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Article

A Parametric Study of the FitzHugh-Nagumo Reaction-Diffusion System

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Abstract

In this article we analyze a dynamical system known as the FitzHugh-Nagumo model, which offers many characteristics of nonlinear systems, such as bifurcation, excitability or limit cycle. The dynamics associated with sets of values of the parameters associated with this model, called excitable, oscillatory and bistable, are analyzed. Then adding a perturbing or diffusive term to the system through the Laplacian, it is studied how these dynamics propagate in a one-dimensional or two-dimensional extended medium, through the definition of a cellular automaton with periodic initial conditions.

Keywords: dynamical systems; nonlinear; cellular automata; FitzHugh-Nagumo; reaction-diffusion

1. Introduction

The Hodgkin-Huxley model, see [3], marked a milestone in its day in the field of neural communication by introducing a quantitative formulation of the electrical signal firing process in terms of a differential equation model.

FitzHugh, [2] and Nagumo, [4] laid the foundation for a simplified version of the original Hodgkin-Huxley model, which describes the dynamics of the neuron in more detail. The FitzHugh-Nagumo (FN) model describes a prototype of an excitable system. Richard FitzHugh proposed the theoretical model and J. Nagumo and others built an equivalent electrical circuit. In FitzHugh's original papers, his model is called the van der Pol oscillator. This is so because the FitzHugh-Nagumo model, for the case $a = b = 0$, reduces to Van der Pol oscillator.

Mathematical Model

There are several versions of the FN model in the literature. Below, we will take the most well-known version, which we will formulate as follows:

$$\begin{aligned}\frac{\partial v}{\partial t} =& F(v,w) = v(v-a)(1-v) - w + I,\\ \frac{\partial w}{\partial t} =& G(v,w) = \epsilon(v - bw),\end{aligned} \tag{1}$$

where the parameters a and b satisfy:

$$-1 \le a \le 1, \quad b > 0.$$

The parameter ϵ takes very small values, so that the variable w is slow compared to a fast v. I is the external stimulus to which the system responds.

The FN model is an example of an excitable oscillator since if the external stimulus I exceeds a certain threshold value, the system exhibits a characteristic curve in phase space, after which the variables v and w will return to their resting value. This behavior is observed in neurons when they are stimulated by an external current.

Our study will initially focus on a parametric analysis for the value of $I = 0$, with the intention of looking for conditions between the values of a and b that determine equilibrium of the system (1) for small, fixed values of ϵ. This analysis will show us that there are three possible cases, *excitability situation, oscillatory situation and bistable situation.* We will then proceed to an analysis when $I > 0$ with the intention of treating I as just another parameter of the system. Finally, we will add a diffusion term so that these situations occur in an extensive medium (spatially), either one-dimensional or two-dimensional. In our study we will present the diffusive problem in a cellular automata framework with periodic initial conditions, also taking into account for the two-dimensional automaton model the Von Neumann neighborhood condition $r = 1$, for a more in-depth reading on cellular automata see [5].

There are many studies on the *FN* model, some more recent ones are for example [6] where a parametric study of the system is carried out, and which we follow in order to compare results. On the other hand, for a reader interested in knowing aspects about the stability and the different bifurcations of this model, see [1], since our study does not contemplate any analysis about the stability of the equilibriums of the system.

2. Study of the FN System Without Diffusion

In this section, we will analyze the case without spatial diffusion. First, we will examine the case in which the FN system does not incorporate external input, that is, $I = 0$, with the intention of studying how the nullclines $F(v, w) = 0$ and $G(v, w) = 0$ vary when the parameters a and b are varied while keeping the parameter ϵ fixed.

2.1. Case $I = 0$

Solving the system formed by $F(v, w) = 0$ and $G(v, w) = 0$ is equivalent in dynamical systems to calculating the equilibrium points (or stationary points) of the associated system of differential equations.

Algebraically, these coincide with the intersection points of $F(v, w) = 0$ and $G(v, w) = 0$, called nullclines.

If we set $I = 0$ in the equation for $F(v, w)$, the intersection points of the nullclines are the solutions of the system:

$$w = v(v - a)(1 - v),$$
$$w = v/b.$$

Then, equating them, we get:

$$vb(v - a)(1 - v) = v,$$

Therefore, we obtain a cubic equation of the following form:

$$bv^3 - (a + 1)bv^2 + (ab + 1)v = 0.$$

Factoring out, we get a quadratic equation of the form:

$$bv^2 - (a + 1)bv + (ab + 1) = 0, \tag{2}$$

Thus, it is established that the solution $v = 0, w = 0$ always exists.

By analyzing the discriminant of the quadratic equation (2), which has the form:

$$\Delta = (a + 1)^2 b^2 - 4b(ab + 1),$$

we determine the different nature of the solutions of (2).

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If the following condition holds:

$$(a+1)^2 b^2 - 4b(ab+1) < 0,$$

which is equivalent to:

$$b < \left(\frac{2}{1-a}\right)^2, \tag{3}$$

then there is only one intersection point or stationary point. In the case of a single intersection point, we can distinguish two different models or situations.

2.1.1. The Intersection Point Lies on the Left Branch of $\mathbf{F}(\mathbf{v}, \mathbf{w}) = \mathbf{0}$

For this case, it must hold that $v_{\text{min}} > 0$, i.e., $a > 0$. Therefore, if the following parameter conditions are simultaneously satisfied:

$$a > 0, \quad b < \left(\frac{2}{1-a}\right)^2, \tag{4}$$

then we have the situation represented in Figure (1)(a).

If we study the extrema of $F(v, w)$, which are the solutions of:

$$\frac{\partial F}{\partial v} = -3v^2 + 2(1+a)v - a = 0,$$

solving this equation gives the following roots:

$$\frac{a}{3} + \frac{1}{3} + \frac{\sqrt{a^2 - a + 1}}{3}, \quad \frac{a}{3} + \frac{1}{3} - \frac{\sqrt{a^2 - a + 1}}{3}$$

Since $v_{\text{min}} > 0$ must hold:

$$a + 1 - \sqrt{a^2 - a + 1} > 0,$$

it must then be $a > 0$.

2.1.2. The Intersection Point Lies in the Central Zone of $\mathbf{F}(\mathbf{v}, \mathbf{w}) = \mathbf{0}$

Then it must be $v_{\text{min}} < 0$, i.e., $a < 0$. Thus, the following conditions must simultaneously hold:

$$a < 0, \quad b < \left(\frac{2}{1-a}\right)^2. \tag{5}$$

We can see in Figure 1(b) the representation of the nullclines for this case.

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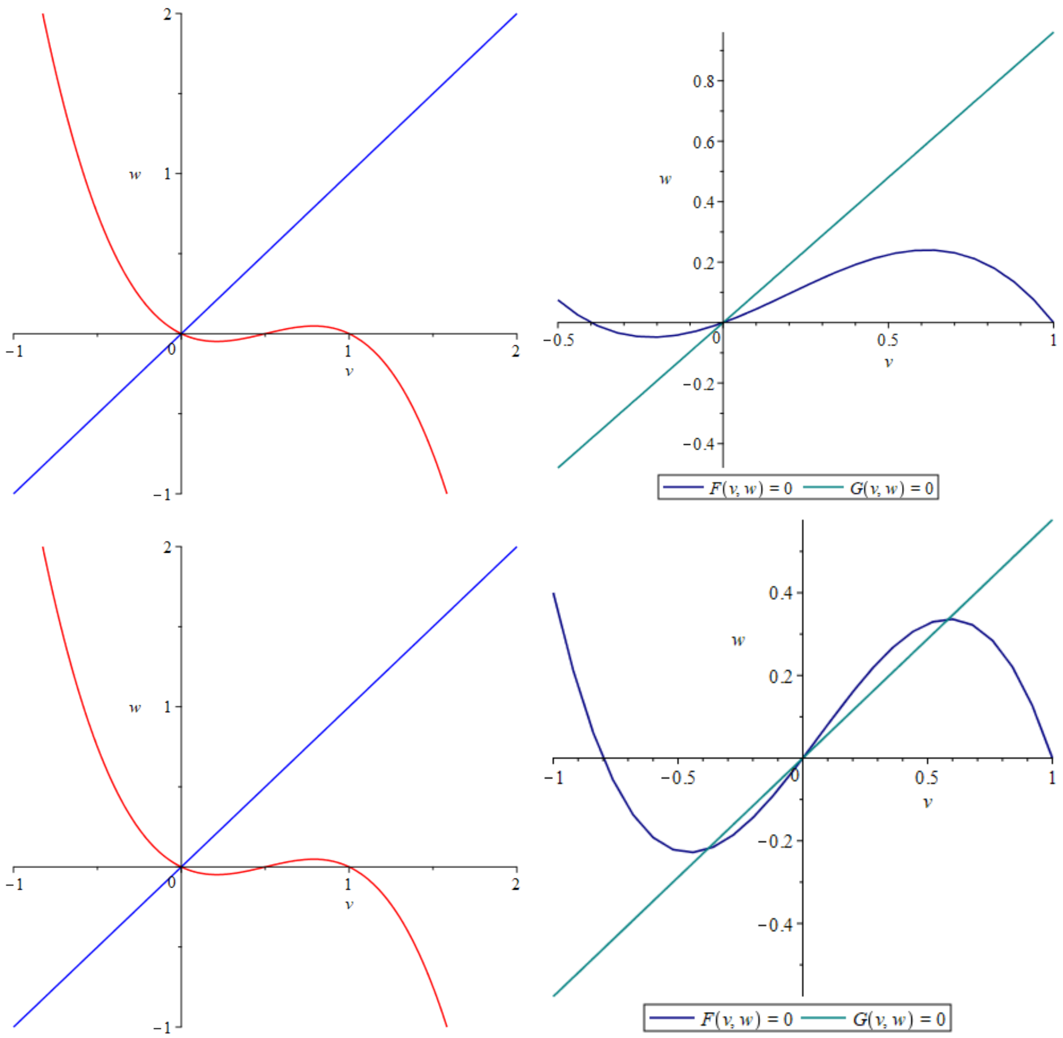

Figure 1. (a) Nullclines for the values $a = 0.5$ and $b = 2$. The intersection point lies on the left branch of $F(v, w) = 0$. (b) Nullclines for $a = -0.7$, $b = 1$. The intersection point lies in the central zone of $F(v, w)$. (c) Nullclines for $a = 0.5$ and $b = 1$. Two intersection points of the nullclines. (d) Nullclines for $a = -0.8$ and $b = 1.734568$. Three intersection points of the nullclines.

2.1.3. Two Intersection Points of the Nullclines

If the following condition between parameters holds:

$$b = \left(\frac{2}{1-a}\right)^2, \tag{6}$$

there are two solutions or two intersection points or stationary points: $(0, 0)$ and the point:

$$v_2 = \frac{1+a}{2}, \quad w_2 = \frac{1+a}{2b}.$$

We can see in Figure (1)(c) the representation of the nullclines.

2.1.4. Three Intersection Points of the Nullclines

If the following condition holds:

$$b > \left(\frac{2}{1-a}\right)^2, \tag{7}$$

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there are three solutions or three intersection points or three stationary points: $(0,0)$ and the following points:

$$v_2 = \frac{1+a}{2} + \sqrt{\left(\frac{1-a}{2}\right)^2 - \frac{1}{b}}, \quad w_2 = \frac{v_2}{b},$$

$$v_3 = \frac{1+a}{2} - \sqrt{\left(\frac{1-a}{2}\right)^2 - \frac{1}{b}}, \quad w_3 = \frac{v_3}{b}.$$

We can see in Figure (1)(d) a representation of this case.

3. Types of Solutions for Each Case

3.1. One Intersection Point

3.1.1. The Intersection Point Lies on the Left Branch of $\mathbf{F}(\mathbf{v},\mathbf{w}) = \mathbf{0}$

This model is called the excitable model (excitability situation). Observing Figure 2, we see a single pulse of a certain amplitude. In the excitable case, there is only one linearly stable state but with a low destabilization threshold. A perturbation that is not too large causes the system to move away from the state and traverse a wide region of the phase space before returning to it.

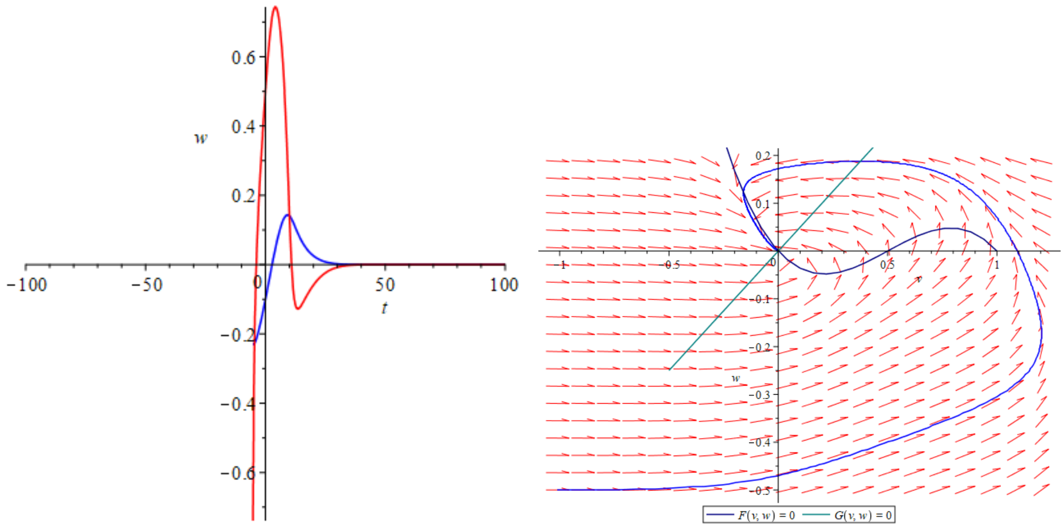

Figure 2. Excitable model with $a = 0.5$, $b = 2$. (a) Solutions $v(t), w(t)$. (b) Phase diagram.

3.1.2. The Intersection Point Lies in the Central Zone of $\mathbf{F}(\mathbf{v},\mathbf{w}) = \mathbf{0}$

When the only stationary state is on the central branch of $F(v,w) = 0$, oscillations of the system occur. This is called the oscillatory situation. In the oscillatory case, we have a stable limit cycle, which would be, in a way, an equivalent response to the sustained emission of pulses in the excitable case. See Figure 3.

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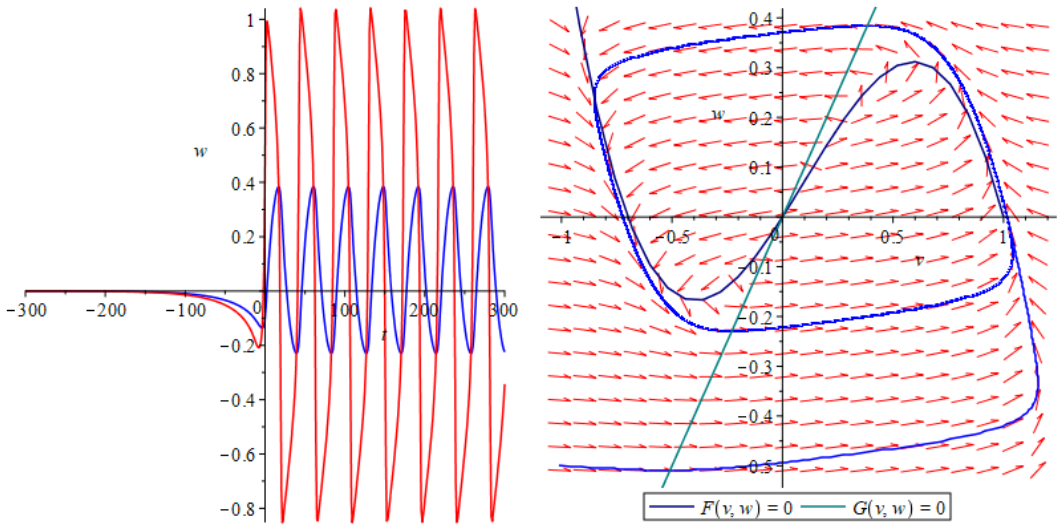

Figure 3. Oscillatory model with $a = -0.7, b = 1$. (a) Solutions $v(t), w(t)$. (b) Phase diagram.

3.2. *Three Stationary Points or Intersection Points*

In this situation, there are three stationary points (bistability situation). In the bistable case, we have two stable states (right and left) and one unstable state (central). As seen in Figure 4, when the system is in one of the stable states, an appropriate perturbation can cause it to be attracted to the other stable state.

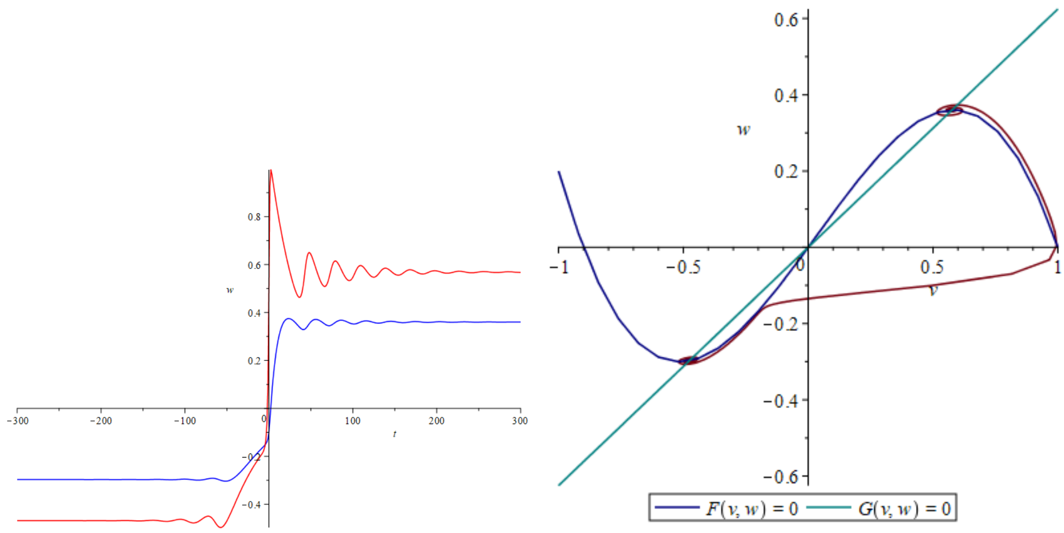

Figure 4. Bistable model with initial conditions $v(0) = 0.5, w(0) = -0.1$. (a) Solutions $v(t), w(t)$. (b) Phase diagram.

3.3. *Case $I > 0$*

We will now implement the previous models when we induce a current $I > 0$ to the system. In this case, if we want to study the system analytically with I as a new parameter, we must analyze the cubic function that appears.

If we set $I > 0$ in the equation for $F(v, w)$, we get the expression:

$$F = v(v - a)(1 - a) - w + I,$$

and for $G(v, w)$:

$$G = \epsilon(v - bw).$$

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The intersection points are the solutions of $F(v,w) = 0$ and $G(v,w) = 0$, i.e.,

$$w = v(v-a)(1-a) + I,$$

$$w = \frac{v}{b}.$$

By substitution, we have the following expression:

$$v(v-a)(1-a) + I - \frac{v}{b} = 0, \quad I > 0.$$

If we define the function:

$$S(v) = v(v-a)(1-v) - \frac{v}{b},$$

then for equilibrium points to exist, it must be $S(v) = -I$. If we differentiate this function and study the associated quadratic, we have three possible cases:

a) If the following holds:

$$s = (1-a)^2 + a - \frac{3}{b} < 0,$$

then $S(v)$ is decreasing, so there is a unique equilibrium point for all values of I.

b) If $s = 0$, $S(v)$ has an inflection point at:

$$v = \frac{1+a}{3}.$$

c) If $s > 0$, then $S(v)$ has a maximum and a minimum:

$$I_M = S\left(\frac{1+a+\sqrt{s}}{m}\right), \quad I_m = S\left(\frac{1+a-\sqrt{s}}{m}\right).$$

Depending on the relationship between I_M, I_m, and I, there will be one, two, or three equilibrium points.

If $I_m < -I$ or $I_m > -I$, there will be one stationary point or equilibrium, i.e., we will be in an excitability situation.

If $I_m < -I < I_M$, there will be three equilibria.

The introduction of the parameter $I > 0$ causes the graph of the nullcline $F(v,w) = 0$ to shift along the ordinate axis. Therefore, the previous models will be modified in their dynamics. For example, as the value of I increases, the excitable model will transition to an oscillatory state, which will return to being excitable at certain values of I. This is called the transition from the excitable regime to the oscillatory regime.

4. Model with Diffusion

Reaction-diffusion systems are mathematical models that describe how one or more substances distributed in space change under the influence of two processes:

1) Diffusion (local movement),

2) Reaction (interaction between v and w).

In previous sections we analyzed the reaction component; we will now examine how the three situations (excitability, oscillatory, or bistable-reaction) manifest in an extended medium (spatially-diffusion). Such a medium is called active and bistable, excitable or oscillatory depending on the dynamics in each case.

We begin by adding the diffusion term through the Laplacian:

$$\begin{aligned} \dot{v} &= D_v \Delta v + v(v-a)(1-v) - w + I, \\ \dot{w} &= D_w \Delta w + \epsilon(v - bw), \end{aligned} \tag{8}$$

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Equations (8) are defined on a certain domain, Ω, with boundary conditions to be specified on its boundary, $\partial\Omega$, and appropriate initial conditions.

4.1. 1D Model

We begin by drastically simplifying the problem (8) by defining it on an interval of length L.

We now consider the system in one dimension, so the equations become:

$$\dot{v} = D_v \frac{\partial^2 v}{\partial x^2} + v(v-a)(1-v) - w + I,$$
$$\dot{w} = D_w \frac{\partial^2 v}{\partial x^2} + \epsilon(v - bw).$$

Using the approximations:

$$\frac{\partial v}{\partial t} \approx \frac{v(\chi, \tau + \Delta\tau) - v(\chi, \tau)}{\Delta\tau},$$
$$\frac{\partial^2 v}{\partial x^2} = \frac{v(\chi - \Delta\chi, \tau) + v(\chi + \Delta\chi, \tau) - 2v(\chi, \tau)}{(\Delta\chi)^2}$$

and similarly for variable w.

We divide the interval into N cells, numbered with an index $i = 1, 2, 3, ..., N$. We discretize the diffusion and reaction terms and define the following quantities:

$$Dif[v(i,\tau)] = v(i+1,\tau) + v(i-1,\tau) - 2v(i,\tau),$$
$$Dif[w(i,\tau)] = w(i+1,\tau) + w(i-1,\tau) - 2w(i,\tau),$$
$$Reac[v(i,\tau)] = v(i,\tau)[(v(i,\tau) - a)(1 - v(i,\tau))] - w(i,\tau) + I,$$
$$Reac[w(i,\tau)] = \epsilon(v(i,\tau) - bw(i,\tau)).$$

The iteration equations become, once initial conditions are defined:

$$v(i,\tau+1) = v(i,\tau) + r_v Dif[v(i,\tau)] + \eta Reac[v(i,\tau)],$$

$$w(i,\tau+1) = w(i,\tau) + r_w Dif[w(i,\tau)] + \eta Reac[w(i,\tau)],$$

where

$$r_v = \frac{D_v \Delta\tau}{(\Delta\chi)^2}, \quad r_w = \frac{D_w \Delta\tau}{(\Delta\chi)^2}, \quad \eta = \Delta\tau$$

To implement our model, we must define boundary conditions. Typically, periodic boundary conditions are considered, such that cell 1 is adjacent to cell N:

$$v(1,\tau) = v(N+1,\tau), \quad w(1,\tau) = w(N+1,\tau)$$

A good number for implementation is $N = 200$ with $\Delta\tau = \eta = 1$, and parameter $\epsilon = 0.005$. Without loss of generality, we can take $I = 0$.

The defined cellular automaton is called discrete, as the set of states is finite. We have a series of states (values of functions $v(i,t)$ and $w(i,t)$) where time and space are discrete.

In the case of an excitable medium, a single pulse propagates. In Figures 5 we observe two pulses traversing the entire space due to periodic boundary conditions. In the oscillatory medium case, we observe a wave train propagating through the entire space, representing the limit cycle - a regular sequence of pulses (see Figures 6). In the bistable medium, a front propagates (see Figures 7).

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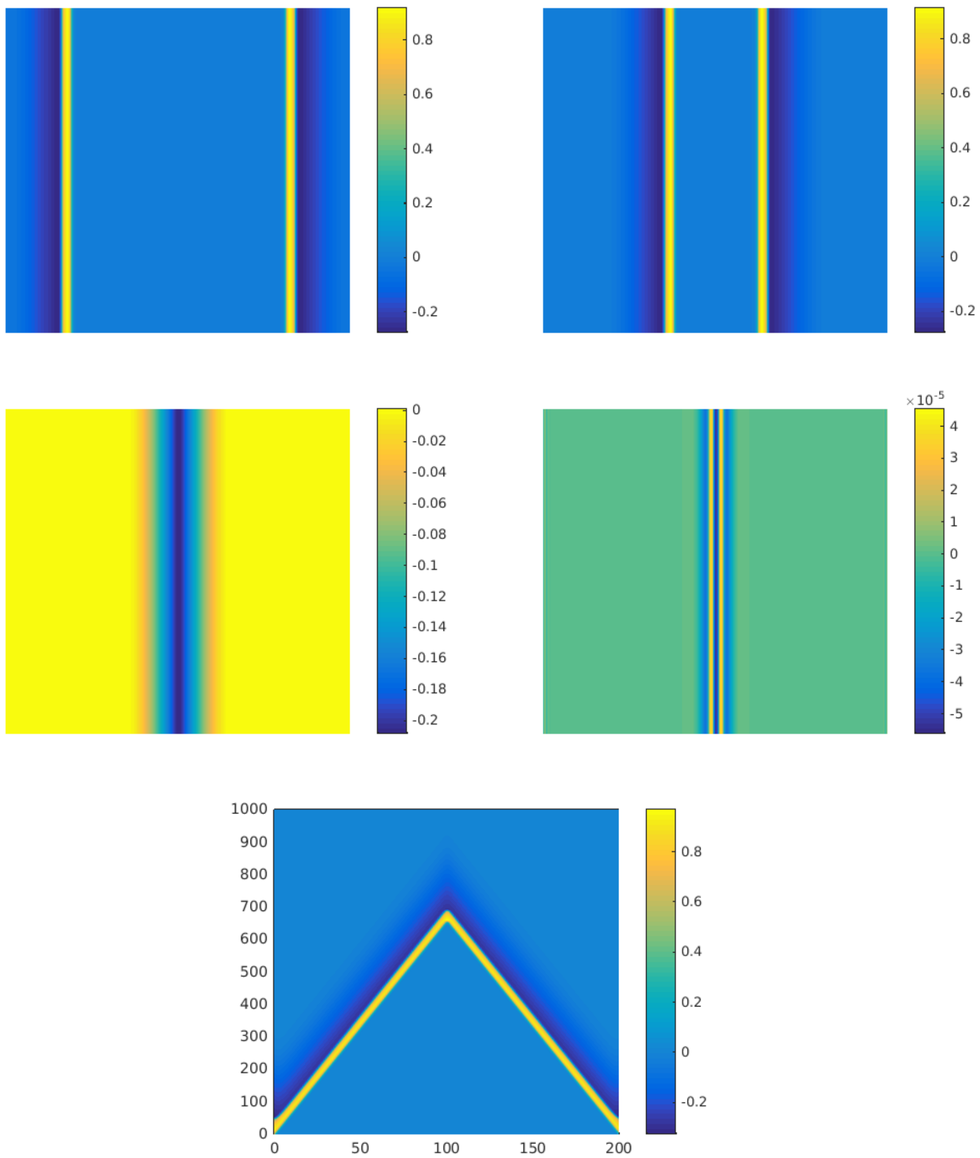

Figure 5. Evolution of states in the cellular automaton. 1D excitable medium with $a = 0.1$ and $b = 0.0001$, $D_v = 0.1$ and $D_w = 0$, (a) $t = 250$, (b) $t = 500$, (c) $t = 750$, (d) $t = 1000$, (e) space-time plot. Initial conditions $v(1) = 1$ and $v(2:200) = 0$, $w(1:200) = 0$. Two pulses traverse the entire space due to periodic boundary conditions.

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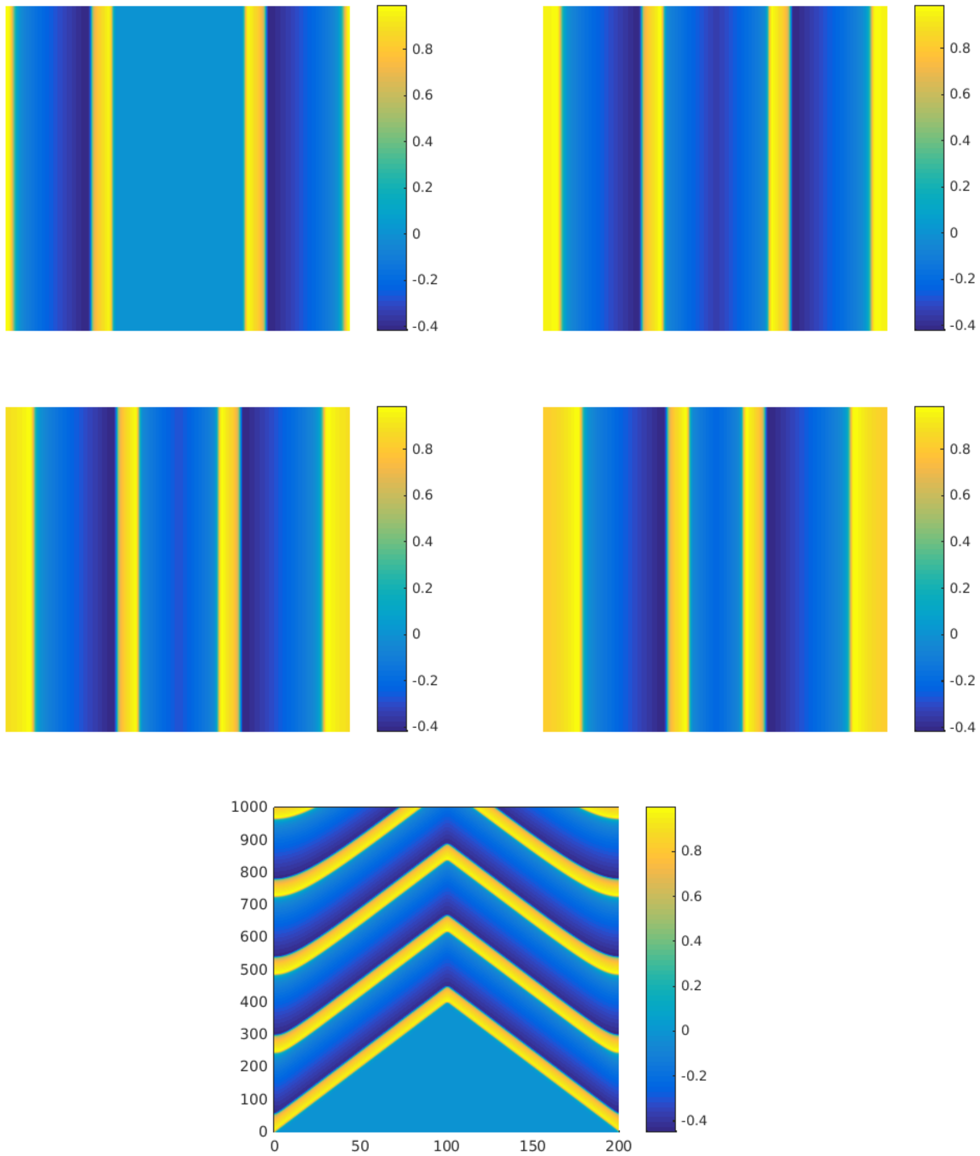

Figure 6. Evolution of states in the cellular automaton. 1D oscillatory medium with $a = -0.1$ and $b = 0.0001$, $D_v = 0.1$ and $D_w = 0$, (a) $t = 250$, (b) $t = 500$, (c) $t = 750$, (d) $t = 1000$, (e) space-time plot. Initial conditions $v(1) = 1$ and $v(2:200) = 0$ and $w(1:200) = 0$. The limit cycle is observed.

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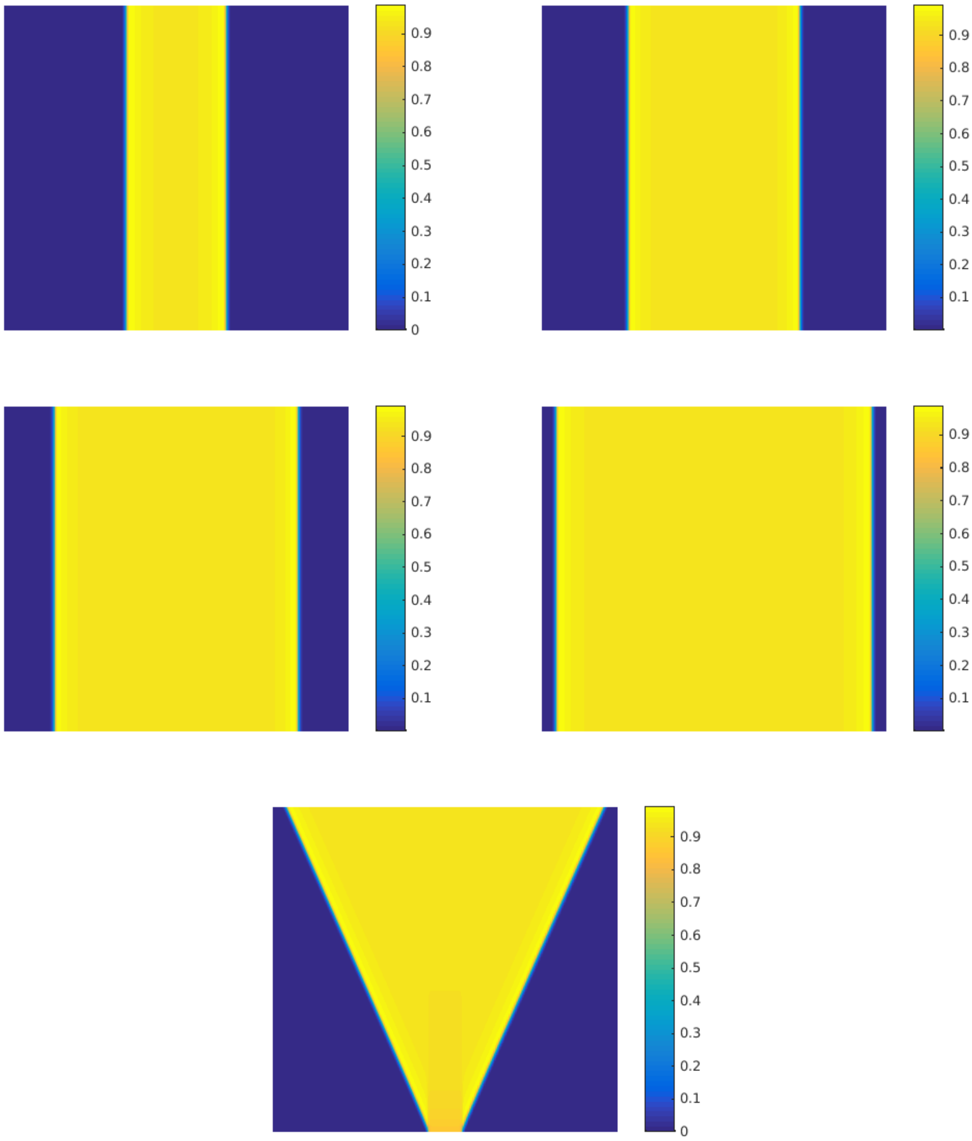

Figure 7. Evolution of states in the cellular automaton. 1D bistable medium with $a = -0.5$ and $b = 10$, $D_v = 0.1$ and $D_w = 0$, (a) $t = 50$, (b) $t = 100$, (c) $t = 150$, (d) $t = 200$, (e) space-time plot. Central initial conditions $v(90:110) = 0.9$ rest 0, $w(90,110) = 0.17$ rest 0.

Links to videos showing the evolution of cellular automaton states are included in Appendix A.

4.2. 2D Model

Let us now consider the system in two dimensions with the Laplacian:

$$\dot{v} = D_v\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) + v(v-a)(1-v) - w + I,$$
$$\dot{w} = D_w\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) + \epsilon(v - bw).$$

In this case, we define a 2D space on a square grid of cells (N,N) - a 2D cellular automaton - with Von Neumann neighborhood conditions $r = 1$. For our study we consider $N = 200$.

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We number the cells with indices $i, j = 1, 2, 3, ..., N$ and define:

$$\begin{aligned}
Dif^x[v(i,j,\tau)] &= v(i+1,j,\tau) + v(i-1,j,\tau) - 2v(i,j,\tau),\\
Dif^y[v(i,j,\tau)] &= v(i,j+1,\tau) + v(i,j-1,\tau) - 2v(i,j,\tau),\\
Dif^x[w(i,j,\tau)] &= w(i+1,j,\tau) + w(i-1,j,\tau) - 2w(i,j,\tau),\\
Dif^y[w(i,j,\tau)] &= w(i,j+1,\tau) + w(i,j-1,\tau) - 2w(i,j,\tau),\\
Reac[v(i,j,\tau)] &= v(i,j,\tau)[(v(i,j,\tau) - a)(1 - v(i,j,\tau))] - w(i,j,\tau) + I,\\
Reac[w(i,j,\tau)] &= \epsilon(v(i,j,\tau) - bw(i,j,\tau)),
\end{aligned}$$

The iteration equations become:

$$v(i,j,\tau+1) = v(i,j,\tau) + r_v^x Dif^x[v(i,j,\tau)] + r_v^y Dif^y[v(i,j,\tau)] + \eta Reac[v(i,j,\tau)],$$

$$w(i,j,\tau+1) = w(i,j,\tau) + r_w^x Dif^x[w(i,j,\tau)] + r_w^y Dif^y[w(i,j,\tau)] + \eta Reac[w(i,j,\tau)],$$

where

$$r_v^x = \frac{D_v \Delta\tau}{(\Delta\chi)^2}, \quad r_w^x = \frac{D_v \Delta\tau}{(\Delta\omega)^2}, \quad \eta = \Delta\tau$$

and

$$r_v^y = \frac{D_w \Delta\tau}{(\Delta\chi)^2}, \quad r_w^y = \frac{D_w \Delta\tau}{(\Delta\omega)^2}.$$

We implement periodic boundary conditions:

$$v(1:200, 1, \tau) = v(1:200, N+1, \tau), \quad v(1, 1:200, \tau) = v(N+1, 1:200, \tau)$$

$$w(1:200, 1, \tau) = w(1:200, N+1, \tau), \quad w(1, 1:200, \tau) = w(N+1, 1:200, \tau)$$

For calculations we consider a square 2D cellular automaton of $(200, 200)$ with $I = 0$.

In the excitable medium case, a single pulse propagates (Figures 8). In the oscillatory medium, a wave train propagates through the entire space, representing the limit cycle (Figures 9). In the bistable medium, a front propagates (Figures 10).

Links to videos showing the temporal evolution of cellular automaton states are included in Appendix A.

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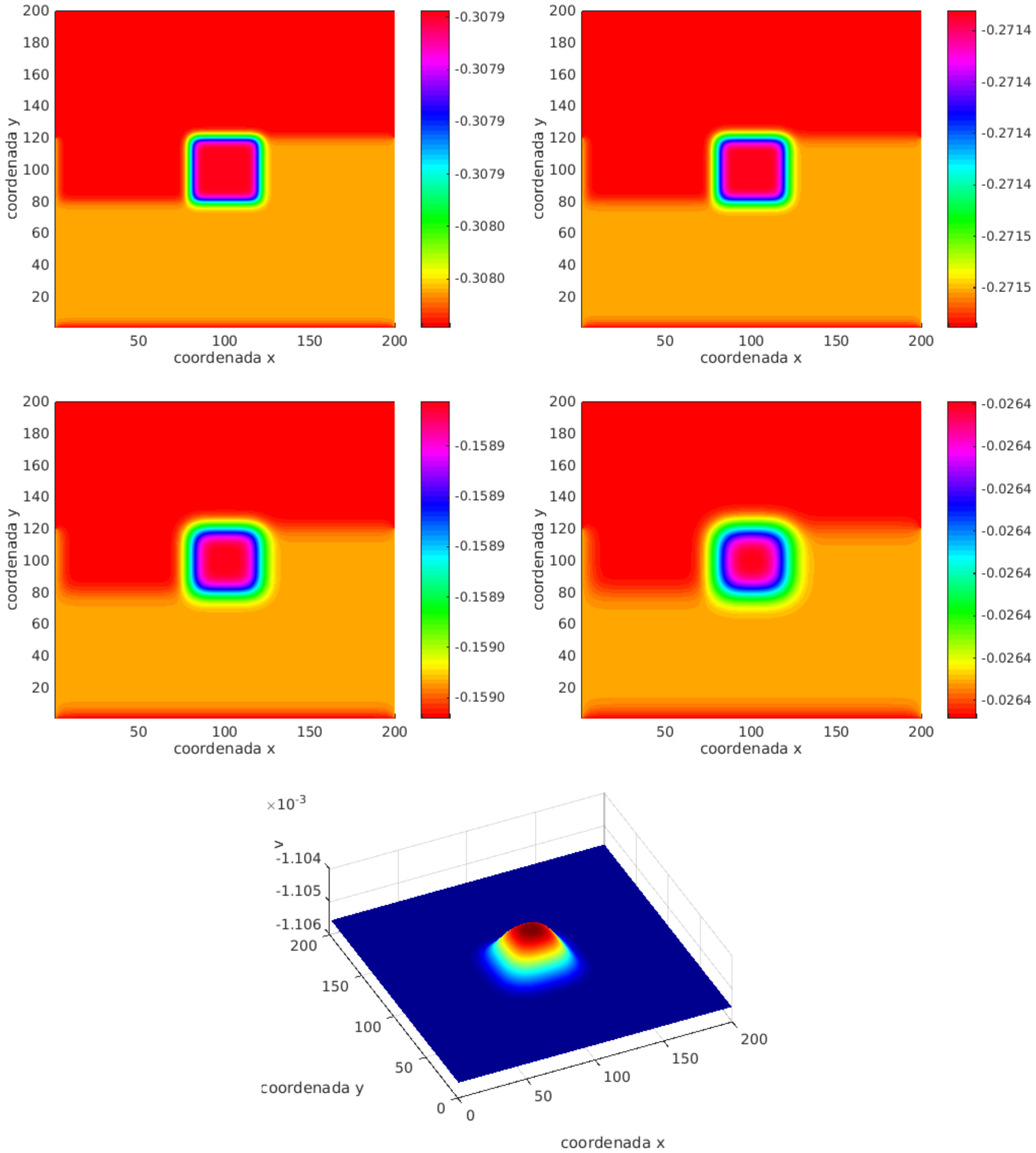

Figure 8. Evolution of states in the cellular automaton. 2D excitable medium, $a = 0.1$, $b = 0.0001$, $D_v^{x,y} = D_w^{x,y} = 0.1$, (a) $t = 50$, (b) $t = 75$, (c) $t = 150$, (d) $t = 250$, (e) surface $z = v(x, y)$ at $t = 250$. Central initial conditions, $v(80:120, 80:120) = 0.95$, rest±0.01 random, $w(1:200, 1:200) = 0$.

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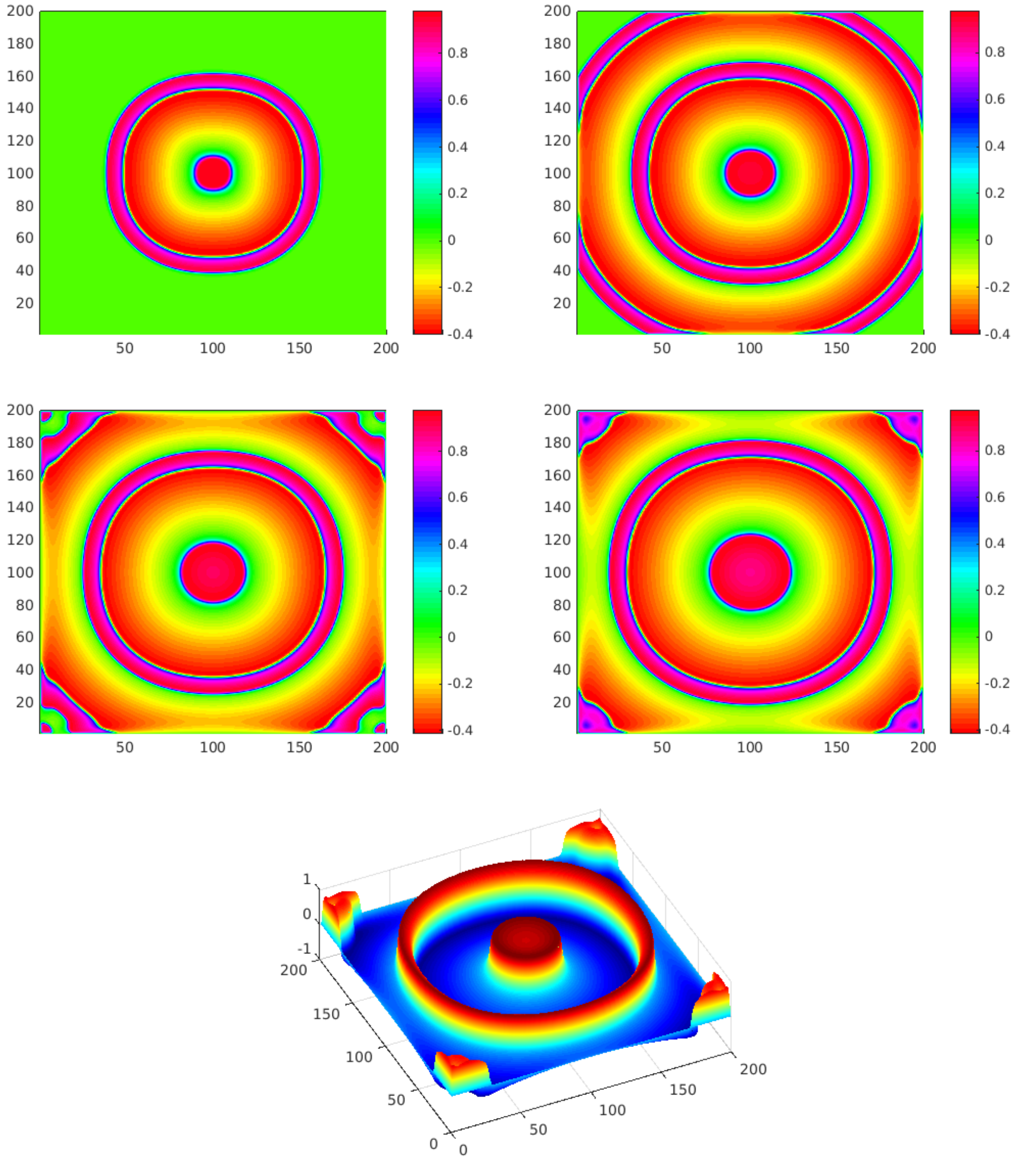

Figure 9. Evolution of states in the cellular automaton. 2D oscillatory medium, $a = -0.1$ $b = 0.0001$, $D_v^{x,y} = D_w^{x,y} = 0.1$, (a) $t = 250$, (b) $t = 500$, (c) $t = 750$, (d) $t = 1000$, (e) surface $z = v(x, y)$ at $t = 1000$. Central initial conditions $v(90 : 110, 90 : 100) = 0.9$ rest 0, $w(1 : 200, 1 : 200) = 0$.

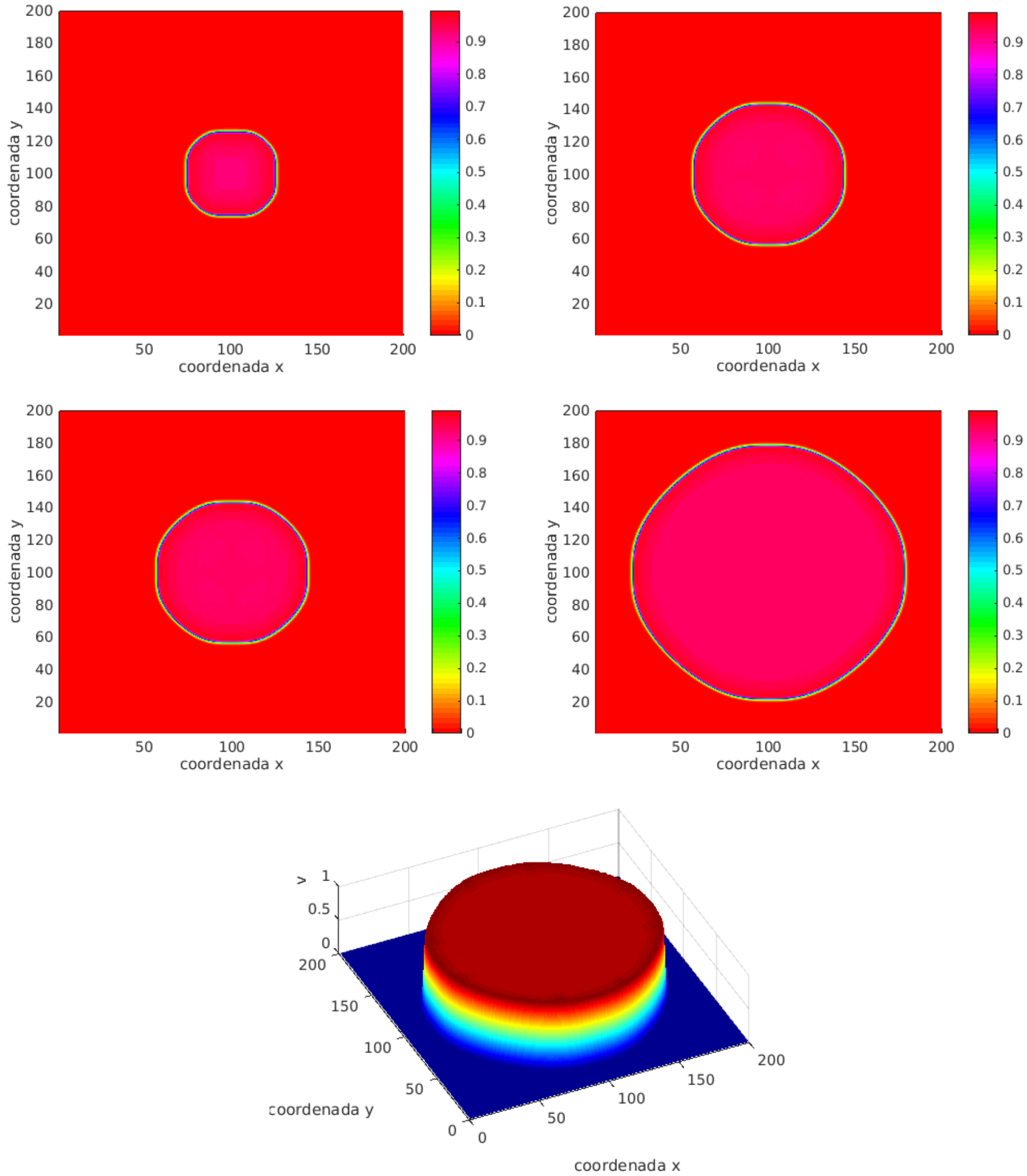

Figure 10. Evolution of states in the cellular automaton. 2D bistable medium, $a = -0.5$, $b = 10$, $D_v^{x,y} = D_w^{x,y} = 0.1$, (a) $t = 50$, (b) $t = 100$, (c) $t = 150$, (d) $t = 200$, (e) surface $z = v(x,y)$ at $t = 200$. Central initial conditions, $v(90:110, 90:110) = 0.9$ rest cells with noise 0.9 ± 0.01 (random), $w(90:110, 90:110) = 0.2$ rest cells zero.

5. Conclusions

This work presents a parametric study of the FitzHugh-Nagumo system. We determine the relationship between model parameters that leads to one, two, or three equilibrium points and their nature. These situations are called excitability, oscillatory, and bistable.

In the excitability case, a reaction-diffusion system can have traveling wave solutions with a profile that takes the form of a curve connecting two constants, called a pulse. Pulses are observed in excitability situations where the system has a single stable equilibrium. The pulse temporarily leaves this state and returns to it.

In the oscillation case, we have a wavefront propagating through the medium, representing the limit cycle. We observe that varying parameter a from -0.1 to 0.1, with $I = 0$ and other parameters fixed, generates a transition from oscillatory to excitable regime.

Fronts are observed in bistability situations: the reaction-diffusion system has two stable constant equilibria and the front evolves from one to the other.

Understanding the parameter relationships that lead to these different dynamics is of great theoretical and experimental utility, as it informs us about the conditions the system must satisfy to exhibit different evolution regimes.

Appendix A Animation Links

1D Excitable Medium
Graphs of $v(t)$ and $w(t)$:
https://drive.google.com/file/d/14rQDtfiNvcsJYth50Iez3rJ8d47szlk6/view?usp=sharing
Evolution of states in the cellular automaton:
https://drive.google.com/file/d/1edjsh8qZYn75JrZWYPWMKhvNfBKbPGN0/view?usp=sharing
1D Oscillatory Medium
Graphs of $v(t)$ and $w(t)$:
https://drive.google.com/file/d/1HYXKmVGSnxv2Ht3U-Celi4qXBGjAf6Sh/view?usp=sharing
Evolution of states in the cellular automaton:
https://drive.google.com/file/d/19-EiTrfJINgmICqPeKpBJhebvHXn_8_y/view?usp=sharing
1D Bistable Medium
Graphs of $v(t)$ and $w(t)$:
https://drive.google.com/file/d/1sVb5KRlstWrn1YoxzfsadTrc6BwtqOCy/view?usp=sharing
Evolution of states in the cellular automaton:
https://drive.google.com/file/d/1MZtoW-RMJf90cZ4OpB_sGSsefsCYODSt/view?usp=sharing
2D Excitable Medium
Evolution of states in the cellular automaton:
https://drive.google.com/file/d/10W2_hgf4Fw8qYDTH7soecQ2tQ6EbcEHl/view?usp=sharing
2D Oscillatory Medium
Evolution of states in the cellular automaton:
https://drive.google.com/file/d/1Vrgoiz6WAvDNgAUb3LCcnIFAOz5D_lLG/view?usp=sharing
2D Bistable Medium
Evolution of states in the cellular automaton:
https://drive.google.com/file/d/1ZV_ncjqaEORCzQ8L63MxE5ZHRhdEpmGy/view?usp=sharing

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Abbreviations

The following abbreviations are used in this manuscript:

FN	FitzHugh-Nagumo
1D	One-dimensional
2D	Two-dimensional

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