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Some Comments on a Time Interval Only Description

H. Hollestelle
Independent researcher, Common Room/Networks Foundation, Bandung 40191, Indonesia

Abstract
The time interval description is a natural way to introduce finite intervals, like finite time intervals. This approach depends on results for radiation propagation from star sources, where properties relate to a propagation surface, which is finite for every realistic event and measurement. In contrast the usual vector approach like for Newton's laws depends on introducing an infinite coordinate system. A time interval only approach necessarily has to start from scratch. Properties for time intervals have to be defined with time intervals. Where the first parts of this paper are devoted to time interval set properties, in the discussion part these are applied to quantities and measurements within astronomy. The introduction provides a survey of results.

Key words: time interval, measurement event, Noether charge, structure constants, differentiation properties, star source radiation propagation, gravitation

1. Introduction and survey of results

A finite time interval relates by itself to measurement, even while with approximately zero interference the situation remains, by approximation, 'free' and without interaction. A realistic description for radiation starting from a 'free' situation tends to be finite, when considering star sources and finite velocities. This is independent of time intervals being infinite, termed Newtonian, meaning Hamiltonian H time independent and noninteracting, and time intervals being finite meaning H time dependent. For infinite time intervals the results for the time interval description are the same as for the usual Newtonian vector description like in [1, Arnold]. Defined is time interval Δt to be the relevant event time interval, the reference interval for any measurement events.
Within a 'dialogue' approach, the Heisenberg view of qm is described among others in [2, Beller]: particle properties like positions or paths can only be accessed from sequences of interactions or measurements. With the Heisenberg view the uncertainty relations are the basis of qm [3, Roos]. According to Beller this is an 'operational' view, which rejects causal space-time description in contrast to several of more 'realistic' views which could emphasize the adequacy of unified concepts of qm (terms by Beller). This discussion is not part of this paper however. Measurements and finite time intervals are interrelated, infinite time intervals are not excluded by the time interval description.

Time interval description and t-quantities and Δt-quantities

In this paper within the time interval description, including time interval equilibrium, applied are results from earlier research [4, Hollestelle] and [5, Hollestelle], that include time interval differentiation, commutation and equilibrium relations, where all time intervals, when H time dependent, are finite and asymmetrical. In the time interval description, a quantity can be a t-quantity, variable with one moment time coordinate t during time interval Δt, or a 'neutral' Δt-quantity, variable with Δt, however invariant during Δt, for instance a time interval average. This means linearity in t resp. linearity in Δt due to the specific definition of time interval equilibrium, that is 'mean velocity theorem' (MVT) equilibrium, different from Lagrangian equilibrium when Hamiltonian H is time dependent. Correspondence for t-quantities with Δt-quantities is discussed in paragraph 2 to 5. This allows a time interval only description, where the one moment time coordinates our left out. Properties for the time interval set, like addition, multiplication and differentiation are discussed in the second part of this paper, paragraph 6 to 10. The third part of this paper includes a discussion of star source radiation and gravitation within a time interval only description among other results, paragraph 11 and 12.

Time intervals and the 'infinite regress' problem

The time interval description includes differentiation to time intervals that depend on time interval boundaries that are themselves one moment time quantities. To find a time interval only description, to relate these boundaries to time interval quantities without being time intervals, this is realized in paragraph 2 to 5. This should resolve the 'infinite regress' problem when defining the Hamiltonian being the 'time interval' version of the Legendre transform of the Lagrangian L. With time intervals instead of one moment time quantities this would mean, due to the derivative in the 'time interval' version, time interval boundaries equal time intervals causing the regress.

Star source radiation and gravitation

The third part of this paper concerns application of the time interval description to star source radiation, paragraph 11, and discussion, paragraph 12. The originating process of star source radiation is not investigated. Assumed is radiation is in 'free motion' without interaction and radiation energy is invariant per time interval due to star source properties with a certain origin mass. Even so, the time intervals related to radiation propagation are always finite, since the wave group-velocity is considered finite. The results for overall propagation energy can be described in terms of gravitational field energy for both zero mass em radiation waves and massive particle waves [4, Hollestelle] and [5, Hollestelle]. Application of time interval only allows an integrated description of these different phenomena, without other assumptions.
Newton's second law relates kinematics and source related forces, applying vectors and space time coordinates, within a one moment time description [6, Goldstein], [7, Newton I]. In this paper equilibrium is due to a multiple of energies rather than actions. This allows to discard the question of inertial forces. The equivalence principle in general relativity (GR) implies equivalence for gravitational energy and geometry, now a result is equivalence is implied for gravitational energy and radiation energy, ie kinetic propagation surface energy due to the em radiation field. This suggests similarly equivalence can be found for radiation energy and geometry. The subject of action 'at a distance' is unresolved within physics. In GR the concept of simultaneous events is proposed, applying a finite velocity of light. In this paper simultaneous radiation energy is the energy of radiation waves 'arriving' within Δt at the finite propagation surface.

A finite propagation surface obviously should be asymmetrical due to its time development and propagation away from the source. In terms of fields this means the surface tends to increase distance from the source due to its geometry and asymmetry, and such purely geometric fields have been studied before, [8, Hollestelle]. Curie's principle seems to be equivalent to Newton's second law, due to asymmetric propagation including an invariant wave group-velocity related development in time, [9, Curie]. Clearly radiation propagation is away from the source where gravitation as a source for movement is directed towards the mass sources.

The essential property for the time interval set is multiplication M [$\Delta t1$, $\Delta t1$] = $\Delta t1$ for any time interval $\Delta t1$, including relevant event time interval Δt. It means the multiplication inverse for Δt is Δt itself, paragraph 3, allowing energies to be equal in value even while time interval Δt and multiplication inverse Δti within the result can still be recovered. In paragraph 12, discussed is for time interval Δt, being finite and asymmetrical, the concepts 'close' or 'remote', meaning in terms of space intervals the mass sources being localizable or not, and in terms of time intervals, 'simultaneous' or 'not simultaneous', with regard to measurements.

The time interval only description for em radiation with zero mass wave particles has some simplicity as advantage and can naturally resume non zero mass wave particles. A time interval equilibrium result is, Δt-quantities are linear in Δt, and the time interval set is closed for multiplication, paragraph 9. Twice differentiation to time interval Δt implies scalar multiplication, similar to wave equations in qm, paragraph 10.

Not all results are linear results. Differentiation is nonlinear with respect to scalar multiplication, paragraph 8. Incident radiation energy can change, due to interaction like the photo-electric effect, depending on interaction event time interval Δt and is nonlinear in the variable #n, the number of wave particles, paragraph 11. Other results include derivation of Noether charges, and structure constants, for time interval equilibrium situations, in terms of commutation properties.

2. Commutation relations and differentiation to time intervals

By discussing commutation relations for one moment time t and differentiation to t, the for operators in field theory usual concept of: 'working to the right' is introduced for one moment time quantity t. A correspondence relating the one moment time set and the time interval set is defined, applying commutation relations and differentiation to time intervals including the 'working to the right' property for time intervals. Correspondence, then, prepares a way to approach the 'infinite regress' problem and to derive time interval only set properties.

For t-quantities, the usual description is one where these depend on one moment time t, and Lagrangian equilibrium. The Newtonian kinematic description is a one moment time description. With Newtonian situations however meant are in this paper H time independent situations, ie situations without external interaction.

The usual commutation is defined with [A1, A2]u = A1(t). A2(t) - A2(t). A1(t), indicated with subscript u, for two t-quantities A1 and A2 at the same one moment time t. With the multiplication . and the usual + and – indications, the following equations are assumed that together define commutation relations for one moment time t. It is assumed that one moment time t is not completely commutation free and multiplication with t is order dependent.

1 t. cn’ = cn. t

1/t. cn = cn’. 1/t

2 [t, cn]u = t. (cn – cn’)

[1/t, cn]u = - (cn – cn’). 1/t

The cn and cn’ are the only commutation quantities related to one moment time, time coordinate, t. From relations 1 it follows the cn and cn’ are neutral quantities.

The commutation equations relate the one moment time t to one moment time differentiation to t: d/dt, including differentiation of cn and cn’ and differentiation of multiplication A1. A2, meaning A1 ‘multiplication’ A2, writing d/dt [A1. A2] for t-quantities A1 and A2. Square brackets indicate d/dt ‘working to the right’ to the bracket. Differentiation d/dt is ‘working to the right’ on every part of a multiplication within the usual first order differential calculus, [1, Arnold], [6, Goldstein], [10, Newton II], with the following distributive, ie symmetrical second associative, property for differentiation:

3 d/dt [A1. A2] = d/dt [A1]. A2 + A1. d/dt [A2] = 2. A1. d/dt [A2] - [A1, d/dt]u. A2

Trivially however essentially in the second part of this equation differentiation d/dt is ‘working to the right’ on A1 or A2 only. The third part is decisive for the interpretation of equations 1 and 2. The differentiation d/dt ‘moving to the right’ leaving, releasing, at each step, and equation 3 means only one step, a quantity to the left, ie at the first step quantity A1, including multiplication with a scalar, that is scalar 2 and a remain, the last part with the minus sign. This is similar to one moment time t or 1/t ‘moving to the right’ leaving to the left cn’ or cn respectively in equations 1.

Within the time interval description, multiplication, addition and differentiation can be applied to t-quantities and to Δt-quantities. From equations 1 and 2 similar equations can be derived for the same cn and cn’ now regarded to be Δt-quantities instead of t-quantities. Assumed is that the described situation is ‘remote’ from H time independent: with time interval Δt = [tb, ta] asymmetrical and finite such that |tb| << |ta|. cn and cn’ are invariant during Δt and can be regarded Δt-quantities: within the time interval description including MVT equilibrium this means they can be described being linear, meaning linear with Δt, and are not described as t dependent, linear or otherwise.

When changing from t-quantity to Δt-quantity perspective, Δt-quantities are regarded part of the time interval set, since they are linear in Δt, and every Δt-quantity can be written equal to some Δt3 = a. Δt. For any Δt-quantity being a time interval, a is scalar number. This is discussed in paragraph 13 part IV and paragraph 8, comment 9, the closure theorem for the time interval set. When the considered Δt-quantity can be different from time intervals regarding dimension and argument a can be a scalar different from a number scalar. Part by part the one moment time parameter t is applied less in the following. The indications M and A are applied for multiplication and addition for time intervals. Equations 10, paragraph 5, are commutation relations for the time interval set and without reference to one moment time t and part of a complete and closed time interval only description.

Comment 1. Differentiation of quantity A1 to one moment time t within the one moment time description: d/dt [A1], reduces to D* [A1] = [1/t, A1]u = (1/t. A1(t) -1. A1(t). 1/t), the usual commutation, for differentiation to one moment time t within the time interval description, disregarding a factor 1/2, [5, Hollestelle]. MVT equilibrium means for any t-quantity A1 = A1(t) linearity in t and D* [A1] = +/- A1/t, the + or - depending on A1 increasing or decreasing with t and A1 positive or negative. It follows D* [A1] remains invariant during Δt. This is part of the t-quantity perspective. Addition A for time intervals is evaluated in paragraph 7, multiplication M for time intervals is evaluated in paragraph 8. The time interval description derivative D*||Δt to time interval Δt of quantity A1 is defined with the time interval commutator, D*||Δ [A1] = [1/t, A1]||Δt = A [1/tb. A1(tb), –1. A1(ta). 1/ta], for time interval Δt = [tb, ta], and relates directly to equations 1 and 2.

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3. Correspondence for one moment time t-quantities with time interval Δt-quantities

The time interval derivative to one moment time t for A1 positive and increasing, is: D* [A1] = A1. 1/t, comment 1. Due to MVT equilibrium D* [A1] remains invariant during Δt. D* [A1] can be regarded a t-quantity as well as a Δt-quantity and this allows for the definition of correspondence for t-quantities with Δt-quantities. Applying averages for Δt of A1 and one moment time t, leaving out subscript ||Δt: < A1. 1/t > = < A1 >/< t > it follows in terms of t-quantities only:

4 A1.1/t = < A1 >/< t >

This relation is valid for any t-quantity A1 and follows from MVT equilibrium and D*[A1]. The average for time interval Δt of parameter t: < t > = < t >||Δt depends on the (a-)symmetry of time interval Δt. The following properties for time interval Δt = [tb, ta] are applied: when H time dependent Δt has finite measure and is asymmetrical with |tb| << |ta|, while for H time independent Δt has infinite measure with |tb| = |ta| each infinite. For asymmetric Δt: parameter t average < t > = 1/2(ta + tb) = t0, with t0 the “multiplication unit” for one moment time parameters during Δt, meaning t0. t = t. t0 = t. Left out are factors 2. Similarly, there is time interval ΔU, the “multiplication unit” for the time interval set.

The average for Δt of A1 is a Δt-quantity and equals < A1 > = < A1 >||Δt = D* [A1]. t0, from equation 4. The derivative of A1 to one moment time t, D* [A1], is similarly a Δt-quantity. The Δt-quantity property of average < A1 >, linearity with Δt, and introducing scalar a and time interval multiplication M, means < A1 > = M [a, Δt] = Δt3. For one moment time t, itself a t-quantity however not a Δt-quantity, one defines correspondence of < t > with time interval ΔU: < t > ~ ΔU. In this way one can define correspondence, indicated with ~, for averages of one moment time quantities corresponding with time interval quantities. A1i indicates with subscript i the multiplication inverse for any A1. The left side of equation 5 is in terms of averages of t-quantities, the right side in terms of time intervals and Δt-quantities:

5 < A1 >/< t > ~ M [Δt3, ΔUi] = M [M [a, Δt], ΔUi]

Comment 2. Correspondence

For the H time dependent asymmetrical situation, averaging for Δt one moment time t, there is < t > = t0, the one moment time ‘multiplication unit’ and different from the ‘multiplication zero’, with t0. t = t. t0 = t, and t0 corresponds with Δt0 in this case being equal to time interval set ‘multiplication unit’ ΔU, and different from the time interval set ‘multiplication zero’ ΔU0: < t >||Δt ~ ΔU.
For the H time independent and symmetrical Δt situation with |tb| = |ta| there is < t > = t0 equals t + -1.t = t0, with t0 in this case both the ‘multiplication zero’ and the ‘addition zero’, corresponding with Δt0 in this case being equal to ΔU0 both the ‘multiplication zero’ and ‘addition zero’ for the time interval set. In this case < t > corresponds with Δt0, being equal to ΔU0, and not equal to ΔU, because of the symmetry of Δt: < t >||Δt ~ ΔU0.
The one moment time t average < t >||Δt = t0 corresponds with the specific time interval termed Δt0, meaning correspondence of one moment time t0 with time interval set ‘multiplication unit’ or ‘multiplication zero’ due to H being time dependent or not.

The corresponding relations for the ‘addition zero’ Δt0 can be applied to describe a transition from non-zero mass to zero mass radiation. The difference in one moment time t average being < t >||Δt ~ Δt0 equal to “multiplication one” or “multiplication zero” for quantities and properties while remaining within physics is interesting in its own.

Comment 3. The defining property for ΔU is: M [ΔU, Δt1] = M [Δt1, ΔU] = Δt1, and for the multiplication inverse with subscript i of Δt1, Δt1i: M [Δt1, Δt1i] = M [Δt1i, Δt1] = ΔU, for any time interval Δt1. One can argue, not regarding uniqueness, ΔU can be identified with Δt, with Δt the relevant event time interval, from M [ΔU, Δt] = Δt while also M [Δt, Δt] = Δt, a relation derived independently for all Δt1 from interpretation in paragraph 7. It also follows Δt0 = Δt is a solution for Δt0. The discussion in paragraph 7 implies some uniqueness properties for Δt.
Comment 4. Where M indicates multiplication, A indicates addition and one defines A [Δt1, Δt0] = A [Δt0, Δt1] = Δt1 and A [Δt1, Δt1iv] = A [Δt1iv, Δt1] = Δt0, for Δt0 ‘addition zero’ for the time interval set and Δt1iv addition inverse with subscript iv for any time interval Δt1. Multiplication with scalar -1 of any time interval Δt1: -1. Δt1, is not necessarily equal to the addition inverse Δt1iv for Δt1, and is not a well-defined time interval when Δt1 is a time interval and Δt asymmetric due to H being time dependent.

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4. Pairs of commutation quantities and pairs of Noether charges

The commutation quantities cn and cn' are t-quantities, invariant during Δt, ie having constant values within their domain (cn-domain or cn'-domain) within Δt, and there is cn(t) = cn and cn'(t) = cn' invariant with t during Δt, where dependence on Δt is described by introducing the positive average scalar density D(cn, Δt). For the Δt average for cn, < cn(t) >||Δt, there is: < cn(t) >||Δt = ∫||Δt dt [cn]. 1/|Δt| = N. D(cn, Δt) with D(cn, Δt) = |cn-domain|/|Δt-domain| a Δt invariant scalar quantity. Applied is the time interval integral to parameter t: I* [2] = ∫||Δt dt [2] = 2. |Δt|, the domain for scalar 2 being the Δt-domain Δt. The t-quantity cn = < cn(t) >||Δt is equal to N only when the cn-domain during Δt has domain measure equal to the Δt-domain measure itself: in this situation there is D(cn, Δt) = |Δt|/|Δt| = 1. It is assumed that this is not always the case due to an equilibrium requirement for cn and cn'.

Similar to D(cn, Δt) defined is D(cn', Δt) for cn' and both densities depend on Δt and on the quantities cn and cn' resp. The equilibrium requirement implies D(cn, Δt) + D(cn', Δt) = 1, both for the same Δt. For two specific time intervals Δt and Δt', when the respective domains equal upper limit 1, there is D(cn', Δt') = D(cn, Δt) = 1. With cn and cn' the same sign, and the addition distributive property valid for quantity N, in terms of neutral t-quantities only one finds:

6 cn + cn' = N. D(cn, Δt) + N. D(cn', Δt) = N

Rewritten with multiplication M and addition A for time intervals one finds the corresponding Δt-quantity perspective. Equation 6 describes a requirement for cn = cn(t) and cn' = cn'(t) where time interval Δt is the variable. Arguments for this requirement are discussed in comment 6, and paragraph 12, part VI. It exists due to cn and cn', being not ordinary constants, rather physical quantities defined within the time interval equilibrium description. In support for the requirement, equation 6 and 7, an argument below includes a theorem for time and space averages.

cn and cn' are 'reciprocal' quantities, a change of perspective to Δt- quantities cn(Δt) and cn'(Δt) and time intervals means the requirement implies A [(cn(Δt), cn'(Δt)] = ΔN, with correspondence < N > = N ~ ΔN, invariant with one moment time t and invariant with Δt. From < t > ~ Δt0 one finds: when < D(cn, Δt) >||Δt ~ ΔU and cn(Δt) = M [ΔN, ΔU] = ΔN, this implies < D(cn', Δt) >||Δt ~ ΔU0 and cn'(Δt) = M [ΔN, ΔU0] = ΔU0, and the other way around while invariant |ΔN| remains the upper limit value for |cn(Δt)| or |cn'(Δt)|. For invariants related to one moment time development referred is to [11, Noether], [12, De Wit, Smith]. The equilibrium requirement in Δt-quantity perspective while Δt remains the variable is:

7 A [cn(Δt), cn'(Δt)] = A [M [ΔN, ΔU], M [ΔN, ΔU0)]] = A [ΔN, ΔU0] = ΔN

Addition A [A1, A2], where A1 and A2 are Δt-quantities and correspond with t-quantity averages, in this paragraph preliminary applied for addition with ΔU and ΔU0, is defined in paragraph 6 and 7. Zero domain is not considered for cn(Δt) or cn'(Δt) since this means they are not time interval related rather one moment time t related. It can be argued ΔN in equation 7 is an approximation for the complete time interval set average < cn(Δt) >||set, equal to the cn(Δt) space average [1, Arnold], considering cn'(Δt) = cn(Δt') where Δt' is the specific time interval different from Δt while |cn(Δt)| and |cn'(Δt')| equal their upper limit value.

The requirement is supported by the second associative property, equation 3, due to the property for the Noether charge NC, D*||Δt [ΔN] = NC. Where from equation 7, ΔN relates to addition, ΔN relates to multiplication similarly. This is discussed in paragraph 12, part I and VI. The neutral t-quantities cn and cn' can be regarded t-Noether charges invariant for parameter t during Δt where the corresponding neutral Δt-quantities cn(Δt) and cn'(Δt) similarly can be regarded Δt-Noether charges. The usual Noether charges, NC, should relate to the relevant event time interval that is decisive for measurements.

<u>Comment 5.</u> *The usual Noether charges NC are the same as the t-Noether charges when limited to the, finite, time interval Δt, the relevant event time interval for measurement and also relevant for deciding on what are constants, ie through measurement. This implies the one moment time description for NC. Otherwise for time interval quantities, the NC are equal to the Δt-Noether charges, similarly limited to finite time interval Δt. The addition of a pair of reciprocal Δt-Noether charges, related through commutation relations for time intervals that correspond with the commutation relations for one moment time t, is invariant with variable Δt, meaning A [cn(Δt), cn'(Δt)] = ΔN is an invariant for the overall time interval set, where in fact cn(Δt) and cn'(Δt) both have the same limit value, and a difference exists only because domain difference depending on the variable, ie Δt. ΔN can be termed a ΔN-Noether charge. ΔN-Noether charges correspond with the usual Noether charges*

NC when unlimited infinite time intervals are considered, for instance non interacting situations with H time independent.

Comment 6. For any relevant event time interval Δt, cn(Δt) and cn'(Δt), being averages and time interval Δt-quantities, resemble for instance < T >||Δt and < V >||Δt, ie the kinetic energy and potential energy quantities being invariant during Δt because of averaging however variable with Δt. For kinetic energy T and potential energy V, both t-quantities and not necessarily invariant during Δt and within the usual one moment time description, Lagrangian equilibrium requires total energy T + V remains invariant during Δt just like < T >||Δt + < V >||Δt. These quantities are in principle independent, however, like for T and V, equilibrium means some relation applies.

5. Solution of the 'infinite regress' problem due to the definition of time interval boundaries

The usual qm description of em radiation, including propagation surface, waves and wave 'group' velocity and de Broglie complementarity for waves and wave particles, applies the one moment time description, and assumes parallel plane waves and a stationary state energy variance, ie parallel 'ray' propagation [13, Sakurai]. In contrast, star source radiation assumes sphere-like propagation.

Re-introduced are results for star source radiation from [4, Hollestelle] and [5, Hollestelle]. The reference space coordinate q, within both the one moment time and time interval description, indicates a coordinate place at the radiation propagation sphere surface such that coordinate origin qc is 'close' to q. It is meant q(ta) = q and q(tb) = qb, where qb indicates the star source coordinate place, for Δt = [tb, ta]. The reference space coordinate q implies a possible measurement place. The line of sight from the star source at place qb to place q at the propagation surface relates to space interval Δq. In this description place q and the 'origin' space coordinate place qc both are regarded to 'move along' with the radiation propagation surface while the star source remains at the invariant place qb in space, with distance |Δq| = |q – qb| increasing with propagation development.

The uncertainty relations continue to be part of qm if only because they relate qm subjects, like elementary particles, photons and electrons, to 'Newtonian' experimental subjects, like for measurements [13, Sakurai]. Assuming de Broglie complementarity derived are uncertainty relations within the time interval description, different from the usual uncertainty relations when the Hamiltonian is time dependent [5, Hollestelle].

Any usual Lorentz transformation TU should leave metric distances unchanged. In [5, Hollestelle] it is argued to introduce new Lorentz transformations TL that leave metric propagation surface measure unchanged since star source radiation energy remains unchanged during propagation while being proportional to the metric measure of the propagation surface.

For time interval Δt = [tb, ta] boundary tb relates by definition, not regarding causality, to boundary ta, with tb = tb[ta]:

8 $\quad 1/tb = - 1/ta\ (1 - (c.\ q(ta))^2)$

The constant c has dimension similar to the multiplication inverse of q, with (c. q) dimensionless. Defined in [4, Hollestelle] are the transformations A and B: A(t) with A(ta) = tb, and B(t) with B(ta) = -1. tb and its reverse B(-1) for which the Hamiltonian remains invariant. Transformation A defines any time interval Δt with [tb, ta] = [A(ta), ta]. A series of possibly many transformations B(-1) transforms Δt to another Δt', and includes a re-scaling of one moment time t, which takes care (c. q(ta))^2 << 1 for equation 8 and transformation A to remain valid for defining Δt with tb and ta resp. negative and positive sign.

Two different solutions exist: q(t) = +/- q0. 1/t0 t and q(t) = +/- q0. t0. 1/t. The factor (c. q0) for q(t) = q0 at t = t0 is an invariant, and equal to multiplication M [c, q0] within the time interval set, where both c and q0 remain invariant during Δt.

Comment 7. Space- and time coordinates q and t transform to q + qL and t + tL resp. when applying a specific Lorentz transformation TL, with qL and tL constant space- and time coordinate values. Transformation B(-1) can be interpreted as a Lorentz transformation TL where t to t' corresponds with Δt to Δt' with boundaries tb' and ta', where q' = q(ta') remains well-defined and 'close' to qc' = q(tc') like q = q(ta) 'close' to qc = q(tc). When |t'/t| = |(t + tL)/t| > 1 and with equation 8 in terms of transformation A, this means that for, the related to A, specific TL: t' = TL(t) is quadratic in t, [5, Hollestelle]. By applying this TL *one derives* for the commutation pair cn(Δt) and cn'(Δt) there is cn'(Δt) = cn(Δt'), starting from cn and cn', equation 1.

Since time interval Δt = [tb, ta] includes one moment time boundaries tb and ta, due to correspondence of t-quantities and Δt-quantities, from equation 8 and comment 7, it follows equations 9.

9a |ta|/|tb| = |cn'(Δt)|/|cn(Δt)|

With correspondence indication ~ the one moment time parameter t-quantities on the left side correspond with time interval Δt-quantities on the right side:

9b ta. tbi ~ M [cn'(Δt), cni(Δt)]

Comment 8. from MVT equilibrium follows the relation D*[q(t)] = +/- q(t)/t, still applying one moment time parameter t even within the time interval description like discussed in relation to equation 8 and comment 1, with two solutions, q(t) linear in t and q(t) linear in 1/t. Due to MVT equilibrium, D* [q] being neutral, independent of t during Δt, q(t) necessarily is linear in t and the linear in 1/t solution can exist only when it is the *same solution*, ie also linear in t.

To evaluate the meaning of the multiplication inverse ti of t-quantity one moment time t itself, started is from the one moment time 'multiplication unit' t0, paragraph 3. For the multiplication inverse for t: ti = 1/t, where ti however not necessarily belongs to the one moment time set, one finds the relation: ti. t = t. ti = t0, which leaves possibly many solutions ti. To give meaning to ti = 1/t, defined is: solution ti is to belong to the one moment time set. In this definition for one moment time t1 and t2, the multiplication t1. t2 = t3 belongs to the one moment time set in agreement with [4, Hollestelle].

For Δt boundaries ta and tb the relation with t0 is: ta + tb = 2t0 and ta. tb = 1/2 (ta^2 + tb^2) approaching to ta. tb = 1/2 ta^2 for |ta| >> |tb|, valid for situations with H time dependent, with Δt highly asymmetric. These one moment time tb and ta properties correspond with the relevant event time interval Δt property M [Δti, Δt] = M [Δt, Δt] = Δt = ΔU, comment 3. This correspondence is applied to derive equation 9.

The overall time interval set needs a new overall description for its Δt-quantity properties. The multiplication inverse Δti for Δt, considered linear in Δt, exists due to the relation M [Δt, Δt] = ΔU, at least a solution is Δti = Δt itself, comment 3. I* and D* within the time interval description refer to integration and differentiation to one moment time t and correspond with I*||Δt and D*||Δt for integration and differentiation to time interval Δt. All Δt-quantities are regarded linear in Δt and can be reduced to time intervals, this is discussed in paragraph 12, part IV. One derives for the Δt-quantity cn(Δt) in the time interval description:

10 I*||Δt [cn(Δt)] = M [Δt, A [cn(Δt), –1. cn'(Δt)]] = A [cn(Δt), – 1. cn'(Δt)]

D*||Δt [cn(Δt)] = M [A [cn'(Δt), –1. cn(Δt)], Δti] = A [cn'(Δt), – 1. cn(Δt)]

These equations are time interval only relations, and do not require one moment parameter t. They include multiplication with Δt resp. Δti and are the time interval equivalent of equations 1, ie they define commutation relations for time intervals and time intervals 'working to the right'. They include Δt and Δti and in this way MVT equilibrium for energies related to I*||Δt and D*||Δt is assured since the results for both in terms of time intervals is the same. In terms of one moment time quantities the results seem opposite since results are similar to those of the one moment time description when H time independent. However, for H time independent, ie a noninteracting situation, both cn and cn' are the same and cn + -1. cn' equals the 'addition zero' for the one moment time set corresponding with ΔU0. *With equations 9 and 10 the 'infinite regress' problem for the definition of time intervals is resolved.*

The meaning of I*||Δt and D*||Δt, integration and differentiation to Δt, starts from MVT equilibrium and the linear in Δt property just like I* and D* with MVT equilibrium and the linear in t property. Derivation of equation 10 includes the commutation versions, comment 1, for I*||Δt and D*||Δt, including one moment time dependence, application of equation 9 and correspondence of t0 with Δt0 = ΔU = Δt for I*||Δt and linearity in Δti for D*||Δt, explaining the occurrence of Δt and Δti within equation 10.

It is assumed the situation is "remote" from H time independent, ie |ta| >> |tb|, and thus |cn'(Δt)| >> |cn(Δt)| due to equation 9. For cn'(Δt) similar equations can be derived. The indication * is for operators, however applied for the above integration and differentiation by exception. From equation 10 one finds similarity for I*||Δt and D*||Δt in agreement with Δti = Δt and comment 8, except for the order of cn(Δt) and cn'(Δt). The order difference can be disregarded in this case since from the correspondence t0 with ΔU = ΔUi it follows I*||Δt [cn(Δt)] = D*||Δt [cn(Δt)], ie the result in terms of time intervals is in fact the same for both integration and

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differentiation, at least for Δt-quantities cn(Δt) and cn'(Δt). Since cn(Δt) is linear in Δt, and any Δt-quantity can be represented linear in cn(Δt), this supports the inference for I*||Δt and D*||Δt that results are exactly similar to equation 10 for all time intervals and Δt-quantities. This is discussed in paragraph 10.

6. Time interval only set properties

Commutation properties are usually defined within a one moment time description and depend on the vector like space time coordinates and quantities that can form combinations like addition or multiplication. In this way differences and change can be included in the one moment time description with differentiation to parameter t. In this paper the time interval description includes both t-quantities and Δt-quantities. However, for a description with time intervals only, specific properties for the time interval set like addition and multiplication are necessary, combinations for time intervals that before did not have meaning yet.

In [4, Hollestelle] two elementary parameters, termed elements, define time interval Δt = [tb, ta] and its boundaries tb an ta within a one-dimensional time concept. It is argued these two elements are such that they agree with the way time can be measured or counted and they are independent of, preliminary to, one moment time t. Any time interval is based on these elements where measurement is possibly counting towards ta in the future to count forward to from tb in the past. The definitions for time interval properties and for elements from [4, Hollestelle] remain valid in this paper. Any time interval includes a part of the past tb belongs to and a part of the future ta belongs to. With tb and ta the one moment time t and vector concept remain within the time interval description. In this paper a time interval only description is proposed, starting with the results from paragraph 5.

7. Addition and some fundamental properties

Within the time interval description defined is addition from the introduction of 'addition zero' time interval Δt0 such that addition of Δt0 with any other time interval Δt1 leaves Δt1 invariant. There is A [Δt1, Δt0] = Δt1. It is plausible, and discussed for Δt0 in comment 4, for any Δt1 there should be Δt1 'addition' Δt1 equals Δt1. When the time interval description regards events and properties within reality and regards these within any time interval, say Δt1, the 'addition zero' Δt0 for the time interval set at least can be relevant event time interval Δt itself. There is A [Δt1, Δt] = Δt1. This depends on the premise that there is only 'one' 'time' for the cosmological universe together and all is in the same 'time' and nothing is late or early. Time intervals do not exist outside themselves, they remain by themselves and don't give extra time from outside themselves to themselves:

11 Δt1 'addition' Δt1, to the same time interval, leaves Δt1 invariant: A [Δt1, Δt1] = Δt1

Δt1 is any time interval within the time interval set. This confirms the interpretation of time interval set addition with domain addition where addition of two identical domains results in the same domain. The time interval description differs in results from the one moment time description when the Hamiltonian H is time dependent [4, Hollestelle], and the relevant event time interval Δt is asymmetric and finite. Time interval Δt includes one moment time boundaries tb and ta: tb differs from –1. ta, while both tb and ta are finite with opposite sign and |tb| < |ta|.

The 'addition inverse' Δtiv for a finite relevant event time interval Δt equals Δt, i.e. A [Δtiv, Δt] equals A [Δt, Δt] = Δt. One finds Δt 'addition' -1. Δt does not equal Δt, where -1. Δt means with opposite signs for the boundaries. The -1. Δt is not the 'addition inverse' for Δt and is not well defined for finite time intervals: it does not belong to the time interval set. One might argue Δt 'addition' -1. Δt equals -1. Δt from the above definition where Δt0 equals Δt. Notice that Δt0 is not a zero-measure time interval in the sense of domain measure equal to zero. The order within addition seems to matter. Addition for any two time intervals Δt1 and Δt2 is indicated with A [Δt1, Δt2].

8. Multiplication and the multiplication closure theorem

Similar to addition A one can define multiplication M, with indication M [Δt1, Δt2] for any two time intervals Δt1 and Δt2, from the introduction of "multiplication unit" ΔU, while the relevant event time interval remains Δt. Like with addition it is not clear immediately what multiplication could mean for the time interval set. Several properties for M are now summarized.

Multiplication properties

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The result of multiplication is assumed to belong to the time interval set: M [Δt1, Δt2] = Δt3 and this implies closure of the time interval set for multiplication M. The resulting Δt3 is a time interval, confirmed by the time interval set "multiplication unit" ΔU relation M [ΔU, Δt1] = Δt1. This is supported by several of the following properties. The multiplication inverse Δt1i for any time interval Δt1 is a time interval itself, including the multiplication property: M [ΔU, Δt1i] = Δt1i. Due to comment 3 the multiplication inverse Δti for Δt equals Δt itself, Δt being the relevant event time interval. M [Δt1, Δt1i] = ΔU for any Δt1, including those Δt1 not equal to Δt, assuming there exists at least one multiplication inverse time interval Δt1i for each Δt1. With similar argument for validity of equation 11 for addition, multiplication M [Δt1, Δt1] = Δt1 is valid for Δt1 any time interval.

Comment 9. Associative properties. The symmetrical first associative, 'series', property for M: M [M [g1, g2], g3] = M [g1, M [g2, g3]] is assumed valid, the symmetrical second associative, 'parallel', property for M: M [g1, M [g2, g3]] = M [M [g1, g2], M [g1, g3]] is not necessarily valid, for gi any three time intervals. The first and second associative property to be similar in validity can be contradictory. The usual derivative of a multiplication applies the second associative property, equation 3. The derivative to one quantity applies commutations in terms of one moment time or time intervals, comment 1. The associative properties are defined for the time interval set, associative properties are known for ordinary sets, for instance for some specific algebras, [14, Jacobson].

For multiplication M the following definition is feasible, relation 12. The integral I*||Δt2 to time interval Δt2 is introduced, just like the usual integral I* to one moment time t, with the integral domain equal to the Δt2-domain:

Comment 10. Multiplication

12 $$M\,[\Delta t1, \Delta t2] = M\,[M\,[< \Delta t1 >||\Delta t2, \Delta t2], \Delta t2] = M\,[< \Delta t1 >||\Delta t, \Delta t2] = I^*||\Delta t2\,[\Delta t1]$$

This definition depends on the order of Δt1 and Δt2. Other relations that possibly are symmetrical in Δt1 and Δt2 are possible however not discussed in this paper. With time interval Δt1 invariant during Δt and Δt1 = < Δt1 >||Δt and Δt = Δt0, there is < Δt1 >||Δt = M [< Δt1 >||Δt, Δt] = M [< Δt1 >||Δt2, Δt2] and < Δt1 >||Δt2 = M [M [< Δt1 >||Δt2, Δt2], Δt2i] = M [M [< Δt1 >||Δt, Δt], M [Δt2i, Δt]] = M [Δt1, Δt2i]. This is the meaning of the second and third part of equation 12, including the average < Δt1 >||Δt2, and the integral to Δt2 by definition I*||Δt2 [Δt1] = M [< Δt1 >||Δt, M [Δt, Δt2i]] = < Δt1 >||Δt2. Applied is comment 3 and the symmetrical first associative property for M, comment 9.

With unchanged multiplication order for Δt1 and Δt2, and with Δt2 = Δt, no changes occur for equation 12 in principle and it immediately follows:

13 $$M\,[\Delta t1, \Delta t] = < \Delta t1 >||\Delta t = I^*||\Delta t\; dt\,[\Delta t1] = \Delta t1$$

The meaning and difference for averaging and integration to time interval Δt2 and to Δt, are clearly indicated from the above equations. When time interval Δt1 and Δt2 do not extend to outside Δt, ie the common domain for Δt1 or Δt2 with Δt equals Δt1 or Δt2, and with one moment time t being continuous within Δt these averages are evident. For Δt1 = ΔU, the 'multiplication unit' for the time interval set, one finds:

14 $$M\,[\Delta U, \Delta t] = I^*||\Delta t\,[\Delta U] = \Delta t$$

15 $$M\,[\Delta U, \Delta ti] = I^*||\Delta ti\,[\Delta U] = \Delta ti$$

The ΔU relations, M [ΔU, Δt] = M [Δt, ΔU] = Δt, comment 3, and M [ΔU, Δt] = ΔU, equation 13, confirm ΔU = Δt and Δti = Δt. From 'multiplication inverse' time interval Δti = Δt one finds independent support for equation 12 assuming the definition for time interval multiplication.

Closed multiplication. Closed multiplication implies M [Δt1, Δt2] = Δt3 is a time interval, and validity of Δt3 = a. Δta, scalar a itself dependent on time interval Δta, implies the solution time interval Δta. Not for all scalars a one finds solutions Δta, at least for a = 1 a solution exists and is Δta = Δt3.

Comment 11. Not all scalar arguments a can be allowed when M [Δt1, Δt2] = a. Δta belongs to the time interval set, where the future domain part measure exceeds the past domain part measure. When Δta is a time interval, -1. Δta is not when H time dependent and Δt asymmetric. For any time interval, equation 8 is the defining and

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necessary relation for one moment time boundaries tb and ta. The existence of Δta depends on Δt-quantities being linear with Δt due to MVT equilibrium, paragraph 12, part IV.

The multiplication result Δt3 can be rewritten with a time interval multiplication Δt3 = a. Δta = M [ca, Δta]. Due to the symmetrical first associative property: ca = M [ca, Δt] = M [ca, M [Δtai, Δta]] = M [M [ca, Δtai], Δta], ie a time interval average multiplication ca = M [< ca >||Δta, Δta] for any time interval ca. Time interval averages for time intervals, like in equation 12, are time intervals themselves. When applying the first associative property with several steps, one finds: M [ca, Δta] = M [M [< ca >||Δta, ai. M [ca, Δta]], Δta] is equal to M [< ca >||Δta, ai. M [ca, M [Δta, Δta]]] = M [< ca >||Δta, ai. M [ca, Δta]] = M [< ca >||Δta, Δta] = ca, and there is ca = Δt3 = a. Δta and Δta = Δt for any allowed scalar a, comment 11. This means a solution is ca = a. Δt and Δt3 = a. Δt and both are linear in Δt.

These solutions follow directly from a. Δta = M [ca, Δta] and property M [Δta, Δta] = Δta. Solutions for any Δta should exist since the time interval set is linear in Δt, ie 1-dimensional. Due to MVT equilibrium these solutions are found by starting from ca = e1. Δt and Δta = e2. Δt, with result Δt3 = e1. e2. Δt linear in Δt and a = e1. Solution Δta = e2. Δt is linear in Δt and includes any scalar e2, meaning in this way any time interval Δta can be a solution. The consideration that any time interval is neutral in Δt and linear in Δt, is discussed in paragraph 12, part IV. For scalar a = 1 solution Δta equals Δt3, and this means a time interval only multiplication exists equal to scalar multiplication at least for a = 1: M [a, Δta] = M [ca, Δta] with solution ca = ΔU = Δt. The scalar a is the resultant for any ca and Δta, however with reference to comment 11.

Comment 12. Closure theorem. The relation Δt3 = a. Δta = M [ca, Δta] depends on the argument scalar a having ‘positive’ sign. For argument a with a ‘negative’ sign the scalar multiplication does not follow the requirements of comment 11. From the first associative property for M with g1 equal to any scalar a one finds linearity with scalar a for multiplication M [Δt, Δta], with Δt3 = a. Δta = a. M [Δt, Δta] = M [a. Δt, Δta], equal to Δt3 = M [ca, Δta] with solution ca = a. Δt linear in Δt. From Δt3 = ca = a. Δt linear in time interval Δt, ie itself a time interval, it follows the closure theorem for any resultant scalar a, with reference to comment 11: *multiplication M is closed within the time interval set.*

Comment 13. The first associative property for differentiation seems valid if only due to the preserved order of the gi in series in comment 9. More precisely, this property is related to the following commutation properties for t-quantities NC(t) that are linear in parameter t due to MVT equilibrium. NC(t) is defined from the neutral t-quantities cn and cn’ with D* [NC(t)] = cn and it follows, equation 1,

16 t|1. NC(t)|2. cn’ = NC(t)|1. t|2

17 1/t|1. NC(t)|2 = NC(t)|1. 1/t|2. cn

The bar with indication |1 or |2 means the t-quantity at this place depends on parameter t with t = t1 or t = t2. Equations 16 and 17 are not definitions, rather they are derived from the defining relation D* [NC(t)] = cn and equation 1 for cn and cn’ from MVT equilibrium within the time interval description. One finds the order of |1 and |2 is preserved when reversing the order of parameter t and quantity NC(t) within the equations due to commutation. For the overall time interval set and within time interval Δt perspective, cn and cn’ correspond with the neutral Δt-quantities cn(Δt) and cn’(Δt), linear dependent on Δt, and confirmed is the differentiation first associative property at least for cn(Δt) and cn’(Δt).

9. Differentiation and the time interval differentiation ‘set rule’

Similar like for multiplication M, paragraph 8, one moment time t can be left out for addition A within the time interval set. Proposed is a definition for A [Δt1, Δt2] including multiplication M [Δt1, Δt2] and differentiation to time interval Δt2, D*||Δt2. In paragraph 9a and 9b this is applied to Δt2 = Δt, equation 21 and 22, where the asymmetrical second associative property is assumed for differentiation to Δt, D*||Δt, evidently similar with one moment time differentiation d/dt, equation 3.

Comment 14. Addition

18 A [Δt1, Δt2] = D*||Δt2 [M [Δt1, Δt2]]

19 A [Δt1, Δt] = D*||Δt [M [Δt1, Δt]] = Δt1

20 A [Δt0, Δt] = D*||Δt [M [Δt0, Δt]] = Δt0

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For Δt equal to Δt0, the Δt0 'addition zero' definition, comment 4, agrees with equation 19 since Δt leaves Δt1 including Δt1 = Δt0 invariant by addition. Equations 19 and 20 result from the identification Δt2 equals Δt like for multiplication, due to the argument from comment 15. Equation 18 can be derived to be valid for Δt1 and Δt2 identified with Δt-quantities cn(Δt) and cn'(Δt). cn(Δt) is linear in Δt due to MVT equilibrium and this supports generalization of equation 18 to the time interval set, discussed in paragraph 12, part IV. The first associative property is applied and cancelling terms are left out. Cancelling terms can be left out due to A [Δt1, Δt] = Δt1.

Comment 15. The definitions for M and A depend on the existence of a real event where the relevant event time interval is Δt. Only with this Δt the properties for Δt0 and ΔU can be considered sensible. Multiplication does not make sense with Δt2 not equal to Δt because it includes a time interval average during Δt2, where however the measurement event time interval, the time interval that is also the reference for averaging, is equal to Δt. Multiplication is included in A due to definition equation 18 and therefore also A depends on relevant event time interval Δt.

9a. The asymmetric second associative property and the one moment time differentiation 'set-rule'

For differentiation of a multiplication g1. g2, with g1 and g2 arbitrary scalar t-quantities, usually applies: d/dt [g1. g2] = c1. d/dt [g1]. g2 + c2. g1. d/dt [g2], in the one moment time description for the derivative to one moment time t, similar to equation 3 when scalars c1 = c2 = 1. When, equation 3, both two parts are valued equal, the symmetric second associative, ie distributive, property is valid for the g t-quantities set.

Assume g1. g2 = a. g3 with g1 = a, an invariant scalar, and g2 any t-quantity. The differentiation 'set-rule' defines the result, to release scalar a: d/dt [a. g3] = c1. d/dt [a]. g3 + c2. a. d/dt [g3] = a. d/dt [g3], in the symmetric one moment time description, equation 3.

9b. The time interval set and the time interval differentiation 'set-rule'

Now the perspective is changed to Δt-quantities and the time interval set. The usual properties for addition and multiplication for one moment time sets are based on the vector interpretation for space time coordinates within the one moment time description. They are however not at all evident for A and M within the time interval set and are not to be applied just like this. The usual properties include: A [a1. g1, a2. g1] equals (a1 + a2). g1, for instance with A [1. g1, 1. g1] = A [g1, g1] = 2. g1, where a1 and a2 are scalar numbers and g1 belongs to the set. These properties decide on the time parameter 'set rule'. These properties do not exist within the time interval set where A [Δt1, Δt1] = Δt1 is essentially different and depending on domain addition instead of vector addition, like for equations 19 and 20 for g1 = Δt1.

Comment 16. The asymmetric second associative property for differentiation to time interval Δt, D*||Δt [M [g1, g2]], with g1 and g2 any two time intervals, is:

21 D*||Δt [M [g1, g2]] = A [c1. M [D*||Δt [g1], g2], c2. M [g1, D*||Δt [g2]]]

with possibly un-equal scalars c1 and c2 for asymmetric differentiation and equal scalars for symmetric differentiation of M [g1, g2]. In the particular case g1 equals a. Δt1 including any scalar a and time interval Δt1, and g2 identified with Δt, from equation 18 and 21, one finds:

22 A [a. Δt1, Δt] = D*||Δt [a. Δt1] = A [c1. M [D*||Δt [a], Δt1], c2. M [a, D*||Δt [Δt1]]]

This is not equal to the differentiation to one moment time 'set rule': d/dt [a. g1] = a. d/dt [g1]. The time interval differentiation 'set rule', equation 23, is derived from equation 18. Applied is, 'addition zero' Δt0 equals Δt and 'multiplication unit' ΔU similarly equals Δt, paragraph 7 and 8. Multiplication with scalar a does not need brackets, M [a. Δt1, Δt] = a. Δt1. The time interval differentiation 'set-rule' is such, that scalar a is placed left and outside of the derivative D*||Δt brackets in equation 22 and not to the right and inside brackets. The 'set rule' is non-trivial since D*||Δt [a], for invariant scalar a, is not necessarily equal to ΔU0 and does not correspond with d/dt [a] equal to the one moment time set 'addition zero' and its 'multiplication zero'. Differentiation provides correspondence for t-quantities with Δt-quantities only when the time interval differentiation 'set rule' is introduced.

Comment 17. The existence of scalar multiplication M [a, Δt2] = a. Δt2, for any Δt2 is assured, in the sense of being equal to certain time interval Δt3, with invariant scalar a, not a time interval. For scalars the usual one moment time indications and definitions for M and A like . and + apply. Due to the closure theorem, comment

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12, for any scalar a with positive sign, the result for M [a. Δt, Δt2] = a. Δt2 = Δt3, is a time interval. With solution Δt = Δt2 and Δt3 belonging to the time interval set, this allows the application of M and A and I*||Δt2 and D*||Δt2 to these Δt3, a linearity or subset property, paragraph 12, part IV. Time interval derivative D*||Δt2 [a] exists independent of this for any scalar a.

It follows from direct evaluation of A and M, applying derivatives D*||Δt [Δt1] = [1/t, Δt1]||Δt, and D* [A1] = [1/t, A1]u, for time interval Δt1 including one moment time boundaries tb and ta and one moment time quantity A1, comment 1, D*||Δt [a. Δt1] does not have to be linear in scalar a and a remain Rest(a) is added, equation 23. This is derived without applying the second associative property constants c1 or c2 for equation 22.

Comment 18. The time interval differentiation 'set rule'

23 D* [a. Δt1] = A [a. D* [Δt1], Rest(a)t]

D*||Δt [a. Δt1] = A [a. D*||Δt [Δt1], Rest(a)||Δt]

Rest(a)t = [1/tb, a]u. Δt = M [D* [a], Δt]

Rest(a)||Δt = M [D*||Δt [a], Δt]

It should be for any 'set rule', scalar a is released from a. Δt1 within the derivative, say D*||Δt [a. Δt1], and returns to the left of the derivative of Δt1 without a, say D*||Δt [Δt1], while remain Rest(a) is added to the right. This is in agreement with the interpretation of D*||Δt 'moving to the right', like one moment time t or time interval Δt, equation 1 and 10, leaving a commutation quantity to the left. The remain Rest(a) is invariant during Δt, and therefore can be considered a t-quantity or a Δt-quantity. Rest(a) is derived by applying equations 4 and 5 and correspondence relations, paragraph 3.

The properties for t0, including t. t0 = t0. t = t for any t, are defined in [4, Hollestelle]. The usual commutation [1/tb, a]u has to be evaluated carefully. For a = 1 there is [1/tb, 1]u equals: tbi. 1 – 1. tbi, with tbi = 1/tb, indeed part of the one moment time set, comment 3. However, this is not necessarily equal to (1 – 1). tbi, the 'multiplication zero' equal to the 'addition zero' for the one moment parameter set, due to nonzero commutation involved, and since the specific scalar 1 is 'multiplication unit' for the scalar set one finds tbi. 1 – 1. tbi = tbi + (tbi)iv only, ie equal to the one moment time 'addition zero' only, when for all t, one moment time addition inverse tiv = -1. t. The usual commutation can be resolved by rewriting it like a time interval commutation that corresponds, equation 4, with time interval differentiation, and regains due to MVT equilibrium zero commutation, comment 1. Applied is time interval operator t* leaves scalar a invariant and the derivation includes a transformation of scalar a by multiplication with parameter t. Operator t* [A1] = M [D*||Δt [M[t, A1]]. Δt] for entity A1, [5, Hollestelle]. In this way it follows [1/tb, a]u equals D* [a].

From equation 4 and comment 4, with one moment time average < t > = t0, D* [a] equals < a >/< t > = a. t0i, for scalar a positive and invariant with t. Assuming H time dependent one moment time 'multiplication unit' t0 corresponds with time interval 'multiplication unit' ΔU. Recall ΔUi = ΔU = Δt0, from comment 3, and Δti equals Δt, from comment 21. It follows D*||Δt [a] and Rest(a)||Δt both equal a. ΔU. Rest(a)||Δt is a time interval related Δt-quantity, implying equation 23 is meaningful as a time interval only relation.

For any scalar a, Rest(a)t = M [a. t0i, Δt] corresponds with a. M [ΔU, Δt] = a. ΔU. This equals Rest(a)||Δt = M [a. ΔU, Δt] = a. ΔU, equal to ΔU for a = 1. In this case, for a = 1, one finds D* [a. Δt1] = a. D* [Δt1] and D*||Δt [a. Δt1] = a. D*||Δt [Δt1] as it should be.

Comment 19. Rest(a)|t, the equation 23 third, right side, part with derivative D* [a], can be derived independently from the second part with the usual commutation [a, 1/tb]u. This means, the second part due to the derivation above, is confirmed by the third part. This is discussed in paragraph 12, part III.

Comment 20. M [ΔU0, Δt1] = ΔU0, with ΔU0 the time interval set 'multiplication zero', corresponding with the scalar set 'multiplication zero'. The domain measure for ΔU0 is zero and ΔU0 = 0. Δt1, for any finite Δt1 including ΔU0 = 0. ΔU. Being of zero domain measure, ΔU0 is not a proper time interval, and resembles a one moment time parameter and does not belong to the time interval set. Time intervals with zero domain measure are not included in the time interval description in this paper.

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<u>*Comment 21.*</u> The multiplication inverse Δti for Δt, is not well defined yet to belong to the time interval set. When M [ΔU, Δt] = Δt and M [Δti, Δt] = ΔU = Δt one finds Δt is a solution for Δti: M [Δt, Δt] = ΔU. The solution Δti = Δt is well defined and belongs to the time interval set.

<u>*9c. The differentiation asymmetric second associative property and arguments c1 and c2*</u>

With c1 and c2 scalars defining the asymmetrical second associative property for differentiation D*||Δt [M [Δt1, Δt2]] with equation 21, one finds, from D*||Δt2 [Δt1] = Δt1 for Δt2 equal to Δt, equation 19, and a = 1,

24 D*||Δt [M [Δt1, Δt]] = A [c1. M [Δt1, Δt], c2. M [Δt1, Δt]]

Within the last part, M [Δt1, Δt] = Δt1, due to Δt = ΔU and Δt = Δt0 for the time interval set.

25 A [Δt1, Δt] = D*||Δt [M [Δt1, Δt]] = A [c1. Δt1, c2. Δt1] = Δt1

This is the time interval set relation equivalent to the one moment time set relation where traditionally c1 = c2 = 1. When both c1 and c2 equal 1 it follows: A [Δt1, Δt1] = Δt1. This is the same result derived from interpretation, paragraph 7. Recall that c1 and c2 are not any scalars, they have the meaning of second associative property arguments for D*||Δt [M [Δt1, Δt2], equation 21, and are possibly asymmetric with c1 different from c2. Independent of equation 22, that depends on the second associative property for multiplication M, comment 9, instead equation 23 introduces Rest(a), without associative property arguments. When both c1 and c2 equal 1 and with Δt1 = Δt, the time interval relation seems similar to the traditional one moment time relation: d/dt [t. t] = c1. (1. t) + c2. (t. 1) = 2. t, however it is not. The time interval set properties differ from the usual one moment time set properties also in this way.

10. Time interval set equilibrium and a theorem for time intervals including wave equations

With the introduction of averages and time interval set commutation to define time interval differentiation and integration to time intervals, one moment time boundaries remain included within the time interval description, comment 1. Time interval set commutation and one moment time t can be left out by defining integration and differentiation to time intervals in terms of M and A only, equation 12. For cn(Δt) and cn'(Δt) equations 26 and 27 are valid due to the results of paragraph 5, and inferred is validity for any two time intervals Δt1 and Δt2.

26 I*||Δt2 [Δt1] = M [Δt1, Δt2]

27 D*||Δt2 [Δt1] = A [Δt1, Δt2]

<u>*Comment 22.*</u> In the time interval description operators are indicated with *, depending on a 'operator quantity' for their interpretation. Operator quantities are indicated with |operator*|, for example the time operator t* relates to quantity |t*| = t. Such operator quantities are not defined for I* and D* and I*||Δt2 and D*||Δt2, and, even while 'working to the right' like operators, these are not proper operators.

It follows from I*||Δt2 and D*||Δt2 having equal results for Δt2 = Δt, equation 10 and discussion, that similarly M and A have equal results in this case, at least for cn(Δt) and cn'(Δt). This does not depend on the perspective: integration or differentiation to one moment time t or to time interval Δt, since M and A are time interval multiplication and addition for both perspectives and, paragraph 9b, with Δt2 = Δt the relevant event time interval, I*||Δt and D*||Δt correspond exactly with I* and D*.

M and A depend on time interval properties resembling common domain and combined domain respectively. The difference for M and A, from I*||Δt [cn(Δt)] and D*||Δt [cn(Δt)], equation 10, depends on the order for cn(Δt) and cn'(Δt) and does not return in the result itself in terms of time intervals. To clarify how I*||Δt and D*||Δt, and M and A can have the same results: this follows from D* [M] = A and D* [M] = M with A = A [Δt1, Δt] = Δt1 and M = M [Δt1, Δt] = Δt1 which is valid for any Δt1 due to the time interval set property Δt = Δt0 and Δt = ΔU. From equations 26 and 27 for cn(Δt) and cn'(Δt) one finds equations 28 to 33.

<u>*Comment 23. theorem.*</u> *Equations I*||Δt2 [Δt1] = M [Δt1, Δt2] and D*||Δt2 [Δt1] = A [Δt1, Δt2] can be applied to any two Δt-quantities or any two time intervals Δt1 and Δt2 from the time interval set.*

The following does not apply the definitions for multiplication or addition. The inference follows directly from the definitions for M and A however these definitions have validity only from assumption. Applied are MVT equilibrium and the time interval derivative. The proof for the closure theorem, comment 12, is slightly different.

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Due to a specific Lorentz transformation TL there is cn'(Δt) = cn(Δt') defining Δt' from Δt. The specific TL and change of Δt to Δt' such that the Hamiltonian H remains unchanged is derived in [4, Hollestelle]. For other changes of Δt to say Δt1, it follows H can be variable. For transformation TL with Δt to Δt', the theorem is valid due to the results of paragraph 5. For all other transformations Δt to Δt1 the linearity of any Δt1 with Δt is assured, even when the linearity constants do not agree with the specific transformation TL and cn(Δt) and cn'(Δt) do not agree with Δt and Δt1. From linearity of Δt1 with Δt it follows the argument for Δt' from Δt is valid for any Δt1 from Δt within the time interval set.
From equation 1 defined are cn(Δt') = cn'(Δt). Meanwhile cn(Δt) and cn(Δt') are linear in the relevant event time interval, ie Δt and Δt' respectively, implying a translation Δt to Δt'. This relation can be reversed unless for instance D*||Δt for cn(Δt) or cn'(Δt) equals the time interval set 'addition zero' ΔU0. In the reverse case Δt' is linear in cn(Δt) and thus Δt' is linear in Δt. In fact, for any time interval, the implied translation from Δt then results in a linear multiplication of Δt.
For any Δt2 the transformation Δt2 from Δt1 is the same as a time interval addition applying a nonspecific Lorentz transformation, ie a transformation without the property H remains unchanged. A translation exists due to this addition, and the theorem does apply for these Δt2 since a translation means a linear multiplication result, due to MVT equilibrium. Similarly, when Δt2 linear in Δt1 a translation does exist, this is discussed in paragraph 12, part IV. Thus, not only for cn(Δt) and cn'(Δt), for any time interval Δt1 and Δt2, and for any Δt-quantities being linear in Δt, the theorem is valid.

An interpretation of change for Δt is necessary. For the changed situation the validity of the following is to be ensured: M' [Δt1, Δt'] = M'' [Δt1, Δt''] = Δt1, ie with the situation change the relevant event time interval Δt' changes to Δt'' while specific properties for M and A do not change and are defined with the relevant event time interval indication Δt, one of these properties Δt = ΔU. It depends on the situation present: before change Δt' = Δt, after change Δt'' = Δt, where situations for Δt' or Δt'' share properties, however not all properties, to assure change.

The no-reverse case.

In this case a change for Δt does not change cn or cn' (recall cn' is defined from equation 1, with invariant Δt): cn(Δt') = cn(Δt'') and cannot be reversed at all. For a change Δt' to Δt'' with Δt' ≠ Δt'', and where for now Δt' = Δt'' is not considered, with Δt' = Δt for the situation before change, and the factor +/-1 not yet specified while +1 for positive increasing cn(Δt') etc, the complete equation for invariant cn(Δt') is A [cn(Δt''), cn(Δt')iv] = M [+/-1. D*||Δt [cn(Δt')], A [Δt'', Δt'iv]] = ΔU0. For D*||Δt [cn(Δt')] = ΔU0 valid, one finds cn(Δt') = cn(Δt'') = ΔU0. The time interval ΔU0 is both 'multiplication zero' and 'addition zero' for the time interval set and has time interval domain measure zero and is not a proper time interval, and cn(Δt) and cn'(Δt) are not well defined.

For D*||Δt [cn(Δt')] = ΔU0 not valid one finds:

I. Above, starting from Δt' = Δt a change to Δt'' = Δt means: A [cn(Δt''), cn(Δt')iv] = M [+/-1. D*||Δt [cn(Δt')], A [Δt'', Δt'iv]] = ΔU0 = Δt0 equal to the time interval "addition zero" since cn(Δt'') = cn(Δt').

II. From I: cn(Δt'') = A [M [+/-1. D*||Δt [cn(Δt')], Δt''], cn(Δt')] = A [M [cn(Δt''), Δt''], cn(Δt'')] = cn(Δt'). These equations follow from Δt' = Δt and Δt'iv = Δt' for the situation before change. Applied is +/-1. D*||Δt [cn(Δt')] = M [cn(Δt'), Δt'] = cn(Δt') for all Δt' including Δt'', ie the validity of equations 26 and 27 for cn and cn' including similarity of results for M and A, depending on equation 10.

III. From II: A [cn(Δt''), cn(Δt')iv] = M [cn(Δt'), A [Δt'', Δt'iv]] = ΔU0. A solution is cn(Δt') = Δt' for all Δt' including Δt'' where Δt' = Δt. When cn(Δt') = M [cn(Δt'), Δt'] one similarly finds the solution cn(Δt') = Δt' from M [Δt', Δt'] = Δt' for all Δt'. This applies the first associative property.

The no-reverse case, with D*||Δt [cn(Δt')] = ΔU0 not valid for any Δt', means cn(Δt') is linear in Δt', and this does not agree with cn(Δt') invariant. *This completes the proof for theorem comment 23.*

28 I*||Δt [cn(Δt)] = A [cn(Δt), A [ΔU, cn(Δt)]] = A [cn(Δt), cn(Δt)]

29 I*||Δt [cn'(Δt)] = M [cn'(Δt), Δt] = A [cn'(Δt), A [ΔU, cn'(Δt)]] = A [cn'(Δt), cn'(Δt)]

30 A [a. Δt1, Δt] = D*||Δt [M [a. Δt1, Δt]] = D*||Δt [a. I*||Δt [Δt1]] = A [a. D*||Δt [I*||Δt [Δt1]], Rest(a)||Δt]

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Equation 30 is valid also with cn(Δt) or cn'(Δt) instead of Δt1. Not necessarily cn(Δt) follows the requirement of time interval asymmetry like Δt itself for situations when H time dependent, and – 1. cn(Δt) is valid however -1. Δt is not. From equation 10 for I*||Δt it follows:

31 a. cn(Δt) = D*||Δt [a. I*||Δt [cn(Δt)]] = A [a. D*||Δt [I*||Δt [cn(Δt)]], Rest(a)||Δt] = A [a. D*||Δt [A [cn(Δt), -1. cn'(Δt)]], Rest(a)||Δt]

a. cn'(Δt) = D*||Δt [a. I*||Δt [cn'(Δt)]] = A [a. D*||Δt [I*||Δt [cn'(Δt)]], Rest(a)||Δt] = A [a. D*||Δt [A [cn(Δt), -1. cn'(Δt)]], Rest(a)||Δt]

For scalar a = 1, Δt1 = cn(Δt) and Δt2 = Δt the remain Rest(a)||Δt is nonzero and equal to cn(Δt). Scalar a can be released from I*||Δt without any nonzero remain, however only released from D*||Δt with 'set rule' equation 23 including remain Rest(a)||Δt. Due to property A [Δt1, Δt1] = Δt1 for any time interval Δt1 including Δt1 = cn(Δt), the remain can be left out and one finds D*||Δt and I*||Δt to be exactly the same.

32 a. cn(Δt) = D*||Δt [a. I*||Δ [cn(Δt)]] = A [a. D*||Δt [I*||Δt [cn(Δt)]], Rest(a)|Δt]

a. cn'(Δt) = D*||Δt [a. I*||Δt [cn'(Δt)]] = A [a. D*||Δt [I*||Δt [cn'(Δt)]], Rest(a)|Δt]

For a = 1 and with Δt-quantity cn(Δt) linear with Δt, one finds:

33 cn(Δt) = D*||Δt [I*||Δt [cn(Δt)]] = A [D*||Δt [I*||Δt [cn(Δt)]], cn(Δt)]

cn'(Δt) = D*||Δt [I*||Δt [cn'(Δt)]] = A [D*||Δt [I*||Δt [cn'(Δt)]], cn'(Δt)]

Wave equations and structure constants

These equations being second order resemble quite closely wave equations. From the discussion below equation 7 it follows ΔN equals both addition or multiplication of the reciprocal pair of commutation quantities. The derivative D*||Δt [ΔN] = A [cn(Δt), cn'(Δt)] = ΔN and similarly the second derivative D*||Δt [D*||Δt [ΔN]] = ΔN and equations 33 apply to ΔN. These equations are due to the derivatives being time interval MVT equilibrium derivatives. Recall ΔN is an invariant for the overall time interval set. ΔN, like any Δt-quantity, is part of the linearity 'subset' for cn(Δt) or cn'(Δt). This is discussed in paragraph 12, part IV.

From combinations of commutations, being multiplications within the generator set, one finds the set structure constants from the second derivatives, [12, De Wit, Smith], [15, Veltman]. Structure constants for the time interval set depend on D*||Δt [D*||Δt [M]], where due to property equation 11 for A and the similar property for M, paragraph 8, multiplication within the generator set equals multiplication within the time interval set, and one finds M = ΔN. The second derivative invariants are Lorentz transformation TL invariants, due to TL being a surface measure preserving transformation, unlike the usual TU. Due to this the overall TL invariant ΔN can be interpreted directly to be a structure constant.

From equations 1 and 10 one finds the pair-like commutation properties of the one moment time set and the time interval set respectively. The commutation constants form a multiple and the existence of an equilibrium overall invariant, in this case ΔN = M [cn(Δt), cn'(Δt)], means for structure constants:

Comment 24. The dimension of the set decides on the number of structure constants to form a set multiple.

Comment 25. For the time interval set the multiple is a pair, due to time and the time interval set, with all time intervals linear in Δt due to closure, being one dimensional.

Paragraph 11. Star source wave propagation

Radiation from a star source described with propagation sphere surfaces is different from radiation with stationary parallel propagation. The metric surface measure, not meaning the usual line element dependent metric, for propagation surfaces is invariant with change of relevant event time interval Δt that indicates time development within the time interval description for qm fields [4, Hollestelle]. This description still assumes de Broglie complementary and depends on the interpretation of radiation existing of waves or wave particles of zero or non-zero mass with a group velocity c(Δt), c_light for the usual *c* for photons. An effort to describe the resulting folded, wrinkled, radiation propagation surface for changing Δt is the following.

Propagation surface A related to radiation time interval Δt, can be described with a regular sphere Q(Δt) that however does not fulfill the above measure invariance. The realistic propagation surface A, can be approximated with surface P, applying a construction from reference [16, Hocking, Young].

For a set of dense disjoint open surface parts, constructed from an indecomposable continuum surface, the overall sum of metric surface part measures can be different from the metric continuum surface measure. The set of parts Pi, i = 1 to n, n the number of parts, can be termed a distribution for the continuum surface Q, and their union termed P. There is mP = ∑mPi, leaving out n, with m indicating metric surface measure. The construction allows mP to be different from mQ. When mA = mP for P an approximation for A, mA can be invariant with Δt, needed to describe propagation surfaces.

Comment 26. The construction of P to describe the propagation wave surface A does not mean it is implied the propagation surface is divided into photon paths Pi. The discussion of propagation starting from the concept of photon paths in qm is not the subject of this paper.

It is possible a distribution exists for indecomposable metric continua with a finite number of metric continua parts. For the propagation distribution the number of parts n is variable with Δt, while for the reference distribution n remains constant and the parts are developed with a variable t˜, where the reference parts Qi represent the construction result for the limit of infinite t˜. This allows for any width factor v, v. mQi = mQ. The propagation surface distribution is described within the time interval perspective. The surface parts Pi depend on, develop with, the relevant time interval Δt, are interpreted to be Δt-quantities, and P includes iteration of additions uPi+1 = A [Pi+1, uPi], and P = uPi, for i = n, where n itself is variable with Δt. For any number n the parts Pi are assumed dense, disjoint and open in Q and mPi = mQ. For all i and all n, the part Pi equals Qi and the metric area mPi(v) = v. mPi, by construction proportional with, ie multiplied with an average part width v, itself possibly depending on n and Δt. It follows for any n, mP(v) = n. v. mQi = n. mQ.

During propagation development when the relevant event time interval Δt changes to Δt', the regular sphere Q similarly changes to Q' with its surface area measure proportional to radius squared. The invariant regular metric cancels from the following.

34 mQ'/mQ ~ (|c'(Δt')|. |Δt'|)^2/(|c(Δt)|. |Δt|)^2

Surface measure mA can be approximated by mP including a metric m for P. The requirement of invariant mA, due to invariant radiation propagation surface energy E, means: mA(Δt')/mA(Δt) = mP(Δt')/mP(Δt) remains invariant and equal to one. With v dependent on n and Δt and with for all Pi(v) the same metric area measure proportional with v, change Δt to Δt' means a change A to A', with n to n' and v to v' accordingly, implying equation 35. From mA'/mA invariance it follows the *metric requirement for maintaining E invariance*:

35 1 = mA'/mA = n'. mQ'/n. mQ

One can change description from Δt to Δt' while n'/n changes proportionally with |Δt'|/|Δt|. For n'/n > 1, this means more 'close' to the star source, or in time interval perspective, more shortly, 'earlier' after emission when |Δt'| < |Δt| from the perspective of a specific Δt, and for n' = 1 a one-part union P. For n'/n < 1, this means more 'remote' from the star source, more 'to the future', 'later' from the specific Δt perspective. In [5, Hollestelle] it is argued the propagation 'surface' metric requirement for A does not apply the linear metric, line element, measures, even when the metric surface measure of a regular sphere traditionally relates to the metric radius measure squared. The parts Pi(v) being 'linear'-like, due to decreasing v, does not imply the ordinary line element. With 'folded, wrinkled' surface is meant a change of n, v, P and A from a situation with n = 1 and mA = mQ, due to Δt to Δt', to n' > 1 and mA' > mQ'.

The other way around when Δt remains invariant, n remains invariant however un-decided. When n is independent it supports a degree of freedom. For any n, each of the n parts Pi is dense in Q and the measure for the complete union P remains mP = n. mQ = mA.
One interpretation is, energy does not change with n, and a change of n implies a symmetry transformation. Any n ≥ 1 is realistic for propagation surface A, approximated with P the union of the parts Pi, to remain a covering for the regular sphere Q. A second interpretation is, implied is a situation with zero temperature. A description for star source cloud radiation exists, including temperature for the source cloud and for the star sources themselves [5, Hollestelle]. One finds NC = (mA)^2 = E^2. A zero temperature means the average space

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interval Δqbi for star source i relative to their average < Δqbi > is near zero which has implications for the symmetry for the star source collective in space.

A is assumed to be an indecomposable continuum. This means no partition includes parts with an interior, even when n equals 1, due to a result from topology [16, Hocking, Young]. This agrees with the concept of propagation surface A being the limit surface for radiation waves during Δt and part of the time interval only description without one moment time however with some structure from the Pi. Simultaneity defined from the complete propagation surface A needs an interpretation, it depends on 'timely' time intervals for radiation measurements, which is assured by Δt, the relevant event time interval, being 'timely'. The term 'timely' is introduced for time intervals, say Δt1, in [4, Hollestelle] meaning: a time interval Δt1 is measurable with one measurement with result |Δt1|. Simultaneous measurements, in the time interval description, are possible within Δt, which agrees with Δt being the relevant event time interval. The concept of change, ie for star source radiation propagation, is linked to finite time intervals Δt and to measurements and H being time dependent. The propagation surface energy is integrated for the inner part of the surface, from the star source to the propagation surface, that is for all energy with simultaneous propagation, all energy 'arriving' at A during Δt, [5, Hollestelle].

Comment 27. Distribution theorem. A 1-dim. space-like radius does not allow a distribution from radius parts. A distribution for the radius or Δq implies a distribution for time interval Δt. The metric requirement is applicable for the 2-dim overall propagation surface A and a distribution for A is possible only because it does not imply a distribution for relevant event time interval Δt.

Star source wave propagation with zero or non-zero mass

When one defines vacuum with total energy H0 value zero, and with kinetic radiation energy E = #n. h. ν, for photon number #n and photon energy h. ν, one can re-write H = H0 + ΔH, to agree with the specific energy quantity ΔH introduced in [4, Hollestelle]. Within the time interval description, a time dependent H = H0 + ΔH = E + V + ΔH is the 'time interval' version of the Legendre transform of Lagrangian L = E – V, with E kinetic energy and V potential energy and ΔH depending on the t-quantity #n, possibly variable due to interaction. The discussion in this paragraph depends on earlier results, applying de Broglie complementarity and the time interval description for star source radiation propagation with relevant event time interval Δt = [tb, ta], and there is for radiation energy E:

36 E(tb) = #n. h. ν = #n. M [h+, Δti]

The function h+ is defined with: h+ = 1/2 (Δ*p. Δ*q + Δ*q. Δ*p), applying the usual one moment time description indication . and + for one moment time t-quantities and the indication Δ* for variances, different from indication Δ for intervals. Even so, one moment time tb and ta occur. Energy E, without interaction and wave function collapse during Δt, is an invariant and neutral Δt-quantity.

The following properties are valid within the time interval description. Δt-quantities can be written linear with Δt due to MVT equilibrium. Δt-quantities are parameter t independent quantities, however can change with a change of Δt. The time interval description, including MVT equilibrium, for t-quantities depends on linearity in t. A t-quantity due to multiplication quadratic in t reduces to a neutral quantity, ie a t^(2) quantity equals a t^(0) Δt-quantity, comment 4. In contrast, a t-quantity, ie linear in t, remains t-quantity due to addition, with linearity implying addition, and remains possibly variable during Δt. A t^(-1) quantity equals a t^(+1) t-quantity, comment 4.

Assume a non-zero mass m1 for material wave particles complementary to radiation waves that emerge from a star source and introduce a certain mass m2. Then it follows E includes multiplication M [m1, m2] and is inverse linear with |Δq| = |c(Δt)|. |Δt| with c(Δt) = M [Δq, Δti], the wave group-velocity, [5, Hollestelle]. This can be interpreted due to its similarity with Newtonian gravitational energy for situations without external interaction. In this energy view propagation wave energy, ie kinetic energy E, equals gravitational energy E due to the above dependences. The meaning of dispersion free propagation energy is unrestrained wave kinetic 'movement', the meaning of gravitation energy is field energy for the propagation surface depending on the star source assuming masses m1 and m2. From the definition for c(Δt), paragraph 11, one finds with Δqi = M [Δti, c(Δt)i] = c(Δt)i,

38 E = #n. A [1/|c(Δt)|, M [m1, m2]], equal to #n. A [1/|c(Δt)|, m2] for m1 'multiplication unit'

39 E = #n. M [1/|Δq|, M [m1, m2]] = #n. D*||Δq [M [m1, m2]] = #n. D*||Δt [M [1/|c(Δt)|, M [m1, m2]]]

With equation 38 and 39 one finds a definition for non-zero mass m2, that can be identified with the source mass. D* or D*||Δt of an invariant does not have to be zero. With 1/|c(Δt)| = D*||Δq [m1] = M [m1, Δqi], there is *c(Δt) equals the multiplication inverse for the propagation wave particle mass density*. There is for photons E = #n. A [Δqi, m2], equation 38, for m1 the 'multiplication unit' mass quantity, and *E can be interpreted to be the source mass apparent density. This suggests wave particle energy h. ν is dependent only on multiplication M [m1, m2] = m2, the source mass.*

For c(Δt) = M [Δq, Δti] invariant, where Δq is not part of the time interval set and c(Δt) is not part of the time interval set, c(Δt) is a Δt-quantity linear in Δt including a non-scalar multiplication, ie Δq. The complete 'volume like' propagation surface A from star source to propagation surface, paragraph 11, depends on Δt and c(Δt). A variable non zero mass m1 or an 'addition zero' mass m1 can both be applied due to m1 and m2 not being scalars rather Δt-quantities where an 'multiplication unit' mass m1 corresponds with Δt0 = ΔU, with both A [m1, m2] = m2 and M [m1, m2] = m2 while E does not reduce to zero. *Then, zero mass wave particles, ie photons, relate to mass m1 being for mass quantities both 'addition zero' and the 'multiplication unit'*, different from a 'multiplication zero' mass in the sense of corresponding to scalar 'multiplication zero' or to ΔU0, and for any mass m2 there is:

40 A [m1, m2] = A [m2, m1] = m2

M [m1, m2] = M [m2, m1] = m2

Including #n the number of photons, non-interacting radiation propagation surface energy E = E(tb) = #n. h. ν is proportional with #n, however with m = ∑i mi = m1, i = 1 to #n, the overall photon mass m remains invariant and equal to "addition zero" m1 for any #n.

A change of radiation energy E during Δt for invariant ν due to external interaction and measurements including wave packet reduction like the photo-electric effect, depends non-linear on the variable #n where #n(tb) = #nb from before interaction: #n. h. ν = + 3/2 (2 #nb – 3)^(-1) (ΔH(ta) - ΔH(tb)) = + 3/2 (2 #nb – 3)^(-1) M [D* [ΔH], Δt], [4, Hollestelle]. This is different from the situation where ν is variable, due to internal interaction with a variable frequency ν and variable wave particle mass and number, however without external interaction and where E = #n. h. ν remains invariant, [5, Hollestelle].
Since wave energy h. ν remains positive it follows a change of sign for the difference ΔH(ta) - ΔH(tb) is possible from a variable #n, #n itself remaining a positive t-quantity, and when #nb is 1 or 2 there is ΔH(ta) - ΔH(tb) is negative or positive. For a situation within the infinite limit of increasing #n, considering ΔH(t) = H(t) – E(tb) and with ΔH(tb) equal to zero, H(t), a t-quantity, equals ΔH(t) apart from the constant E(tb), and the difference ΔH(ta) - ΔH(tb) = H(ta), a t-quantity, is proportional with #n. E.

Paragraph 12. Discussion

The time interval only description provides an alternative to the one moment time description and to vector calculus to describe differences and change. A time interval only description and solution for the 'infinite regress' problem, is presented in paragraph 2 to 5. Some results that depend on time intervals only were already obtained in [4, Hollestelle] and [5, Hollestelle]: one result is gravitational energy and radiation propagation energy for both zero and nonzero mass wave particles can be described integrated in one approach.
The description in this paper depends on time interval commutation quantities. Due to the considered set dimension, these quantities form multiples, for the time interval set, multiples are pairs of quantities. Where the number of degrees of freedom increases due to these multiples, equilibrium induces constraints and reduction of degrees of freedom. The time interval equilibrium requirement means invariance, for the multiplication of all quantities within a multiple, in this way introducing Noether charges, and in the same way, structure constants, for the relevant set. In this paper the relevant set is the time interval set, a one-dimensional set.
Radiation propagation in the usual qm field theory setting applies fields that extend to all infinite spacetime. This is due to radiation interpreted being parallel and stationary. Infinite spacetime is not very suitable for the definition of measurement events. When a star source is assumed, radiation propagation includes sphere surface limits unlike the parallel situation. Measurement and interaction, with H time dependent, allows for the introduction of a finite relevant event time interval. In the following discussed are some of the results.

I. The equivalence principle and simultaneous descriptions

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Assuming a situation without interaction, radiation energy E, interpreted in paragraph 11 and 12 with kinetic radiation energy, is an invariant. Radiation propagation implies dispersion free noninteracting radiation complementary with free moving wave particles, ie de Broglie complementarity. According to general relativity and the equivalence principle, this kinematic energy is 'equal' to gravitational energy when considering a kinetic energy space time coordinate system accelerated in the gravitational energy space time coordinate system and it is not possible to decide from measurement [6, Goldstein], [18, Einstein]. Like reciprocal quantities, paragraph 3, defined is a simultaneous pair of energies, Es, kinetic, and Ee, gravitational. To follow the arguments for reciprocal quantities the energy pair is related to each other by a reference transformation. The energies Es and Ee have the same invariant value each from their own reference, not meaning the respective space time coordinate system, rather like complementarity of wave and wave particle within qm. A relation for Es and Ee references differs from the usual Newtonian one equating kinematics and dynamics, since radiation and gravitation are considered from energy, not action. Due to MVT equilibrium, I*||Δt and D*||Δt being 'equal' is assured, since the results for both in terms of time intervals are the same, equation 10.

From equation 38 and 39, Es = #n. A [1/|c(Δt)|, A [m1, m2]], and Ee = #n. M [1/|c(Δt)|, M [m1, m2]]. The Es reference is 'addition' or differentiation, the Ee reference is 'multiplication' or integration. Both results agree with Newtons second law from differentiation of equations 38 and 39, Es due to differentiation to Δt and coordinate Δq being positive and decreasing and Ee due to differentiation to Δq, [4, Hollestelle], [6, Goldstein] and [7, Newton I]. In particular differentiation to resp. Δt or Δq implies opposite sign. The mass quantities for both equations 38 and 39 being regarded identical is the basis for the interpretation of matter properties and of the equivalence principle [1, Arnold], [17, Newton III], [18, Einstein]. Now it is tried to apply the arguments of the description of reciprocal quantities, paragraph 3.

Following the discussion below equation 8 for a pair of Noether charges, introduced is a simultaneous reference transformation TR, where pair-part for Ee is the transform of pair-part for Es and vice versa. Still, energies Es and Ee are Δt-quantities that allow application of A, M and D*||Δt while remaining within the same pair-part.

The proposed pair transformation TR implies for the energies: A [Es, Δt] to M [Ee, Δt], from addition to multiplication and reverse, where evidently the value for Es and Ee remains the same. Both pair-parts depend on energy invariance, valid for propagation surface energy due to noninteraction and for gravitation energy due to the time interval property D*||Δt [Ee] = Ee.

With D*||Δt [Es] = Es and D*||Δt [Δt] = Δt, since Δt = ΔU there is Es = M [Δt, Es] = M [Es, Δt] and it follows Es = D*||Δt [M [Es, Δt]] = A [c1. M [Es, Δt], c2. M [Es, Δt]] = A [c1. Es, c2. Es]. A reference transformation results in Ee = M [c1. Ee, c2. Ee]. This also means: D*||Δt [Es] = A [c1. Es, c2. Es] and D*||Δt [Ee] = M [c1. Ee, c2. Ee].

The pair Es and Ee relate due to the specific reference transformation, just like a multiple of reciprocal quantities that relate due to a certain equilibrium requirement that can be described with a transformation, say cn(Δt) to cn'(Δt) and a specific TL, such that cn(Δt') = cn'(Δt).

<u>Comment 23.</u> *The time interval densities for cn(Δt) and cn'(Δt) decide on the second associativity factors c1 and c2 for differentiation D*||Δt [M] of a multiplication M = [A1, A2], like for instance M = Ee proportional with M [m1, m2], due to D*||Δt [cn(Δt)] = A [c1. cn(Δt), c2. cn(Δt')] for Δt-quantity cn(Δt), one pair-part of the multiple cn(Δt) and cn'(Δt).*
The m1 and m2 represent the Δt and Δt' sources and star source for Ee respectively. The differentiation constants c1 and c2 are recognized to relate to multiple Δt-quantities like Δt-Noether charges due to their multiplication being invariant, in this case equal to ΔN, and themselves are properties changing with Δt, ie propagation and time development.

It is inferred that differentiation as an expression for geometry is directly related to source interaction like gravitational energy. Since differentiation is part of the usual wave equation description of qm, the inference means geometry and gravitation can be integrated with qm through the second associative factors for differentiation. Indeed, one way to derive qm wave functions is from Noether charges.
This inference is a time interval only result. It is expected that similarly differentiation can be generalized to space-time intervals and second associativity property factors related to Noether charges that describe properties changing with space-time intervals. However, there is a difference for time intervals and space intervals, already encountered in [4, Hollestelle] and [5, Hollestelle]: time intervals and time do not allow to 'turn backwards' where 3-dimensional space intervals do, and, the propagation sphere surface requirement of invariant metric

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area only emerges from Δt, not from Δq. This difference is also discussed when introducing structure constants, paragraph 10, and the distribution theorem, paragraph 11, comment 27.

II. Radiation energy and wave particle number

The time interval description of time development applies MVT equilibrium instead of Lagrangian equilibrium. A multiplication of with q and p corresponding quantities, within the context of the time interval description of star source radiation, recovers the uncertainty relation h+ = 1/2 (Δ*p. Δ*q + Δ*q. Δ*p), paragraph 12, reducing to Planck's constant h for H time independent [5, Hollestelle].

Resuming the de Broglie complementary wave energy h. ν, one finds: M [h+, Δti] = M [M [Δp, Δq], Δti] = M [h, ν] and without interaction the invariant radiation energy equals Es = #n. h. ν for #n wave particles or photons. It is argued Δ* variances are equal to Δ variations except for a minus sign, and when including interaction, the variable t-quantity is #n even while the relevant event time interval Δt = [tb, ta] starts or ends with a noninteracting situation. Then, radiation energy Es(ta) = #n. Es(tb) for Es(tb) = #n. h. ν depends nonlinear on #n however linear on frequency ν, paragraph 11. *This relation can provide independent confirmation from measurements for the de Broglie Einstein relation for the interdependence of kinetic energy and frequency, E = h. ν for wave particle and wave respectively*.

III. The 'set rule' and Stokes theorem. The meaning of 'close' and 'remote'

Comment 24. Scalar multiplication M [Δt1, Δt2] = Δt3 = a. Δta can be evaluated by applying transformation T1 and 'scale' transformation T2, termed B and C in [4, Hollestelle] and discussed below equation 8. T1 includes the specific transforms T1 [tb] = tb and |T1 [t0]| << |t0| and T1 [ta] = -1. tb, and T2 is a scale transformation transforming Δt = [tb, ta] to T2 [Δt] = [C. tb, C. ta] with C a scalar and boundary parameters t to T2 [t] = C. t = t0. (T2 [t0])^(-1). t, for a certain given T2 [t0], except for t = t0, since t0 is not a possible boundary parameter. T1 and T2 transform time intervals to time intervals while remaining within the time interval set. Δtb within commutator [1/tb, A]||Δtb, is the result of repeated application of T1 and T2 to Δt = [tb, ta].

From equation 23 for 'set rule' remain Rest(a)|t, the third part is derived independently and confirms the second part: derivative D* [a] = D*||Δt [a] equals the ordinary commutator [1/tb, a]u. This indicates that situations 'remote' from and 'close' to with respect to situations with H time independent are also more similar than the terms suggest. The situations 'remote' from and 'close' to Newtonian, with Newtonian meaning: with H time independent, can be related by interpretation from MVT equilibrium of the commutators [1/t, a]||Δt and [1/t, a]u for any quantity a and any Δt.

A change of the relevant time interval Δt from Δt' to Δt'' corresponds with a change of the interval boundaries. The time interval derivative to one moment time parameter tb then can be defined to be equal to [1/tb, a]||Δtb with Δtb = [tb', tb''] and similarly for ta with Δta = [ta', ta'']. Recall that these Δtb, and Δta, are for all tb' and tb'' well defined time intervals, due to T1 and T2 transforming proper time intervals to proper time intervals. One derives [1/tb, a]||Δtb is equal to [1/t, a]||Δt, and thus corresponds with D*||Δt [a]. For this to confirm Stoke's theorem, in a time interval set version, it should be D*||Δt [a] = a. This is valid for a equal to a multiplication like ΔN, paragraph 10.

The derivatives D* [a] and D*||Δt [a] being equal is discussed below 'set rule' equation 23. One also derives [1/tb, a]||Δtb ~ 1/2 [1/tb, a]u with both correspondences only valid for 'close' to Newtonian, H time independent, situations. It follows D*||Δt [a] ~ 1/2 [1/tb, a]u, which both are proper quantities, for 'close' to Newtonian situations meaning Δt is 'close' to, ie asymptotically, symmetrical, while the independent confirmation, comment 19, of this correspondence for both expressions from equation 23 is derived for all situations, H time dependent or time independent. This implies some questions concerning what means 'close' to time independent for H and what means 'close' to symmetrical for Δt. In the description of star source radiation 'close' to or 'remote' from symmetrical for Δt means 'remote' from or 'close' to for the propagation surface from the star source [5, Hollestelle]. The above relativity of the concept of measure and asymmetry in terms of the relevant time interval Δt can be an introduction to space time itself and to the resolution of the problem of action at a distance.

IV. Scalar multiplications and scale transformations. Linear subsets within the time interval set. Closure theorem

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From M [Δt1, Δt2] = Δt3, per definition a time interval, and since any time interval is neutral in Δt, there is assumed Δt3 is linear in Δt: Δt3 = e. Δt for some scalar e. Transformation T2 with scale factor C = a provides meaning for scalar multiplication with a time interval result: a. Δta = Δt3 = M [Δt1, Δt2] with a. Δta = T2 [Δta] remains a proper, ie well defined, time interval. Now write Δta itself linear in Δt. With ca = ΔU and a = 1 multiplication Δt3' = M [ΔU, Δta] = e'. Δt = Δta. All a. Δta can be expressed with time intervals e. Δt linear in Δt. By choosing Δta = Δt one finds specific ca and a where M [Δt1, Δt2] = e. Δt = ca.

Then a. Δta = M [ca, Δta] and a. Δta = a. M [ΔU, Δta] = M [a. ΔU, Δta]. This means multiplication with 'multiplication unit' ΔU or with ca both leave the linear subset for Δta invariant. Indeed, it follows M [Δt1, Δt2] is linear in Δta. The linear subset for Δta is the part of the time interval set defined from, time interval or scalar, multiplication with Δta. It follows M [Δt1, Δt2] = a. e'. Δt = e. Δt with scalar e = a. e' and part of the linear subset for Δt. When Δt2 equals Δt there is M [Δt1, Δt] = Δt1 and this confirms the assumption: any time interval Δt1 from the time interval set can be written like a multiplication including Δt, and is linear in Δt, one of the properties for M, paragraph 8. This confirms in terms of linear subsets the time interval set closure theorem, comment 12.

V. Linear and non-linear events

Introduced are two different quantities within the time interval description, t-quantities and Δt-quantities. It is possible to relate non-linear events, including Δt-quantities like Noether charges that depend on multiplication properties, with linear events, with t-quantities like one moment time t coordinates that depend on addition properties, by applying transformations 'working to the right' like multiplication with t or 1/t. This means, non-linear events are related to linear events within the time interval description by applying one of these transformations.

VI. Structure constants

Usually, structure constants are defined from the multiplication properties for the set or group elements writing these like exponentials of generator set elements. From assuming the canonical property for the set elements, it follows the commutator for two generator elements can be assumed to be linear within the generator set, ie gives another generator set element for result, including a scalar multiplication. These scalars are termed structure constants. Taking care of the multiplication Taylor series depends on writing out all higher order ordinary commutators of generators that should reduce to first order ordinary commutators which provides a linear result, depending on the structure constants for the set. For the time interval set, multiplication includes a linear result with M [Δt1, Δt2] = Δt3 = a. Δta providing the linearity constants and in this case this is enough to find the structure constants, without applying the generator set.

For any set, structure constants themselves are independent of the set representation. Indeed, for the time interval set the structure constants are independent of the number of necessary and different Δta, ie whether the subset of Δta is reducible or not. Similarly, the Δta subset is not completely determined by structure constant properties. There being only one independent subset for the time interval set, confirms the time interval set being 1-dimensional, and the existence of only one independent structure constant. A structure constant is to relate to the essential properties or quantities of the set, in this case these are the group velocity c(Δt) and the differentiation arguments c1 and c2, that together introduce the specific relations for space-time and time development, like propagation.

There is c(Δt) = M [Δq, Δti] = D*||Δt [Δq] = [Δt, Δq]||Δt is equal to ordinary commutator [Δt, Δq]u which is independent of t within Δt. For Δq a neutral Δt-quantity, M [c(Δt), Δt] = c(Δt) = M [M [c(Δt), Δti], Δt] provides the linearity constant for Δt, and ordinary commutator [Δt, Δq]u = M [M [c(Δt), Δti], Δt] with M [c(Δt), Δti] = c(Δt) a structure constant for the time interval set. However, c(Δt) does not have scalar dimension.

When measuring c(Δt) the time interval Δt is the relevant event time interval, which also is the relevant event time interval for measuring Δq. This can be resolved with the quantity h+. One of the assumptions for non-interaction star source wave propagation [4, Hollestelle] is |Δ*q| = |Δq| and |Δ*p| = |Δp| and this implies, h+ is a Δt-quantity linear in |Δq|. |Δp|, and due to being equal to M [E, Δt], linear in Δt, paragraph 12. E = M [h+, Δti] for E invariant during propagation and without interaction. Both these relations are equal to ordinary commutator relations and both h+ and Δq are linear in Δt since they are Δt-quantities. It follows E = M [h+, Δti] = [Δt, h+]u and this ordinary commutator equals E = [E, Δt] = M [M[E, Δti], Δt] with M[E, Δti] = E a structure constant for the time interval set like c(Δt). E and h+ are only introduced to find that from quantities with

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different dimensions the structure constant for a set remains the same, the number of structure constants depending just on the dimension of the set, and the time interval set is 1-dimensional.

Interaction can be included with h+ = h+(Δt) or with Δq = Δq(Δt) and time interval differentiation to Δt without applying parameter t. The wave velocity c(Δt) and energy E, Δt-quantities, can be interpreted to be structure constants disregarding dimension. Now introduce the following invariant velocity and energy quantities c_/|c_| and h. ν/|h. ν| that render dimensionless multiplication results with c(Δt) and E respectively. *It follows sc = M [c(Δt), (c_/|c_|)i] and se = M [E, (h. ν/|h. ν|)i] are structure constants for the time interval set.*

The time interval set is different from the usual groups considered in relation with structure constants, ie within elementary particle field theory. One difference is the canonical property usually assumed for these groups to derive the multiplication results for the generators, which however is not valid for the time interval set, due to the existence of the non-zero remain Rest(a). The structure constants being provided by the addition quantities ΔN, it is inferred that there is only one pair of cn(Δt), cn'(Δt), since equation 1 includes indeed only one corresponding pair cn and cn', and there is only one independent structure constant for the time interval set itself, with cn(Δt) and cn'(Δt) Δt-quantities. This means sc = se, they are same structure constant. Similar arguments are valid for interval quantities with a dimension different from the time interval set, for instance Δq-quantities and the space interval set. Due to equation 23 the remain Rest(a) is included in Noether charge NC, that can be derived by assuming both the overall time interval and space interval averages for cn(Δt) are the same, [1, Arnold] and equal to ΔN. One finds the time interval derivative to Δt of the complete space integral ΔN to be NC = A [M [c(Δt), ΔN], c(Δt)], depending on the first associative property for multiplication and applying similarity for M and A, paragraph 10. This is equal to the time interval derivative of overall time interval average ΔN = M [cn(Δt), cn(Δt')], ie equal to D*||Δt [M [cn(Δt), cn(Δt')]] = A [a. D*||Δt [Δta], Rest(a)|Δt] = NC, for a = c(Δt) and Δta = ΔN and Rest(a)|Δt = c(Δt). From paragraph 10, D*||Δt [ΔN] = ΔN meaning ΔN = NC and NC = M [c(Δt), NC] with solutions c(Δt) = NC or c(Δt) equals 'multiplication unit' ΔU.

<u>Comment 25.</u> *the non-linear property of the time interval set is included in Noether charge NC, with NC = sc. M [c_/|c_|, ΔNi] where sc the structure constant for the time interval set depending on c(Δt), and with NC = M [Rest(c(Δt)), ΔN] = M [c(Δt), ΔN].*

<u>Comment 26.</u> considered only are time intervals with domain measure non zero. Not considered are time intervals with the domain measure zero.

<u>VII. Equation-sign and the addition commutator</u>

The equation A [ΔU0, Δt0] = Δt0 from ΔU0 = (1 - 1). Δt0 = A [Δt0, -1. Δt0] seems contradictory, unless one considers comment 20 for ΔU0, and indicates there is an extra freedom when quantities can be subject to 'moving to the other side of the equation-sign' by multiplication with scalar -1. It means introducing an addition commutator. When commutation quantities are added the related multiple changes to include these quantities and similar for the Noether charge and structure constants, and the number of degrees of freedom. The equation-sign side seems to matter and have different value, depending on the order of the involved time intervals. This is subject for another paper, however the time interval set seems to be a good start to investigate this. A similar freedom seems to reside with Newton's laws and equilibrium definitions.

The confusion exists when one applies: -1. Δt0 to mean Δt0iv within the above addition A, and interpreting addition to mean a difference due to scalar -1, and can be avoided by 'moving to the other side of the equation-sign'. Differences are part of the one moment time description since this description depends on vectors. However, in the time interval only description the addition inverse is defined from A [Δt2, Δt2iv] = Δt0 for any time interval Δt2.

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