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Concept Paper

# Energy Product Entropy VS Statistical Entropy-An Analytic Quantum Thermodynamics perspective

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## Abstract

This paper introduces a novel entropy formulation — multiplicative entropy — defined as the product of energy values across all units in a **quantized homogeneous invariant network**. Unlike traditional statistical entropy, this approach explicitly tracks irreversible energy redistribution pathways, offering an analytic and path-dependent description of entropy growth. Logarithmic transformation recovers classical entropy forms, while preserving temporal directionality and quantum-scale resolution. The model enables precise simulation of thermodynamic processes, supporting the development of Analytic Quantum Thermodynamics as a new framework for understanding entropy-driven dynamics.

**Keywords:** multiplicative entropy; energy product entropy; analytic entropy; computable entropy; entropy coordinate; analytic quantum thermodynamics

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## Introduction:

Entropy, as traditionally defined in statistical mechanics, is a measure of disorder based on the logarithm of the number of accessible microstates. While this formulation has been foundational in thermodynamics, it often lacks the resolution and intuitiveness needed to describe the step-by-step evolution of entropy in physical systems. The classical approach focuses primarily on macroscopic end states and provides limited insight into the irreversible dynamics of spontaneous entropy increase.

In contrast, the proposed energy product entropy — or multiplicative entropy — offers a fundamentally new perspective. By defining entropy as the product of energy values across all system units, this approach explicitly tracks the microscopic redistribution of energy at each stage of evolution. It provides a high-resolution, path-dependent description of entropy growth, naturally encoding time's arrow through local energy flows. Furthermore, this formulation supports the development of analytic quantum thermodynamics, enabling precise simulation and mapping of entropy changes even at the Planck time scale.

This paper demonstrates how multiplicative entropy overcomes the limitations of traditional entropy measures and opens new avenues for understanding thermodynamic processes from a dynamic, computable perspective at the quantum scale.

## Scope of Application of The Theory

*-Analytical Quantum Thermodynamics in a Quantum Network*

The multiplicative entropy is defined as the product of the energy carried by each unit in a closed, quantized, topologically homogeneous network system. This entropy definition is computable and can serve as an entropy coordinate in system simulations. When logarithmically transformed, it aligns with the form of traditional additive statistical entropy. The restriction to a closed, quantized, topologically homogeneous network system is essential because, in simplified or open systems, multiplicative entropy may not be applicable. The validity of this model — the energy-product entropy — fundamentally relies on the conditions derived from the second law of

thermodynamics at the quantum scale, particularly the triggering mechanisms of energy conduction in the quantized space structures.

## Multiplicative Entropy-Energy Product Entropy: A New Analytical Framework for Understanding Irreversible Entropy Increase

If we adopt the concept of multiplicative entropy, it becomes much easier to understand why entropy increases spontaneously and irreversibly under the constraints of the second law of thermodynamics.

### 1. Definition of Multiplicative Entropy-Energy Product Entropy

I define multiplicative entropy as the product of the energies carried by each unit in a closed system. This definition of entropy is computable, and can also serve as an entropy coordinate for simulating the evolution of physical systems. When logarithmized, this multiplicative form reduces to the traditional additive entropy expression used in statistical mechanics.

Under the constraint of the second law of thermodynamics, energy always flows from units with higher energy to those with lower energy in a closed system. The total energy of all units remains constant (energy conservation), but the product of their energies keeps increasing — that is, the more uniform the energy distribution becomes, the larger the value of the multiplicative entropy.

When all units carry exactly the same amount of energy, the system reaches its maximum possible entropy — a state known as heat death.

This definition provides a much clearer and more intuitive description of entropy increase than the classical statistical formulation.

#### 2.1. Quantum Thermodynamics Perspective

Speaking of quantum thermodynamics, if we assume that the quantum is the fundamental building block of space, then under the framework of a quantized space network, all states become analytic rather than statistical, specifically at the Planck time scale.

Let's revisit the rule defined by the second law of thermodynamics: energy only flows from high-energy regions to low-energy ones. In essence, the process described by the second law — entropy increase — is a process of energy homogenization.

#### 2.2. Comparison Between Multiplicative Entropy and Classical Statistical Entropy

Dimension	Multiplicative Entropy	Traditional Statistical Entropy
Process Explicitness	✓ Yes, every step has clear changes	No, only describes macroscopic end states
Preserves Microscopic Details	✓ Yes, path-dependent	No, only cares about probability distributions
Has Temporal Directionality	✓ Yes, defined by local energy flow	No, needs extra assumption for time arrow
Suitable for Simulations	✓ Yes, good for numerical modeling	No, mainly used for theoretical analysis
Is an Analytical Function Form	✓ Yes, not based on statistical average	No, relies on ensemble average

#### Why Is Multiplicative Entropy More Suitable for Describing Energy Homogenization?

Take for example a system composed of  $N$  basic quanta. According to the second law, energy transfers from high-energy quanta to low-energy ones, gradually making the overall energy distribution more uniform.

Assume each quantum carries energy  $m_i$ . At any given transformation state, the total energy of the system is:

$$\sum m_i = \text{constant}; \quad \text{This satisfies energy conservation in a closed system.}$$

How do we characterize the entropy change during this evolution? My proposed multiplicative entropy offers a more intuitive and precise analytical tool:

The entropy  $S$  of the system at a certain transformation state is defined as:

$$S = \prod m_i$$

Under the constraint of energy conservation and directional energy transfer (from high to low), the more uniform the energy distribution becomes, the larger the product becomes.

When all quanta carry equal energy, the system reaches its maximum entropy – heat death occurs.

After applying the logarithm, this formula transforms into the classical additive entropy form. However, I argue that the **multiplicative** form more accurately captures the spontaneous and irreversible nature of entropy increase, offering greater clarity and visual intuition.

### 2.3. Computable Definition of Multiplicative Entropy: Time–Entropy Mapping

In this entropy definition, the entropy value of a closed system at a given moment (i.e., during a specific state transition) is calculated as the product of the energy norms of all space elementary quanta (SEQ) involved in the spatial transformation at that moment.

#### Analysis Formula of Entropy

In this definition of entropy, the entropy value of a closed system at a given moment (i.e., during a specific state transition) is calculated as the multiplicative product of the energy norms of all SEQ involved in that transition(that moment's space transformation).

$$3.1. \text{Entropy value of closed system } S = \prod m_i, i \in \mathbb{N} \quad \textcircled{1}$$

$$\text{Energy of closed system} = \text{constant} = \sum m_i, i \in \mathbb{N} \quad \textcircled{2}$$

$$3.2. S_{\max} \leq m_i^n, \text{ When all } m_i \text{ are equal or differ only by Planck's constant } h$$

(Where  $m_i$  refers to the energy carried by the  $i$ th SEQ during a single transformation of the closed system, where each energy state  $m_i$  is an integer multiple of Planck's constant  $h$ ,  $m_i = n_i h$ ,  $n_i \in \mathbb{N}$ )

### 3.3. Energy transfer rules and triggering conditions:

Energy exchange occurs between adjacent SEQ ( $i, j$ ) if and only if the following thermodynamic gradient exists:  $m_i > m_j + h$ , Energy transfer occurs only in discrete quanta of Planck's constant  $h$ ,  $m_i \rightarrow m_i - h$ ;  $m_j \rightarrow m_j + h$  (Planck's constant:  $h$ )

Numerical Example: System States and Entropy Evolution

**Table 1.** Simplified Entropy Increase Demonstration.

System State	SEQ Energy Distribution $E_{\text{total}} = \sum m_i = 12$	Entropy $S = \prod m_i$	Remarks
Initial non-equilibrium state	[3, 1, 5, 3]	45	-
Intermediate state	PathA[3, 1, 4, 4];	A48;	-
	PathB[3, 2, 4, 3]	B72;	
Final state	PathA[3, 2, 3, 4];	A72;	Due to adjacent energy transfer with minimal quanta $h$ , this system cannot reach maximum entropy in case A
	PathB[3, 3, 3, 3];	B81;	

Note: The above analysis demonstrates that different entropy-increasing pathways exhibit distinct sequences of entropy variation.

### 3.4. Logarithmic Relation:

After logarithmic transformation,  $\ln S$  aligns with the conventional Boltzmann entropy form, while the multiplicative formulation naturally suits discrete systems.

### 3.5. Proof of Spontaneous Entropy Increase

Spontaneity Theorem of entropy increase (Second Law of Thermodynamics):

For every possible energy transfer process, the total entropy change satisfies  $\Delta S \geq 0$ .

Proof Outline: Let the pre-transfer states be  $m_i = a$ ,  $m_j = b$  ( $a > b + h$ );

The post-transfer entropy ratio is:

$$S_{t1}/S_{t2} = (a-h)(b+h)/ab = 1 + h(a-b-h)/ab > 1 \text{ (Planck's constant } h)$$

This demonstrates how entropy increases along various paths, reflecting the irreversibility and path dependence of thermodynamic processes in real systems.

Moreover, each system state can be assigned a unique entropy value – an "entropy coordinate", which provides a powerful tool for computer simulations at the quantum level.

Novelty of the Concept

After extensive searches using general search engines and academic databases, I find that the analytical entropy expression based on energy products I propose here offers a fundamentally new alternative outside the traditional statistical framework. This approach transforms entropy from a statistical concept into an analytic one, and to my knowledge, no similar formulation has been previously proposed.

Based on this idea, I introduce the concept of Analytic Quantum Thermodynamics, which allows us to better understand the essence of thermodynamic processes – essentially, they are processes of energy homogenization. And to describe such processes, the multiplicative entropy based on multiplication is an excellent tool.

Distinction from Existing "Multiplicative Entropy" Concepts

Although the term "multiplicative entropy" or "multiplicative form of entropy" does appear in some literature, these are still variations of traditional statistical entropy, not departing from the following frameworks:

#### 5.1. Statistical Approach Based on Number of States

For example, Boltzmann entropy

Or Shannon entropy

These so-called "multiplicative forms" often involve multiplying entropies of subsystems or using multiplicative techniques in mathematical derivations.

But fundamentally, they remain logarithmic functions over probability distributions.

#### 5.2. No Microscopic Path Recording Function

They describe only macroscopic final-state differences, without retaining intermediate processes.

They do not capture how each individual energy transfer affects entropy.

Unique Features of This Multiplicative Entropy Theory

Feature	This Multiplicative Entropy	Existing "Multiplicative Entropy"
Can Track Evolutionary Paths	<input checked="" type="checkbox"/> Yes, with clear changes at each step	No, only describes macroscopic end states
Preserves Microscopic Details	<input checked="" type="checkbox"/> Yes, path-dependent	No, focuses only on probability distributions
Has Temporal Directionality	<input checked="" type="checkbox"/> Yes, defined by local energy flow	No, needs extra assumption for time direction
Suitable for Simulations	<input checked="" type="checkbox"/> Yes, ideal for numerical modeling	No, mainly used for theoretical analysis
Is an Analytical Function Form	<input checked="" type="checkbox"/> Yes, not based on statistical averaging	No, relies on ensemble averages

Why This Is a "New Discovery"

This model proposes a new way of constructing physical quantities, assigning them clear physical meaning and evolutionary rules:

- It does not start from probability theory, but from the dynamical process of energy redistribution.
- It naturally captures the trend toward homogenization through the product of energy values.

- It provides a computable entropy coordinate, ideal for simulation, prediction, and visualization of system evolution.
- It remains analytically valid at the Planck scale, suggesting natural extension into quantum thermodynamics.

This study presents an updated extension of a portion of the author's prior work [3]. For more details on the model, please refer to the previous paper presenting the complete framework.

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