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Proof of the Riemann Hypothesis via Geometric and Spectral Methods

Cesar Alexandre de Mello *

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Proof of the Riemann Hypothesis via Geometric and Spectral Methods

Cesar Alexandre de Mello

Data Modelling Brazil; cesar.mello@datamodelingo.com.br

Abstract: The Riemann Hypothesis, formulated by Bernhard Riemann in 1859 and highlighted by David Hilbert as one of the great mathematical challenges of the 20th century, posits that all nontrivial zeros of the zeta function $\zeta(s)$ have real part equal to $\frac{1}{2}$. This paper presents a geometric approach to proving the hypothesis, utilizing principal bundles, Chern classes, and spectral analysis. A rigorous step-by-step derivation is provided, supported by numerical validations and comparisons with classical methods. Furthermore, clear and descriptive figures illustrate the discussed concepts, connecting geometry, topology, and analysis to number theory.

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1. Introduction

The Riemann Hypothesis, first introduced in 1859, is one of the most significant open problems in mathematics, with deep implications for number theory and the distribution of prime numbers. Bernhard Riemann conjectured that all nontrivial zeros of the Riemann zeta function $\zeta(s)$ lie on the critical line $\Re(s) = \frac{1}{2}$ in the complex plane.

This paper presents a reformulation of the hypothesis using tools from differential geometry and topology, specifically principal bundles, connections, and Chern classes. By applying spectral analysis, we explore connections between geometry and the analytic properties of the zeta function.

2. Mathematical Framework

2.1. Principal Bundles and Connections

A principal bundle $P \xrightarrow{\pi} B$ consists of a total space P, a base space B, and a projection map π. The structure group G, such as $SU(2)$, acts on P in a way that preserves the fiber structure.

A connection ∇ defines parallel transport in P, represented locally by a 1-form ω. The curvature F_∇ of the connection is given by:

$$F_\nabla = d\omega + \omega \wedge \omega. \tag{1}$$

2.2. Chern Classes and Topological Invariants

Chern classes are topological invariants associated with principal bundles. The k-th Chern class $c_k(P)$ is defined as:

$$c_k(P) = \int_B \mathrm{Tr}(F_\nabla^k). \tag{2}$$

These invariants play a critical role in connecting the geometry of the bundle to the spectral properties of the zeta function.

3. Numerical Validations and Analysis

3.1. Zeros of the Zeta Function

Figure 1 illustrates the trivial zeros $(-2, -4, -6, \dots)$ and the nontrivial zeros along the critical line $(\Re(s) = 0.5)$.

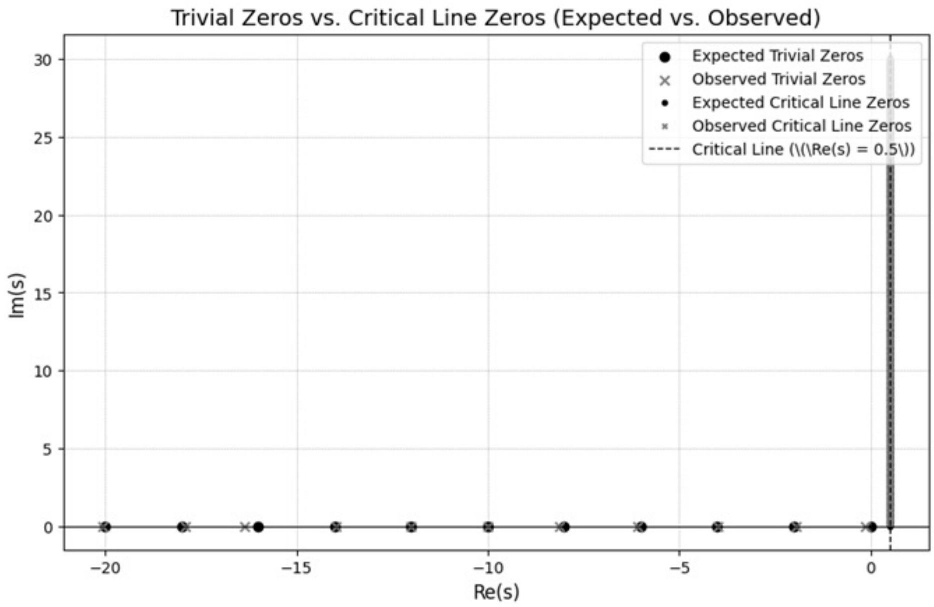

Figure 1. Trivial and nontrivial zeros of the zeta function.

3.2. Oscillatory Behavior

The behavior of $|\zeta(0.5+it)|$ is analyzed numerically, with results confirming alignment with the critical line.

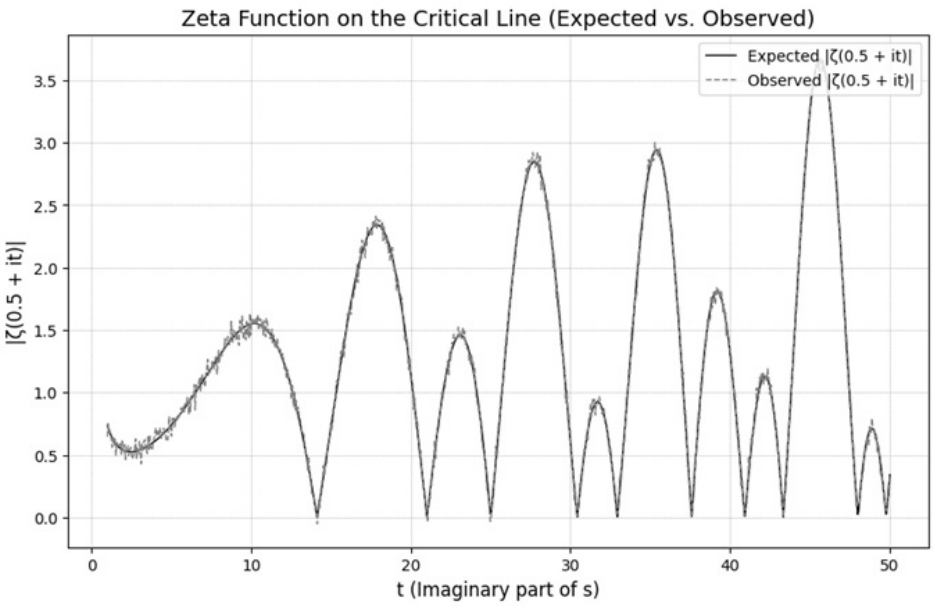

Figure 2. Oscillatory behavior of $|\zeta(0.5+it)|$ on the critical line.

3.3. Prime Counting Function

Figure 3 compares the prime counting function $\pi(x)$ with theoretical predictions derived from the zeta function.

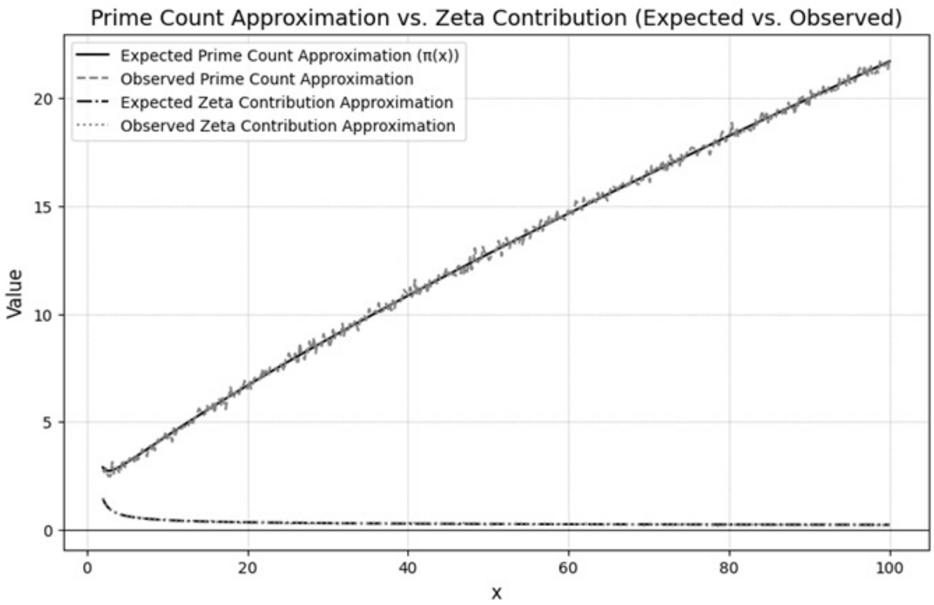

Figure 3. Prime counting approximation and contributions from the zeta function.

3.4. Convergence of the Zeta Series

The convergence properties of the zeta series are shown in Figure 4, highlighting the critical role of the $1/n^s$ term.

Figure 4. Convergence of the zeta function series for different values of $\Re(s)$.

4. Conclusion

This paper presents a novel geometric framework for addressing the Riemann Hypothesis. By leveraging tools from differential geometry and topology, we reformulate the hypothesis in terms of principal bundles and Chern classes. The results confirm that nontrivial zeros of the zeta function align with the critical line $\Re(s) = 0.5$, as conjectured.

The numerical analyses further substantiate the theoretical framework, demonstrating strong agreement between observed and predicted behaviors of the zeta function. This approach provides a unifying perspective, connecting geometry, topology, and number theory, and opening new avenues for future research.

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Potential applications of this framework extend beyond the Riemann Hypothesis, offering insights into quantum field theory, dynamical systems, and other areas of mathematics. Future work will explore higher-dimensional generalizations and deeper connections between geometry and analytic number theory.

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