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Validation of Analytical Results for Counter Current Flow in Square Channels Separated by a Membrane in a Hemodialysis Module Using Experimental Module Results

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Abstract

Counter-current flow in channels separated by a membrane was studied by several scientists and researchers. The current study aims to analytically simulate and describe the distribution of pressure, volume flow rate, and velocity in square channels separated by a membrane. Consequently, the current study was executed using 1-D analytical solutions to achieve several significances: avoiding the experimental tests' execution, reducing the effort of module design for expensive and time-consuming tasks, and easy observation of the variations in pressure, volume flow rate, and velocity. The 1-D analytical solution directly simulated the flow in square channels separated by a membrane to solve the continuity and Darcy's Law, where pressure, volume flow rate, and velocity are calculated. The experiment module results were used to validate the results of the 1-D analytical solutions. The results of the current study found that the pressure decreases from the inlet to the outlet of the channel, and the horizontal velocity decreases from the inlet to the middle of the channel length and increases to the outlet of the channel. The result of the 1-D analytical solutions is characterized by acceptance accuracy compared with experimental module results. Consequently, the 1-D analytical solutions can be used to explore and display the distribution of pressure, volume flow rate, and velocity in square channels separated by a membrane with acceptance accuracy to examine the hemodialysis prototype module efficiency and performance before fabrication.

Keywords: counter-current flow; membrane; hemodialysis; 1-D analytical solution; experimental module results

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1. Introduction

1.1. Background

The analytical solution of the flow between two porous parallel plates is reported by few researchers. The Navier-Stokes (N-S) equations had been solved to obtain a complete description of the fluid flow of one-dimensional incompressible steady-state laminar flow in a channel having a rectangular cross section. One side of the cross section, representing the distance between the porous walls, is much smaller than the other with two equally porous walls, which leads to detailed expressions for the dependence of the velocity components and the pressure [1]. An analytical solution for the pressure drop in a rectangular slit with constant wall velocity being proportional to the trans-membrane pressure difference (constant wall permeability) and a tube with porous (permeable) walls for the constant wall permeability was reported. The pressure drops in crossflow membrane modules are a function of wall permeability, channel dimension, axial position, and fluid properties [2]. The analytical solution of one-dimensional incompressible fluid flow between two porous parallel plates was reported. There is a crossflow (vertical velocity) along the y-direction due to the walls being porous. The analytical solution was found by applying the continuity equation and N-S equations with specific assumptions and boundary conditions. The solution depends on the cross-stream Reynolds number [3]. The 1-D analytical solution model for the two fluids flowing in parallel or counterflow through passages separated by a permeable wall was derived. It uses the continuity equation for the flow in the channel and Darcy's law for the flow through the membrane to determine the mass transfer for the counter-current flow where counter-current could improve the uniformity of mass flux distribution. The flow rate decreases from the inlet to some intermediate distance and then increases back to the outlet in counter-current [4].

In membrane filtration processes where a combined free and porous flow occurs, N-S equations are used to model free flows, while Darcy's law is used to model flows in porous media with small porosity. A combination of free flow and flow through porous media occurs, and the continuity of the flow field regime across the interface between laminar flow and the porous region can be modeled by coupling Darcy's law and the N-S equations. Recent studies modeled flow in hollow fiber membrane modules. Hemodialyzers are cylindrical modules filled with several thousand (~8000–16,000) hollow polymeric fibers and commonly used as part of hemodialysis therapy devices. The typical sizes of hemodialyzer modules (containing thousands of fibers) range from 1 to 4 cm in diameter, from 9 to 90 μm in membrane (fiber) thickness, from 200 to 750 μm in membrane (fiber) diameter, and from 15 to 35 cm in length [4–16].

1.2. Performance, Efficiency, Removal, and Clearance of Hemodialysis

Membrane processes are used in technical and medical applications for the separation of waste from valuable substances [17]. Hollow fiber membrane modules are manufactured depending on the fiber material used and the type of impurities, which are designed to separate [18]. The effect of all the dialyzer-related factors on solute clearance has not been reported in the literature, making it difficult to optimize dialyzer design to maximize solute removal from patient's blood [13,19]. A crossflow process in which fluid is removed from the blood by setting a suitable transmembrane pressure (convective mass transfer) and by setting a suitable concentration (diffusive mass transfer) is hemofiltration (HF) and hemodialysis (artificial kidney) [17]. The efficiency and performance of hemodialysis operation depend on both module design and clinical procedures [10]. The typical operating conditions are achieved when inlet blood and dialysate flow rates are 300 mL/min and 500 mL/min [14,15,20]. The removal efficiency of a single pass for low-molecular-weight metabolites like urea would therefore be about 65%, assuming that convection is the primary mechanism of mass transfer [5]. Membrane separation owing to the merits of lower energy consumption, simplicity, high efficiency, and easy operation and control [21]. The ideal dialysis membranes should achieve the following optimum characteristics [21–23].

1. Sufficient mechanical, physical (withstand pressure), chemical, and thermal stability (withstand sterilization process).
2. Large surface area
3. Optimal biocompatibility (hydrophilic/hydrophobic).
4. The support layer is highly porous, and the thin active separation layer (high solute fluxes) ensures high permeability for efficient removal of toxins from the blood.
5. High and macro porosity (high hydraulic permeability).
6. Narrow and uniform pore size distribution (sharp molecular weight cut-off (MWCO)) and exhibit high selectivity to prevent the loss of beneficial proteins
7. No back diffusion (from the dialysate to the blood).
8. Minimal surface roughness (reduce interaction with blood)
9. Exhibit excellent hemocompatibility and cytocompatibility (prevent protein adsorption, platelet adhesion, blood coagulation, complement activation, and hemolysis).
10. Prevent the entry of any possible bacterial contaminants, such as endotoxins, from the dialysis fluid into the bloodstream to prevent adverse effects on patient health.

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1.3. Objectives

The primary objectives of the current study are to physically describe the flow patterns that develop under counter-current conditions and to demonstrate the advantages of 1-D analytical solutions to examine the proposed design before fabricating and to optimize the membrane contactors. Consequently, the 1-D analytical solutions ultimately save time and cost compared with expensive experimental prototyping.

The main objective of the current study is to analytically simulate and describe the distribution of pressure, volume flow rate, and velocity in square channels separated by a membrane. The 1-D analytical solution is characterized by the following advantages.

1. A 1-D analytical solution can model and visualize the flow condition with acceptable accuracy compared to measuring with physical equipment and avoid the execution of experimental tests, which face challenges in tiny channels and membranes.
2. A 1-D analytical solution reduces the expense and time-consuming nature of physical modeling and experimental tests and identifies potential problems like backflow before the manufacture of costly physical modules.
3. The 1-D analytical solution helps to observe variations in pressure and horizontal velocity in the model in an easy way, but in real experiments, it is very difficult to observe.

2. Materials and Methods

2.1. Experimental Module (Hoskins)

Membrane-integrated (75*75*10 μm) microfluidic chips were fabricated to be used in the experimental module (Hoskins). Burst pressure testing was conducted on microfluidic chips to examine the structural robustness of the fabricated devices. A steady input flow rate (syringe) (dyed water at a rate of 100 μL/min) with three of four inlet/outlet ports sealed produced an exponential increase in pressure over time. The results showed that the chips withstood internal pressures averaging 1.27 MPa. Flow testing from ~35 μL/min to ~345 μL/min confirmed stable operation in 75 μm square channels, with no leakage and minimal flow resistance up to ~175 μL/min without deviation from the predicted behavior in the 75 μm. They experimentally examined the fabricated membrane for robust pressure with different volume flow rates in the model, which is 3050 μm*210 μm (two channels (75 μm each, a membrane of 10 μm, and a solid wall around the membrane of 25 μm) [24], as shown in Figure 1.

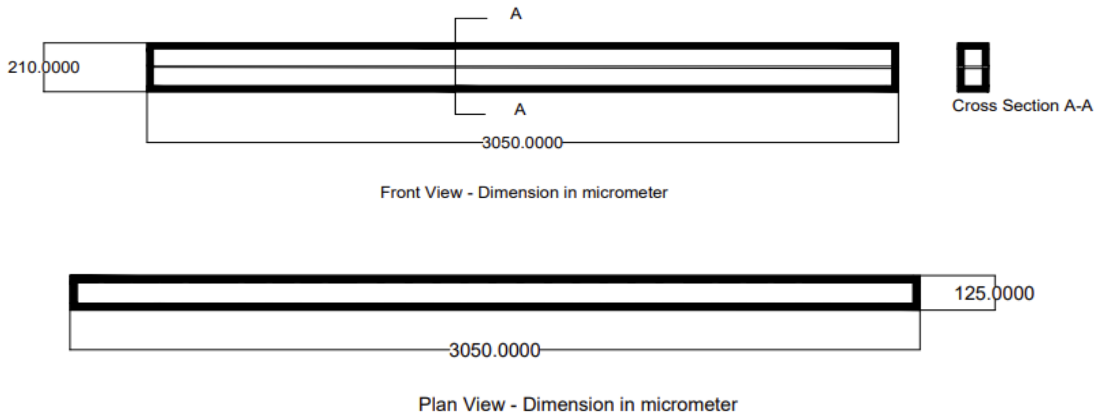

Figure 1. Experimental Module (Hoskins) (Front and Plan View).

2.2. Hemodialysis Module

The current study reviewed the limitations in the hemodialysis process and suggested the criteria that lead to selecting the best module to be used with perfect performance and efficiently

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removing pollutants from blood with a high clearance percentage and reducing the time required to complete the process. Clearance is defined as the blood volume that should be completely purified in the time unit to yield a given solute removal rate [14].

$$CL = (Q_{Bi}\, C_{Bi} - Q_{Bo}\, C_{Bo}) / C_{Bi} \quad (1)$$

where:

- Q_{Bi} and Q_{Bo} are the blood flow rates at the hemodialyzer's inlet and outlet.
- C_{Bi} and C_{Bo} are the inlet and outlet solute concentrations in the blood.

The experimental module (Hoskins) with its dimensions is not suitable to use in hemodialysis unless the module is modified and fabricated to the current study module with length (25 cm) and 300 numbers and compact in hemodialyzers (rectangular modules (25 modules in x-axis and 12 modules in y-axis) = (25*0.0125 cm) * (12*0.0210 cm) = 0.3125 cm * 0.252 cm = **0.07875cm^2**) with inlet and outlet for blood and dialysis; it may improve the hemodialysis process efficiency (clearance) and reduce the time required to treat patient blood from urea.

The current study module will need **37 min** for 3000 ml and **92 min** for 7500 ml, with an average of about **60 min (1 hr)** with u = 0.097 m/s in the module. The comparison between recent studies and the experimental module (Hoskins) with the suggested current study module (hemodialysis module) is presented in Table 1 and Figure 2.

Table 1. Comparison of hemodialysis module between recent studies, experimental module (Hoskins), and current study.

Module parameter	**Recent studies**	**Experimental module (Hoskins)**	**Current study module**
Length (cm)	15 - 35	0.3	25
Membrane thickness (µm)	9 – 90	10	10
Height (Diameter (D) or Side) (µm)	Circular 200-230	Rectangular (width*height) 75*(75+10+75)	Rectangular (width*height) 75*(75+10+75)
# of modules in x-section	8000-16000	1	300
Flow rate (Q) of blood mL/min	300	35-345 (Dyed water)	300
Flow rate (Q) of dialysis mL/min	500	35-345 (Dyed water)	500

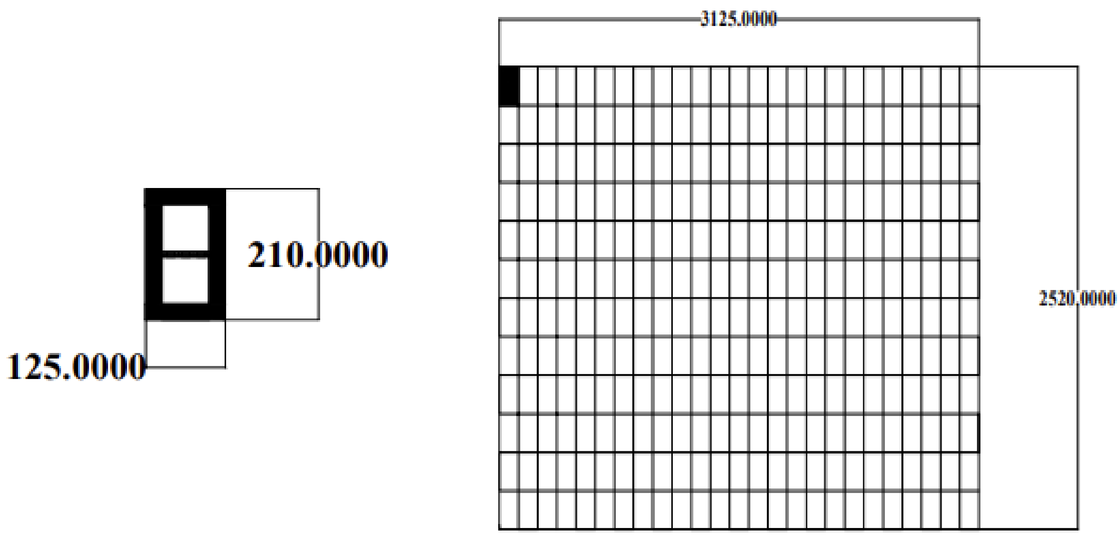

Figure 2. Current Study Hemodialysis Module Cross Section (y, z) with Length (x) (Dim. in µm).

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3. Results

The experimental module (Hoskins) results for counter-current flow in square channels separated by a membrane are tabulated and figured in the following sections [24].

3.1. Pressure Results

The pressure is measured for syringe inlets 1 and 2 (upper and lower channel inlets), membrane chip outlet-1 and 2, and upper and lower channel outlets 1 and 2 in the experimental module (Hoskins), and it follows the empirical equations [24] as shown in Table 2 and Figure 3.

The pressure for the syringe inlets 1 and 2 (upper and lower channel inlets)

$P = 0.0406Q$ (2)

The pressure for the membrane chip outlets 1 and 2

$P = - 0.00005Q^2 + 0.0491Q - 0.3561$ (3)

The pressure for the upper and lower channel outlets 1 and 2

$P = -0.00005Q^2 + 0.0523Q-0.5086$ (4)

where

P = Pressure (kpa)

Q = Inlet volume flow rate (μL/min)

Table 2. Pressure in experimental module (Hoskins) with constant wall permeability.

Inlet volume flow rate (Q)	**Pressure (P) (KPa)**		
Q (μL/min)	**Syringe inlets 1 and 2 (upper and lower channel inlets)**	**Membrane chip outlets 1 and 2**	**Upper and lower channel outlets 1 and 2**
35	1.421	1.30115	1.26065
70	2.842	2.8359	2.9074
100	4.06	4.0539	4.2214
175	7.105	6.70515	7.11265
250	10.15	8.7939	9.4414
345	14.007	10.63215	11.58365

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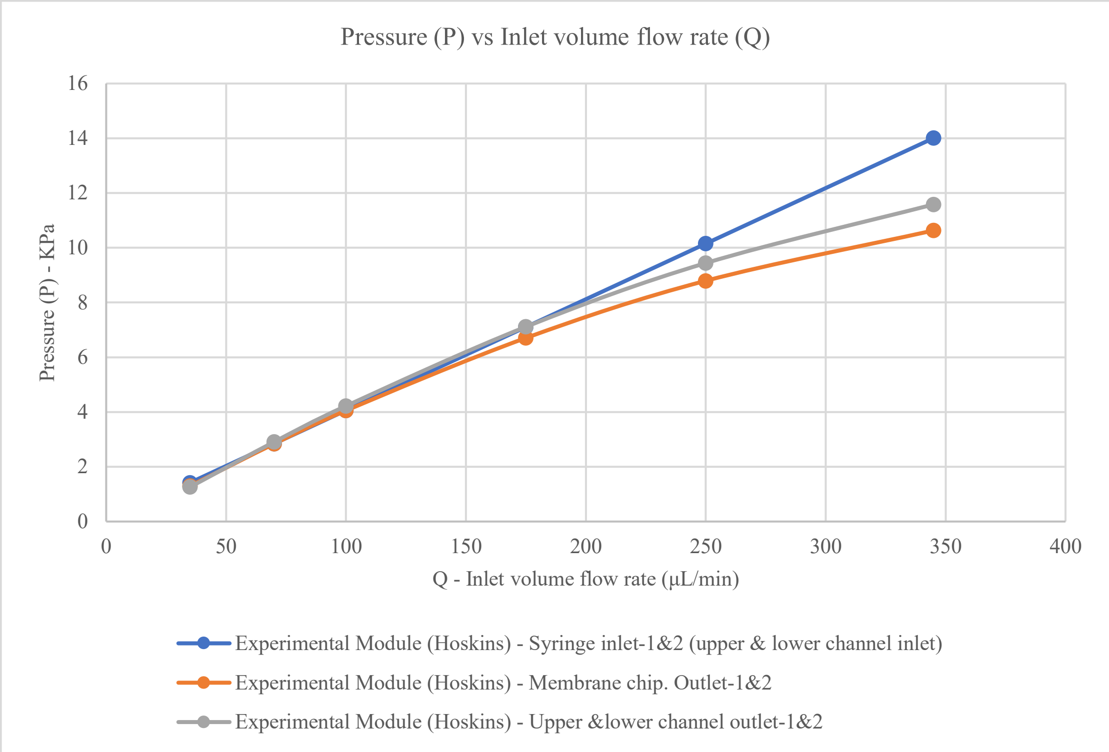

Figure 3. Pressure in experimental module (Hoskins) with constant wall permeability (module parameters in the text).

3.2. Velocity Results

The horizontal velocity is measured for syringe inlets 1 and 2 (upper and lower channel inlets), membrane chip outlets 1 and 2, and upper and lower channel outlets 1 and 2 in the experimental module (Hoskins), and it follows the empirical equations [24], as shown in Table 3 and Figure 4.

The horizontal velocity for the syringe inlets 1and 2 (upper and lower channel inlets)

u(m/s) = 0.003Q+4E-16 (5)

The horizontal velocity for the membrane chips. outlets 1 and 2

u(m/s) = -3E-06Q²+ 0.0036Q-0.026 (6)

The horizontal velocity for the upper and lower channels outlets 1 and 2

u(m/s) = -4E-06Q²+ 0.0038Q-0.0372 (7)

where

u = horizontal velocity (m/s)

Q = Inlet volume flow rate (μL/min)

Table 3. Horizontal velocity in experimental module (Hoskins) with constant wall permeability.

	Horizontal velocity (u) (m/s)		
Inlet volume flow rate (Q)	**Experimental module (Hoskins)**		
Q (μL/min)	**Syringe inlets 1 and 2 (upper and lower channels inlets)**	**Membrane chips. outlets 1 and 2**	**Upper and lower channels outlets 1 and 2**
35	0.105	0.096325	0.0909
70	0.21	0.2113	0.2092
100	0.3	0.304	0.3028
175	0.525	0.512125	0.5053
250	0.75	0.6865	0.6628

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345	1.035	0.858925	0.7977

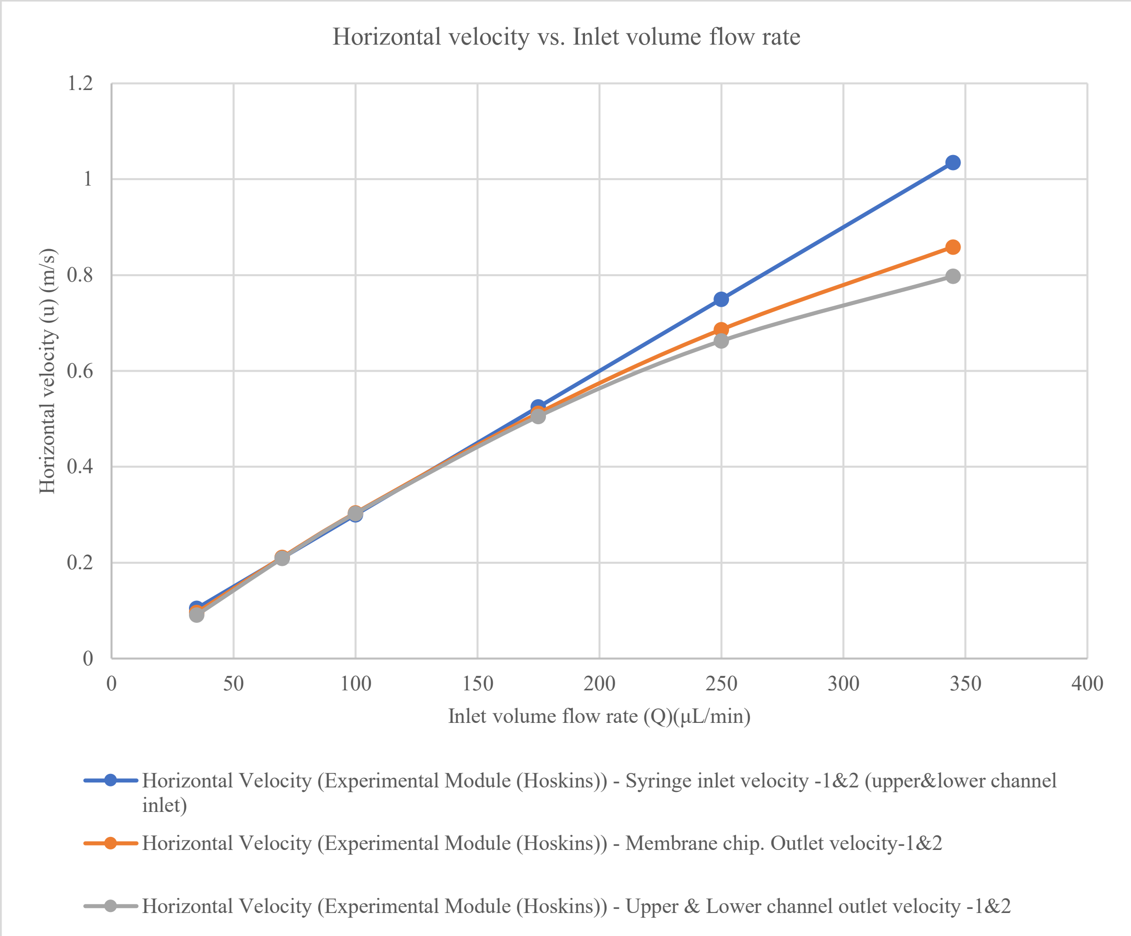

Figure 4. Horizontal velocity in experimental module (Hoskins) with constant wall permeability (module parameters in the text).

4. Comparison Between the Results of Analytical Solutions (Hagen-Poiseuille, Babu, 1-D (Ciofalo)) and the Experiment Module (Hoskins)

4.1. Calculation of Permeability (k), Porosity, and Hydraulic Permeability (Lp) for Experiment Module (Hoskins)

4.1.1. Calculation of the Permeability (k)

The experiment module (Hoskins) uses Hagen-Poiseuille (HP) equation for a square cross-section channel [24].

$\Delta P = 0.63 * 8\mu QL / \pi\ (a/2)^4$

where (a) = the height and width of the square channel (m).

The mean velocity in the channel can be calculated from Hagen-Poiseuille (HP) equation.

$\mathbf{u_{mean/HP} = [(dp/dx)\ \pi\ (a^2)] / 80.64\ \mu}$

The mean velocity in the membrane can be calculated from Darcy's law.

$\mathbf{u_{mean/\ Darcy} = (k/\mu)\ \partial p/\partial x}$

By assuming, $\mathbf{u_{mean/HP} = u_{mean/\ Darcy}}$

$\mathbf{(dp/dx)\ \pi\ (a^2) / 80.64\ \mu = (k/\mu)\ \partial p/\partial x}$

$\mathbf{K = \pi\ a^2 / 80.64}$

A = h = 7.5E-05m

k = π (7.5E-05^2) / 80.64 = **2.19E-10 m²**

4.1.2. Calculation of Porosity

Fabricated membranes were made (thickness = 10 μm and area = 75 × 75 μm) with pores 3.73 μm in diameter and very high porosities that reached up to **60%**. The hole is square with side w (3.73 μm) + wall (1 μm) = 4.73 μm [24].

Porosity (ϕ) = volume of voids/total volume

Number of holes = (3000/4.73) * (75/4.73) = 10057 holes

Volume of one hole = 3.73 * 3.73 * 10 = 139.129 μm^3

***Porosity* (ϕ)** = volume of voids/total volume = (10057 * 139.129)/3000 * 75 *10 = **0.622**

4.1.3. Calculation of the Hydraulic Permeability (Lp)

The general equation that relates the change in volume flow rate in the channel with the pressure and flux through the membrane is

$dQ/dx = w * q(x) = w * L_p * P_i$

To calculate the hydraulic permeability (L_p), the parameters can be used from the experimental module (Hoskins) [24].

Inlet volume flow rate (Q_i) = 345 μL/min = 5.75E-09 m^3/s

Outlet volume flow rate (Q_o) = 270 μL/min = 4.50E-09 m^3/s

Channel length (L) = 3E-03 m

Channel width (w) = 7.5E-05 m

Inlet pressure (P_i) = 14.145 kPa

$dQ/dx = w * q(x) = (Q_i - Q_o) /L = w * q(x)$

((5.75E-09 -4.50E-09) μL/min) /3E-03 = 7.5E-05 * q(x), q(x) = 5.56E-03 m/s

Lp = q(x)/P_i = 5.56E-03/14145 = **3.93E-07 m/Pa.s**

The permeability (k), porosity, and hydraulic permeability (Lp) parameters of the experimental module (Hoskins) are shown in Table 4.

Table 4. permeability (k), Porosity, and hydraulic permeability (Lp) parameters of the experimental module (Hoskins).

Parameters	Permeability (k) m^2	Porosity (ϕ) -	Hydraulic permeability (Lp) m/Pa.s
Experimental Module	2.19E-10	0.622	3.93E-07

The experimental module (Hoskins) was examined by applying inlet volume flow rate Q and measuring the outlet volume flow rate, then from the volume flow rate, they calculated the mean velocity Q/A. They calculated the pressure gradient through the module length. To validate 1-D analytical solutions, the current study used the experiment module results [24].

For the experiment module, the theoretical results are calculated for the square solid channel without the membrane's effect using the Hagen-Poiseuille equation, while the experimental results are gathered from the real fabricated experiment module of counter-current flow in channels separated by a membrane. For more comparison, the theoretical results are also calculated for the square solid channel without the membrane's effect using equations (8) [3]. The 1-D analytical solutions (Ciofalo) of counter-current flow in channels separated by a membrane were used with specific assumptions [4]. The pressure (P), volume flow rate (Q), and horizontal velocity comparison for the theoretical and experimental results of the experiment module with 1-D analytical (theoretical) are shown in Table 5 and Figures 5 and 6.

$P = (28.46\mu\ Q_i\ L)/ (h^4)$ (8)

where: P = Pressure (KPa)

Q_i = Inlet volume flow rate (m^3/s)

L = Channel length (m) = 0.003 m

h = Channel height (m) = 75E-06 m

μ = Dynamic viscosity (Pa.s) = 0.001 Pa.s

The comparison between the theoretical results (Hagen-Poiseuille, Babu, and Ciofalo) and experimental module (Hoskins) results is shown in Table 5.

Table 5. Comparison between the theoretical results (Hagen-Poiseuille, Babu, and Ciofalo) and experimental module results (Hoskins).

Methods	**Analytical (Hagen-Poiseuille)**	**Analytical (Babu)**	**Experiment module (Hoskins)**		**1-D analytical solution (Ciofalo)**	
Limitation	without membrane		with membrane			
Input Data	Theoretical Results	Theoretical Results	Experimental Results		Theoretical Results k = 2.19E-10 m², Lp=3.93E-07 m/Pa.s	
Q = volume flow rate	Calculated	Calculated	Measured	Measured	Calculated	Calculated
Q_i (μL/min)	P_i (KPa)	P_i (KPa)	Q_o (μL/min)	P_o (KPa)	Q_o (μL/min)	P_i (KPa)
35	1.435	1.5739	35	1.26065	28.41	1.287
70	2.87	3.1477	70	2.9074	56.82	2.574
100	4.1	4.4967	100	4.2214	81.33	3.684
175	7.175	7.8693	175	7.11265	142.04	6.434
250	10.25	11.2419	218	9.4414	202.91	9.192
345	14.145	15.5138	270	11.58365	280.02	12.684

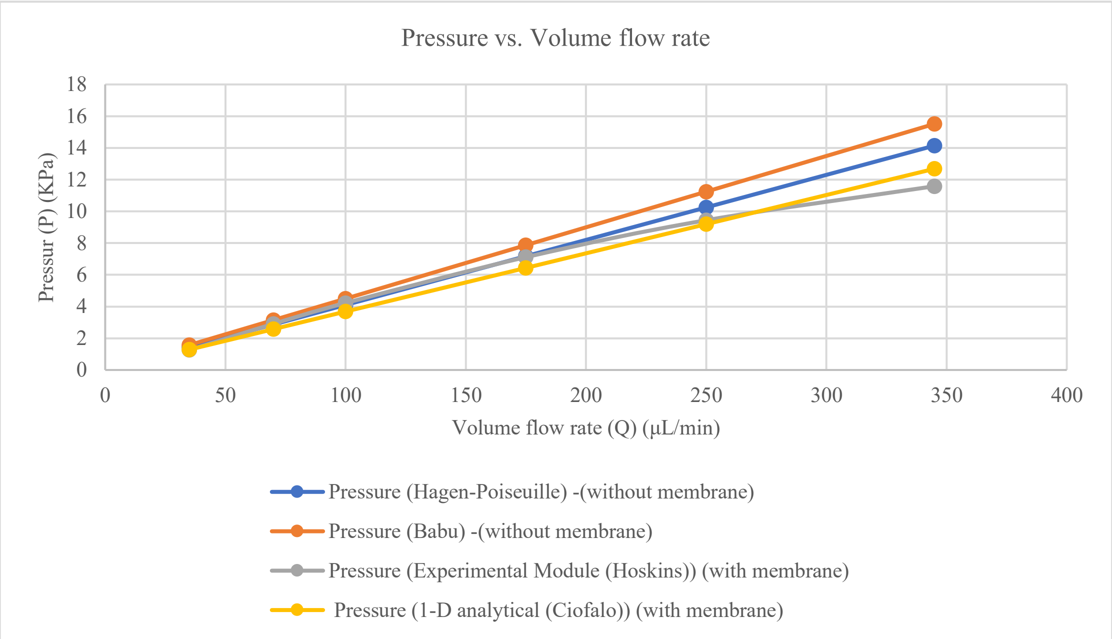

Figure 5. Comparison of pressure between the theoretical results (analytical) (Hagen-Poiseuille, Babu, and Ciofalo) and experimental module (Hoskins) results (constant wall permeability) (module parameters in the text).

Table 6. Comparison of horizontal velocity between experimental module (Hoskins) and 1-D analytical solution (Ciofalo).

Horizontal velocity (u) (m/s)

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Inlet volume flow rate (Q)	Experimental module (Hoskins)			1-D Analytical Solution (Ciofalo)	
Q_i (μL/min)	Syringe inlets 1 and 2 (upper and lower channels inlets)	Membrane chips. outlets 1 and 2	Upper and lower channels Outlets 1 and 2	Upper and lower channels inlets 1 and 2	Upper and lower channel outlets 1 and 2
35	0.105	0.096325	0.0909	0.103612	0.084096
70	0.21	0.2113	0.2092	0.207224	0.168193
100	0.3	0.304	0.3028	0.296625	0.240756
175	0.525	0.512125	0.5053	0.518059	0.420482
250	0.75	0.6865	0.6628	0.740089	0.600693
345	1.035	0.858925	0.7977	1.021314	0.828949

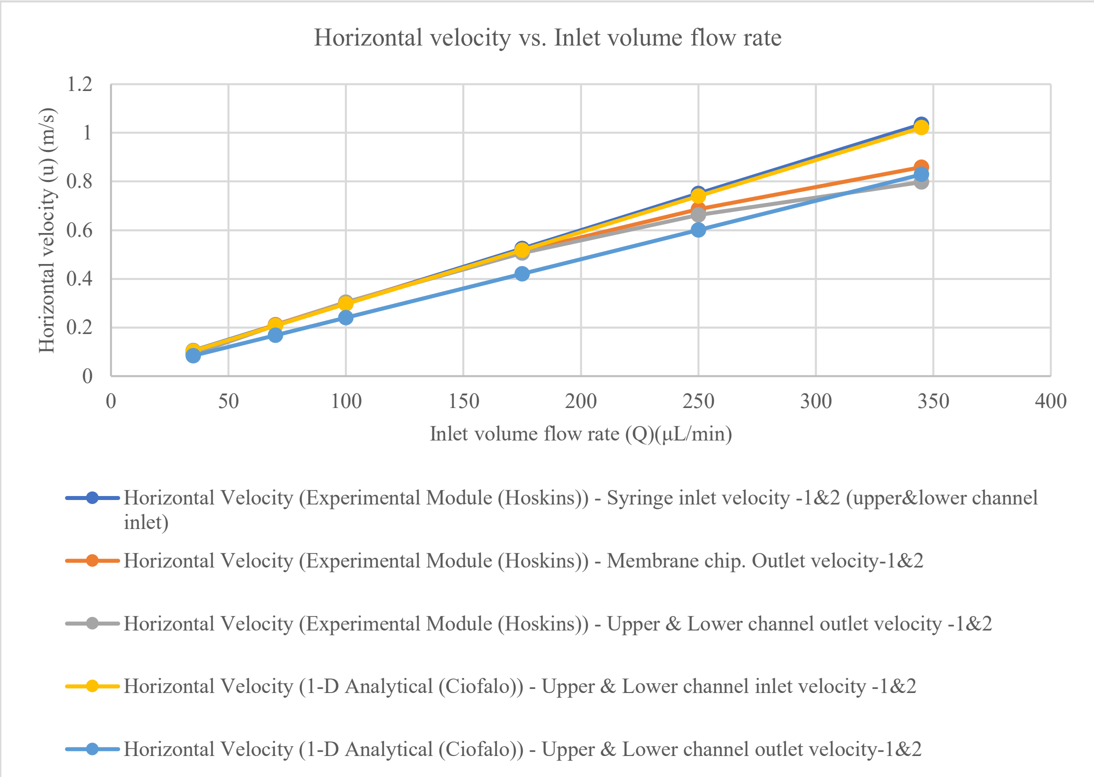

Figure 6. Comparison of horizontal velocity between experimental modules (Hoskins) with impermeable walls and 1-D analytical solution (Ciofalo) with constant wall permeability (module parameters in the text).

5. Analysis and Discussion

The experimental module (Hoskins) found that the membrane-integrated chips exhibited outlet flow asymmetries greater than 10%, indicating active fluid transfer across the membrane and highlighting flow-dependent permeability. The theoretical results are calculated for a square solid channel without the membrane's effect using the Hagen-Poiseuille equation and equation (8) [4]. The results show slight differences because the effect of the horizontal velocity variation is in the y and z directions (du/dy and du/dz) for the rectangular cross-section channel that was studied [3]. The experimental module (Hoskins) measured the flow parameters for counter-current flow in the channel separated by a membrane. The 1-D analytical solution for counter-current flow in the channel separated by a membrane with specific assumptions was derived [4]. The experimental module (Hoskins) results and the 1-D analytical solutions (Ciofalo) are slightly different but within the accepted percentage.

The results in Table 5 and Figures 5 and 6 show the following remarks:

1. There is a difference of about 10% between the theoretical results calculated by the experimental module (Hoskins) (Hagen-Poiseuille equation) and the Babu expression for a solid square channel. This difference is due to the method used to solve the 2nd order of differential equations and the assumption. The experimental module (Hoskins) used Hagen-Poiseuille equations that were modified from circle section to square section, while Babu used a Fourier series solution to solve the 2nd order of differential equations for rectangular sections.
2. There is a difference between the experimental module (Hoskins) results and the (1-D) analytical solution (Ciofalo) model for flow in square channels separated by a membrane. This difference is due to the experimental module (Hoskins) using accurate devices to measure the pressure and volume flow rate, despite the difficulty of measuring these parameters in tiny micrometer channels. The (1-D) analytical solution (Ciofalo) model uses analytical solutions with specific assumptions that affect the results.
3. The experimental results of the pressure, volume flow rate, and horizontal velocity measured in the experimental module (Hoskins) coincide with the theoretical results for specific volume flow rate (175 μL/min) and then deviate, decreasing as volume flow rate increases while the (1-D) analytical (Ciofalo) solution increases as volume flow rate increases at a constant rate.

6. Conclusion

The results of the current study coincide with the acceptance accuracy of the experiment module (Hoskins) results for counter-current flow in square channels separated by a membrane. Therefore, the experiment module (Hoskins) results can be used to validate the (1-D) analytical solution (Ciofalo). Consequently, the 1-D analytical solution (Ciofalo) for counter-current flow in square channels separated by a membrane can be used to explore and display the distribution of pressure, volume flow rate, and velocity with acceptable accuracy to examine the hemodialyzer prototype module efficiency and performance before fabricating to save time and cost without performing experimental tests. The current study prepared an EXCEL sheet to calculate pressure, horizontal velocity, and volume flow rate for any module dimensions and fluid properties to examine any module before fabricating.

The current study proposed a hemodialyzer module where the cross section of the experiment module (Hoskins) can be used with some modifications (increasing the length (from 3 mm t0 25 cm) and the number of channels (300) (upper (blood) and lower (dialysis)) and increasing the velocity in the lower channel (dialysis) because the blood flow rate is (300 mL/min) and the dialysis flow rate is 500 ml/min. The module may improve the hemodialysis process efficiency and reduce the time required to treat patient blood from urea. The average amount of blood in human ranges from 3000 mL to 7500 mL.

The final dimensions and parameters of a proposed hemodialyzer module (shown in Table 1 and Figure 2) in the current study are

1. Length = 25 cm
2. Height = 0.252 cm
3. Width = 0.3125 cm
4. No. of modules (the experiment module (Hoskins)) = 300
5. Blood velocity (u = 0.097m/s).
6. Dialysis velocity (u = (5/3) 0.097m/s = 0.1617m/s).

The limitations that should be addressed for the module (hemodialyzers) are in the following points.

1. The flood flow rate is 300 ml/min from the humans' body through hemodialysis.
2. The average human's blood ranges from 3000 mL to 7500 mL; its needs from 10 to 15 min to be treated in hemodialysis.
3. The dialysis flow rate is 500 ml/min, which means the channel cross section should be larger than the blood channel by 5/3, or the velocity should be increased by 5/3.

4. The surface area of the membrane should be maximized (in length and width) to increase the area and time of the contact between blood and dialysis solute.
5. The typical sizes of a module (hemodialyzer) range from 2 to 5 cm in diameter and from 15 to 35 cm in length.
6. Hemodialysis is a microfiltration process that works with pressure difference of 1000 to 20000 Pa for pores of 0.05-10 μm and a thickness of 10-150 μm in a porous membrane. The working pressure between the two channels makes mass transfer between blood and dialysis through the membrane.

The criteria that need to be satisfied for a proper dialysis are assessed by positive clinical outcomes and laboratory tests, and the ideal dialysis membranes should achieve the optimum characteristic, ensure effective removal of waste products and excess fluid while maintain overall patient health. Criteria include well-controlled blood pressure, urea Clearance (at least 1.2), and reduction (65%) for thrice-weekly hemodialysis. Performance depends on blood pressure and Q (blood with urea) in = Q (clean blood) out. The solute clearance is the main performance parameter addressed in hemodialysis.

Author Contributions: Conceptualization, R.P.S.; methodology, R.P.S. and A.A.; validation, R.P.S. and A.A.; resources, R.P.S.; investigation, A.A.; data curation, A.A.; formal analysis, R.P.S. and A.A.; writing—original draft preparation, A.A.; writing—review and editing, R.P.S. and A.A.; visualization, A.A.; supervision, R.P.S.

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Nomenclature
P pressure ($N/m^2 = kg/m.s^2$ = Pascal)
U, **u** horizontal velocity (m/s)
L channel length (m)
h = 2H channel height (m)
w channel width (m)
Q flow rate (m^3/s)
K permeability (m^2)
Lp hydraulic permeability (m/Pa.s)
u, v, and w velocity components in the x, y, and z directions (m/s)
Greek letters
μm micrometer
ϱ Density (Kg/m^3)
μ Dynamic Viscosity (kg/m.s)
ν = μ/ϱ Kinematic Viscosity (m^2/s)
ϕ Porosity (-)

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