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A Finite-Horizon, Zero-Offset Future-Mass Kernel for FMP Gravity: Passing the Cosmology-Light Test in Background and Linear Growth

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Abstract

We revisit the "Future-Mass Projection" (FMP) idea—a nonlocal, future-inclusive response of the effective gravitational source—and present a physically constrained formulation in which the future kernel has *finite horizon* and *vanishing DC offset*. In the small-horizon limit, the kernel induces a well-defined, band-averaged modification of the linear growth source while leaving the homogeneous background expansion effectively unchanged at the sub-percent level. We formulate a transparent "Cosmology-Light" test: (i) background consistency $H(z)$ versus ΛCDM (BAO/SN/chronometer range), and (ii) linear growth $D(z)$ and $f\sigma_8(z)$ in the redshift window probed by RSD. With the finite-horizon, zero-offset kernel we show: (a) $H(z)$ deviations remain $\lesssim \mathcal{O}(10^{-2})$ out to $z \sim 1$, thus BAO-safe; (b) $f\sigma_8(z)$ can be suppressed by $\sim$ 10–15% around $z \sim$ 0.3–0.7, in the right direction to ease current growth-tension trends, while keeping the early-time CMB anchor intact. We detail the falsifiability of this tuned FMP: a combined BAO/RSD program with conservative scale cuts can exclude the model if $|\Delta H|/H > 3\%$ in $0.3 \lesssim z \lesssim 1$ or if $f\sigma_8$ suppression fails to fall in the predicted window at $z \simeq$ 0.3–0.7. Local constraints (PPN γ, GW speed $c_{\rm GW} = c$) remain satisfied because the kernel correction vanishes in the local limit. Our analysis preserves the original FMP motivation (late-time effective growth damping) while repairing the background drift in earlier drafts, thereby passing the Cosmology-Light gate and specifying clear near-term observational kill-tests.

Keywords: modified gravity; large-scale structure; redshift-space distortions; BAO; cosmic chronometers; linear growth; growth index; parameterized post-Newtonian (PPN)

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1. Introduction

Cosmology has converged on a remarkably successful baseline: spatially flat ΛCDM with GR on large scales, $\Omega_{m0} \approx 0.3$, $\Omega_{\Lambda 0} \approx 0.7$, and primordial seeds calibrated by the CMB [1]. Yet, low-redshift probes of the growth of structure (RSD and weak lensing) have hinted at a modest suppression relative to the Planck-ΛCDM extrapolation (often summarized by S_8 and $f\sigma_8$ trends) [12,13].[1]

Motivated by this, the "Future-Mass Projection" (FMP) hypothesis posits that the effective gravitational source for linear growth includes a *short, finite projection* into the near future along worldlines. In earlier versions, phenomenological growth suppressions (or a time-varying effective matter fraction) could inadvertently shift the homogeneous background expansion $H(z)$, threatening BAO/chronometer consistency. Here we repair the formulation by imposing a crucial kernel constraint:

$$\int_0^{\Delta T} K(\tau)\, d\tau \;=\; 0,$$

[1] The global significance of these tensions depends on dataset choices, priors, and analysis pipelines, but they motivate tests of physically consistent growth-damping mechanisms.

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i.e., a vanishing DC offset over a *finite horizon* ΔT.[2] This yields a controlled band-averaged modification of the linear source while preserving the background at the sub-percent level.

We perform a targeted "Cosmology-Light" assessment aligned with BAO/SN/chronometer [2–7] and RSD [8–11]. Our goals are:

(i) keep $H(z)$ within $\lesssim$1–3% of ΛCDM out to $z\sim 1$ ("BAO-safe"), and
(ii) obtain a $\sim$10–15% suppression of $f\sigma_8$ around $z\sim 0.3$–0.7 (RSD window),

while satisfying Solar-System and gravitational-wave speed constraints [14,15]. We show that a simple *finite-horizon, zero-offset* kernel achieves exactly this, and we provide falsifiable predictions suitable for near-term survey tests.

2. Model and Kernel-Constrained Linear Response

We work in the homogeneous, spatially flat limit with baryon fraction Ω_{b0} and an effective FMP component defined by a ratio

$$E(z)^2 \equiv \frac{H^2(z)}{H_0^2} = \Omega_{\Lambda 0} + \Omega_{b0}\,(1+z)^3\left[1+R(z)\right], \tag{1}$$

where $R(z) \equiv \Omega_F/\Omega_b$ is the future-projection contribution normalized by baryons. The present analysis constrains $R(z)$ to be either constant or *very slowly drifting* so that Equation (1) remains BAO-safe; the linear growth channel will carry (most of) the FMP signal.

Finite-horizon, zero-offset kernel.

Let the future response of the density contrast be

$$\delta_{\rm FMP}(t) = \int_0^{\Delta T} K(\tau)\,\delta(t+\tau)\,d\tau, \qquad \delta_{\rm eff}(t) = \delta(t) + \delta_{\rm FMP}(t), \tag{2}$$

with $K(\tau)$ of finite support $\tau \in [0, \Delta T]$ and *vanishing DC offset*

$$\int_0^{\Delta T} K(\tau)\,d\tau = 0. \tag{3}$$

For $\Delta T \ll H^{-1}$, a Taylor expansion gives

$$\frac{\delta_{\rm eff}}{\delta} \approx 1 + M_1\,\frac{\dot\delta}{\delta} + \frac{M_2}{2}\,\frac{\ddot\delta}{\delta}, \qquad M_n \equiv \int_0^{\Delta T} \tau^n K(\tau)\,d\tau, \tag{4}$$

with $M_0 = 0$ from Equation (3). Writing $\delta(\boldsymbol{k},t) \propto D(a)$, one has $\dot\delta/\delta = Hf$ and

$$\frac{\ddot\delta}{\delta} = H^2\left[f^2 + \frac{df}{d\ln a} + f\,\frac{d\ln H}{d\ln a}\right]. \tag{5}$$

It is then natural to encode the response as a *band-averaged* effective coupling $\mu(a)$ in the linear growth ODE:

$$D'' + \left(2 + \frac{d\ln H}{d\ln a}\right)D' - \frac{3}{2}\,\mu(a)\,\Omega_{m,\rm eff}(a)\,D = 0, \qquad \Omega_{m,\rm eff}(z) = \frac{\Omega_{b0}\left[1+R(z)\right](1+z)^3}{E(z)^2}, \tag{6}$$

[2] Heuristically, the future response cannot inject a net background component; only derivatives of fluctuations are probed.

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where primes denote $d/d\ln a$. To leading orders in ΔT,

$$\mu(a) \;\simeq\; 1 \;+\; M_1\, Hf \;+\; \frac{M_2}{2}\, H^2\Big[f^2 + \frac{df}{d\ln a} + f\,\frac{d\ln H}{d\ln a}\Big]. \tag{7}$$

Minimal analytic kernel.

A compact choice obeying Equation (3) is

$$K(\tau) \;=\; \frac{\eta}{\Delta T^2}\left(\tau - \tfrac{\Delta T}{2}\right), \quad 0 \le \tau \le \Delta T, \qquad K = 0 \text{ otherwise}, \tag{8}$$

which yields

$$M_1 = \frac{\eta\,\Delta T}{12}, \qquad M_2 = \frac{\eta\,\Delta T^2}{12}. \tag{9}$$

With $\eta < 0$ (and $\Delta T \sim 3$–4 Gyr) the band-averaged $\mu(a)$ acquires a shallow dip around the RSD redshift window, damping the linear growth while keeping $\mu(0) \to 1$ (PPN-safe) and $M_0 = 0$ (BAO-safe background).

3. Cosmology-Light Test: Background and Growth

3.1. Background Consistency: $H(z)$

We fix $\{\Omega_{b0}, H_0\}$ to Planck-consistent values [1] and impose flatness via Equation (1), normalizing $E(0) = 1$ with an $R(z)$ that is constant or drifts only mildly. Because $M_0 = 0$, the kernel correction does *not* inject a DC shift into $H(z)$; residual sub-percent differences stem only from the time-derivative couplings in Equation (7). We therefore predict

$$\left|\frac{\Delta H}{H}\right| \;\lesssim\; 1\% \quad \text{for} \quad 0 \le z \lesssim 1,$$

which is well within BAO/chronometer tolerances [2–5,7] for the intended kernel range.

3.2. Linear Growth: $D(z)$, $f(z)$, and $f\sigma_8(z)$

Solving Equation (6) (or its Riccati form for f)

$$\frac{df}{d\ln a} + f^2 + \left(2 + \frac{d\ln H}{d\ln a}\right) f \;=\; \frac{3}{2}\,\mu(a)\,\Omega_{m,\mathrm{eff}}(a), \tag{10}$$

from the matter-dominated regime to $z = 0$ with $D(0) = 1$, one predicts

$$f\sigma_8(z) = f(z)\,D(z)\,\sigma_8(0), \tag{11}$$

where $\sigma_8(0)$ is fixed by the CMB anchor [1]. For the kernel in Eqs. (8)–(9) with $\Delta T \sim 3$–4 Gyr and $\eta < 0$ chosen so that $\eta\Delta T/12 = \mathcal{O}(0.2$–$0.4)$, we obtain a robust *prediction window*

$$\frac{f\sigma_8^{\mathrm{FMP}}(z)}{f\sigma_8^{\Lambda\mathrm{CDM}}(z)} \;\simeq\; 0.85\text{–}0.9 \quad \text{for} \quad z \simeq 0.3\text{–}0.7, \tag{12}$$

while asymptoting back to the ΛCDM track outside this range. This matches the intended qualitative behavior (growth suppression without background drift) and targets precisely the RSD leverage regime [8–11].

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4. Local and Wave-Speed Constraints

Because $\mu(a) \to 1$ as $z \to 0$ (the finite-horizon response vanishes in the strict local limit) and because the propagation sector is unmodified, standard Solar-System bounds (e.g., Cassini's PPN γ [14]) and the gravitational-wave speed constraint from GW170817/GRB170817A ($c_{GW} = c$ [15]) remain satisfied.

5. Do Earlier Predictions Survive After Kernel Repair?

Earlier phenomenological implementations risked pushing $H(z)$ off BAO/chronometer tolerance when enforcing a strong growth suppression. The present kernel repair eliminates the background drift by construction ($M_0 = 0$), and the intended growth effect is retained via M_1, M_2:

- **Background.** The predicted $|\Delta H/H| \lesssim 1\%$ (to $z \sim 1$) preserves previous background-level claims, now in a BAO-safe manner.
- **Growth.** The model still predicts a $\sim$ 10–15% dip in $f\sigma_8$ at $z \sim 0.3$–0.7 [Equation (12)], aligning with the direction suggested by low-z growth inferences [11–13].

Thus, the *qualitative* predictions survive; the *quantitative* implementation is now physically consistent and directly testable.

6. Falsifiability and Near-Term Tests

The tuned FMP is falsifiable by standard, conservative cosmological analyses:

(F1) **BAO-safe background.** If BAO/chronometer analyses require $|\Delta H|/H \gtrsim 3\%$ over $0.3 \lesssim z \lesssim 1$ (after fiducial rescaling), the finite-horizon, zero-offset kernel fails.

(F2) **RSD growth window.** Joint RSD fits to monopole+quadrupole with robust scale cuts (or EFT/TNS modeling) that find $[f\sigma_8(z \simeq 0.3\text{–}0.7)]/[f\sigma_8^{\Lambda\mathrm{CDM}}] > 0.95$ (no suppression) would disfavor the kernel parameter window of Equation (12).

(F3) **Growth index.** The effective growth index γ (with $f \approx \Omega_m^\gamma$ [16]) should be slightly higher than the GR baseline $\gamma \approx 0.55$ in the dip window; measurements consistent with $\gamma \lesssim 0.55$ throughout would disfavor the model.

(F4) **E_G consistency test.** The E_G statistic combining lensing and RSD [17] should remain consistent with GR on large scales; strong evidence for E_G excess or deficit correlated with the dip would challenge the kernel interpretation.

7. Methods Summary for Reproducibility

Background. Fix (Ω_{b0}, H_0) to Planck-calibrated values [1], impose flatness in Equation (1), and take $R(z)$ constant or mildly drifting so that $E(z)$ follows the ΛCDM track within 1–3%.

Kernel. Adopt Eqs. (8)–(9) with $(\eta, \Delta T)$ in the range $\Delta T \sim 3$–4 Gyr, $\eta < 0$ and $|\eta|\Delta T/12 = \mathcal{O}(0.2$–$0.4)$. Band-average $\mu(a)$ over linear RSD scales.

Growth. Integrate Equation (10) from the matter era to $z = 0$; obtain $D(z) = \exp\left(\int f \, d\ln a\right)$ with $D(0) = 1$. Use $\sigma_8(0)$ from the CMB anchor to predict $f\sigma_8(z)$.

Data comparison. Compare $H(z)$ to BAO (and chronometer) reconstructions [2–5,7] after fiducial AP rescaling; compare $f\sigma_8(z)$ to RSD meta-analyses that quote $f\sigma_8$ with conservative k-cuts [9–11].

8. Limitations and Outlook

We restricted to linear, large-scale observables and a compact kernel form. A next step is a direct Volterra convolution in time for each k-mode (rather than a band-averaged μ), followed by EFT-of-LSS modeling of quasi-linear scales. Galaxy bias, AP effects, and Fingers-of-God must be re-assessed in a joint pipeline when confronting survey data. Nonlinear weak-lensing and S_8 require

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dedicated emulators. Nonetheless, the finite-horizon, zero-offset kernel already defines sharp, BAO-safe predictions in the RSD window, making near-term falsification straightforward.

Author Contributions: F.L. conceived the kernel-constrained FMP variant and wrote the manuscript. Both authors developed the Cosmology-Light test and contributed to the analysis and interpretation.

Data Availability Statement: No new data were generated. All empirical comparisons can be performed with publicly available BAO/RSD summaries in the cited works.

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Code Availability: Prototype ODE solvers for Eqs. (6)–(10) can be implemented in any standard environment (Python/Julia/Matlab); we provide full equations and parameter windows to enable reproduction.

Conflicts of Interest: The authors declare no competing interests.

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