Article

q-SUMUDU TRANSFORMS OF PRODUCT OF GENERALIZED BASIC HYPERGEOMETRIC FUNCTION AND APPLICATION

V.K. Vyas [1], Ali A. Al -Jarrah[1] and S.D. Purohit[2,]

[1]Sur University College,
P. O. Box -440, P.C.-411,
Sur, Sultanate of Oman;
vkvyas@suc.edu.om,
aljarrah@suc.edu.om

[2] Department of HEAS (Mathematics),
Rajasthan Technical University,
Kota 324010, Rajasthan, India;
sunil_a_purohit@yahoo.com

* Correspondence: *vkvyas@suc.edu.om*; Tel.: +968-925-95-194

Abstract: The prim objective of commenced article is to determine q-sumudu transforms of a product of unified family of q-polynomials with basic (or q-) analogue of fox's H-function and q-analog of I-functions. Specialized cases of the leading outcome are further evaluated as q-sumudu transform of general class of q-polynomials and q-sumudu transforms of the basic analogues of Fox's H-function and I-functions.

Keywords: q-analogue of Sumudu transforms; q-analogue of hypergeometric functions, general class of q-polynomials, Fox's H-function; basic analogue of I-function.

---

1. Introduction

The q-calculus is a one of most interesting research field in current instant, usually due to its importance in the area of quantum physics and mathematical sciences. On the other hand, integral transform is one of the major tools to solve differential equations. Laplace, Fourier, Mellin and Hankel are frequently using for the same. The integral transform of Sumudu type was introduce through Watugala [1] in 1993, and he put forward to obtain the result of ordinary differential equations in problems of control engineering. Nowadays, the Sumudu transform is also an important integral transform to solve ordinary differential equation. The major influence of the Sumudu transform is that it is useful to obtain solution of problems beyond resorting to a different frequency domain, by cause of it conserve scale and unit properties.

In 2003, Belgacem et al. [2] gives explanatory observation for the Sumudu transform, and investigated number of fundamental properties of it. Albayrak et al. [3] gives q-analogue of the Sumudu transform, and they also obtained q-Sumudu transforms of certain special functions, including q-polynomials, see particularly [4].

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 24 October 2018 doi:10.20944/preprints201810.0548.v1

In the present paper, we aimed at to evaluate the q-Sumudu transforms for a product of general class of q-polynomials and basic analogue of some generalized special functions. Special cases of our main results have also been discussed.

2. Preliminaries

For our investigation we need the q-analog of Sumudu transform, introduced by Albayrak et al. [3], as follow:

$$S_q\{f(t);s\}=\frac{1}{(1-q)s}\int_0^s E_q(\frac{q}{s}t)f(t)\,d_qt\ ,\ s>0, \tag{2.1}$$

preamble to the collection of functions

$$A=\{f(t)|\exists M,\tau_1,\tau_2>0,|f(t)|<ME_q(|t|/\tau_j),t\in(-1)^j\times[0,\infty)\} \tag{2.2}$$

and

$$\boldsymbol{S}_q\{f(t);s\}=\frac{1}{(1-q)s}\int_0^\infty e_q(-\frac{1}{s}t)f(t)\,d_qt\ ,\ s>0, \tag{2.3}$$

provided the functions belongs to the set

$$B=\{f(t)|\exists M,\tau_1,\tau_2>0,|f(t)|<Me_q(|t|/\tau_j),t\in(-1)^j\times[0,\infty)\} \tag{2.4}$$

On the other hand, the q-version of exponential series are defined by

$$e_q(t)=\sum_{n=0}^{\infty}\frac{x^n}{(q;q)_n}=\frac{1}{(t;q)_\infty}\qquad |t|<1 \tag{2.5}$$

and

$$E_q(x)=\sum_{n=0}^{\infty}\frac{(-1)^n q^{n(n-1)/2}x^n}{(q;q)_n}=(t;q)_\infty\qquad (t\in C). \tag{2.6}$$

The basic improper integration cf. [5, 6], are defined as

$$\int_0^x f(t)\,d_qt=x\ (1-q)\sum_{k=0}^{\infty}q^k f(xq^k)\ , \tag{2.7}$$

$$\int_0^{\infty/A} f(x)\,d_qx=(1-q)\sum_{k\in Z}\frac{q^k}{A}f(\frac{q^k}{A}) \tag{2.8}$$

By using result of (2.7) and (2.8), the q-Sumudu transforms perhaps expressed as

$$S_q\{f(t);s\}=(q;q)_\infty\sum_{k=0}^{\infty}\frac{q^k f(sq^k)}{(q;q)_k} \tag{2.9}$$

and

 doi:10.20944/preprints201810.0548.v1

$$\boldsymbol{S}_q\{f(t);s\} = \frac{s^{-1}}{\left(-\frac{1}{s};q\right)_\infty} \sum_{k\in Z} q^k f\left(q^k\right)\left(-\frac{1}{s};q\right)_k . \tag{2.10}$$

For our purpose, we suppose α is real or complex and $|q|<1$, then the q-shifted factorial is expressed as under (see [7])

$$(a;q)_n = \begin{cases} 1 & ; \quad n=0 \\ (1-a)(1-aq)\cdots(1-aq^{n-1}) & ; \quad n\in\mathrm{N}, \end{cases} \tag{2.11}$$

and its natural extension is

$$(a;q)_\alpha = \frac{(a;q)_\infty}{(aq^\alpha;q)_\infty}, \quad \alpha\in\mathrm{C}. \tag{2.12}$$

For $n=\infty$ the definition (2.1) remains usful as a convergent infinite by-product, provided $|q|<1$, as under:

$$(a;q)_\infty = \prod_{j=0}^{\infty}(1-aq^j). \tag{2.13}$$

Moreover, the (basic) q-analog of the binomial (power) function $(x\pm y)^n$ cf. Ernst [8], is given by

$$(x\pm y)^{(n)} \equiv (x\pm y)_n \equiv x^n(\mp y/x;q)_n = x^n\sum_{k=0}^{n}\begin{bmatrix} n \\ k\end{bmatrix}_q q^{k(k-1)/2}(\pm y/x)^k \tag{2.14}$$

so that

$$\underset{q\to 1^-}{Lt}\ (x\pm y)^{(n)} = (x\pm y)^n, \tag{2.15}$$

where the q-version of binomial coefficient is given as:

$$\begin{bmatrix} \alpha \\ k\end{bmatrix}_q = \frac{(q^{-\alpha};q)_k}{(q;q)_k}(-q^\alpha)^k q^{-k(k-1)/2} \quad (k\in\mathrm{N}, \alpha\in\mathrm{C}). \tag{2.16}$$

Consider $f(x) = \sum\limits_{n=-\infty}^{+\infty} A_n x^n$ be a power series in x, defined over a bounded sequence of real or complex numbers, (cf.[7]) thereupon we have

$$f[x\pm y]_q = \sum_{n=-\infty}^{+\infty} A_n\, x^n(\mp y/x;q)_n . \tag{2.17}$$

Further, the q-gamma function is defined as follows: (cf. [6])

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 24 October 2018 doi:10.20944/preprints201810.0548.v1

$$\Gamma_q(\alpha) = \int_0^{1/(1-q)} x^{\alpha-1} E_q(q(1-q)x) d_q x \qquad (\alpha > 0) \tag{2.18}$$

$$\Gamma_q(\alpha) = K(A;\alpha) \int_0^{\infty/A(1-q)} x^{\alpha-1} E_q(-(1-q)x) d_q x \qquad (\alpha > 0), \tag{2.19}$$

where

$$K(A;t) = A^{t-1} \frac{\left(-q/A;q\right)_\infty}{\left(-q^t/A;q\right)_\infty} \cdot \frac{(-A;q)_\infty}{\left(-Aq^{1-t};q\right)_\infty} \qquad (t \in R). \tag{2.20}$$

For the variable t, the above function $K(A;t)$ gives the subsequent relation:

$$K(x;t+1) = q^t K(x;t). \tag{2.21}$$

Here the q-gamma function can also written in following form

$$\Gamma_q(\alpha) = \frac{(q;q)_\infty (1-q)^{1-\alpha}}{(q^\alpha;q)_\infty} = \frac{(1-q)_{\alpha-1}}{(1-q)^{\alpha-1}} = \frac{(q;q)_{\alpha-1}}{(1-q)^{\alpha-1}}, \tag{2.22}$$

where $\alpha \neq 0,-1,-2,\cdots$.

Now, we lead by looking back on a system of q-polynomials $f_{n,N}(x;q)$ in terms of a bounded complex sequence $\{S_{n,q}\}_{n=0}^{\infty}$, given as (cf. Srivastava and Agarwal [9])

$$f_{n,N}(x,q) = \sum_{j=0}^{[n/N]} \begin{bmatrix} n \\ N_j \end{bmatrix} S_{j,q} x^j \qquad (n = 0,1,2\ldots) \tag{2.23}$$

fixed up with positive integer N.

By virtue of the Mellin-Barnes category q-contour integral, Saxena and Kumar [10] made known a basic analog of the I-function as under:

$$I_{A_i,B_i}^{m,n}\left[x;q \,\middle|\, \begin{matrix} (a_j,\alpha_j)_{1,n}, (a_{ji},\alpha_{ji})_{n+1,A_i} \\ (b_j,\beta_j)_{1,m}, (b_j,\beta_j)_{m+1,B_i} \end{matrix}\right]$$

$$= \frac{1}{2\pi\omega} \int_C \frac{\prod_{j=1}^{m} G(q^{b_j-\beta_j s}) \prod_{j=1}^{n} G(q^{1-a_j+\alpha_j s})\, \pi\, x^s}{\sum_{i=1}^{r} \left\{ \prod_{j=m+1}^{B_i} G(q^{1-b_{ji}+\beta_{ji}s}) \prod_{j=n+1}^{A_i} G(q^{a_{ji}-\alpha_{ji}s}) \right\} G(q^{1-s}) \sin \pi s} ds, \tag{2.24}$$

where $0 \le m \le B_i; 0 \le n \le A_i$; $i = 1,2,\cdots,r$; r is finite; $\omega = \sqrt{-1}$; and

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 24 October 2018 doi:10.20944/preprints201810.0548.v1

$$G(q^a) = \left\{ \prod_{n=0}^{\infty} (1 - q^{a+n}) \right\}^{-1} = \frac{1}{(q^a;q)_\infty}.$$

Also $\alpha_j, \beta_j, \alpha_{ji}, \beta_{ji}$ are real and positive and a_j, b_j, a_{ji}, b_{ji} are arbitrary numbers of complex type.

The contour of integration C runs from $-i\infty$ to $+i\infty$ chosen so that all the poles of $G(q^{b_j - \beta_j s})$; $1 \le j \le m$, are to its right, and those of $G(q^{1-a_j+\alpha_j s})$, $1 \le j \le n$, are to its left and at least some $\varepsilon > 0$ distance away from the contour C. If $\mathrm{Re}[s \log(x) - \log \sin \pi s] < 0$, for huge amount of $|s|$ on the contour, in other words if $|\arg x| < \pi$, the basic integral defined above converges. It may be observe that the contour of integration C can be replaced by other suitably indented contours parallel to the imaginary axis.

It is readable to note that as $r = 1, A_1 = A; B_1 = B;$ definition (2.24) yields the basic equivalent (q-analog) of the Fox's *H*-function due to Saxena *et al.* [11], namely

$$H_{A,B}^{m,n}\left[x;q \,\middle|\, \begin{matrix} (a,\alpha) \\ (b,\beta) \end{matrix} \right] = \frac{1}{2\pi\omega} \int_C \frac{\prod_{j=1}^{m} G(q^{b_j - \beta_j s}) \prod_{j=1}^{n} G(q^{1-a_j+\alpha_j s})\, \pi\, x^s}{\prod_{j=m+1}^{B} G(q^{1-b_j+\beta_j s}) \prod_{j=n+1}^{A} G(q^{a_j - \alpha_j s}) G(q^{1-s}) \sin \pi s}\, ds, \tag{2.25}$$

Again, if we consider $\alpha_i = \beta_j = 1, \forall\, i$ and j in the definition (2.25), it reduces to a basic analog of the Meijer's *G*-function defined by Saxena *et al.* [11], namely

$$H_{A,B}^{m,n}\left[x;q \,\middle|\, \begin{matrix} (a,1) \\ (b,1) \end{matrix} \right] \equiv G_{A,B}^{m,n}\left[x;q \,\middle|\, \begin{matrix} a_1, \cdots, a_A \\ b_1, \cdots, b_B \end{matrix} \right]$$

$$= \frac{1}{2\pi\omega} \int_C \frac{\prod_{j=1}^{m} G(q^{b_j - s}) \prod_{j=1}^{n} G(q^{1-a_j+s})\, \pi\, x^s}{\prod_{j=m+1}^{B} G(q^{1-b_j+s}) \prod_{j=n+1}^{A} G(q^{a_j - s}) G(q^{1-s}) \sin \pi s}\, ds, \tag{2.26}$$

where $0 \le m_1 \le B, 0 \le n_1 \le A$ and $\mathrm{Re}[s \log(x) - \log \sin \pi s] < 0$.

Moreover, if we take $n = 0,\ m = B$ in the definition (2.26), we obtain the basic analog of *E*-function (MacRobert's function) as (cf [12]):

 doi:10.20944/preprints201810.0548.v1

$$G_{A,B}^{B,0}\left[x;q\left|\begin{matrix}a_1,\cdots,a_A\\ b_1,\cdots,b_B\end{matrix}\right.\right]=\mathrm{E_q}[\mathrm{B};\mathrm{b_j}:\mathrm{A};\mathrm{a_j}:x]. \tag{2.27}$$

For remarkable fundamental properties, along with numerous applications of the Meijer's G-function or Fox H-functions, one is allowed to refer the research treatise by Mathai and Saxena [13], [14] and Mathai et al. [15].

3. Main Results

In this segment, we found q-Sumudu transforms for a product of the universal system of q-polynomials and q-analog of the H- function and I-functions. We state the following theorems:

Theorem 3.1:- If $\{S_{n,q}\}_{n=0}^{\infty}$ be a bounded complex sequence, let $m_1, n_1; A, B$ be positive integers such that $0\le m_1\le B, 0\le n_1\le A$ and N be an arbitrary positive integer. Then the following q-Sumudu transform holds:

$$S_q\left\{x^{\lambda}f_{n,N}(x,q)H_{A,B}^{m_1,n_1}\left[x^k;q\left|\begin{matrix}(a_1,\alpha_1),\dots,(a_A,\alpha_A)\\(b_1,\beta_1),\dots,(b_B,\beta_B)\end{matrix}\right.\right];s\right\}$$

$$=\frac{s^{\lambda}}{G(q)}\sum_{j=0}^{[n/N]}\begin{bmatrix}n\\N_j\end{bmatrix}S_{j,q}s^j\,H_{A+1,B}^{m_1,n_1+1}\left[s^k;q\left|\begin{matrix}(-\lambda-j,k),(a_1,\alpha_1),\dots,(a_A,\alpha_A)\\(b_1,\beta_1),\dots,(b_B,\beta_B)\end{matrix}\right.\right];s\quad,\ k>0 \tag{3.1}$$

provided $\mathrm{Re}[s\log(x)-\log\sin\pi s]<0$.

Proof: On making use of definitions (2.23) and (2.25), the LHS, let L, of the main result (3.1) can be represented as

$$L=S_q\left\{x^{\lambda}\sum_{j=0}^{[n/N]}\begin{bmatrix}n\\N_j\end{bmatrix}S_{j,q}x^j\frac{1}{2\pi i}\int_C\frac{\prod_{j=1}^{m}G(q^{b_j-\beta_j s})\prod_{j=1}^{n}G(q^{1-a_j+\alpha_j s})\ \pi\,(x^k)^z}{\prod_{j=m+1}^{B_i}G(q^{1-b_{ji}+\beta_{ji}s})\prod_{j=n+1}^{A_i}G(q^{a_{ji}-\alpha_{ji}s})\,G(q^{1-s})\sin\pi s}ds\right\}$$

or

$$L=\frac{1}{2\pi\,\omega}\int_C\frac{\prod_{j=1}^{m}G(q^{b_j-\beta_j s})\prod_{j=1}^{n}G(q^{1-a_j+\alpha_j s})\ \pi}{\prod_{j=m+1}^{B_i}G(q^{1-b_{ji}+\beta_{ji}s})\prod_{j=n+1}^{A_i}G(q^{a_{ji}-\alpha_{ji}s})\,G(q^{1-s})\sin\pi s}S_q\left(x^{j+\lambda+kz}\right)ds.$$

Upon using the known result due [3], namely

$$S_q(x^{\alpha-1})=s^{\alpha-1}(1-q)^{\alpha-1}\Gamma_q(\alpha),$$

the above expression reduce to

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 24 October 2018 doi:10.20944/preprints201810.0548.v1

$$L=\frac{s^{\lambda}}{2\pi i}\int_C \frac{\prod_{j=1}^{m}G(q^{b_j-\beta_j s})\prod_{j=1}^{n}G(q^{1-a_j+\alpha_j s})\;\pi\,(s^k)^z(1-q)^{j+\lambda+ks}\,\Gamma_q(j+\lambda+ks+1)}{\prod_{j=m+1}^{B_i}G(q^{1-b_{ji}+\beta_{ji}s})\prod_{j=n+1}^{A_i}G(q^{a_{ji}-\alpha_{ji}s})\,G(q^{1-s})\sin\pi s}.$$

The desired right-hand side of (3.1) may be obtained by further simplification, as under

$$S_q\left\{x^{\lambda}f_{n,N}(x,q)\,H^{m_1,n_1}_{A,B}\left[x^k;q\left|\begin{matrix}(a_1,\alpha_1),\ldots,(a_A,\alpha_A)\\(b_1,\beta_1),\ldots,(b_B,\beta_B)\end{matrix}\right.\right];s\right\}$$

$$=\frac{s^{\lambda}}{G(q)}\sum_{j=0}^{[n/N]}\begin{bmatrix}n\\N_j\end{bmatrix}S_{j,q}s^j\,H^{m_1,n_1+1}_{A+1,B}\left[s^k;q\left|\begin{matrix}(-\lambda-j,k),(a_1,\alpha_1),\ldots,(a_A,\alpha_A)\\(b_1,\beta_1),\ldots,(b_B,\beta_B)\end{matrix}\right.\right];s\quad,\ k>0$$

provided $\mathrm{Re}[s\log(x)-\log\sin\pi s]<0$.

In similar fashion, we derive another result as under:

Theorem 3.2. Consider $\mathrm{Re}(\mu)>0$ and $\mathrm{Re}[s\log(x)-\log\sin\pi s]<0$, then the q-Sumudu transform for a product of q-analog of I-function and q-polynomials family $f_{n,N}(x;q)$ is given by the subsequent formula:

$$S_q\left\{x^{\lambda}f_{n,N}(x,q)I^{m,n}_{A_i,B_i}\left[\rho x^k;q\left|\begin{matrix}(a_j,\alpha_j)_{1,n},(a_{ji},\alpha_{ji})_{n+1,A_i}\\ \\(b_j,\beta_j)_{1,m},(b_j,\beta_j)_{m+1,B_i}\end{matrix}\right.\right]\right\}$$

$$=s^{\lambda}f_{n,N}(s,q)I^{m,n+1}_{A_i+1,B_i}\left[\rho x^k;q\left|\begin{matrix}(-j-\lambda,k),(a_j,\alpha_j)_{1,n},(a_{ji},\alpha_{ji})_{n+1,A_i}\\ \\(b_j,\beta_j)_{1,m},(b_j,\beta_j)_{m+1,B_i}\end{matrix}\right.\right],$$

(3.2)

where $0\le m\le B_i;0\le n\le A_i$; $i=1,2,\cdots,r$; r is finite, $|q|<1$, $\{S_{n,q}\}_{n=0}^{\infty}$ be a bounded complex sequence and λ is any arbitrary.

Proof: On making use of definitions (2.23) and (2.24), the left hand side (say L) of the main result (3.2) becomes

$$L=S_q\left\{x^{\lambda-1}\sum_{j=0}^{[n/N]}\begin{bmatrix}n\\N_j\end{bmatrix}S_{j,q}x^j\,\frac{1}{2\pi\,\omega}\int_C\frac{\prod_{j=1}^{m}G(q^{b_j-\beta_j s})\prod_{j=1}^{n}G(q^{1-a_j+\alpha_j s})\;\pi\,(\rho x^k)^s}{\sum_{i=1}^{r}\left\{\prod_{j=m+1}^{B_i}G(q^{1-b_{ji}+\beta_{ji}s})\prod_{j=n+1}^{A_i}G(q^{a_{ji}-\alpha_{ji}s})\right\}G(q^{1-s})\sin\pi s}ds\right\}$$

$$L=\sum_{j=0}^{[n/N]}\begin{bmatrix} n \\ N_j \end{bmatrix} S_{j,q}\, s^j \frac{1}{2\pi\,\omega}\int_C \frac{\prod_{j=1}^{m} G(q^{b_j-\beta_j s})\prod_{j=1}^{n} G(q^{1-a_j+\alpha_j s})\ \pi\,\rho^s}{\sum_{i=1}^{r}\left\{\prod_{j=m+1}^{B_i} G(q^{1-b_{ji}+\beta_{ji}s})\prod_{j=n+1}^{A_i} G(q^{a_{ji}-\alpha_{ji}s})\right\} G(q^{1-s})\sin \pi s} S_q\left(x^{j+\lambda+ks}\right) ds$$

Again using known result

$$S_q(x^\alpha)=s^\alpha(1-q)^\alpha\,\Gamma_q(\alpha+1),$$

the above expression reduce to

$$L=\sum_{j=0}^{[n/N]}\begin{bmatrix} n \\ N_j \end{bmatrix} S_{j,q}\, s^j \frac{x^{\lambda-1}}{2\pi\,\omega}\int_C \frac{\prod_{j=1}^{m} G(q^{b_j-\beta_j s})\prod_{j=1}^{n} G(q^{1-a_j+\alpha_j s})\ \pi\,(x^k)^z(1-q)^{j+\lambda+ks-1}\,\Gamma_q(j+\lambda+ks)}{\sum_{i=1}^{r}\left\{\prod_{j=m+1}^{B_i} G(q^{1-b_{ji}+\beta_{ji}s})\prod_{j=n+1}^{A_i} G(q^{a_{ji}-\alpha_{ji}s})\right\} G(q^{1-s})\sin \pi s}.$$

By simplification of above relation, we easily obtain RHS of the result (3.2).

4. Extraordinary Cases

In the indicated segment, we shall deal with certain particular cases of our main sequel. For example, if we set $r=1, A_1=A;$ and $B_1=B,$ in the main result (3.2), it yields to result (3.1).

Also, if we fixed $\alpha_i=\beta_j=1, \forall\, i$ and j in the result (3.1), we arrive at the coming result:

$$S_q\left\{x^\lambda f_{n,N}(x,q)G_{A,B}^{m,n}\left[x^k;q\left|\begin{matrix} a_1,\cdots,a_A \\ b_1,\cdots,b_B\end{matrix}\right.\right];s\right\}$$

$$=\frac{s^\lambda}{G(q)}\sum_{j=0}^{[n/N]}\begin{bmatrix} n \\ N_j \end{bmatrix} S_{j,q}\, s^j\, G_{A+1,B}^{m\ ,n\ +1}\left[s^k;q\left|\begin{matrix} -\lambda-j,k;a_1,..a_A, \\ b_1,\ldots,b_B\end{matrix}\right.\right];s\ \ ,k>0 \qquad (4.1)$$

By conveying particular values to the sequence $\{S_{n,q}\}_{n=0}^{\infty}$, our main result (Theorem 3.1) can be brought to bear certain q-Sumudu transforms involving orthogonal q-polynomials and the basic analog of Fox's H-function. To illustrate the same, we deal with the following cases.
By setting N=1, we have

$$f_{n,1}(x,q)=L_n^{(\alpha)}(x,q)$$

Thereupon, the result (3.1) yields to

$$S_q\left\{x^\lambda L_n^{(\alpha)}(x,q)\,H_{A,B}^{m_1,n_1}\left[x^k;q\left|\begin{matrix}(a_1,\alpha_1),\ldots,(a_A,\alpha_A) \\ (b_1,\beta_1),\ldots,(b_B,\beta_B)\end{matrix}\right.\right];s\right\}$$

 doi:10.20944/preprints201810.0548.v1

$$= \frac{s^{\lambda}}{G(q)} \sum_{j=0}^{n} S_{j,q} s^{j} H_{A+1,B}^{m_1,n_1+1}\left[s^{k};q \left| \begin{matrix} (-\lambda - j,k),(a_1,\alpha_1),\ldots,(a_A,\alpha_A) \\ (b_1,\beta_1),\ldots,(b_B,\beta_B) \end{matrix} \right. \right];s \quad ,k>0\,. \tag{4.2}$$

A detailed account of various hypergeometric orthogonal q-polynomials can be found in the research monograph by Koekoek *et al.* [16]. Therefore, one can derive similar type of results by taking into consideration the definitions of the q-polynomials given in [16]. We conclude with the remark that by suitably assigning values to the bounded sequence $\{S_{n,q}\}_{n=0}^{\infty}$, the q-image formulas given by the relations (3.1) and (3.2) being of general nature, and will lead to several q-Sumudu transforms for the product of orthogonal q-polynomials and the basic analogue of the generalized functions.

Acknowledgments: The author(s) would like to acknowledge that this paper has been financially supported by Sur University College Research Funds

References

1. G. K. Watugala, Sumudu transform: a new integral transform to solve differential equations and control engineering problems, Int. J. Math. Educ. Sci. Technol., 24 (1993), 35-43.
2. F.B.M. Belgacem, A.A. Karaball and S.L. Kalla, Analytical investigations of the Sumudu transform and applications to integral production equations, Mathematical Problems in Engineering, 3 (2003), 103–118.
3. D. Albayrak, S.D. Purohit, F. Ucar, On q-analogues of Sumudu transforms, An. St. Univ. Ovidius Constanta, Ser. Mat vol. 21(1), (2013), 239-260.
4. D. Albayrak, S.D. Purohit, F. Ucar, On q-Sumudu transforms of certain q-polynomials, Filomat 27(2), (2013), 411–427
5. F.H. Jackson, The application of basic numbers to Bessel's and Legendre's functions, Proc. London Math. Soc. 2 (2) (1905), 192–220
6. A. De Sole, V.G. Kac, On Integral representations of q-gamma and q-beta functions (English, Italian summary) , Atti Accad. Naz.Lincei Cl. Sci. Fis. Mat. Natur. Rend. Lincei (9) Mat. Appl. 16 (2005), 11–29.
7. G. Gasper and M. Rahman, Basic Hypergeometric Series, Encyclopedia of Mathematics and Its Applications, 35, Cambridge University Press, Cambridge and New York. 1990.
8. T. Ernst, A method for q-calculus, J. Nonlinear Math. Phys. 10(4) (2003), 487-525.
9. H. M. Srivastava and A. K. Agarwal, Generating functions for a class of q-polynomials, Ann. Mat. Pure Appl. 154 (1989), 99-109.
10. R.K. Saxena and R.K. Kumar, A basic analogue of the generalized H-function, Le Matematiche, Vol. L(1995), Fasc. II, 263-271.
11. R.K. Saxena, G.C. Modi and S.L. Kalla, A basic analogue of Fox's H-function. Rev. Tec. Ing. Univ. Zulia, 6 (1983), 139-143.
12. R.P. Agarwal, A q-analogue of MacRobert's generalized E-function, Ganita, 11 (1960), 49–63.
13. A.M. Mathai and R.K. Saxena, Generalized Hypergeometric Functions with Applications in Statistics and Physical Sciences. Lecture series in Math. No. 348, Springer-Verlag, Heidelberg, 1973.
14. A.M. Mathai and R.K. Saxena, The H-function with Applications in Statistics and other Disciplines, John Wiley & Sons. Inc. New York, 1978.
15. A.M. Mathai, R.K. Saxena, and H.J. Haubold, The H-function Theory and Applications, Springer, New York, 2010.
16. R. Koekoek, P. A. Lesky, and R. F. Swarttouw, Hypergeometric Orthogonal Plynomials and their q-Analogues, Springer Monographs in Mathematics. Springer-Verlag, Berlin, 2010.