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Relaxation and Transient Processes in Disordered LC Systems

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Abstract

This work studies relaxation processes and transient currents in a disordered multi-component LC system, consisting of inductive and capacitive reactances (non-dissipative elements). At first glance, relaxation processes seem impossible in non-dissipative systems such as LC systems. Nevertheless, at the percolation threshold, we obtain the exact solution for the multi-component system, consisting of two different types of inductors and two different capacitors with random displacement and connections. It was shown that the effective conductivity of such a non-dissipative disordered system has a real value. This means that relaxation and transient currents arise in this problem. The relaxation times for these processes are established for both low and high frequencies. A physical interpretation of the obtained results is given.

Keywords: inductances; capacitors; dissipation; relaxation time; asymptpotics; low frequences; high frequences

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1. Introduction.

As is well known, the problem of relaxation and transient processes in conducting media is closely connected with the problem of the conductivity of the media [1–3], because the conductivity of a conducting medium

σ is determined by its relaxation time τ :

$$\sigma = \frac{q^2 n\tau}{m} \quad (1)$$

where n and m are the particle concentration and mass, respectively, and q is an electric charge.

In the case of a disordered, multi-component LC system, which is composed of reactances from inductors and capacitors (non-dissipative elements) with random connections between them, it initially seems that no relaxation or transient processes are possible due to the non-dissipative character of the elements. Nevertheless, due to disorder in the system, caused by random placement and connections of reactances, finite dissipation arises at the percolation threshold. As a result, relaxation and transient processes appear in the LC systems.

To study this problem, the exact method developed in [4–6] was used. This method is based on the symmetry of two-dimensional equations relative to rotational or dual symmetry. Below we apply this method, which is described in the Appendix in more detail, to calculate the effective conductivity of a discrete LC system consisting of randomly connected inductances and capacitances of two types with different values.

The paper is structured as follows. In Section 2 we calculate the effective conductivity of a disordered multi-component LC system. In Section 3 we study the transient currents at short and long times. Section 4 concludes the paper. The details of the calculations are provided in the Appendix.

2. Effective Conductivity of Many-Phase System Consisting of Randomly Connected L and C at Percolation Threshold.

Let us consider the conductivity of a four-component medium consisting of randomly placed inductors of two kinds and capacitors of two kinds (non-dissipative elements) with different values (a four-color chessboard) at equal concentrations of inductors p_1 and capacitors p_2 : $p_1 = p_2 = 1/4$

At the percolation threshold $p_c = 1/2$ we need to use the rotational transformations (A2) with coefficients $a = c = 0$. Therefore, according to formulae (A7) and (A8) the following expression for the effective conductivity is obtained:

$$Y^e(0) = \sqrt{\frac{Y_1 Y_3 (Y_2 + Y_4) - Y_2 Y_4 (Y_1 + Y_3)}{Y_1 + Y_3 - Y_2 - Y_4}} \quad (1)$$

The key feature of this solution is that the expression yields a real value. This result is unusual and interesting because a medium consisting of inductances and capacitances—elements with purely imaginary impedances—nevertheless exhibits a real effective conductivity at the percolation threshold. The analogous result was first obtained in [4], see also [6].

Let us analyze this formula (1) in the case when $Y_1 = i\omega L_1$, $Y_3 = i\omega L_2$ and $Y_2 = 1/i\omega C_1$, $Y_4 = 1/i\omega C_2$. After some transformations, the following expression is obtained:

$$Y^e(0) = \sqrt{\left(\frac{L_1 + L_2}{C_1 + C_2}\right)\left(\frac{1 + \omega^2/\Omega_1^2}{1 + \omega^2/\Omega_2^2}\right)} \quad (2)$$

Here we introduce the following notations: $\Omega_1^2 = \left(\frac{C_1 + C_2}{1/L_1 + 1/L_2}\right)$ and $\Omega_2^2 = \left(\frac{L_1 + L_2}{1/C_1 + 1/C_2}\right)$.

At low frequencies $\omega << \Omega_1, \Omega_2$ the following formula is obtained:

$$Y^e(0, \omega \to 0) \approx \sqrt{\frac{L_{eff}}{C_{eff}}} = \sqrt{\left(\frac{L_1 + L_2}{C_1 + C_2}\right)} \quad (3)$$

In the opposite case at high frequencies $\omega >> \Omega_1, \Omega_2$ another expression is obtained:

$$Y^e(0, \omega \to \infty) \approx \sqrt{\frac{L_{eff}}{C_{eff}}} = \sqrt{\frac{1/C_1 + 1/C_2}{1/L_1 + 1/L_2}} \quad (4)$$

And at intermediate frequencies the value depends on the relation between frequencies $\omega, \Omega_1, \Omega_2$.

In the case $\Omega_1 << \omega << \Omega_2$ we obtain from (2):

$$Y^e(0) \approx \frac{1}{\omega C_{eff}} = \frac{1}{\omega \sqrt{C_1 C_2}} \quad (5)$$

For clarity, these results are presented graphically in Figure 1.

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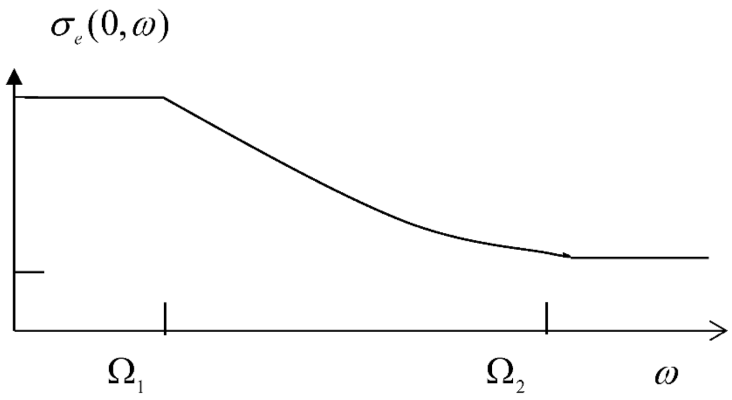

Figure 1. Dependence of the effective conductivity on the frequency.

In the opposite case $\Omega_2 << \omega << \Omega_1$ we obtain from (2) – see Figure 2 as well:

$$Y^e(0) \approx \omega L_{eff} = \omega\sqrt{L_1 L_2} \quad (6)$$

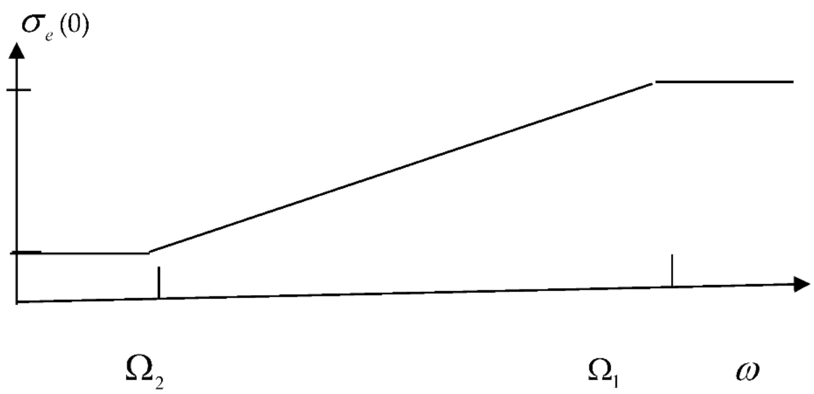

Figure 2. Dependence of the effective conductivity on the frequency.

Transient Currents in LC System.

As is well known, in an LC system consisting of inductors (L) and capacitors (C), only oscillatory phenomena are possible. However, because a finite resistance exists in the investigated system at the percolation threshold – see formula (2), one can expect that transient currents and relaxation processes will appear:

$$I(t) = I_0 \exp\left(-\frac{t}{\tau_{eff}}\right) \quad (7)$$

Thus, the problem reduces to finding the value of the effective relaxation time τ_{eff}. Let us study these currents in two limiting cases: at low and high frequencies, as described by the formulae (3) and (5).

In the system with inductance L and resistivity $R = \sqrt{L/C}$ the relaxation time is equal to: $\tau_L = L/R = L/\sqrt{L/C} = \sqrt{LC}$; in the system with capacitor C and the same effective resistivity R the relaxation time is equal to: $\tau_C = RC = C\sqrt{L/C} = \sqrt{LC}$. We therefore assume these quantities are equal within the system under investigation:

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$$\tau_{eff} = L_{eff} / R = RC_{eff} \quad (8)$$

The exact relation between two effective characteristics L_{eff} and C_{eff} is obtained:

$$L_{eff} / C_{eff} = R^2 \quad (9)$$

As a result, in the case of low frequencies $\omega \to 0$ we obtain:

$$\frac{L_{eff}}{C_{eff}} = \frac{L_1 + L_2}{C_1 + C_2} \quad (10)$$

If we assume that at low frequencies the effective inductor L_{eff} is equal to $L_{eff} = L_1 + L_2$, which means that the main current paths in the disordered system are provided by the series-connected inductors and the parallel-connected capacitors $C_{eff} = C_1 + C_2$ - see Figure 3

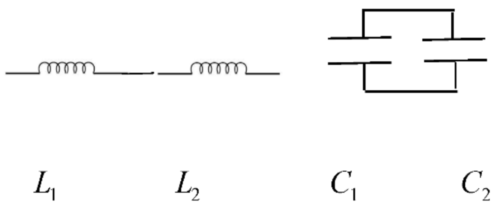

Figure 3. The current paths are provided by series-connected inductors and by capacitors with parallel connection.

Thus, the effective relaxation time at low frequencies is:

$$\tau_{eff}(\omega \to 0) = \sqrt{(L_1 + L_2)(C_1 + C_2)} \quad (11)$$

And in the case of high frequencies $\omega \to \infty$, we obtain the following relation:

$$\frac{L_{eff}}{C_{eff}} = \frac{1/C_1 + 1/C_2}{1/L_1 + 1/L_2} \quad (12)$$

If we assume that at high frequencies the effective inductor L_{eff} is equal to $1/L_{eff} = 1/L_1 + 1/L_2$, this means that in this limiting case (high frequencies) the main current paths in the disordered system are provided by the parallel-connected inductors and series-connected capacitors $1/C_{eff} = 1/C_1 + 1/C_2$.

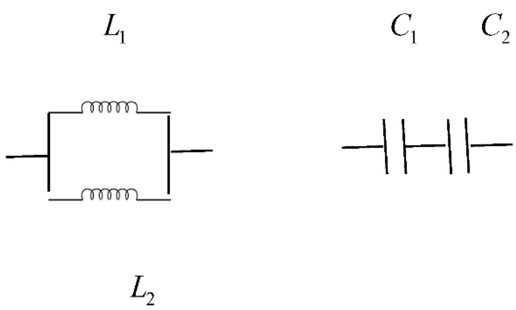

Figure 4. The current paths are provided by inductors with parallel connection and by capacitors with series connection.

Thus, the effective relaxation time at high frequencies is:

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$$1/\tau_{eff}(\omega\to\infty)=\sqrt{(1/L_1+1/L_2)(1/C_1+1/C_2)} \quad (13)$$

It is easy to see that in the case of the two-component LC system, when $L_1=L_2$ and $C_1=C_2$, the formulae (11) and (13) reduce to a single expression [3]:

$$\tau_{eff}(\omega\to 0)=\tau_{eff}(\omega\to\infty)=\sqrt{LC} \quad (14)$$

4. Discussion.

In this work, we have studied current flow in a two-dimensional system of randomly connected circuit elements. It has been shown that the effective conductivity is constant, i.e., independent of phase concentration, and is equal to either the capacitive reactance or the inductive reactance—see formula (3). However, at the point of the percolation transition, a finite effective resistance appears in a system of randomly connected circuits $R=\sqrt{\frac{L}{C}}$. The emergence of a dissipative state with finite resistance can be understood as follows. Suppose the initial phase through which the current flows consists of inductors; consequently, all the energy is concentrated in the inductive elements. In order for current to flow through another phase, the capacitive phase, it is necessary to transfer this energy to the capacitances, and for this it is necessary to excite local oscillations in the circuits, during which energy will be transferred from the inductors to the capacitances. In other words, the transition from one non-dissipative state of the system, described by conduction, to another non-dissipative state, described by a different value, is possible only through the excitation of the circuit, i.e., through the dissipative state at the point of topological transition. In our opinion, the obtained results can be used to study the properties of multi-component metamaterials.

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Conflicts of Interest: "The authors declare no conflicts of interest.".

Appendix A. The Exact Method for Studying Disordered Conducting Media: The Dykhne Approach

As is well known, a two-dimensional conducting medium is described by the DC equations and Ohm's law:

$$div\,\vec{j}=0,\quad curl\,\vec{e}=0,\quad \vec{j}=\sigma\vec{e} \quad (A1)$$

where $\vec{j}$ and $\vec{e}$ are the local electric current and electric field, and σ is the conductivity of the medium. These equations have an internal symmetry – the invariance with respect to linear rotational transformations [6–8]:

$$\vec{j}=b[\vec{n},\vec{e}'],\quad \vec{e}=d[\vec{n},\vec{j}'] \quad (A2)$$

Here $\vec{n}$ is a unit vector normal to the plane.

In the general case of an anisotropic medium, for example, in a magnetic field, Ohm's law has a tensor form:

$$\vec{j}=\hat{\sigma}\vec{e} \quad (A3)$$

Here $\hat{\sigma}$ is a two-dimensional conductivity tensor with components σ_{xx} and σ_{xy}. Therefore, in this case, to describe the dual symmetry of two-dimensional anisotropic media we use generalized linear transformations of rotation:

$$\vec{j}=a\vec{j}'+b[\vec{n},\vec{e}'],\quad \vec{e}=c\vec{e}'+d[\vec{n},\vec{j}'] \quad (A4)$$

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In the transformed system, primed Ohm's law also has a tensor form. It is more convenient to reformulate the problem described above using complex variables with the imaginary unit $i = \exp(i\pi/2)$ as the local rotation by the angle $\pi/2$. This means that we use the following notations:

$$j = j_x + ij_y,\; e = e_x + ie_y \quad \text{(A5)}$$

and complex conductivity [9,10]:

$$Z = X + iY \quad \text{(A6)}$$

After applying transformations (4), we obtain the relations between the components of the initial and primed systems in the general case:

$$X' = \frac{X(ac+bd)}{(Xd)^2 + (Yd+a)^2} \quad \text{(A7)}$$

$$Y' = \frac{X^2cd + (cY-b)(dY+a)}{(Xd)^2 + (Y^2d+a)^2} \quad \text{(A8)}$$

In the case of capacitors and inductors:

$$X = 0,\; Y_C = -1/\omega C \text{ or } Y_L = \omega L$$

Let us apply this method to study the multi-component disordered system, consisting of randomly connected inductances of two types L_1, L_2 and capacitances of two types C_1, C_2.

There are three possible transformations of the symmetry, according to permutations of the components of different types and the corresponding relations of duality for the effective conductivity [17].

1) First permutation symmetry of the system: an interchange of reactance components with odd and even numeration by positions:

$$Y_1'' = Y_2,\; Y_2'' = Y_1 \quad (1\leftrightarrow 2)\ \text{(A9)}$$

2) The second symmetry of the system: an interchange of reactances with odd and even numeration in its positions and change of the direction of normal vector $\vec{n}$ as:

$$Y_1'' = Y_2,\; Y_2'' = Y_1,\; (1\leftrightarrow 2) \text{ and } \vec{n} \to -\vec{n} \quad \text{(A10)}$$

3) The third symmetry of the system: the only change of the direction of normal vector $\vec{n}$ as $\vec{n} \to -\vec{n}$

$$Y_1'' = Y_1,\; Y_2'' = Y_2,\; (1\leftrightarrow 1,\, 2\leftrightarrow 2) \text{ and } \vec{n} \to -\vec{n} \quad \text{(A11)}$$

Here, the coefficients a_3, b_3, d_3 are determined by the conditions (3).

$$a_3 = -c_3 = 1,\; b_3 = \frac{2Y_1Y_2}{Y_1+Y_2},\; d_3 = \frac{2}{Y_1+Y_2} \quad \text{(A12)}$$

In this case, the primed system is macroscopically equivalent to the original and as a result the effective conductivity is determined by the equation:

$$d_3Y_e(\delta)^2 + b_3 - 2a_3Y_e(\delta) = 0 \quad \text{(A13)}$$

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