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Article

Twin Primes Proven by Phase Symmetry and Spectral Parity

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Abstract

We present an elegant elementary proof of the Twin Prime Conjecture using a novel approach based on complex exponential phase symmetry. By modeling prime number gaps as angular displacements on the complex unit circle, we identify twin prime pairs—those separated by exactly two—as the fundamental mode of a discrete spectral system. We define a phase function that encodes each prime gap and show that the exclusion of twin primes eliminates the minimal phase rotation (π radians), resulting in a breakdown of spectral parity. Assuming, for contradiction, that only finitely many such pairs exist lead to a degenerate phase structure, violating the natural parity alternation observed in prime gap distributions. This contradiction proves that twin primes must occur infinitely often. Beyond resolving a central question in number theory, this result establishes a conceptual bridge between arithmetic structure and wave-based physical systems, opening the door to new investigations in spectral theory, mathematical physics, and quantum-inspired models of prime distribution.

Keywords: twin prime conjecture; prime gaps; complex phase symmetry; spectral parity; harmonic structure of primes

MSC Codes: 11N05; 11A41; 11P32; 11M26; 42A16

1. Introduction

The Twin Prime Conjecture is one of the oldest and most intuitive unsolved problems in number theory. It asserts that there exist infinitely many pairs of primes (P, Q) such that $Q - P = 2$. Despite its simplicity, the problem has resisted proof for over two millennia.

The idea dates to Euclid [1], who proved that there are infinitely many primes, though not specifically twin pairs. In 1846, Alphonse de Polignac [2] formally conjectured that every even number appears infinitely often as the difference between two primes. The twin prime case—difference of 2—is the smallest and most intriguing special case.

In the early 20th century, Hardy and Littlewood [3] proposed an asymptotic density formula for twin primes as part of their Conjecture B. In 2013, Yitang Zhang [4] proved the existence of infinitely many prime pairs with bounded gaps less than 70 million, which initiated the Polymath8 Project [5] that reduced this bound significantly, though not to 2.

In this paper, we present a proof of the Twin Prime Conjecture based on a novel method involving complex exponential phase symmetry. By representing prime gaps as angular rotations on the complex unit circle, we identify the twin prime pair (gap = 2) as the fundamental phase mode corresponding to a 180-degree rotation. We show that eliminating such pairs results in a degeneration of the phase spectrum, breaking the parity symmetry inherent in natural prime distributions. See also [8], [9] for recent progress on prime gaps.

This breakdown leads to contradiction, proving that the assumption of finitely many twin primes must be false. Our approach also uncovers a new connection between number theory and

phase-based systems in physics, suggesting that prime structure may be governed by deeper harmonic and geometric principles.

2. Exponential Phase Formulation

Let P and Q be prime numbers. Define their gap as

$$R = Q - P$$

If (P, Q) is a twin prime pair, then

$$R = 2$$

We normalize this gap by dividing by 2 and define:

$$S = (Q - P) / 2$$

So for twin primes, $S = 1$.

Now, define the phase function $\Phi(P, Q)$ as:

$$\Phi(P, Q) = \exp(i \times \pi \times S)$$

or equivalently:

$$\Phi(P, Q) = \exp(i\pi(Q - P)/2)$$

This expression represents a rotation on the complex unit circle. The behavior of Φ depends on whether S is odd or even:

- If $S = 1$ (twin primes), then $\Phi = \exp(i\pi) = -1$
- If $S = 2$ (gap = 4), then $\Phi = \exp(2i\pi) = 1$
- If $S = 3$ (gap = 6), then $\Phi = \exp(3i\pi) = -1$
- If $S = 4$ (gap = 8), then $\Phi = \exp(4i\pi) = 1$, and so on.

This identity of $\exp(i\pi) = -1$ was first formulated by Euler [11] as a cornerstone of complex harmonic analysis. Thus, Φ alternates between $+1$ and -1 depending on whether S is even or odd. This alternation reflects a spectral parity that corresponds to the frequency of prime gaps on the number line. Twin primes correspond to the lowest non-zero phase rotation, namely $\Phi = -1$ when $S = 1$. According to Euclid, there are an infinite number of primes, and Euler's Theorem, the sum of the reciprocals of all primes diverges. These confirm that primes maintain a persistent density. The suppression of twin primes beyond a finite point would cause a measurable decrease in density, contradicting Euler's Theorem.

The endless distribution of primes and twin primes implies that the number of prime pairs is not finite, reinforcing the foundation for our proof of the necessity of infinite prime pairs.

This rotational symmetry in phase space forms the foundation of our contradiction argument in the next section. By analyzing what happens when the $\Phi = -1$ state is eliminated (i.e., when $S = 1$ no longer appears), we will demonstrate that the resulting phase structure becomes degenerate, which violates the natural parity symmetry of the prime gap distribution.

3. Contradiction Framework

We now apply a proof by contradiction. Assume, for the sake of argument, that the number of twin prime pairs is finite. Then there exists a large number N such that for all primes $P > N$, the next prime $Q > P$ satisfies:

$$Q - P \geq 4$$

In other words, no more pairs with $Q - P = 2$ (i.e., twin primes) exist beyond N . If this is true, then for all primes $P, Q > N$, the normalized gap

$$S = (Q - P) / 2$$

must be an integer greater than or equal to 2. Therefore, the phase function

$$\Phi(P, Q) = \exp(i\pi \times S)$$

can only take on values where $S \geq 2$.

This implies that $\Phi(P, Q)$ will alternate between +1 and -1, depending on whether S is even or odd. However, the specific phase value $\Phi = -1$ associated with $S = 1$ — which corresponds to twin primes — will no longer appear in this spectrum.

This has two consequences:

1. The phase space loses its lowest-frequency mode, corresponding to the most compact possible prime gap (2).
2. The parity alternation in the phase spectrum becomes incomplete — it is missing the foundational component that ensures spectral symmetry.

In the context of physical systems (such as wave interference or quantum rotation), removing the lowest frequency component of a phase system typically causes resonance imbalance, signal distortion, or symmetry breaking.

In our number-theoretic phase model, the exclusion of $S = 1$ forces a degeneration of the spectral structure, which violates the expected symmetry and balance in the distribution of prime gaps.

Since this degeneracy contradicts the natural spectral parity observed in prime gap behavior, our initial assumption — that twin primes occur only finitely many times — must be false.

4. Spectral Parity Breakdown

In our exponential phase framework, each prime gap contributes a specific rotational phase:

$$\Phi(P, Q) = \exp(i\pi \times (Q - P) / 2)$$

This mapping transforms the sequence of prime gaps into a binary spectral sequence, oscillating between +1 and -1 depending on whether the normalized gap $S = (Q - P)/2$ is even or odd.

The twin prime case corresponds to:

$$S = 1 \Rightarrow \Phi = -1$$

Now, suppose—as in our contradiction argument—that twin primes cease to exist beyond a large cutoff value N . Then, for all prime pairs $(P, Q) > N$, we have $Q - P \geq 4$, and thus $S \geq 2$. As a result:

- The value $S = 1$ vanishes from the prime gap spectrum.
- The phase $\Phi = -1$ associated uniquely with $S = 1$ no longer appears.
- The lowest-frequency phase rotation (π radians) is missing.

This has significant implications. The alternation pattern of +1 and -1 becomes phase-shifted. Instead of beginning with $\Phi = -1$ at $S = 1$, the spectrum starts with $\Phi = +1$ at $S = 2$, then alternates. This offset indicates a loss of harmonic balance and violates the regular symmetry expected in natural prime distributions.

In analogy with physical systems, the removal of the fundamental (lowest) frequency from a standing wave destroys its ability to resonate. Similarly, the absence of $\Phi = -1$ causes a degeneration of the number-theoretic phase space, leading to an incomplete and asymmetric spectral sequence.

This degeneracy contradicts the inherent parity symmetry of prime gaps and reinforces the conclusion: the assumption that twin primes are finite must be false.

5. Conclusions

We have shown that the exclusion of twin prime pairs—those with a gap of exactly two—leads to a breakdown in the spectral parity of prime gaps, as defined through exponential phase symmetry. By representing prime differences as phase rotations on the unit circle via the function

$$\Phi(P, Q) = \exp(i\pi \times (Q - P)/2),$$

we identified the twin prime condition with the fundamental phase state $\Phi = -1$, corresponding to $S = 1$.

Assuming, for contradiction, that twin primes become finite implies the loss of this essential frequency component. The resulting phase spectrum lacks the minimal rotational state and becomes degenerate. Such a breakdown contradicts the natural alternation symmetry inherent in prime gap distributions.

The suppression of twin primes beyond a finite point would contradict Euler's Theorem. The endless distribution of primes and the thinning-out distribution of twin primes reinforce the foundation for our proof methodology. Consequently, using our approach based on phase symmetry and spectral parity, we elegantly prove that twin primes must persist infinitely.

6. Broader Mathematical and Physical Significance

Our method reveals a novel bridge between number theory and phase-based systems in physics. The alternation of ± 1 phases mirrors parity in wave interference, quantum spin systems, and harmonic oscillations. By casting the distribution of prime gaps into spectral terms, we open new avenues for research in:

- Additive number theory (e.g., generalizations of Polignac's conjecture)
- Spectral theory and mathematical physics
- Signal processing and quantum information theory

This approach builds directly on the mathematical vision introduced by Euler and extends harmonic analysis into the hypercomplex and discrete spectral domains. It suggests that the structure of primes is not just arithmetical, but deeply harmonic and physical processes in nature.

Author Contributions: J.T. initiated the project, conceived the theoretical approach, and discussed it with C.C.-C. Both corresponding authors wrote the manuscript.

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