E-05	not conclusive

This behavior can be explained by the influence of the level of in-plane constraints on the value of principal stresses. In the case of plane strain, the stress in the thickness direction are equal $\sigma_3 = (\sigma_1 + \sigma_2)\nu$, where ν is Poisson's ratio, σ_3 is the stress in thickness direction, and σ_1, σ_2 are the stresses in the other directions (i.e. in the specimen plane) and therefore the maximum shear stress depends on the stresses σ_1 and σ_2 . In the case considered, on the crack growth path, the stress level σ_2 depends on the SIF value while σ_1 is equal to the T-stress. As can be seen in Fig. 5 value of maximum shear stress depends on the T-stress. The lower is the T-stress value the higher is maximum shear stress.

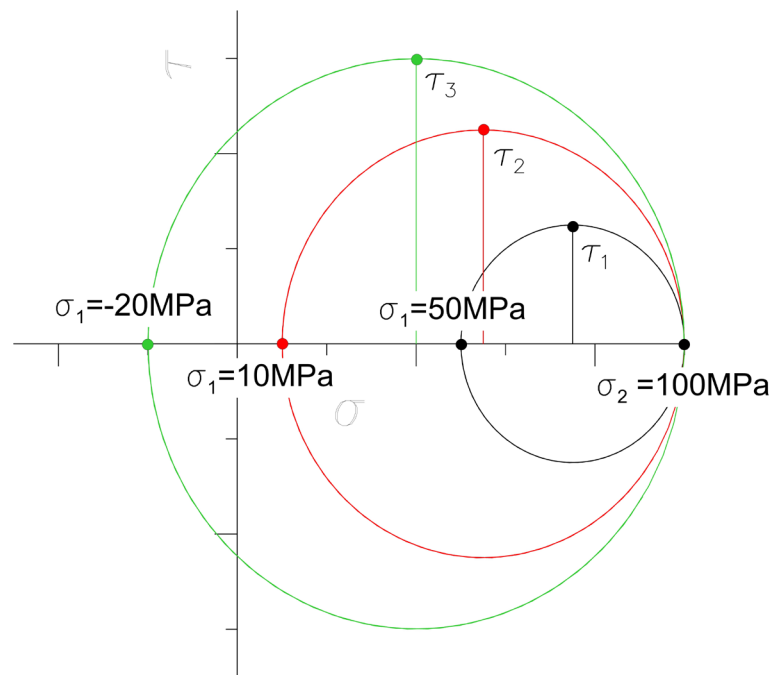


Figure 5. Mohr's circles for selected values of T-stress.

5. Conclusions

Tests were carried out in which specimens with different levels of the T-stress were loaded with repeating cycles of the same amplitude of SIF. Different crack growth rates were obtained for each geometry for two levels of loading. These rates are correlated with the level of in-plane constraints described by T-stress values. The lower the T-stress the greater the fatigue crack growth rate. The results indicate the need to introduce a quantity characterizing the in-plane constraints into the crack growth function as it has a strong influence on the behavior of the fatigue crack.

Author Contributions: Conceptualization, J.G. and U.J-G.; methodology, J.G.; software, J.G. and U.J-G.; validation, J.G., U.J-G. and S.L.; formal analysis, J.G. and U. J-G.; investigation, S.L. and J.G.; resources, J.G.; data curation, J.G.; writing—original draft preparation, J.G. and U. J-G.; writing—review and editing, J.G. and U.J-G.; visualization, J.G. and U.J-G.; supervision, J.G.; project administration, S.L.; funding acquisition, J.G. and U.J-G. All authors have read and agreed to the published version of the manuscript.

Funding: This research received no external funding.

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: Not applicable.

Acknowledgments: Not applicable.

Conflicts of Interest: The authors declare no conflicts of interest.

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