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Article

Information Theory of Gravity (ITG): A Modified Entropic Gravity Using the Mass-Energy-Information Equivalence Principle

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Abstract: We presented a modified form of Emergent Gravity (EG), using the Holographic Principle and Vopson's Mass-Energy-Information Equivalence Principle (MEIEP). We use MEIEP to distinguish the type of information contained between a black hole and an ordinary gravitating object. We have shown that there is a way to have a model of EG that allows for the First Law of Thermodynamics to be violated at the Planck and Quantum level with a consequence that can be negligible in stellar scale and "corrective" in galactic scale, at the Macroscopic level. Combining the correction imposed by Special Relativity, the model gave results similar to General Relativity without necessarily going geometric in interpretation. Lastly, we have found a way to resolve the problem of quantum decoherence, where quantum entanglement can be unified with gravity.

Keywords: emergent gravity; MOND; modified newtonian gravity; Vopson's Energy-Matter-Information Equivalence Principle

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1. Introduction

Despite all the criticisms and oppositions, Entropic Gravity (EG), also known as Emergent Gravity, remains an intriguing theoretical framework that proposes that gravity is not a fundamental interaction, but rather a phenomenon that emerges from more fundamental processes. It gained traction over the years, particularly in Statistical Mechanics and Thermodynamics, and spawned so many modifications. The variations and extensions of the entropic gravity concept explore ideas such as the use of graviton mass [1], non-standard or deformed entropy functions [2], modified gravitational coupling [3], and modified Equipartition Law [4], among others [5]. Some even have suggested a way to relate it to a quantum interpretation [6] and consider its cosmological implications [2,7]. A study also investigated potential signatures of emergent gravity in current gravitational wave data, suggesting avenues for experimental validation [8]. The recent review of EG theory [9] suggests that emergent gravity models face criticisms and challenges, highlighting the need for more rigorous validation and theoretical underpinning. In this study, a new theoretical underpinning to modify the standard EG approach is suggested by not relying solely on the Holographic Principle. Our paper attempts to answer some of the major criticisms of EG, most of which are rooted in the approach used by Verlinde in his seminal work on EG. In his seminal paper [10], Verlinde conjectured that ordinary surfaces are holographic screens that obey the First Law of Thermodynamics (TFLT). He primarily used in his paper the equation,

$$F\Delta r = T\Delta S \tag{1}$$

where Δr is the distance of the test particle from the holographic screen, T is the temperature in the screen, and ΔS is the change in entropy S. Verlinde argued that as $\Delta r \to 0$, i.e., as the test particle touches the screen and increases the entropy, it induces gravitational force as a kind of entropic force [11]. In his analogy, a test particle that enters a gravitational field is likened to a polymer molecule that enters a region immersed in a thermal bath where, in such a condition, it gives rise to an entropic force. This analogy was heavily criticized in several papers after Verlinde's publication [12,13]. In the

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paper of Wang and Braunstein [14], it was shown that for surfaces away from horizons, TFLT fails and therefore undermines the key thermodynamic assumption of the EG program. Related to this criticism is the problem of making any EG model relativistic in form since any attempt to derive Einstein's equations was again criticized with their use of TFLT. For example, T. Wang [15] said that the inclusion of energy-momentum conservation severely restricts a wide class of potential modifications of entropic gravity, and concluded that ''the modified entropic gravity models....if not killed, should live in a very narrow room to assure the energy-momentum conservation". TFLT is problematic for EG because there is an apparent violation of the conservation of energy or mass if one is to associate entropy with mass. As stated by the Area Theorem, two or more entropies, when added up, can produce a much larger entropy than the sum. This must also be true with mass since it is related to entropy via the equation $S = 4\pi M^2$. We will show how this apparent violation of mass-energy conservation is not necessarily problematic but can be useful in an EG model. Finally, we will attempt to resolve here the idea that entropic processes break quantum coherence at some temperature scale [16]. This, however, was argued by others to have loopholes [17], but the problem must still be resolved to align any EG model with Quantum Mechanics. The resolution of this problem could be an avenue for a new approach to solve the Unification Problem.

The paper is organized as follows: In Sections 2 and 3, we reviewed the Holographic Principle and Vopson's Mass-Energy-Information Equivalence Principle. Then, in Section 4, a modified Newtonian Gravity equation was derived, which was modified further by deriving its relativistic form and showing its consistency with Special Relativity. Also discussed in Section 4 are the following topics: Modification of Kepler's Third Law, Derivation of the Gravitational Redshift, Mercury's Perihelion Shift, and Light Deflection around the Sun, just like all the predictions of General Relativity (GR). In Section 5, we showed that the model predicts the MOND equation, the Tully-Fisher Relation, and the External Field Effect. In Section 6, the model was shown to derive GR as a special case and emerges as an approximation of the model at the macroscopic level. Lastly, in Section 7, we showed a possible solution to the quantum decoherence problem.

2. Beyond Holographic Principle

In retrospect, the Holographic Principle came from the initial conjecture of Bekenstein [18] in the 1970s that there must be an analogy between Black Hole Physics and Thermal Physics. Using such analogy, the laws of Black Hole Mechanics that were later developed suggest that the surface area, A, is proportional with the entropy, S, by at least up to some multiplicative constants, γ, i.e., $S = \gamma A$. Subsequently, the value for γ was established [19] to be related to the Planck length, l_p, i.e., $\gamma = \in / l_p^2$, such that,

$$S_{bh} = A/4\pi l_p^2 = A/A_p = N_a = r^2/l_p^2 = (N_l)^2 \tag{2}$$

where we set here $\in$ to $1/4\pi$ while others [19] set its value to $1/4$. From this, the Holographic Principle was formulated, stating that the quantity N_a or the number of cells in the holographic screen is a measure of entropy within the volume of space enclosed by the holographic screen. Consequently, since $r_s = 2GM/c^2$, the relation of entropy with mass can be derived from Eqn. (2)

$$S_{bh} = \frac{GM^2}{\hbar c} = \left(\frac{M}{M_p}\right)^2 = \left(\frac{E}{E_p}\right)^2 = (N_m)^2 \tag{3}$$

where, M_p and E_p, are the Planck mass and Planck energy, respectively. This relation is a simple case since other quantities, such as the electric charge and the spin, can also increase the entropy of a black hole. However, if only the mass-entropy relation is to be considered, the question that a typical EG model is set to answer is: ''Can we still use this result in a non-black hole setting?". To answer this question, we are motivated by the fact that, in a non-black hole setting, the change in the strength of gravity is proportional to the mass density, ρ, as expressed explicitly in the Poisson Equation,

$$\nabla^2 \phi = 4\pi G\rho \tag{4}$$

where the scalar potential ϕ gives the strength of gravity. In the relativistic case, the density ρ becomes the density and flux of energy and momentum of the gravitational system. With this in mind, we aim to use not only the quantity, $N_a = (N_l)^2$, but also its counterpart for mass or energy, $(N_m)^2$, such that the magnitude of gravity will be in terms of the mass-energy density of the system. Hence, we suggest here the use of the quantity,

$$(N_d)^2 = \frac{(N_m)^2}{(N_l)^2} = \left(\frac{M/r}{M_p/l_p}\right)^2 = \left(\frac{\rho_l}{\rho_{pl}}\right)^2 \tag{5}$$

to serve as a measure of the density of information or entropy of the system. The role of the mass is already integrated with our approach at the very start. With Verlinde's approach, he introduced the mass by equating $E = mc^2$ with the Equipartition Theorem, then relating it to N_a to derive Newton's equation. Here, the quantity N_d will be used at the very start as a fundamental ratio related to the strength of gravity. It is defined by the linear density $\rho_l = M/r$, and the Planck (linear) density $\rho_{pl} = M_p/L_p = \sqrt{c^4/G^2}$. In this way, the intrinsic connection of matter and space in the description of gravity was maintain by combining the mass ratio N_m, as a measure of entropy within the gravitating matter, and the quantity N_a as a measure of entropy in the surrounding space. Our model posits that the resulting density ratio is proportional to the magnitude of gravity, i.e.,

$$N_F = \frac{F}{F_p} \sim \frac{S_m}{S_l} = \frac{(N_m)^2}{(N_l)^2} = (N_d)^2 \tag{6}$$

where the Planck Force, $F_p = c^4/G$, is added to make everything unitless. For a black hole, $S_m = S_l$ within the event horizon; thus, the force of gravity is equal to the Planck force from which it is impossible to escape. Outside of the event horizon, $S_m \leq S_l$ since the distance from the singularity or the center of gravity is greater than the Schwarzschild radius r_s. For ordinary matter, there is also a distinction from the entropy related to space with the entropy related to mass since there are variations of the density from the center of gravity to the edge of the gravitational system, i.e., at the end of the so-called Hill's Sphere. This happens since, within the volume of the gravitational system, different types of particles are enclosed that serve as mediums in which information is stored. Within ordinary gravitating matter, particles are bounded by non-gravitational fields where the binding energy will add up to the total mass of the gravitating matter, while particles within the vicinity are "free" particles bounded only by the gravitational field except if the gravitating matter also has electric charge. Furthermore, the gravitating matter and the particles in the vicinity occupy two different volumes. This is also true for a black hole since a singularity is not a point but a region that occupies a volume [20]. These differences should be considered to define the quantity of information we associate with the particles. In addition, two types of temperature should be used to define the entropies. The first type is the temperature associated with particles within the vicinity of the source of gravity. The other one is the temperature associated with particles within the gravitating matter or at the singularity. In most EG models, this difference was never considered. Here, since we relate the magnitude of gravity with the entropy, these differences should be considered to have a more accurate calculation for the magnitude of gravity.

3. Vopson's Equivalence Principle

Another major departure of our model from Verlinde's original approach is how to relate the mass with temperature. Our model will not use the Energy Equipartition Theorem,

$$E = \frac{1}{2}Nk_bT = Mc^2 \tag{7}$$

which most EG models that follow after Verlinde also used. One author, for example, made some "Debye correction" but, in essence, still uses the Energy Equipartition Law [4]. Here, we used the

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equation of Vopson [21] that extended the so-called Landauer's Principle [22]. It relates the energy with the temperature by using the Shannon formula for entropy, i.e.,

$$E = k_b \cdot T \cdot \ln(\Omega) = N \cdot k_b \cdot H(X) \cdot \ln(2) = Mc^2 \tag{8}$$

where the information here corresponds to observing N set of events X such that $H(X)$ is the information entropy function and k_b is the Boltzmann constant. The number of possible states or information-bearing microstates, Ω, is compatible with the macro-state: $\Omega = 2^{NH}$. The temperature T is related to the total kinetic energy of real particles that comprises the gravitating matter in which bits of information can be stored for each particle. This is Vopson's MEIEP, and there is a convincing reason to adopt this as a measure of entropy for matter. Just recently, Vopson used MEIEP to quantify the amount of information contained in the visible matter in the entire Universe [23]. He also proposed a way to experimentally verify MEIEP via a particle annihilation experiment [24]. To combine MEIEP with the Holographic Principle, the equation of MEIEP must be written first in Planck scale units. To do this, we simplify first Eqn.(8) as follows,

$$M = N\left(\frac{k_b}{c^2}\right) \cdot T \ln(2) \tag{9}$$

where we consider the simple case of a "digital information" with $X = \{0, 1\}$, such that $H(X) = 1$. Notice that the constant, $\frac{k_b}{c^2}$, serves as a constant of proportionality and a conversion factor. To make the equation above unitless and in Planck scale units, we insert other constants such that by unit analysis,

$$\frac{k_b}{c^2} = \sqrt{\frac{k_b^2}{c^4}} = \sqrt{\frac{1}{\frac{c^4}{k_b^2}}} = \sqrt{\frac{\frac{\hbar c}{G}}{\frac{\hbar c^5}{Gk_b^2}}} = \frac{\sqrt{\frac{\hbar c}{G}}}{\sqrt{\frac{\hbar c^5}{Gk_b^2}}} = \frac{M_p}{T_p} \tag{10}$$

where T_p is the Planck temperature. Substituting this in Eqn.(9) and rearranging, we have,

$$\frac{M}{M_p} = N\left(\frac{T}{T_p}\right) \ln(2) = N_m \tag{11}$$

This is just the unitless version of Vopson's equation but in Planck scale units. It is not applicable for a black hole since, at the singularity, particles stop moving, and the temperature is absolute zero. Thus, we suggest here that the expression above must be applicable only for ordinary matter that is not confined within the singularity of a black hole. In this note, our approach differs from the one used by Verlinde and others, where they used equations applicable only at the quantum level but could be incompatible with the Holographic Principle written in Planck scale units. In our approach, everything is in Planck scale units, which is applicable not only at the Planck scale but can be shown to be applicable also at the quantum and macroscopic scale. To show this, we consider the total mass, M, at the macroscopic level, which can be expressed in terms of the smallest unit of mass to contain the smallest possible unit of information at the quantum level, i.e., $M = N_{mb} M_{bit}$. This will yield us,

$$\frac{M_{bit}}{M_p} = \left(\frac{T}{T_p}\right) \ln(2) = N_{mb} \tag{12}$$

To get the magnitude of gravity that can be associated to a single bit of matter, we square first Eqn.(12), then divide it by $(N_l)^2$ where the distance r is within the Compton scale,

$$\frac{F_{bit}}{F_p} \sim \frac{(N_{mb})^2}{(N_l)^2} = \left(\frac{\rho_{bit}}{\rho_{pl}}\right)^2 = (N_d)^2 \tag{13}$$

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This is similar to our previous calculation of the magnitude of gravity for a black hole with r equal to or greater than the Schwarzschild radius. For the type of particles that can store information, Vopson [21] postulated that information can only be stored in stable particles with non-zero rest mass. On the other hand, the force carrier bosons can only transfer information in a waveform. Vopson estimated that each particle in the observable universe contains 1.509 bits of information [23]. In the same paper, he added that ''information could also be stored in other forms, including on the surface of the space-time fabric itself, according to the holographic principle". This is essentially what the present paper is suggesting. The two known methods of storing information, spacetime and matter, will be combined here to describe gravity. This was already hinted by Vopson when he conjectured the possible role of information in the Dark Matter Problem and even called Information as the ''Fifth State of Matter" [21]. Now, using Eqn. (11), we can calculate the smallest amount of information at the Planck scale where $T = T_p$. This is by setting $M = N_p M_p$ which yields us,

$$N_p = \ln 2 \approx 0.693 \tag{14}$$

This is about one unit smaller than the estimated smallest amount of information for a quantum particle. The force of gravity at the Planck length distance would then be,

$$F \approx (0.693)^2 F_p = (0.48) F_p \tag{15}$$

or half of the value of the Planck force.

4. Gravity in Stellar Scale

4.1. Modified Newtonian Gravity

In this section, we will now consider the case of applying the model at the macroscopic level like in a 2-body system with mass, $M = M_1 + M_2$. We will also apply it to a galaxy to show that our model is consistent with GR and other models like MOND. Note that the dimensionless form of Newton's Law of Gravity in terms of Planck scale units can be expressed as follows,

$$N_F = \frac{F}{F_p} = \frac{\left(\frac{M_1}{M_p}\right)\left(\frac{M_2}{M_p}\right)}{\left(\frac{r}{l_p}\right)\left(\frac{r}{l_p}\right)} = \frac{N_1 N_2}{N_a} \tag{16}$$

However, rewriting it in this form does not mean it is now applicable at the Planck scale or in the case of a strong gravitational field. The equation above is still applicable only for weak fields. It must be modified for cases when the magnitude of gravity is relatively large such that Newton's equation will become a special case when gravity is relatively weak. In our model, the amount of information stored within the gravitating matter will be used, and it is quantified via Vopson's MEIEP. The amount of information in a 2-body system in terms of the masses of two gravitating objects is represented by the sum,

$$N_m = N_1 + N_2 = \frac{M}{M_p} + \frac{m}{M_p} \tag{17}$$

where, M and m, are the masses of the two gravitating objects. The quantities, $N_1 = M/M_p$, and $N_2 = m/M_p$, can be used to calculate the maximum possible density of information that can be stored for each gravitating matter. As a rule, a purely information-theoretic approach to gravity should be that the magnitude of gravity is dependent solely on the amount of information that resides in space and matter within a gravitational system. If gravity is strong enough to generate virtual particles in the vicinity, those particles must be included in the description of entropy within the system. Hence, what we mean by a ''purely information-theoretic approach" is that the magnitude of gravity F should only be dependent or quantifiable by the value of N_m and N_a. The former represents the amount of information that resides in a gravitating matter, and the latter represents the amount of information within the space in the vicinity of gravitating matter. We can combine the two quantities and come

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up with a measure of information density given by $N_d = N_m/\sqrt{N_a}$ such that the square of it to be proportional to N_F, that is,

$$N_F = \frac{F}{F_p} = \epsilon(N_d)^2 = \epsilon\frac{(N_m)^2}{N_a} = \epsilon\left(\frac{N_m}{N_l}\right)^2 \tag{18}$$

for some unitless constant of proportionality ϵ. Thus, for a 2-body system, we have,

$$F = \frac{2c^4}{G}\frac{N_1 N_2}{N_l^2}\epsilon + \frac{c^4}{G}\left(\frac{N_1^2 + N_2^2}{N_l^2}\right)\epsilon = F_{NG} + F_{HG} \tag{19}$$

where we can set $\epsilon = 1/2$ and show that

$$F_{NG} = \frac{2c^4}{G}\frac{N_1 N_2}{r^2/L_p^2}\epsilon = \hbar c\frac{Mm}{r^2 M_p^2} = G\frac{Mm}{r^2} \tag{20}$$

is the familiar Newtonian Gravity (NG). Hence, the quantity F gives us the magnitude of gravity just like in Newton's theory, however, it has an additional quantity,

$$F_{HG} = \frac{c^4}{2G}\left(\frac{N_1^2 + N_2^2}{N_l^2}\right) \tag{21}$$

which can be interpreted as an excess magnitude of gravity or a ''Hidden Gravity''(HG), in addition to what is already given by the Newtonian equation of gravity. To simplify, we can rewrite Eqn. (19) as follows,

$$F = G\frac{Mm}{r^2}\epsilon + G\left(\frac{M^2 + m^2}{r^2}\right)\epsilon \tag{22}$$

which can be simplified further into,

$$F = G\frac{Mm}{r^2}f\epsilon \tag{23}$$

where $f = f(M/m) = 1 + \left(\frac{M}{m} + \frac{m}{M}\right)$ is a correction term that simplifies into $f \approx 1 + M/m$, if $m << M$.

4.2. Relativistic Form of the Model

The most commonly used approach in introducing a relativistic new theory of gravity is to generalize the Einstein-Hilbert action, $S = \int \sqrt{-g}Rd^4x$, by imposing additional parameters into the action, such as scalar, vector, tensor, and spinor fields, and then making it conformally invariant to produce a new field equation for gravity. One of the well-known examples of such an approach is the Tensor–Vector–Scalar (TeVeS) gravity theory by Bekenstein [25] as a relativistic generalization of the MOND paradigm of Milgrom [26]. This Lagrangian approach will not be used here since the model presented here will focus more on the relation of gravity with the information density rather than on its energy density. Both densities will be shown, however to be interrelated. Instead, the relativistic approach here primarily uses Special Relativity (SR). It will be shown here that the consequences of SR are enough to derive the predictions made in GR without necessarily going geometric in the mathematical formalism. In addition, it will be shown that the model can derive not only Newton's theory of gravity in a weak field as a postdiction, but it can also describe the effect of gravity in a strong field as its prediction. Like in GR, the model can also show that it can explain Mercury's anomalous orbital precession at its perihelion. The same thing is true for the case of the bending of starlight when it grazes near the Sun and with the gravitational redshift effect. Thus, in the succeeding subsections, we will show how our model can also predict the outcome of the so-called ''classical tests'' that had been known to pass by GR with flying colors. The main difference, however, is that all of these will be

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accomplished by not necessarily going geometric in the formalism and interpretation of how gravity operates in Nature.

In a frame of reference where the Sun is at rest and of a planet moving towards it from aphelion (i.e. at slow velocity) to the perihelion, the rest energy of the Sun is given by $E = M_0 c^2 \gamma$ of the Sun and the Kinetic Energy of the planet is $E_k = m_0 c^2(\gamma - 1)$ using Special Relativity. The function f can be approximated further as follows,

$$f \approx 1 + \frac{M_0}{m_0} = 1 + \frac{E/c^2\gamma}{E_k/c^2(\gamma - 1)} \tag{24}$$

$$\approx 1 + \alpha\frac{v^2}{c^2} = 1 + \alpha\beta^2 \tag{25}$$

where $\alpha = \frac{E/2}{E_k}$, $\beta = v/c$ and $\gamma = 1/\sqrt{1 - v^2/c^2}$. At the frame of reference where the relative velocity v is zero or its value is relatively small in comparison to the speed of light c, such that $\beta \approx 0$, then Eq.(23) becomes the usual Newtonian gravity equation. On the other hand, at perihelion, where β attains a non-zero value, we can set $\alpha \to 1$, such that Eqn. (23) becomes,

$$F \approx \frac{GM_0 m_0}{r^2}\left[1 + \frac{v^2}{c^2}\right] \tag{26}$$

The immediate consequence of this is a modified Kepler's Third Law,

$$P^2 \approx \frac{4\pi^2}{GM_0}\left[1 + \frac{v^2}{c^2}\right]^{-1} r^3 \tag{27}$$

by combining Eqn. (26) with the equation for centripetal force $F_c = m_0 v^2/r = (m_0/r)(2\pi r)^2/P^2) = m_0 r/P^2$ where P is the orbital period for a simplest path of a circular orbit. The equation above becomes Kepler's Third Law for a slow-moving object. This implies that the model is consistent with all known experiments made to test Newton's theory.

4.3. Horizon Mass and Gravitational Redshift

The result of the previous section leads to the notion of irreducible mass, M_{irr}, and the horizon mass, M_h,

$$\frac{v^2 r}{G} \approx M_0\left[1 + \frac{v^2}{c^2}\right] = M_{irr}\left[1 + \frac{v^2}{c^2}\right] = M_h(r) \tag{28}$$

where the rest mass M_0 acts as the irreducible mass, M_{irr}, and M_h is the horizon mass. The horizon mass is dependent on where the observer is. At infinity or at the edge of the Hill's Sphere, where the gravitational influence is too weak and therefore the test object's linear velocity ($v = \omega r$) or angular velocity, ω, is so small such that $v^2/c^2 \approx 0$, which gives us $M_h = M_0$. However, in the case of a strong gravitational field, we rewrite Eqn. (24),

$$f = 1 + \frac{M_0}{m_0} = 1 + \frac{E/c^2\gamma}{E_k/c^2(\gamma - 1)} \tag{29}$$

$$= 1 + \frac{E}{E_k}(1 - \sqrt{1 - \beta^2}) \tag{30}$$

such that we have,

$$M_h(r) = \frac{v^2 r}{G} = M_0\left[1 + \frac{E}{E_k}\left(1 - \sqrt{1 - \beta^2}\right)\right] \tag{31}$$

Instead of considering the possiblity of $E_k \to E/2$, we consider the case of $E_k \to E$ and $\beta \to 1$. This can only happen for a black hole at the event horizon, which gives us,

$$M_h = 2M_{irr} \tag{32}$$

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Thus, for black holes, the horizon mass becomes twice the irreducible mass. This statement is known in Black Hole Physics as the Horizon Mass Theorem [27]. This concept of the mass of the gravitating object being observer-dependent or an asymptotic mass will be used in the succeeding section. For the observable mass, it must be the difference between the horizon mass and the irreducible mass, which then gives the total energy contained within a radius at coordinate r,

$$E(r) = (M_h - M_0)c^2 \tag{33}$$

$$= M_0 c^2 \left(\frac{E}{E_k}\right)\left(1 - \sqrt{1-\beta^2}\right) \tag{34}$$

Setting $E_k \to E/2$ and $v^2 = 2M_0G/r$, which can be substituted on the righthand side of the equation to yield,

$$E(r) = 2M_0c^2\left(\frac{E/2}{E_k}\right)\left[1 - \sqrt{1-\beta^2}\right] \tag{35}$$

$$= \frac{rv^2c^2}{G}\left[1 - \sqrt{1 - \frac{2GM_0}{rc^2}}\right] \tag{36}$$

In the case of a black hole,

$$E(r) = \frac{rc^4}{G}\left[1 - \sqrt{1 - \frac{2GM_0}{rc^2}}\right] \tag{37}$$

since at the event horizon $v = c$ where M_0 is the mass of the black hole observed at infinity as $r \to \infty$. This was already derived in the seminal work [27] on the horizon mass of a black hole. The equation above was also known to emerge in GR studies trying to find the exact energy expression of a Schwarzschild black hole in the quasilocal energy approach [28], in the so-called teleparallel equivalent formulation of general relativity [29], and the gravitational redshift approach [30]. Our approach, however, is simpler than the derivation using GR. Now, setting $E_0 = \frac{r_0c^4}{G}$ as energy observed at $r = r_0$ and using the Planck equation, $E = hf$, we yield an expression for the gravitational redshift,

$$\frac{\Delta E}{E_0} = \frac{v_\infty}{v_0} = \frac{f_{obs} - f_{emit}}{f_{emit}} = \sqrt{1 - \frac{2GM_0}{rc^2}} \tag{38}$$

where v_∞ and f_{obs} are the observed velocity and frequency at infinity, while f_{emit} and v_0 are the initial frequency emitted and velocity, respectively.

4.4. Perihelion Shift of Mercury

In the previous section, we have clarified that the observed mass, say of a star, is position-dependent relative to the observer. With this, we can now discuss the prediction of the model regarding the precession shift of the planet Mercury, which is similar to what GR had predicted. Historically, the so-called ‘‘anomalous precession shift of Mercury", being one of the predictions of GR, is contested by some authors even in recent years. They argue that the correction applied to Newton's equation by Einstein was not a direct consequence of his geometrization of gravity in his GR but simply from what was previously derived from Special Relativity [31]. This relatively unknown controversy is rooted in the fact that there is a slight error in the Einstein calculation from Schwarzchild's metric to derive the correct formula for Mercury's perihelion shift [32,33]. Nowadays, this error in the calculation does not in any way tarnish the reputation of GR since there are other ways now to derive Mercury's perihelion shift using GR. However, such so-called ‘‘derivation" based on GR [34] is still on ‘‘shaky ground" or founded on assumptions which are usually not explicitly stated in the derivation, especially in the algebraic approach of the derivation like in the work of Taylor and Wheeler [35]. Hence, in the end, the status quo is that the validity of GR is almost unquestioned. The consensus is that any new theory that wants to deviate from GR is said to be very "tight", i.e., it must be able to explain all of the so-called

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successes of GR and not just one part of it. In this section, we focus on achieving this by showing first that our model can also derive the precession shift of Mercury and the deflection of light using the modified Newtonian gravity equation given by Eqn. (26). Our approach is similar to the earlier work of Kou [36], but with some corrections and clarifications in Kou's original work where he did not consider the variation in the observed mass of the Sun relative to a moving planet. We start with the standard Newtonian dynamical theory of planetary motion for comparison, which is given by two equations below,

$$\frac{d\varphi}{dt} = \frac{L}{r^2} \tag{39}$$

$$\frac{1}{2}\left(\frac{dr}{dt}\right)^2 = E + \frac{GM}{r} - \frac{L^2}{2r^2} \tag{40}$$

These equations show the planet's angular momentum and energy conservation laws, respectively, wherein the first equation, L is the angular momentum and φ is the angle of the angular polar coordinate while in the second equation, E is the energy of the system and r is the distance of the planet. In our model, using Eqn. (26), it will lead to the modification of the last equation, i.e.,

$$\frac{1}{2}\left(\frac{dr}{dt}\right)^2 = E + \frac{GM}{r}\left(1 + \frac{v^2}{c^2}\right) - \frac{L^2}{2r^2} \tag{41}$$

where v is the planet's velocity toward the host star in a stellar system. This velocity can be written as a combination of tangential and radial velocity,

$$v^2 = v_{tan}^2 + v_{rad}^2 = \left(\frac{dr}{dt}\right)^2 + \left(r\frac{d\varphi}{dt}\right)^2 \tag{42}$$

$$= \left(\frac{dr}{dt}\right)^2 + \left(\frac{L}{r}\right)^2 \tag{43}$$

Using this, we can write Eqn.(41) as follows,

$$\left(\frac{1}{2} - \frac{GM}{rc^2}\right)\left(\frac{dr}{dt}\right)^2 = E + \frac{GM}{r} + \frac{GM}{c^2}\frac{L^2}{r^3} - \frac{L^2}{2r^2} \tag{44}$$

To simplify, we set $\frac{1}{2} >> \frac{GM}{rc^2}$, and ignore the $\frac{GM}{rc^2}$ term at the left-hand side of the equation which yields us,

$$\left(\frac{1}{2}\right)\left(\frac{dr}{d\varphi}\frac{d\varphi}{dt}\right)^2 = \left(\frac{L^2}{2}\right)\left(\frac{1}{r^2}\frac{dr}{d\varphi}\right)^2 \tag{45}$$

$$= E + \frac{GM}{r} + \frac{GM}{c^2}\frac{L^2}{r^3} - \frac{L^2}{2r^2} \tag{46}$$

by subsituting Eqn.(39) in the equation. Taking the derivative of both sides of the equation in terms of φ and noting that $\frac{d}{d\varphi}\left(\frac{1}{r}\right) = -\frac{1}{r^2}\frac{dr}{d\varphi}$ where $r = r(\varphi)$, we have,

$$\frac{L^2}{2}\frac{d^2}{d\varphi^2}\left(\frac{1}{r}\right) + \frac{L^2}{r} = GM + \frac{3GML^2}{r^2c^2} \tag{47}$$

By setting $u = GM/r$, and substituting this to the last equation, we finally have the expression,

$$\frac{d^2u}{d\varphi^2} + u = \left(\frac{GM}{L}\right)^2 + \frac{3u^2}{c^2} \tag{48}$$

This result is similar to the relativistic correction of General Relativity to the Newtonian formula by a factor of $3u^2/c^2$. In the case of Mercury and the Sun, $u^2 = 2.3 \times 10^6\ m^2/s^2$, where we plugged in the measured value of the distance of Mercury from the Sun at aphelion, $r = 5.79 \times 10^{10}$m, and we initially set the mass to be $M = 2 \times 10^{27}$kg, to get the value for u such that the order of the correction factor is,

$$\frac{3u^2}{c^2} \sim 10^{-4} \tag{49}$$

Note that in the Newtonian formula,

$$\frac{d^2u}{d\varphi^2} + u = \left(\frac{GM}{L}\right)^2 \tag{50}$$

the solution of this is given by,

$$u = \left(\frac{GM}{L}\right)^2 (1 + e\cos\theta) \tag{51}$$

Since $3u^2/c^2$ is very small, the Newtonian solution above can be considered as the zero-order solution of Eqn. (48). Substituting it to Eqn.(48), we have,

$$\begin{aligned}\frac{d^2u}{d\varphi^2} + u = &\left(\frac{GM}{L}\right)^2 + \frac{3}{c^2}\left(\frac{GM}{L}\right)^4 \\ &+ \frac{6}{c^2}\left(\frac{GM}{L}\right)^4 (e\cos\varphi) + \frac{3}{c^2}\left(\frac{GM}{L}\right)^4 (e\cos\varphi)^2\end{aligned} \tag{52}$$

Since for the case of Mercury, the eccentricity is very small ($e << 1$) and $\left(\frac{GM}{L}\right)^2 >> \frac{3}{c^2}\left(\frac{GM}{L}\right)^4$, we can consider the others terms negligible and rewrite Eqn.(48) to be,

$$\frac{d^2u}{d\varphi^2} + u \approx \left(\frac{GM}{L}\right)^2 + \frac{6}{c^2}\left(\frac{GM}{L}\right)^4 (e\cos\varphi) \tag{53}$$

where the right-hand side is all just constants. To find the solution, we use the ansatz: $u = u_1 + u_2$ to have two separate equations:

$$\frac{d^2u_1}{d\varphi^2} + u_1 = \left(\frac{GM}{L}\right)^2 \tag{54}$$

$$\frac{d^2u_2}{d\varphi^2} + u_2 = \frac{6}{c^2}\left(\frac{GM}{L}\right)^4 (e\cos\varphi) \tag{55}$$

This will give us two solutions:

$$u_1 = \left(\frac{GM}{L}\right)^2 (1 + e\cos\theta); \tag{56}$$

$$u_2 = \frac{3}{c^2}\left(\frac{GM}{L}\right)^4 (e\varphi\sin\varphi) \tag{57}$$

which can be combined into,

$$u = \left(\frac{GM}{L}\right)^2 \left[1 + e\cos\theta + \frac{3}{c^2}\left(\frac{GM}{L}\right)^2 e\varphi\sin\varphi\right] \tag{58}$$

to serve as solution to Eqn.(53). We can simplify the expression inside the square bracket by factoring out e and using the auxiliary angle formula:

$$a\sin\varphi + b\cos\varphi = \sqrt{a^2 + b^2}\cos(\varphi + \theta) \tag{59}$$

where $a = 1$, $b = \frac{3}{c^2}\left(\frac{GM}{L}\right)^2$. Then, using the following approximations:

$$\theta = arctan\left(\frac{a}{b}\right) \approx \frac{a}{b} = \frac{1}{b}; \;\; b^2 \approx 0; \;\; \frac{1}{1+x} \approx 1 - x \tag{60}$$

Finally, we have the expression,

$$u \approx \left(\frac{GM}{L}\right)^2 \left\{1 + e\cos\left[1 - 3\left(\frac{GM}{cL}\right)^2\right]\varphi\right\} \tag{61}$$

as the solution for the planet orbit equation that came out of the modified Newtonian equation of the model. Looking at this, one can see that after one turn $\varphi = 2\pi$, that planet can't return as its orbit will precess a small angle. The cycle T of the orbit will no longer be at $\varphi = 2\pi$ but,

$$T = \frac{2\pi}{1 - 3\left(\frac{GM}{cL}\right)^2} \tag{62}$$

Each time the planet will orbit around the perihelion, it will turn an angle,

$$\varphi_n = \frac{2n\pi}{1 - 3\left(\frac{GM}{cL}\right)^2} \approx 2n\pi\left[1 + 3\left(\frac{GM}{cL}\right)^2\right] \tag{63}$$

From this, we can compute,

$$\Delta\varphi_n = (\varphi_{n+1} - \varphi_n) - 2\pi \tag{64}$$

$$= 6\pi\left[\frac{GM}{cL}\right]^2 = 6\pi\left[\frac{GM}{c \cdot r \cdot v}\right]^2 \tag{65}$$

where we use $L = r \cdot v$, such that the quantity in the square bracket is unitless. Substituting now the orbital velocity at perihelion, 6.9×10^4m/s, $G = 6.67 \times 10^{-11} Nm^2/kg^2$, the perihelion distance $r = 4.6 \times 10^{10}$m and setting the mass at perihelion to be $M = 2 \times 10^{30}$kg, we finally have,

$$\Delta\varphi_n = 6\pi \cdot \left(\frac{6.67 \cdot 2}{2.9 \cdot 4.6 \cdot 6.9} \times \frac{10^{-11} \cdot 10^{30}}{10^8 \cdot 10^{10} \cdot 10^4}\right)^2 \tag{66}$$

$$= 3.96 \times 10^{-7} = 0.81'' \approx 0.1'' \tag{67}$$

which means for every Mercury's year (88 days), there are $0.1''$ turn of the precession angle, giving us the total precession angle,

$$\sum \Delta\varphi_n = 41.5'' \approx 42'' \tag{68}$$

within a century on Earth. This is relatively close to the observed approximate value of 43 arcseconds, considering the margin of error and some rounding off to whole numbers of some of the values in the calculations.

4.5. Deflection of Light

Another so-called test of GR is the deflection of light, which is usually observed nowadays in the gravitational lensing of galaxy clusters. This can also be predicted by the model in the case of a light grazing around the Sun, as already shown in the works of Kou [36] on Modified Newton's Law. Using again the modified orbital or dynamical equations,

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$$\frac{d\varphi}{dt} = \frac{L}{r^2} \tag{69}$$

$$\frac{1}{2}\left(\frac{dr}{dt}\right)^2 = E + \frac{GM}{r}\left(1 + \frac{v^2}{c^2}\right) - \frac{L^2}{2r^2} \tag{70}$$

Combining both equations by expanding $\frac{dr}{dt} = \frac{dr}{d\varphi}\frac{d\varphi}{dt}$, setting $v = c$ and using $dr = -r^2 d\left(\frac{1}{r}\right)$, we get,

$$\frac{L^2}{2}\left[\frac{d\left(\frac{1}{r}\right)}{d\varphi}\right] = E - \frac{L^2}{2r^2} + \frac{2GM}{r} \tag{71}$$

Getting the derivative and multiplying with $\frac{GM}{L^2}$ on both sides of the equation and defining $u = GM/r$, the equation above can be simplified as follows,

$$\frac{d^2u}{d\varphi^2} + u = 2\left(\frac{GM}{L}\right)^2 \tag{72}$$

where $L = Rc$, R being the radial of the Sun. To find the approximate solution, we make the right-hand side of the equation negligible by defining $\tilde{u} = 1/r$ such that $u = GM\tilde{u}$ and dividing both sides with GM, yielding us,

$$\frac{d^2\tilde{u}}{d\varphi^2} + \tilde{u} = \frac{2GM}{(Rc)^2} \tag{73}$$

The approximate solution of this is a line,

$$\tilde{u}_0 = R^{-1}\cos\varphi \tag{74}$$

while the general solution is a line with a deflection term,

$$\tilde{u} = \tilde{u}_0 + \frac{2GM}{(Rc)^2} = R^{-1}\cos\varphi + \frac{2GM}{(Rc)^2} \tag{75}$$

For a straight line, it can be extended to infinity ($r \to 0$), which makes $\tilde{u} \to 0$ while the azimuth angle is $\varphi = \pm\frac{\pi}{2}$. We can reverse this by setting that at infinite where $\tilde{u} \to 0$, and we set $\varphi = \pm\left(\frac{\pi}{2} + \theta\right)$ giving us,

$$0 = R^{-1}\cos\left[\pm\left(\frac{\pi}{2} + \theta\right)\right] + \frac{2GM}{(R^2c^2} \tag{76}$$

$$= R^{-1}\sin\theta + \frac{2GM}{R^2c^2} \tag{77}$$

From this, we can derive for θ,

$$\sin\theta \approx \theta = \frac{2GM}{c^2R} \tag{78}$$

which finally gives us the light deflection angle,

$$\Delta\varphi = 2\theta \approx \frac{4GM}{c^2R} \tag{79}$$

For solar mass 1.988×10^{30} kg, and radial $R = 6.955 \times 10^8$m, we can predict that,

$$\Delta\varphi = \frac{4 \cdot 6.67 \cdot 1.988}{(2.9979)^2 \cdot 6.995} \times \frac{10^{-11} \cdot 10^{30}}{10^{16} \cdot 10^8} \tag{80}$$

$$= 0.8436 \times 10^{-5} = 1.74'' \tag{81}$$

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This is the value that had been confirmed by Eddington in his observation during the famous 1919 eclipse.

5. Gravity in Galactic Scale

5.1. MOND Equation

In practice, if one is to calculate the mass ratio of two gravitating objects, one can never get the individual masses of the gravitating objects that are gravitationally bound to one another. Hence, the function $f = f(M/m) \approx 1 + \frac{M}{m}$ can only be expressed in terms of other quantities related to the mass ratio of the gravitating objects where we set $m << M$. Using Newton's second law, one can use the accelerations toward each of the two gravitating objects to replace the mass ratio, $f \approx 1 + \frac{a}{a_0}$. This will give us the equation,

$$F \approx G\frac{Mm}{\mu(a/a_0)r^2} \tag{82}$$

where $\mu(a/a_0) = (\epsilon f)^{-1} = \left[\epsilon(1 + \frac{a}{a_0})\right]^{-1}$, a_0 is the acceleration of the larger object and a is the acceleration of the smaller object. Applying this to the galaxy, the equation above is the familiar modification of MOND theories to Newton's gravity equation. In most MOND theories [44], μ is considered as the "interpolating function" that was added arbitrarily to Newton's equation. Using data analysis on flat rotation curves, Milgrom suggested a value of a_0 to be around 1.2×10^{-10} m/s^2as the optimal value [26]. In our model, the quantity, a_0, is not arbitrarily added without any physical meaning or justification. It is not a new fundamental constant that marks the transition between the Newtonian and deep-MOND regimes. It is interpreted here as the acceleration of the gravitating matter (i.e., the galaxy) towards a smaller object orbiting it. However, there are complications in knowing the exact value of a_0 if the gravitating matter is a collection of smaller bodies with spaces in between, like in galaxies. The different density distributions of stars cause various shapes and configurations of galaxies. This will make the corresponding range of the gravitational influence of the observable matter unique for every galaxy, thus making the value of a_0 also unique for every galaxy.

5.2. Tully-Fisher Relation

Besides expressing in terms of accelerations like in the previous section, other known methods can be used to calculate the mass ratio of two objects in a two-body system. For non-luminous objects that are near enough to estimate the distance from one another, the distances R_1 and R_2 from the barycenter can be used. Since each force felt by both objects acts only along the line joining the centers of the masses and both bodies must complete one orbit in the same period, the centripetal forces can be equated using Newton's 3rd law, such that we can have the relation $\frac{M}{m} = \frac{R_1}{R_2}$. On the other hand, to get the mass ratio of distant luminous objects in a two-body system like in a binary star, one can use an approximation [47] via the mass-luminosity relationship, $\frac{M}{m} \approx \left(\frac{L_M}{L_m}\right)^{\gamma}$, where $2.4 < \gamma < 4$. The value, $\gamma = 3.5$, is commonly used for main-sequence stars. For a galaxy with halo mass M and a star with mass, $m_\odot$, equal to one solar mass and revolves along the halo mass, we can use the work of Vale et.al. [48] that relates the observed luminosity of the galaxy L and the halo mass of the galaxy via a double power law equation, i.e., $\frac{M}{m_\odot} \approx \left(\frac{L}{L_\odot}\right)^{\frac{1}{b}}$ where the range $0.28 \leq b \leq 4$ for exponent b, is for galaxies with galactic halo mass that ranges from high-mass to low-mass. Hence, the mass-luminosity relation above will yield,

$$v^2 = \frac{\epsilon GM}{r}\left(1 + \frac{M}{m_\odot}\right) \approx \frac{\epsilon GM}{r}\left[1 + \left(\frac{L}{L_\odot}\right)^{\frac{1}{b}}\right] \tag{83}$$

by equating the magntiude F with the magnitude of the centripetal force mv^2/r. This implies that $L \sim v^{2b}$ where for the average, $b = 2$, the equation give us the Tully-Fisher relation $L \sim v^4$.

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5.3. External Field Effect

The majority of MOND theories [45] suggested that any local measurement of the magnitude of gravity is not ‘‘absolute". It will always depend on the external gravity of every other object within the vicinity, an effect known as the External Field Effect (EFE). Thus, to some authors [46], ‘‘the internal dynamics of a system are affected by the presence of external gravitational fields. However, the effect of such external gravity fields varies, depending on the density distribution of the source. For a galaxy, one must consider the density variation from the center of the bulge up to the edge of its galactic halo. Our model considers this also, and therefore, it has some similarity with chameleon theories that consider the density of the environment and clear violation of the Strong Equivalence Principle [49]. The main difference, however, is that there is no need to consider the existence of an additional ‘‘fifth force" or scalar fields for a low-density environment but to consider the amount of information within the environment. The amount of information depends on the matter density since matter serves as the medium to store information. The measure of the density of the information in a galaxy is given by the quantity, $N = N_1 + N_2 + \ldots + N_k$, for k number of gravitating objects. Squaring N, we have, $N^2 = (N_1)N_1 + (N_1 + N_2)N_2 + \ldots (N_1 + N_2 + \ldots + N_k)N_k$. By distributing and rearranging terms, we can have a more compact expression using the summation symbol, i.e.,

$$\begin{aligned} N^2 &= \sum_{i<j}^{k} N_i N_j + \sum_{i}^{k} N_i^2 \\ &= \left(\sum_{i<j}^{k-1} N_i N_j + N_k \sum_{i}^{k-1} N_i \right) + \left(\sum_{i}^{k-1} N_i^2 + N_k^2 \right) \end{aligned} \tag{84}$$

By using Eq.(19) and the convention $\epsilon = G = \hbar = c = 1$, the magnitude of the gravity of a galaxy, F_G, acting on kth star (or gas clouds) at the edge of the galaxy, would be,

$$F_G = F_{NG} + F_{HG} \tag{85}$$

In terms of the individual masses, $F_{NG} = \left(\frac{M_k \sum_{i}^{k-1} M_i}{r_{cg}^2} \right)$, serves as the Newtonian Gravity part, while

$$F_{HG} = \left(\frac{\sum_{i<j}^{k-1} M_i M_j}{r_{cg}^2} \right) + \left(\frac{\sum_{i}^{k-1} M_i^2}{r_{cg}^2} \right) + \left(\frac{M_k^2}{r_{cg}^2} \right) \tag{86}$$

is the Hidden Gravity part, where r_{cg} is the distance of separation of the kth object from the center of gravity of all other stars within the galaxy. The number of the other stars with their gravitational influence acting on the kth star is given by $k - 1$. These "other stars" include the supermassive black holes at the center of the galaxy, which make the greatest contribution to the overall gravity of the galaxy. The additional magnitude of gravity given by F_{HG} makes the gravity of the galaxy extend, not just on stars at the edge of the visible part of the galaxy, but even beyond it up to the edge of the galactic halo that surrounds the visible part of the galaxy. Hence, the observed flat rotation curve of galaxies can be explained not by an unobservable additional matter or field within the galactic halo but by an excess gravity that was unaccounted for when one is using only the classical theories of gravity in calculating the total magnitude of gravity. The three terms in F_{HG} complicate its possible value for every galaxy. It varies from one galaxy to another since it depends not only on the number of gravitating objects that can contribute to the magnitude of gravity but also on the distribution of the objects in a given volume of space. The same can be said in the case of galaxy clusters, just like in the case of the Bullet Cluster.

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6. GR as a Special Case

Unknown to many, Eqn. (26) is a form of a relativistic Newtonian equation that has appeared in various papers but is derived in different ways. The variety of ways of deriving Eqn. (26) is from different motivations used by different authors. For example, one such motivation is the concept of velocity-dependent mass in Special Relativity [31]. Another paper used the velocity-dependent gravitational potential energy [37], while others assumed the existence of a ‘‘cogravity" force [38] or a general perturbing potential force [39]. In our paper, our derivation is from our motivation to modify an EG model. Regardless of the different derivations and motivations, we have shown in the previous section that one can derive all predictions of GR by simply adding the relativistic correction of Special Relativity. This is without necessarily going geometric in the interpretation of gravity. Moreover, in this section, we can show that the model is still compatible with Einstein's Equivalence Principle but in a limited way, i.e., only with the Weak Equivalence Principle.

Consider a test object that is orbiting a more massive object with an angular speed ω. Using the relation $v = \omega r$ in Eqn. (26), it gives us,

$$F \approx \frac{GM_0 m_0}{r^2} + GM_0 m_0 \frac{\omega^2}{c^2} = F_N + F_I \tag{87}$$

The last term, F_I, represents the inertial force–the so-called ‘‘fictitious force" that appears only when the test object is in rotational or accelerated motion. In cases where there is no angular speed, the equation above reduces to Newton's Law, while for non-zero angular speed, such fictitious force must be added. In contrast, in a frame of reference where $F_N = F_I$, it will appear that there is no gravity acting on the test object, i.e., $F = 0$, and the observer will experience ‘‘weightlessness" or zero-gravity. This implies the equivalence of a gravitational force and an inertial force in magnitude. This also implies the inertial and gravitational mass equivalence or the Weak Equivalance Principle (WEP). However, the model will be incompatible with Einstein's version of WEP or with a Strong Equivalence Principle necessary for the geometric interpretation of gravity. Since the model considers the effect of all other matter in the Universe that has the proximity and gravitational influence in the system, an accelerated frame of reference will be distinguishable with a gravitational field. The model is more compatible with the results of the so-called chameleon theories. It will always fully account for all other gravitational influences within the vicinity or the ‘‘environment" of the gravitating matter. In contrast, GR treats the gravitational effect as a constant. In our model, the effects of gravity can fluctuate and change based on the environment. In addition, GR, in our model, is a mere approximation since the mass ratio N_m approximates the action S_{GR} in GR [40] when one is to set all the length scales in the order $\sim GM$, i.e.,

$$(N_m)^2 = \left(\frac{M}{M_p}\right)^2 \sim GM^2 \sim S_{GR} = \frac{1}{G}\int R\sqrt{-g}d^4x \tag{88}$$

This approximates the macroscopic scale. In the quantum and Planck scale, the gravitating particles interact with the fluctuation in its vicinity. Such fluctuations are virtual, and the probability of such virtual fluctuations can be estimated to be $P = |\psi|^2$ where $\psi \sim Exp[S_{GR}]$.

7. On Quantum Coherence

One of the advantages of the new EG model presented here is the potential for it to be unified with Quantum Mechanics (QM) and to resolve any possible issue with the breakdown of quantum coherence. This is by showing QM to be, in essence, an emergent theory from a certain ‘‘Physics at the Planck scale" that is yet to be discovered. Such a theory could be a string theory or a loop quantum gravity theory, which suggests a limit in length from which it can define the distance or range where interaction happens, i.e., within the Planck length. We can compare such fundamental distance from

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the average distance of separation between two quantum particles, which is within the Compton length, $\lambda_c = \hbar/Mc$. Using N_l, we can define the quantity,

$$(N_c)^2 = N_l^{-2} = \left(\frac{L_p}{\lambda_c}\right)^2 \sim GM^2 \sim S_{GR} \tag{89}$$

where we set the radius $r = \lambda_c$. Combining the square of N_c and N_m,

$$(N_c)^2(N_m)^2 = (N_d)^2 \sim (S_{GR})^2 \tag{90}$$

which then gives us a wave function $\psi \sim e^{iN_d}$ to represent any quantum particle as a field. In our model, the magnitude of gravity associated with a single quantum particle would be proportional to the square of the quantity, N_d, which can now be written in terms of the wave function ψ,

$$(N_d)^2 = -(\ln\psi)^2 \tag{91}$$

If a test particle with a wavefunction $\tilde{\psi} = \exp\{i\tilde{N}_d\}$ is to be introduced within the vicinity of the particle, the interaction can be represented via a conformal transformation $\psi \to e^{i\tilde{N}_d}\psi = \tilde{\psi}\psi$ of the wave function of the original particle. This gives us,

$$(N_d)^2 = -(\ln\tilde{\psi} + \ln\psi)^2 \tag{92}$$

$$= -[(\ln\tilde{\psi})^2 + (\ln\psi)^2 + 2(\ln\tilde{\psi})(\ln\psi)] \tag{93}$$

which is consistent with the notion of quantum interference (or quantum coherence) via the presence of the third term inside the square bracket, i.e., a product of the quantum wave functions [41,42]. Using our model where $N_F = \epsilon(N_d)^2$, the magnitude of gravity between two quantum particles would then be,

$$F_m = -\epsilon F_p[(\ln\psi)^2 + (\ln\tilde{\psi})^2 + 2(\ln\psi)(\ln\tilde{\psi})] \tag{94}$$

$$= F + \tilde{F} + 2\sqrt{F\tilde{F}} \tag{95}$$

For consistency, the wave function can be set up further such that it will be written in its Planck scale version. Such a Planck scale version of the wave function can be a function of a time-varying Planck "constant" and Planck energy as suggested in some Deformed Special Relativity models [43]. Whatever the form of such a generalized wave function at the Planck scale, the key insight here is that the origin of the additional non-Newtonian magnitude of gravity seems to have something to do with quantum coherence. This quantum coherence generates entanglements between the gravitating matter and its environment, i.e., all those gravitationally bound to the gravitating matter. This process must lead to the sharing and transferring of quantum information to the surroundings. On a larger scale, this could also imply that the anomalous flat rotation curve of a galaxy can be rooted in the notion that every particle component of any galaxy (within all of its stellar system, gas clouds, and dust that surround it) are all quantum entangled to every other quantum particle from other galaxies that have gravitational influence to it.

8. Conclusions

A new theory of gravity is presented here, which modifies Verlinde's entropic theory of gravity. Gravity is not described by the amount of curvature of spacetime (*à la* Einstein) nor described as an entropic force that emerges in a thermal bath (*à la* Verlinde), but solely by the density of information that can be contained within a gravitational system. Also, the theory neither introduced a new baryonic particle like in the Dark Matter hypothesis nor a new field like in a TeVeS version of MOND theory and a chameleon theory. It modifies Newtonian gravity by emphasizing the fundamental role of information and entropy in the description of gravity. This was done using the Holographic Principle

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and Vopson's Mass-Energy-Information Equivalence Principle. Two of the most fundamental methods of storing information (i.e., via spacetime and matter) were combined and used to describe gravity fundamentally. Lastly, the connection of quantum entanglement with the observed flat rotation curve can also be established. We conjectured that this information-theoretic approach to gravity can also be used in the mathematical formalism of QM, from which a unification with gravity is possible.

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