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[Hsi-Jen Yeh](#)*

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Article

A Unified View of Physical Regularization: A Proper-Time Filter Function Approach

Hsi-Jen James Yeh

Massachusetts Maritime Academy, Buzzards Bay, MA 02532, USA; drjyeh@gmail.com

Abstract

The emergence of ultraviolet (UV) divergences in quantum field theory (QFT) is a direct consequence of singular behavior at short distances, mathematically represented by the lower limit of the Schwinger proper-time integral. While various regularization schemes exist, their physical motivations often appear disparate. This paper presents a unified framework that classifies physical regularization schemes by introducing a regulating filter function, $g(\tau)$, into the Schwinger proper-time integrand. We demonstrate that distinct physical postulates for modifying Planck-scale physics—such as a minimum proper time, a discretized spacetime path, or modified spacetime dimensionality—can be rigorously mapped to a specific choice for $g(\tau)$. We analyze the connection between the mathematical properties of this filter function (e.g., continuity, analyticity) and the physical nature of the resulting regularized field theory, showing how discontinuous or non-analytic filters lead to non-local integro-differential equations, while smooth, analytic filters lead to local, higher-derivative theories. This approach provides a powerful pedagogical tool for understanding and comparing physical regularization methods and their implications for the structure of spacetime.

Keywords: quantum field theory; regularization; renormalization; schwinger proper time; non-local field theory; planck scale physics; UV divergence; feynman path integral

1. Introduction: The Problem of Ultraviolet Divergences

Quantum Field Theory (QFT) stands as the most successful framework for describing the fundamental interactions of nature, with its predictions, particularly within Quantum Electrodynamics (QED), confirmed to extraordinary precision [1]. Despite this success, a foundational challenge persists: the calculation of quantum corrections through loop diagrams is plagued by ultraviolet (UV) divergences. These infinities arise from the integration over arbitrarily high momenta of virtual particles, a mathematical consequence of treating particles as ideal points interacting in a continuous spacetime. The standard solution to this problem is the procedure of renormalization [2,3], where these infinities are systematically absorbed into a redefinition of the theory's fundamental parameters.

To manage the infinities during intermediate calculations, a regularization scheme must be employed. Methods such as Pauli-Villars regularization [4] and dimensional regularization [5] have proven to be powerful mathematical tools. However, they are widely regarded as unphysical artifices, offering little insight into the physical reason for the absence of infinities in nature. This has motivated the search for *physical* regularization schemes—modifications to the theory's fundamental postulates that are rooted in a plausible physical principle. Such principles often emerge from considerations of quantum gravity, suggesting that the smooth spacetime of classical relativity is an approximation that breaks down at the Planck scale. Postulates such as a minimum physical length [6–8], a maximum acceleration [9], or the effects of spacetime foam [10] all point towards a universe where the point-like singularities of standard QFT are naturally resolved.

This paper proposes a unified framework to understand and classify these physical regularization schemes. We demonstrate that the Schwinger proper-time representation of the Feynman propagator [11,12] provides a universal arena for implementing such modifications. We introduce the concept

of a *proper-time filter function*, $g(\tau)$, which modifies the Schwinger integrand and serves as a direct mathematical encoding of the underlying physical postulate. By analyzing the mathematical properties of $g(\tau)$, we can directly deduce the physical nature of the resulting regularized theory, providing a clear and powerful bridge between Planck-scale physics and observable field theory.

2. The Schwinger Proper-Time Framework

The Schwinger proper-time representation provides the ideal mathematical setting for our analysis. It recasts the Feynman propagator, the central object in perturbative QFT, as an integral over a single parameter, the proper time τ . For a scalar particle of mass m , the propagator in momentum space is given by:

$$\Delta_F(p) = \frac{i}{p^2 - m^2 + i\epsilon} = i \int_0^\infty d\tau e^{-i\tau(p^2 - m^2 + i\epsilon)} \quad (1)$$

This representation has a profound physical interpretation. The path integral for the propagator can be decomposed into a sum over all possible proper-time durations of the particle's worldline. The integral in Equation (1) is precisely this sum. The UV divergences in loop calculations can be traced directly to the lower limit of this integral, the $\tau \rightarrow 0$ region, which corresponds to contributions from paths of infinitesimal proper length and, consequently, infinite momentum.

2.1. The Filter Function Formalism

We propose that any physical regularization scheme that aims to tame the $\tau \rightarrow 0$ behavior can be expressed as a modification of the Schwinger integrand by a regulating filter function, $g(\tau)$. The modified propagator, $\Delta_F^{\text{mod}}(p)$, is then given by the general form:

$$\Delta_F^{\text{mod}}(p) = i \int_0^\infty d\tau g(\tau) e^{-i\tau(p^2 - m^2)} \quad (2)$$

The standard, unregularized theory corresponds to the trivial case where $g(\tau) = 1$. For a regularization to be effective, $g(\tau)$ must satisfy the following general conditions:

1. $\lim_{\tau \rightarrow 0} g(\tau) = 0$ (To suppress the UV divergence)
2. $\lim_{\tau \rightarrow \infty} g(\tau) \rightarrow 1$ (To recover the correct infrared, low-energy behavior)

This framework is powerful because the choice of $g(\tau)$ is not arbitrary; it is a direct mathematical consequence of the chosen physical principle. In the following sections, we will derive the specific form of $g(\tau)$ for several distinct physical postulates and analyze the resulting field theories.

3. Classification of Physical Regularization Schemes

We now analyze several distinct physical postulates for modifying Planck-scale physics and derive their corresponding filter functions.

3.1. Method 1: Minimum Proper Time (Integral Cutoff)

Physical Principle:

Based on the postulate of a minimum length for any physical worldline, this method posits that no process can occur over a proper-time interval shorter than a universal minimum, τ_{min} . For physically motivated reasons, this is often identified with the Planck time, $t_P = \sqrt{\hbar G/c^5}$. This principle restricts the path integral to only those paths with total proper time $T \geq \tau_{\text{min}}$, as developed in [13].

Filter Function:

This hard cutoff is represented by the Heaviside step function.

$$g(\tau) = \Theta(\tau - \tau_{\text{min}}) \quad (3)$$

Field Equation Impact:

The modified propagator is $\Delta_F^{\text{mod}}(p) = (p^2 - m^2)^{-1} e^{-i\tau_{\min}(p^2 - m^2)}$, which introduces powerful exponential suppression. The resulting field theory is fundamentally non-local at the Planck scale, corresponding to an integro-differential equation of motion that smears out interactions that would otherwise be point-like.

3.2. Method 2: Minimum Proper Time (Discretized Path)

Physical Principle:

An alternative formulation of the minimum proper time evolution is that a particle's worldline consists of a series of discrete steps. The fundamental duration $\tau_{\min}$ is also associated with the Planck time. The total proper time is quantized, $\tau_n = n \cdot \tau_{\min}$ for $n = 1, 2, \dots$, turning the path integral into a discrete sum, a scheme detailed in [14].

Filter Function:

This discretization is encoded by a sum of Dirac delta functions.

$$g(\tau) = \tau_{\min} \sum_{n=1}^{\infty} \delta(\tau - n\tau_{\min}) \quad (4)$$

Field Equation Impact:

The modified propagator becomes a geometric series, $\Delta_F^{\text{mod}}(p) \propto (1 - e^{-i\tau_{\min}(p^2 - m^2)})^{-1}$. This filter is non-analytic and produces a non-local theory on the order of the Planck scale which the short-distance structure of spacetime.

3.3. Method 3: Smooth Damping (Higher Derivatives)

Physical Principle:

Instead of forbidding short paths, one can smoothly penalize them. This is the logic behind higher-derivative theories, first explored by Pais and Uhlenbeck [15] and later applied to gravity by Stelle [16]. The presence of higher-order kinetic terms acts to smooth out divergences. The simplest implementation is Pauli-Villars regularization [4], which introduces a heavy "ghost" particle of mass M .

Filter Function:

The Pauli-Villars scheme is equivalent to a smooth, analytic filter.

$$g(\tau) = (1 - e^{-iM^2\tau}) \quad (5)$$

Field Equation Impact:

This yields the propagator $\Delta_F^{\text{mod}}(p) = i(p^2 - m^2)^{-1} - i(p^2 - M^2)^{-1}$. The smoothness of $g(\tau)$ leads to a local, higher-derivative field equation: $(\square + m^2)(\square + M^2)\phi(x) = 0$. This locality, however, comes at the cost of introducing an unphysical ghost state, a central challenge for such theories [17,18].

3.4. Method 4: Power-Law Suppression (Anomalous Spacetime Dimension)

Physical Principle:

In some quantum gravity scenarios, such as Asymptotic Safety [19,20] or Horava-Lifshitz gravity [21], spacetime exhibits a scale-dependent ("running") dimensionality. At high energies (short distances), the spectral dimension of spacetime can reduce. As argued by Padmanabhan [22], this anomalous diffusion of particles at short scales modifies the density of states in the proper-time path integral.

Filter Function:

This modification corresponds to weighting the integrand with a power of τ .

$$g(\tau) \propto \tau^\alpha \quad (\alpha > 0) \quad (6)$$

Field Equation Impact:

The propagator becomes $\Delta_F^{\text{mod}}(p) \propto (p^2 - m^2)^{-(\alpha+1)}$. Like the Pauli-Villars method, this analytic filter leads to a local, higher-derivative field equation: $(\square + m^2)^{\alpha+1}\phi(x) = 0$. Once again, the analyticity of the filter results in a local but ghost-ridden theory.

3.5. Method 5: Action Cost (Exponential Non-Locality)

Physical Principle:

One can introduce an energetic penalty for short-time evolution. This is the basis of modern non-local field theories, which are constructed to be ghost-free from the outset. Early ideas by Yukawa [23] were developed into consistent theories by authors like Moffat and Krasnikov [24,25].

Filter Function:

This approach corresponds to an exponential filter function, where Λ is the scale of non-locality.

$$g(\tau) = e^{-1/(\Lambda^2\tau)} \quad (7)$$

Field Equation Impact:

This yields a propagator suppressed by a non-analytic exponential factor. The corresponding wave operator contains an infinite number of derivatives, $D(p) \propto (p^2 - m^2)e^{(p^2 - m^2)/\Lambda^2}$, leading to a non-local integro-differential equation. The non-analyticity of $g(\tau)$ (an essential singularity at $\tau = 0$) ensures the theory is free of ghost states [26].

4. Analysis and Discussion

The classification in the previous section reveals a deep and elegant connection between the mathematical nature of the filter function $g(\tau)$ and the physical structure of the resulting regularized QFT. We can summarize our findings with a powerful conjecture:

- *Non-Analytic Filters $\rightarrow$ Non-Local Theories:* Filter functions that are non-analytic at $\tau = 0$ (e.g., a step function, a sum of deltas, or a function with an essential singularity) result in field theories that are fundamentally non-local. Their equations of motion are integro-differential and are generally free of unphysical ghost states.
- *Analytic Filters $\rightarrow$ Local, Higher-Derivative Theories:* Filter functions that are analytic at $\tau = 0$ (e.g., the Pauli-Villars filter or a power-law) result in field theories that are local but contain higher-order derivatives. These theories are often simpler to handle mathematically but typically suffer from the presence of ghost particles with negative norm.

This connection presents a clear choice: a physically well-behaved (ghost-free) but mathematically complex non-local theory, or a mathematically simpler local theory that contains unphysical states. The filter function formalism provides a powerful "bottom-up" approach to constructing UV-complete field theories. It begins with the standard framework of QFT and introduces a modification, encoded by $g(\tau)$, motivated by a specific quantum gravity principle. This allows for a systematic exploration of different Planck-scale hypotheses within a single, unified language.

5. Conclusions

We have introduced a unified framework for understanding and classifying physical regularization schemes in quantum field theory based on a proper-time filter function, $g(\tau)$, which modifies the Schwinger integrand. We have shown that a variety of distinct physical postulates for modifying

Planck-scale physics—a minimum proper time, a discretized worldline, a scale-dependent spacetime dimension, or exponential non-locality—can be rigorously mapped to a specific mathematical choice for $g(\tau)$.

Our analysis reveals a profound link between the mathematical properties of the filter function and the physical nature of the resulting theory. Non-analytic filters lead to ghost-free but non-local integro-differential equations. In contrast, analytic filters lead to local, higher-derivative field theories that are often plagued by unphysical ghost states.

This filter function formalism serves as a powerful pedagogical tool, unifying seemingly disparate physical ideas into a single, coherent mathematical structure. It clarifies the fundamental trade-offs involved in constructing a UV-finite theory of fundamental interactions and provides a clear roadmap for exploring the phenomenological consequences of different models of Planck-scale physics. By focusing on the proper-time representation, we can directly probe the consequences of modifying the quantum particle's history, offering a physically intuitive bridge between the known world of the Standard Model and the frontiers of quantum gravity.

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References

1. D. Hanneke, S. Fogwell, and G. Gabrielse, *Phys. Rev. Lett.* **100**, 120801 (2008).
2. J. Collins, *Renormalization* (Cambridge University Press, 1984).
3. M. E. Peskin and D. V. Schroeder, *An Introduction to Quantum Field Theory* (Addison-Wesley, 1995).
4. W. Pauli and F. Villars, *Rev. Mod. Phys.* **21**, 434 (1949).
5. G. 't Hooft and M. Veltman, *Nucl. Phys. B* **44**, 189 (1972).
6. H. S. Snyder, *Phys. Rev.* **71**, 38 (1947).
7. D. Amati, M. Ciafaloni, and G. Veneziano, *Phys. Lett. B* **216**, 41 (1989).
8. L. J. Garay, *Int. J. Mod. Phys. A* **10**, 145 (1995).
9. E. R. Caianiello, *Lett. Nuovo Cimento* **32**, 65 (1981).
10. J. A. Wheeler, *Ann. Phys.* **2**, 604 (1957).
11. J. Schwinger, *Phys. Rev.* **82**, 664 (1951).
12. R. P. Feynman, *Phys. Rev.* **80**, 440 (1950).
13. H.-J. J. Yeh, *Minimum Proper-Time Evolution and Affine Propagation: A Lorentz-Invariant Cutoff for QED and Quantum Gravity*, Zenodo (2024), doi:10.5281/zenodo.16881998.
14. H.-J. J. Yeh, *Discretized Proper-Time Path Integral and the Affine Propagator: A Lattice-Based Regularization Scheme*, Zenodo (2024), doi:10.5281/zenodo.16904537.
15. A. Pais and G. E. Uhlenbeck, *Phys. Rev.* **79**, 145 (1950).
16. K. S. Stelle, *Phys. Rev. D* **16**, 953 (1977).
17. T. D. Lee and G. C. Wick, *Nucl. Phys. B* **9**, 209 (1969).
18. B. Grinstein, D. O'Connell, and M. B. Wise, *Phys. Rev. D* **77**, 025012 (2008).
19. S. Weinberg, in *General Relativity: An Einstein Centenary Survey*, edited by S. W. Hawking and W. Israel (Cambridge University Press, 1979).
20. M. Reuter, *Phys. Rev. D* **57**, 971 (1998).
21. P. Horava, *Phys. Rev. D* **79**, 084008 (2009).
22. T. Padmanabhan, *Phys. Rev. D* **81**, 124040 (2010).
23. H. Yukawa, *Phys. Rev.* **77**, 219 (1950).
24. J. W. Moffat, *Phys. Rev. D* **41**, 1177 (1990).
25. N. V. Krasnikov, *Theor. Math. Phys.* **73**, 1184 (1987).
26. T. Biswas, E. Gerwick, T. Koivisto, and A. Mazumdar, *Phys. Rev. Lett.* **108**, 031101 (2012).

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