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Article

Plithogenic Vertex Graph and Neutrosophic Vertex HyperGraph

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Abstract

A Plithogenic Set extends the classical fuzzy, intuitionistic, and neutrosophic paradigms by assigning attribute-based membership and contradiction values to elements, and the same ideas naturally extend to graph-based structures such as the Plithogenic Graph. Neutrosophic sets, in turn, represent elements with independent degrees of truth, indeterminacy, and falsity on the unit interval, thus handling incomplete, inconsistent, and ambiguous information effectively. This paper investigates the Plithogenic Graph, the Plithogenic Vertex Graph, and the Plithogenic Edge Graph. In addition, it examines the Intuitionistic Fuzzy Vertex Graph and Edge Graph, the Neutrosophic Vertex HyperGraph and Neutrosophic Edge HyperGraph, and the Neutrosophic Vertex SuperHyperGraph and Neutrosophic Edge SuperHyperGraph.

Keywords: plithogenic set; plithogenic graph; neutrosophic graph; neutrosophic set

1. Preliminaries

This section presents the fundamental concepts and definitions that underpin the discussions in this paper. Throughout this paper, all structures and sets are assumed to be finite. Moreover, the graph is assumed to be undirected and simple.

1.1. Intuitionistic Fuzzy Graph

An intuitionistic fuzzy graph assigns to each vertex and edge a pair of degrees (membership and nonmembership), together with a hesitation margin, allowing one to model relational uncertainty more expressively [1–5]. It is well known that this framework generalizes the classical fuzzy graph [6,7].

Definition 1 (Intuitionistic Fuzzy Graph). [8] Let $G = (V, E)$ be a finite, simple, undirected (crisp) graph with $E \subseteq \binom{V}{2}$. An intuitionistic fuzzy graph (IFG) on G is a pair

$$(A, B),$$

where

- $A = (\mu_A, \nu_A)$ assigns to each vertex $v \in V$ two numbers $\mu_A(v), \nu_A(v) \in [0, 1]$ (membership and nonmembership) with

$$0 \leq \mu_A(v) + \nu_A(v) \leq 1, \quad \pi_A(v) := 1 - \mu_A(v) - \nu_A(v) \in [0, 1].$$

- $B = (\mu_B, \nu_B)$ assigns to each edge $e = \{u, v\} \in E$ two numbers $\mu_B(e), \nu_B(e) \in [0, 1]$ with

$$0 \leq \mu_B(e) + \nu_B(e) \leq 1, \quad \pi_B(e) := 1 - \mu_B(e) - \nu_B(e) \in [0, 1].$$

These assignments satisfy the standard consistency (compatibility) constraints for every $e = \{u, v\} \in E$:

$$\mu_B(\{u, v\}) \leq \min\{\mu_A(u), \mu_A(v)\}, \quad \nu_B(\{u, v\}) \geq \max\{\nu_A(u), \nu_A(v)\}.$$

Example 1 (Real-world IFG: Supply-chain collaboration network). Three regional warehouses $V = \{A, B, C\}$ consider joint replenishment. An undirected edge $\{u, v\} \in E$ indicates that the pair (u, v) can collaborate (e.g., share transportation or inventory). Uncertainty comes from fluctuating demand, carrier reliability, and contractual constraints.

Graph. Let

$$G = (V, E), \quad V = \{A, B, C\}, \quad E = \{\{A, B\}, \{B, C\}, \{A, C\}\}.$$

Vertex grades. Interpret $\mu_A(v)$ as the readiness of warehouse v to collaborate (higher is better) and $\nu_A(v)$ as the incompatibility of v with collaboration (higher is worse). Hesitation is $\pi_A(v) = 1 - \mu_A(v) - \nu_A(v)$.

v	$\mu_A(v)$	$\nu_A(v)$	$\pi_A(v)$
A	0.80	0.10	0.10
B	0.60	0.20	0.20
C	0.50	0.30	0.20

Edge grades. Interpret $\mu_B(\{u, v\})$ as the pairwise feasibility of a joint plan between u and v , and $\nu_B(\{u, v\})$ as the pairwise opposition (e.g., contractual or capacity frictions). Hesitation is $\pi_B(e) = 1 - \mu_B(e) - \nu_B(e)$. Choose

e	$\mu_B(e)$	$\nu_B(e)$	$\pi_B(e)$
$\{A, B\}$	0.55	0.25	0.20
$\{B, C\}$	0.45	0.35	0.20
$\{A, C\}$	0.48	0.40	0.12

Consistency (IFG constraints). For every $e = \{u, v\} \in E$ we verify

$$\mu_B(\{u, v\}) \leq \min\{\mu_A(u), \mu_A(v)\}, \quad \nu_B(\{u, v\}) \geq \max\{\nu_A(u), \nu_A(v)\}.$$

Indeed:

$$\begin{aligned} \{A, B\} : \mu_B &= 0.55 \leq \min(0.80, 0.60) = 0.60, & \nu_B &= 0.25 \geq \max(0.10, 0.20) = 0.20; \\ \{B, C\} : \mu_B &= 0.45 \leq \min(0.60, 0.50) = 0.50, & \nu_B &= 0.35 \geq \max(0.20, 0.30) = 0.30; \\ \{A, C\} : \mu_B &= 0.48 \leq \min(0.80, 0.50) = 0.50, & \nu_B &= 0.40 \geq \max(0.10, 0.30) = 0.30, \end{aligned}$$

and $0 \leq \mu_B(e) + \nu_B(e) \leq 1$ for each edge.

Warehouse pairs with higher readiness and lower incompatibility (e.g., $\{A, B\}$) exhibit larger μ_B and smaller ν_B , signaling easier collaboration. Pairs facing capacity mismatches or policy frictions (e.g., $\{A, C\}$) show larger ν_B and smaller μ_B . The hesitation terms π_A, π_B capture unknowns such as seasonal spikes or carrier volatility. Thus (A, B) is a favorable link, (B, C) is moderate, and (A, C) is the riskiest partnership under current information. This constitutes an intuitionistic fuzzy graph model of a supply-chain collaboration network.

1.2. Neutrosophic Set and Graph

Neutrosophic sets model elements via independent truth, indeterminacy, and falsity degrees in $[0, 1]$, accommodating incomplete, inconsistent, and ambiguous information effectively [9–12]. A Neutrosophic Set generalizes the notions of Fuzzy Sets [13–15] and Intuitionistic Fuzzy Sets [16,17].

Neutrosophic graphs represent vertices and edges with truth, indeterminacy, and falsity memberships, enabling uncertain, contradictory, or incomplete modeling in networks[18–22]. Related notions include the Quadripartitioned Neutrosophic Graph [23–25], the Pentapartitioned Neutrosophic Graph [26–29], the Bipolar Neutrosophic Graph [30–32], and the Neutrosophic Directed Graph [33–35], among others.

Definition 2 (Neutrosophic Graph (single-valued)). [22] Let V be a finite vertex set and $E \subseteq \binom{V}{2}$ an undirected edge set. A (single-valued) neutrosophic graph on (V, E) is a pair

$$G = (A, B),$$

where

- $A = (T_A, I_A, F_A)$ assigns to each $v \in V$ its truth-, indeterminacy-, and falsity-memberships $T_A(v), I_A(v), F_A(v) \in [0, 1]$ with

$$0 \leq T_A(v) + I_A(v) + F_A(v) \leq 3 \quad (\forall v \in V),$$

- $B = (T_B, I_B, F_B)$ assigns to each $e = \{u, v\} \in E$ its truth-, indeterminacy-, and falsity-memberships $T_B(e), I_B(e), F_B(e) \in [0, 1]$ with

$$0 \leq T_B(e) + I_B(e) + F_B(e) \leq 3 \quad (\forall e \in E),$$

and the vertex-edge compatibility constraints hold for every edge $e = \{u, v\} \in E$:

$$T_B(\{u, v\}) \leq \min\{T_A(u), T_A(v)\}, I_B(\{u, v\}) \geq \max\{I_A(u), I_A(v)\}, F_B(\{u, v\}) \geq \max\{F_A(u), F_A(v)\}.$$

A neutrosophic vertex graph assigns to each vertex degrees of truth, indeterminacy, and falsity, thereby inducing compatible edge evaluations to effectively represent uncertainty [36]. A neutrosophic edge graph assigns truth, indeterminacy, and falsity degrees directly to each edge, with corresponding constraints on vertices to capture relational uncertainty. These structures are known to generalize fuzzy vertex[37,38]/edge graphs[39,40].

Definition 3 (Neutrosophic Vertex Graph). [36] Let V be a finite vertex set and $E \subseteq \binom{V}{2}$ be (crisp) edges. A neutrosophic vertex graph on (V, E) consists of vertex assignments

$$A = (T_A, I_A, F_A) : V \rightarrow [0, 1]^3, \quad 0 \leq T_A(v) + I_A(v) + F_A(v) \leq 3 \quad (\forall v \in V).$$

When one wishes to induce edge grades from the vertex information, the canonical edge relation $B_A = (T_{B_A}, I_{B_A}, F_{B_A})$ on E is defined by

$$T_{B_A}(\{u, v\}) := \min\{T_A(u), T_A(v)\}, I_{B_A}(\{u, v\}) := \max\{I_A(u), I_A(v)\}, F_{B_A}(\{u, v\}) := \max\{F_A(u), F_A(v)\}.$$

This choice is consistent with the vertex-edge bounds in the neutrosophic graph definition above.

Definition 4 (Neutrosophic Edge Graph). Let V be a finite vertex set and $E \subseteq \binom{V}{2}$ be (crisp) edges. A neutrosophic edge graph on (V, E) assigns to each edge $e \in E$ the triple

$$B = (T_B, I_B, F_B) : E \rightarrow [0, 1]^3, \quad 0 \leq T_B(e) + I_B(e) + F_B(e) \leq 3 \quad (\forall e \in E).$$

If vertex grades $A = (T_A, I_A, F_A)$ are also specified, they must be compatible with the edges in the sense that, for every $e = \{u, v\} \in E$,

$$T_B(\{u, v\}) \leq \min\{T_A(u), T_A(v)\}, I_B(\{u, v\}) \geq \max\{I_A(u), I_A(v)\}, F_B(\{u, v\}) \geq \max\{F_A(u), F_A(v)\}.$$

1.3. Plithogenic Set and Graph

A Plithogenic Set [41–45] models elements with attribute-based membership and contradiction functions, extending fuzzy[14,46], intuitionistic[16,47–49], and neutrosophic sets[9,50]. A plithogenic graph assigns attribute-based membership vectors and contradiction functions to vertices or edges, generalizing fuzzy, intuitionistic, and neutrosophic models [51–54].

Definition 5 (Plithogenic Set). [41,55] Let S be a universal set and $P \subseteq S$ a nonempty subset. A Plithogenic Set is a quintuple

$$PS = (P, v, Pv, pdf, pCF),$$

where

- v is an attribute,
- Pv is the set of possible values of the attribute v ,
- $pdf : P \times Pv \rightarrow [0, 1]^s$ is the Degree of Appurtenance Function (DAF),¹
- $pCF : Pv \times Pv \rightarrow [0, 1]^t$ is the Degree of Contradiction Function (DCF).

The DCF satisfies, for all $a, b \in Pv$,

$$\text{Reflexivity: } pCF(a, a) = 0, \quad \text{Symmetry: } pCF(a, b) = pCF(b, a).$$

Here $s \in \mathbb{N}$ is the appurtenance dimension and $t \in \mathbb{N}$ the contradiction dimension.

Definition 6 (Plithogenic Graph). (cf. [53,57]) Let $G = (V, E)$ be a crisp (simple, undirected) graph with $E \subseteq \{\{x, y\} : x, y \in V, x \neq y\}$. A plithogenic graph is a pair

$$PG = (PM, PN),$$

where the vertex and edge components are specified as follows.

Vertex component.

$$PM = (M, \ell, ML, adf, aCf),$$

with

- $M \subseteq V$ a chosen vertex subset;
- ℓ an attribute attached to vertices;
- ML the set of possible values of ℓ ;
- $adf : M \times ML \rightarrow [0, 1]^s$ the vertex DAF;
- $aCf : ML \times ML \rightarrow [0, 1]^t$ the vertex DCF.

Edge component.

$$PN = (N, m, ML', bdf, bCf),$$

with

- $N \subseteq E$ a chosen edge subset;
- m an attribute attached to edges;
- ML' the set of possible values of m ;
- $bdf : N \times ML' \rightarrow [0, 1]^s$ the edge DAF;
- $bCf : ML' \times ML' \rightarrow [0, 1]^t$ the edge DCF.

All inequalities in $[0, 1]^k$ are interpreted componentwise. Fix $s, t \in \mathbb{N}$. The following axioms are required.

(A1) **Edge-vertex compatibility (appurtenance bound).** For all $\{x, y\} \in N$ and $a, b \in ML$,

$$bdf(\{x, y\}, (a, b)) \leq \min\{adf(x, a), adf(y, b)\}. \quad (1)$$

(A2) **Contradiction consistency (edge vs. vertices).** For all $(a, b), (c, d) \in ML'$,

$$bCf((a, b), (c, d)) \leq \min\{aCf(a, c), aCf(b, d)\}. \quad (2)$$

¹ In the literature, DAF is defined in slightly different ways: some variants use powerset-valued constructions, others the simple cube $[0, 1]^s$. We adopt the latter (classical) form here; cf. [56].

(A3) *Reflexivity and symmetry of DCF.*

$$\text{aCf}(u, u) = 0, \quad \text{aCf}(u, v) = \text{aCf}(v, u) \quad (\forall u, v \in ML),$$

$$\text{bCf}(u, u) = 0, \quad \text{bCf}(u, v) = \text{bCf}(v, u) \quad (\forall u, v \in ML').$$

When $s = t = 1$, all maps are scalar-valued in $[0, 1]$ and (1)-(2) are scalar inequalities.

1.4. Hypergraphs and Superhypergraphs

A *hypergraph* extends an ordinary graph by permitting each edge to join an arbitrary nonempty subset of vertices; this allows multiway (higher-order) relationships to be modeled within a single framework [6,58–61]. A *SuperHyperGraph* captures hierarchical interaction patterns by iterating the powerset construction to a prescribed depth n , thereby organizing vertices and edges across multiple levels [45,62–67].

Definition 7 (Base set). A base set (or ground set) is a fixed finite set S from which all subsequent objects are formed:

$$S = \{x \mid x \text{ belongs to the chosen domain}\}.$$

All constructions below ultimately draw their elements from S .

Definition 8 (Powerset). [68,69] For a set X , its powerset is

$$\mathcal{P}(X) = \{A : A \subseteq X\}.$$

We also use the nonempty powerset $\mathcal{P}^*(X) := \mathcal{P}(X) \setminus \{\emptyset\}$.

Definition 9 (Hypergraph [59,70]). A hypergraph is a pair $H = (V(H), E(H))$ with $V(H) \neq \emptyset$ and $E(H) \subseteq \mathcal{P}^*(V(H))$. Throughout, both $V(H)$ and $E(H)$ are finite.

Example 2 (Hypergraph: Project Teams and Tasks). Consider a company with employees $V = \{a, b, c, d\}$. Each task may require any number of employees. Model the assignment by the hypergraph

$$H = (V, E), \quad E = \{e_1 = \{a, b\}, e_2 = \{b, c, d\}, e_3 = \{a, d\}\}.$$

Here, each hyperedge e_i is the (nonempty) set of employees assigned to task i : task 1 needs $\{a, b\}$, task 2 needs $\{b, c, d\}$, and task 3 needs $\{a, d\}$. Because hyperedges may contain more than two vertices, this representation naturally captures multi-person collaborations that a simple graph (pairwise edges only) cannot express.

Definition 10 (n -th powerset). [68,71–74] For a set X , define $\mathcal{P}_1(X) = \mathcal{P}(X)$ and, for $n \geq 1$,

$$\mathcal{P}_{n+1}(X) = \mathcal{P}(\mathcal{P}_n(X)).$$

When excluding the empty set, write $\mathcal{P}_n^*(X) = \mathcal{P}_n(X) \setminus \{\emptyset\}$.

Example 3 (n -th powerset: Meal Planning from Ingredients). Let the ingredient set be $X = \{\text{rice}, \text{fish}\}$. The first powerset is

$$\mathcal{P}_1(X) = \mathcal{P}(X) = \{\emptyset, \{\text{rice}\}, \{\text{fish}\}, \{\text{rice}, \text{fish}\}\},$$

whose elements can be interpreted as recipes (ingredient bundles). The second powerset

$$\mathcal{P}_2(X) = \mathcal{P}(\mathcal{P}_1(X))$$

consists of all menus (collections of recipes), e.g.

$$\{\{\text{rice}\}\}, \quad \{\{\text{fish}\}, \{\text{rice}, \text{fish}\}\}, \quad \mathcal{P}_1(X), \text{ etc.}$$

If one excludes the empty set, then $\mathcal{P}_1^*(X) = \mathcal{P}_1(X) \setminus \{\emptyset\} = \{\{\text{rice}\}, \{\text{fish}\}, \{\text{rice}, \text{fish}\}\}$, and similarly $\mathcal{P}_2^*(X) = \mathcal{P}_2(X) \setminus \{\emptyset\}$. This hierarchy reflects real planning: level 1 groups ingredients into recipes, while level 2 groups recipes into menus (e.g., for a day or event).

Definition 11 (*n*-SuperHyperGraph). [45,75–77] Fix a finite base set V_0 and $n \in \mathbb{N}_0$. Define $\mathcal{P}^0(V_0) = V_0$ and $\mathcal{P}^{k+1}(V_0) = \mathcal{P}(\mathcal{P}^k(V_0))$ for $k \geq 0$. An *n*-SuperHyperGraph is a pair

$$\text{SHG}^{(n)} = (V, E), \quad V \subseteq \mathcal{P}^n(V_0), \quad E \subseteq \mathcal{P}^*(V),$$

whose elements of V are the *n*-supervertices and whose members of E are nonempty sets of supervertices (the *n*-superedges).

Example 4 (*n*-SuperHyperGraph: Consortia of Project Teams ($n = 2$)). Let the base set of individual researchers be $V_0 = \{A, B, C\}$. Then $\mathcal{P}^1(V_0) = \mathcal{P}(V_0)$ is the set of all teams, and $\mathcal{P}^2(V_0) = \mathcal{P}(\mathcal{P}(V_0))$ is the set of consortia of teams. Define the supervertex set

$$V = \{v_A = \{\{A\}\}, v_B = \{\{B\}\}, v_{AB} = \{\{A\}, \{B\}\}\} \subseteq \mathcal{P}^2(V_0),$$

and superedges (joint milestones that require multiple consortia)

$$E = \{e_1 = \{v_A, v_{AB}\}, e_2 = \{v_B, v_{AB}\}\} \subseteq \mathcal{P}^*(V).$$

The pair $\text{SHG}^{(2)} = (V, E)$ is a 2-SuperHyperGraph: elements of V are supervertices (consortia of teams), and each $e_i \in E$ is a nonempty set of such supervertices (a superedge) that must coordinate to deliver a milestone. This models hierarchical collaboration: individuals $\rightarrow$ teams $\rightarrow$ consortia, with superedges capturing multi-consortia interactions typical in large research programs.

1.5. Neutrosophic *n*-Superhypergraph

A single-valued neutrosophic hypergraph is a hypergraph constructed from a single-valued neutrosophic graph [11,78–82], and a single-valued neutrosophic superhypergraph is a superhypergraph constructed from a single-valued neutrosophic graph. A single-valued neutrosophic hypergraph and a neutrosophic *n*-superhypergraph are given as follows [45,83].

Definition 12 (Single-Valued Neutrosophic Hypergraph). (cf.[79,84–87]) Let $V = \{v_1, \dots, v_n\}$ be a finite vertex set and let $E = \{E_i\}_{i=1}^m$ be a family of nontrivial single-valued neutrosophic subsets of V such that

$$V = \bigcup_{i=1}^m \text{supp}(E_i).$$

Each neutrosophic hyperedge E_i is given by

$$E_i = \{(v, T_{E_i}(v), I_{E_i}(v), F_{E_i}(v)) : v \in V\},$$

where

$$T_{E_i}, I_{E_i}, F_{E_i} : V \longrightarrow [0, 1] \quad \text{satisfy} \quad 0 \leq T_{E_i}(v) + I_{E_i}(v) + F_{E_i}(v) \leq 3 \quad \forall v \in V.$$

Then $H = (V, E)$ is called a single-valued neutrosophic hypergraph.

Example 5 (Single-Valued Neutrosophic Hypergraph). Let $V = \{v_1, v_2, v_3\}$ and let $E = \{E_1, E_2\}$, where each E_i is a single-valued neutrosophic subset of V . Define, for every $v \in V$, the triples $\langle T_{E_i}(v), I_{E_i}(v), F_{E_i}(v) \rangle \in [0, 1]^3$ with component sum ≤ 3 :

$$\begin{aligned} E_1 : \quad & \langle T_{E_1}(v_1), I_{E_1}(v_1), F_{E_1}(v_1) \rangle = (0.80, 0.10, 0.10), \\ & \langle T_{E_1}(v_2), I_{E_1}(v_2), F_{E_1}(v_2) \rangle = (0.60, 0.20, 0.20), \\ & \langle T_{E_1}(v_3), I_{E_1}(v_3), F_{E_1}(v_3) \rangle = (0, 0, 0); \\ E_2 : \quad & \langle T_{E_2}(v_1), I_{E_2}(v_1), F_{E_2}(v_1) \rangle = (0.30, 0.20, 0.50), \\ & \langle T_{E_2}(v_2), I_{E_2}(v_2), F_{E_2}(v_2) \rangle = (0, 0, 0), \\ & \langle T_{E_2}(v_3), I_{E_2}(v_3), F_{E_2}(v_3) \rangle = (0.90, 0.05, 0.05). \end{aligned}$$

Then $V = \{v_1, v_2, v_3\} = \text{supp}(E_1) \cup \text{supp}(E_2)$, so the coverage condition holds. Thus $H = (V, E)$ is a single-valued neutrosophic hypergraph. (One may interpret E_1, E_2 as two “tasks” with graded truth/indeterminacy/falsity of each vertex’s involvement.)

Definition 13 (Neutrosophic n -Superhypergraph). (cf. [45,83]) Let V_0 be a finite base set and define recursively

$$\mathcal{P}^0(V_0) = V_0, \quad \mathcal{P}^{k+1}(V_0) = \mathcal{P}(\mathcal{P}^k(V_0)) \quad (k \geq 0).$$

An n -Superhypergraph is a pair $\text{SHT}^{(n)} = (V, E)$ with

$$V \subseteq \mathcal{P}^n(V_0), \quad E \subseteq \mathcal{P}(V).$$

A Neutrosophic n -Superhypergraph on (V, E) is a tuple

$$(V, E, T_V, I_V, F_V, T_E, I_E, F_E),$$

where

- $T_V, I_V, F_V : V \rightarrow [0, 1]$ assign to each n -supervertex $v \in V$ its truth, indeterminacy, and falsity degrees, with

$$0 \leq T_V(v) + I_V(v) + F_V(v) \leq 3 \quad (\forall v \in V).$$

- $T_E, I_E, F_E : E \times V \rightarrow [0, 1]$ encode a neutrosophic incidence (membership) of v into a superedge $e \in E$, subject to

$$0 \leq T_E(e, v) + I_E(e, v) + F_E(e, v) \leq 3 \quad (\forall e \in E, v \in V),$$

the vertex–domination (componentwise) constraints

$$T_E(e, v) \leq T_V(v), \quad I_E(e, v) \leq I_V(v), \quad F_E(e, v) \leq F_V(v),$$

and the support condition

$$v \notin e \implies T_E(e, v) = I_E(e, v) = F_E(e, v) = 0.$$

Thus T_V, I_V, F_V describe the intrinsic neutrosophic status of supervertices, while T_E, I_E, F_E specify how those supervertices contribute neutrosophically to each superedge.

Example 6 (Neutrosophic 2-Superhypergraph). Let the base set be $V_0 = \{a, b\}$. Then $\mathcal{P}^1(V_0) = \{\{a\}, \{b\}, \{a, b\}\}$ and $\mathcal{P}^2(V_0) = \mathcal{P}(\mathcal{P}^1(V_0))$. Choose the supervertex set

$$V = \{v_a = \{\{a\}\}, \quad v_b = \{\{b\}\}, \quad v_{ab} = \{\{a\}, \{b\}\}\} \subseteq \mathcal{P}^2(V_0),$$

and superedges

$$E = \{ e_1 = \{v_a, v_{ab}\}, e_2 = \{v_b, v_{ab}\} \} \subseteq \mathcal{P}(V).$$

Assign neutrosophic vertex degrees (each component in $[0, 1]$, sums ≤ 3):

$$\langle T_V(v_a), I_V(v_a), F_V(v_a) \rangle = (0.70, 0.20, 0.10),$$

$$\langle T_V(v_b), I_V(v_b), F_V(v_b) \rangle = (0.60, 0.20, 0.20),$$

$$\langle T_V(v_{ab}), I_V(v_{ab}), F_V(v_{ab}) \rangle = (0.90, 0.05, 0.05).$$

Define edge incidences only on vertices that belong to each superedge (support condition), and make them componentwise dominated by the corresponding vertex degrees:

$$\begin{aligned} \text{on } e_1 = \{v_a, v_{ab}\}: \quad & \langle T_E(e_1, v_a), I_E(e_1, v_a), F_E(e_1, v_a) \rangle = (0.60, 0.15, 0.05), \\ & \langle T_E(e_1, v_{ab}), I_E(e_1, v_{ab}), F_E(e_1, v_{ab}) \rangle = (0.85, 0.05, 0.05); \end{aligned}$$

$$\begin{aligned} \text{on } e_2 = \{v_b, v_{ab}\}: \quad & \langle T_E(e_2, v_b), I_E(e_2, v_b), F_E(e_2, v_b) \rangle = (0.55, 0.15, 0.15), \\ & \langle T_E(e_2, v_{ab}), I_E(e_2, v_{ab}), F_E(e_2, v_{ab}) \rangle = (0.80, 0.05, 0.05). \end{aligned}$$

For any $e \in E$ and $v \in V \setminus e$, set $T_E(e, v) = I_E(e, v) = F_E(e, v) = 0$. By construction,

$$T_E(e, v) \leq T_V(v), \quad I_E(e, v) \leq I_V(v), \quad F_E(e, v) \leq F_V(v),$$

and each incidence triple sums to ≤ 3 . Therefore $(V, E; T_V, I_V, F_V; T_E, I_E, F_E)$ is a neutrosophic 2-superhypergraph. (One may view v_a, v_b, v_{ab} as single/team units, and e_1, e_2 as coalitions with graded participation.)

2. Review and Main Results

We now present the results established in this paper.

2.1. Intuitionistic Fuzzy Vertex Graph

An intuitionistic fuzzy vertex graph assigns to each vertex membership and nonmembership degrees with hesitation, inducing compatible edge assessments automatically.

Definition 14 (Intuitionistic Fuzzy Vertex Graph). Let $G = (V, E)$ be as above. An intuitionistic fuzzy vertex graph (IFVG) on G specifies only vertex grades

$$A = (\mu_A, \nu_A) : V \rightarrow [0, 1]^2, \quad 0 \leq \mu_A(v) + \nu_A(v) \leq 1 \quad (\forall v \in V),$$

with hesitation $\pi_A(v) = 1 - \mu_A(v) - \nu_A(v)$.

When one wishes to derive edge grades from the vertex information, the canonical induced edge assignment $B_A = (\mu_{B_A}, \nu_{B_A})$ on E is defined by

$$\mu_{B_A}(\{u, v\}) := \min\{\mu_A(u), \mu_A(v)\}, \quad \nu_{B_A}(\{u, v\}) := \max\{\nu_A(u), \nu_A(v)\},$$

which automatically satisfies $0 \leq \mu_{B_A} + \nu_{B_A} \leq 1$ and the IFG compatibility inequalities.

Example 7 (Intuitionistic Fuzzy Vertex Graph (IFVG)). Let $G = (V, E)$ with $V = \{a, b, c\}$ and $E = \{\{a, b\}, \{b, c\}\}$. Assign vertex grades

$$\mu_A(a) = 0.80, \nu_A(a) = 0.10 \quad (\pi_A(a) = 0.10),$$

$$\mu_A(b) = 0.60, \nu_A(b) = 0.20 \quad (\pi_A(b) = 0.20),$$

$$\mu_A(c) = 0.30, \nu_A(c) = 0.50 \quad (\pi_A(c) = 0.20),$$

so $0 \leq \mu_A(v) + \nu_A(v) \leq 1$ for all $v \in V$. The canonical induced edge grades are

$$\mu_{B_A}(\{u, v\}) = \min\{\mu_A(u), \mu_A(v)\}, \quad \nu_{B_A}(\{u, v\}) = \max\{\nu_A(u), \nu_A(v)\}.$$

Hence

$$\begin{aligned} \{a, b\} : \quad & \mu_{B_A} = \min(0.80, 0.60) = 0.60, \quad \nu_{B_A} = \max(0.10, 0.20) = 0.20, \quad \pi_{B_A} = 0.20; \\ \{b, c\} : \quad & \mu_{B_A} = \min(0.60, 0.30) = 0.30, \quad \nu_{B_A} = \max(0.20, 0.50) = 0.50, \quad \pi_{B_A} = 0.20, \end{aligned}$$

and in each case $0 \leq \mu_{B_A} + \nu_{B_A} \leq 1$. Thus (A, B_A) defines an IFVG on G .

2.2. Intuitionistic Fuzzy Edge Graph

An intuitionistic fuzzy edge graph labels every edge with membership and nonmembership degrees under hesitation, constraining vertex grades for consistency.

Definition 15 (Intuitionistic Fuzzy Edge Graph). Let $G = (V, E)$ be as above. An intuitionistic fuzzy edge graph (IFEG) on G specifies only edge grades

$$B = (\mu_B, \nu_B) : E \rightarrow [0, 1]^2, \quad 0 \leq \mu_B(e) + \nu_B(e) \leq 1 \quad (\forall e \in E),$$

with hesitation $\pi_B(e) = 1 - \mu_B(e) - \nu_B(e)$.

If vertex grades $A = (\mu_A, \nu_A)$ are also provided, then (A, B) must satisfy, for all $e = \{u, v\} \in E$,

$$\mu_B(\{u, v\}) \leq \min\{\mu_A(u), \mu_A(v)\}, \quad \nu_B(\{u, v\}) \geq \max\{\nu_A(u), \nu_A(v)\}.$$

Example 8 (Intuitionistic Fuzzy Edge Graph (IFEG)). Let $G = (V, E)$ with $V = \{a, b, c\}$ and $E = \{\{a, b\}, \{b, c\}\}$. Specify edge grades

$$\begin{aligned} \{a, b\} : \quad & \mu_B = 0.55, \nu_B = 0.25 \quad (\pi_B = 0.20), \\ \{b, c\} : \quad & \mu_B = 0.35, \nu_B = 0.45 \quad (\pi_B = 0.20), \end{aligned}$$

so $0 \leq \mu_B(e) + \nu_B(e) \leq 1$ for all $e \in E$. Provide vertex grades

$$\mu_A(a) = 0.70, \nu_A(a) = 0.20; \quad \mu_A(b) = 0.60, \nu_A(b) = 0.25; \quad \mu_A(c) = 0.40, \nu_A(c) = 0.45,$$

each satisfying $0 \leq \mu_A(v) + \nu_A(v) \leq 1$. Then the IF edge-vertex compatibility holds:

$$\begin{aligned} \{a, b\} : \quad & \mu_B = 0.55 \leq \min(0.70, 0.60) = 0.60, \quad \nu_B = 0.25 \geq \max(0.20, 0.25) = 0.25; \\ \{b, c\} : \quad & \mu_B = 0.35 \leq \min(0.60, 0.40) = 0.40, \quad \nu_B = 0.45 \geq \max(0.25, 0.45) = 0.45. \end{aligned}$$

Hence (A, B) defines a consistent IFEG on G .

2.3. Plithogenic Vertex Graph

A plithogenic vertex graph attaches attribute based memberships to vertices with contradiction function, generating edge memberships via signed aggregation rules.

Definition 16 (Plithogenic Vertex Graph (PVG)). Let $G = (V, E)$ be a finite simple undirected graph. Fix

- an attribute alphabet ML for vertices;
- a sign map $\sigma : ML \rightarrow \{+1, -1\}$ indicating, for each attribute value $a \in ML$, whether higher membership is favorable (+1) or unfavorable (-1);
- a degree of appurtenance function (DAF)

$$\text{adf} : V \times ML \longrightarrow [0, 1]^s,$$

assigning to each vertex $x \in V$ and attribute value $a \in ML$ an s -tuple of membership degrees (component-wise order on $[0, 1]^s$);

- a degree of contradiction function (DCF)

$$\text{aCf} : ML \times ML \longrightarrow [0, 1]^t,$$

with $\text{aCf}(u, u) = 0$ and $\text{aCf}(u, v) = \text{aCf}(v, u)$ (componentwise).

The canonical induced edge DAF is the map

$$\text{bdf} : E \times ML \rightarrow [0, 1]^s, \quad \text{bdf}(\{x, y\}, a) := \begin{cases} \min\{\text{adf}(x, a), \text{adf}(y, a)\}, & \sigma(a) = +1, \\ \max\{\text{adf}(x, a), \text{adf}(y, a)\}, & \sigma(a) = -1, \end{cases}$$

where $\min / \max$ act componentwise on $[0, 1]^s$. The structure

$$PG_V = (V, E; ML, \sigma; \text{adf}, \text{aCf}; \text{bdf})$$

is called a Plithogenic Vertex Graph (PVG). (Other t -norm / t -conorm pairs may replace $\min / \max$; we fix these to connect with neutrosophic graphs.)

Example 9 (Plithogenic Vertex Graph (PVG): Team formation). Let $G = (V, E)$ with $V = \{x, y, z\}$ and $E = \{\{x, y\}, \{y, z\}\}$. Take a vertex attribute alphabet

$$ML = \{\text{rel}, \text{risk}\}, \quad \sigma(\text{rel}) = +1 \text{ (higher is favorable)}, \quad \sigma(\text{risk}) = -1 \text{ (higher is unfavorable)}.$$

Work with $s = t = 1$. Set a symmetric vertex DCF $\text{aCf}(\text{rel}, \text{rel}) = \text{aCf}(\text{risk}, \text{risk}) = 0$, $\text{aCf}(\text{rel}, \text{risk}) = \text{aCf}(\text{risk}, \text{rel}) = 0.7$. Give vertex DAF values (componentwise scalars in $[0, 1]$):

$$\begin{aligned} \text{adf}(x, \text{rel}) &= 0.90, & \text{adf}(x, \text{risk}) &= 0.20, \\ \text{adf}(y, \text{rel}) &= 0.70, & \text{adf}(y, \text{risk}) &= 0.40, \\ \text{adf}(z, \text{rel}) &= 0.50, & \text{adf}(z, \text{risk}) &= 0.60. \end{aligned}$$

The canonical induced edge DAF is

$$\text{bdf}(\{u, v\}, a) = \begin{cases} \min\{\text{adf}(u, a), \text{adf}(v, a)\}, & a = \text{rel}, \\ \max\{\text{adf}(u, a), \text{adf}(v, a)\}, & a = \text{risk}. \end{cases}$$

Hence

$$\begin{aligned} \{x, y\} : \text{bdf}(\cdot, \text{rel}) &= \min(0.90, 0.70) = 0.70, & \text{bdf}(\cdot, \text{risk}) &= \max(0.20, 0.40) = 0.40; \\ \{y, z\} : \text{bdf}(\cdot, \text{rel}) &= \min(0.70, 0.50) = 0.50, & \text{bdf}(\cdot, \text{risk}) &= \max(0.40, 0.60) = 0.60. \end{aligned}$$

Thus $PG_V = (V, E; ML, \sigma; \text{adf}, \text{aCf}; \text{bdf})$ is a PVG that aggregates vertex “reliability” and “risk” into edge memberships via the signed rule.

Theorem 1 (PVG generalizes the Neutrosophic Vertex Graph). Every single-valued Neutrosophic Vertex Graph $A = (T_A, I_A, F_A) : V \rightarrow [0, 1]^3$ (with canonical induced edges $T_{B_A}(\{x, y\}) = \min\{T_A(x), T_A(y)\}$, $I_{B_A}(\{x, y\}) = \max\{I_A(x), I_A(y)\}$, $F_{B_A}(\{x, y\}) = \max\{F_A(x), F_A(y)\}$) is a special case of a PVG.

Proof. Given (V, E) and (T_A, I_A, F_A) , define a PVG by

$$ML = \{T, I, F\}, \quad \sigma(T) = +1, \quad \sigma(I) = \sigma(F) = -1,$$

take $s = t = 1$, set the vertex DAF

$$\text{adf}(x, T) = T_A(x), \quad \text{adf}(x, I) = I_A(x), \quad \text{adf}(x, F) = F_A(x),$$

and choose any symmetric DCF with $\text{aCf}(u, u) = 0$ (its values are immaterial for this embedding). By the PVG definition,

$$\text{bdf}(\{x, y\}, T) = \min\{T_A(x), T_A(y)\} = T_{B_A}(\{x, y\}),$$

$$\text{bdf}(\{x, y\}, I) = \max\{I_A(x), I_A(y)\} = I_{B_A}(\{x, y\}), \quad \text{bdf}(\{x, y\}, F) = \max\{F_A(x), F_A(y)\} = F_{B_A}(\{x, y\}).$$

Thus the PVG reproduces exactly the neutrosophic vertex graph (both vertex and canonically induced edge degrees), proving that NVGs are PVGs with the above specialization. $\square$

2.4. Plithogenic Edge Graph

A plithogenic edge graph assigns attribute based memberships to edges with contradiction function, optionally bounded by vertex memberships for coherence.

Definition 17 (Plithogenic Edge Graph (PEG)). Let $G = (V, E)$ be a finite simple undirected graph. Fix

- an attribute alphabet ML' for edges;
- a sign map $\sigma' : ML' \rightarrow \{+1, -1\}$;
- an edge DAF

$$\text{bdf} : E \times ML' \longrightarrow [0, 1]^s;$$

- an edge DCF

$$\text{bCf} : ML' \times ML' \longrightarrow [0, 1]^t,$$

with $\text{bCf}(u, u) = 0$ and $\text{bCf}(u, v) = \text{bCf}(v, u)$ (componentwise).

Optionally, if vertex degrees $\text{adf} : V \times ML' \rightarrow [0, 1]^s$ are given on the same alphabet, we require the compatibility bounds for every $e = \{x, y\} \in E$ and $a \in ML'$:

$$\sigma'(a) = +1 : \text{bdf}(e, a) \leq \min\{\text{adf}(x, a), \text{adf}(y, a)\}, \quad \sigma'(a) = -1 : \text{bdf}(e, a) \geq \max\{\text{adf}(x, a), \text{adf}(y, a)\},$$

all componentwise in $[0, 1]^s$. The structure

$$PG_E = (V, E; ML', \sigma'; \text{bdf}, \text{bCf})$$

is called a Plithogenic Edge Graph (PEG).

Example 10 (Plithogenic Edge Graph (PEG): Network links). Let $G = (V, E)$ with $V = \{u, v, w\}$ and $E = \{\{u, v\}, \{v, w\}\}$. Use the edge attribute alphabet

$$ML' = \{\text{bw}, \text{lat}\}, \quad \sigma'(\text{bw}) = +1 \text{ (more bandwidth is better)}, \quad \sigma'(\text{lat}) = -1 \text{ (more latency is worse)}.$$

Take $s = t = 1$ and set a symmetric edge DCF with $\text{bCf}(\text{bw}, \text{bw}) = \text{bCf}(\text{lat}, \text{lat}) = 0$, $\text{bCf}(\text{bw}, \text{lat}) = \text{bCf}(\text{lat}, \text{bw}) = 0.6$. Assign edge DAF values

$$\begin{aligned} \{u, v\} : \text{bdf}(\cdot, \text{bw}) &= 0.80, \quad \text{bdf}(\cdot, \text{lat}) = 0.40, \\ \{v, w\} : \text{bdf}(\cdot, \text{bw}) &= 0.60, \quad \text{bdf}(\cdot, \text{lat}) = 0.50. \end{aligned}$$

Optionally provide vertex DAF on the same alphabet to check compatibility:

$$\begin{aligned} \text{adf}(u, \text{bw}) &= 0.90, \quad \text{adf}(v, \text{bw}) = 0.80, \quad \text{adf}(w, \text{bw}) = 0.60, \\ \text{adf}(u, \text{lat}) &= 0.30, \quad \text{adf}(v, \text{lat}) = 0.40, \quad \text{adf}(w, \text{lat}) = 0.50. \end{aligned}$$

Then the PEG bounds hold componentwise:

$$\begin{aligned} \{u, v\} : \text{bdf}(\text{bw}) &= 0.80 \leq \min(0.90, 0.80) = 0.80, & \text{bdf}(\text{lat}) &= 0.40 \geq \max(0.30, 0.40) = 0.40; \\ \{v, w\} : \text{bdf}(\text{bw}) &= 0.60 \leq \min(0.80, 0.60) = 0.60, & \text{bdf}(\text{lat}) &= 0.50 \geq \max(0.40, 0.50) = 0.50. \end{aligned}$$

Thus $PG_E = (V, E; ML', \sigma'; \text{bdf}, \text{bCf})$ is a coherent PEG describing link quality using bandwidth (positive) and latency (negative).

Theorem 2 (PEG generalizes the Neutrosophic Edge Graph). *Every single-valued Neutrosophic Edge Graph $B = (T_B, I_B, F_B) : E \rightarrow [0, 1]^3$ (with optional vertex grades $A = (T_A, I_A, F_A)$ satisfying the usual compatibility $T_B(\{x, y\}) \leq \min\{T_A(x), T_A(y)\}$, $I_B(\{x, y\}) \geq \max\{I_A(x), I_A(y)\}$, $F_B(\{x, y\}) \geq \max\{F_A(x), F_A(y)\}$) is a special case of a PEG.*

Proof. Fix $ML' = \{T, I, F\}$ and $s = t = 1$, and define

$$\sigma'(T) = +1, \quad \sigma'(I) = \sigma'(F) = -1.$$

Set the edge DAF by identification:

$$\text{bdf}(e, T) = T_B(e), \quad \text{bdf}(e, I) = I_B(e), \quad \text{bdf}(e, F) = F_B(e), \quad (e \in E),$$

and choose any symmetric DCF bCf with $\text{bCf}(u, u) = 0$. If vertex grades A are given, define

$$\text{adf}(x, T) = T_A(x), \quad \text{adf}(x, I) = I_A(x), \quad \text{adf}(x, F) = F_A(x)$$

(on the same alphabet), so that the PEG compatibility bounds become

$$\begin{aligned} \text{bdf}(e, T) &\leq \min\{\text{adf}(x, T), \text{adf}(y, T)\}, & \text{bdf}(e, I) &\geq \max\{\text{adf}(x, I), \text{adf}(y, I)\}, \\ \text{bdf}(e, F) &\geq \max\{\text{adf}(x, F), \text{adf}(y, F)\}, & (e = \{x, y\}), \end{aligned}$$

which are exactly the neutrosophic edge–vertex constraints. Hence every neutrosophic edge graph is realized as a PEG under this specialization. $\square$

2.5. Neutrosophic Vertex HyperGraph

A Neutrosophic Vertex HyperGraph assigns each vertex neutrosophic truth, indeterminacy, and falsity degrees, while hyperedges connect subsets, and incidences respect bounds derived from vertices componentwise.

Definition 18 (Neutrosophic Vertex HyperGraph). *Let $H = (V, E)$ be a hypergraph. A Neutrosophic Vertex HyperGraph on H is a tuple*

$$\text{NVH}(H) = (V, E; T_V, I_V, F_V; T_{inc}, I_{inc}, F_{inc}),$$

where

- $T_V, I_V, F_V : V \rightarrow [0, 1]$ are vertex neutrosophic grades satisfying

$$0 \leq T_V(v) + I_V(v) + F_V(v) \leq 3 \quad (\forall v \in V),$$

- $T_{inc}, I_{inc}, F_{inc} : E \times V \rightarrow [0, 1]$ are incidence grades with

$$v \notin e \implies T_{inc}(e, v) = I_{inc}(e, v) = F_{inc}(e, v) = 0,$$

and the (componentwise) vertex-domination constraints

$$T_{inc}(e, v) \leq T_V(v), \quad I_{inc}(e, v) \leq I_V(v), \quad F_{inc}(e, v) \leq F_V(v) \quad (\forall e \in E, \forall v \in V).$$

A common canonical choice is $T_{inc}(e, v) = T_V(v)\mathbf{1}_{\{v \in e\}}$, $I_{inc}(e, v) = I_V(v)\mathbf{1}_{\{v \in e\}}$, $F_{inc}(e, v) = F_V(v)\mathbf{1}_{\{v \in e\}}$.

Example 11 (Task teams as a vertex hypergraph). Let $V = \{A, B, C\}$ be employees and $E = \{\{A, B\}, \{B, C\}, \{A, B, C\}\}$ be feasible coalitions. Set

$$\langle T_V, I_V, F_V \rangle(A) = (0.9, 0.05, 0.05),$$

$$\langle T_V, I_V, F_V \rangle(B) = (0.7, 0.2, 0.1),$$

$$\langle T_V, I_V, F_V \rangle(C) = (0.6, 0.3, 0.1),$$

and take the canonical incidence $T_{inc}(e, v) = T_V(v)\mathbf{1}_{\{v \in e\}}$ (similarly for I, F). This produces an NVH(H): each coalition (hyperedge) inherits the vertex grades of its members, while absent members contribute zero.

Theorem 3 (The vertex model generalizes hypergraphs and neutrosophic vertex graphs). Let NVH(H) be as in Definition 18.

(a) (To hypergraphs) Every hypergraph $H = (V, E)$ embeds faithfully into a Neutrosophic Vertex HyperGraph by setting, for all $v \in V$ and $(e, v) \in E \times V$,

$$T_V(v) = 1, \quad I_V(v) = 0, \quad F_V(v) = 0, \quad T_{inc}(e, v) = \mathbf{1}_{\{v \in e\}}, \quad I_{inc}(e, v) = F_{inc}(e, v) = 0.$$

(b) (To neutrosophic vertex graphs) If $H = (V, E)$ is 2-uniform (i.e. every $e \in E$ has $|e| = 2$), then NVH(H) reduces to a neutrosophic vertex graph by keeping the same vertex triples (T_V, I_V, F_V) and viewing E as the usual edge set of a simple graph.

Proof. (a) The stated assignments satisfy $T_V + I_V + F_V = 1 \leq 3$ and, for all (e, v) , $v \notin e \Rightarrow T_{inc} = I_{inc} = F_{inc} = 0$ by definition. If $v \in e$, then $T_{inc}(e, v) = 1 \leq T_V(v) = 1$ and $I_{inc}, F_{inc} = 0 \leq I_V(v), F_V(v) = 0$, so the vertex-domination constraints hold. Forgetting the neutrosophic labels recovers exactly the original (V, E) , hence the embedding is faithful.

(b) When every hyperedge has cardinality 2, we may regard H as a simple undirected graph on V with edge set $E \subseteq \binom{V}{2}$. Keeping the vertex assignments (T_V, I_V, F_V) gives a neutrosophic vertex graph. The incidence maps $T_{inc}, I_{inc}, F_{inc}$ are consistent with (and bounded by) the vertex labels, so they do not alter the usual neutrosophic vertex graph structure; one may take the canonical choice $T_{inc}(e, v) = T_V(v)$, etc., for $v \in e$. $\square$

2.6. Neutrosophic Edge HyperGraph

A Neutrosophic Edge HyperGraph assigns to each hyperedge neutrosophic truth, indeterminacy, and falsity degrees, leaving vertices crisp, modeling uncertain multiway relations and connectivity across subsets.

Definition 19 (Neutrosophic Edge HyperGraph). Let $H = (V, E)$ be a hypergraph. A Neutrosophic Edge HyperGraph on H is a tuple

$$\text{NEH}(H) = (V, E; T_E, I_E, F_E),$$

where $T_E, I_E, F_E : E \rightarrow [0, 1]$ assign to each hyperedge $e \in E$ its neutrosophic truth-, indeterminacy-, and falsity-degrees, with

$$0 \leq T_E(e) + I_E(e) + F_E(e) \leq 3 \quad (\forall e \in E).$$

(Optionally, if vertex neutrosophic labels T_V, I_V, F_V are also given, one may impose the compatibility bounds $T_E(e) \leq \min_{v \in e} T_V(v)$, $I_E(e) \geq \max_{v \in e} I_V(v)$, $F_E(e) \geq \max_{v \in e} F_V(v)$, but these are not required in the basic definition.)

Example 12 (Project difficulty as an edge hypergraph). Let the same (V, E) as above be given. Define edge grades as perceived project feasibility:

$$\langle T_E, I_E, F_E \rangle(\{A, B\}) = (0.8, 0.1, 0.1),$$

$$\langle T_E, I_E, F_E \rangle(\{B, C\}) = (0.6, 0.25, 0.15),$$

$$\langle T_E, I_E, F_E \rangle(\{A, B, C\}) = (0.7, 0.2, 0.1).$$

Then $\text{NEH}(H)$ records, for each coalition, its truth/uncertainty/falsity degree as a single triple attached to the hyperedge.

Theorem 4 (The edge model generalizes hypergraphs and neutrosophic edge graphs). Let $\text{NEH}(H)$ be as in Definition 19.

(a) (To hypergraphs) Every hypergraph $H = (V, E)$ embeds into a Neutrosophic Edge HyperGraph by setting, for all $e \in E$,

$$T_E(e) = 1, \quad I_E(e) = 0, \quad F_E(e) = 0.$$

(b) (To neutrosophic edge graphs) If $H = (V, E)$ is 2-uniform, then giving edge labels $(T_E, I_E, F_E) : E \rightarrow [0, 1]^3$ yields exactly a neutrosophic edge graph on the simple graph (V, E) .

Proof. (a) The assignments satisfy $T_E + I_E + F_E = 1 \leq 3$ for each e . Discarding the neutrosophic information recovers the original hypergraph (V, E) , showing a faithful embedding.

(b) When $|e| = 2$ for all $e \in E$, the pair (V, E) is a simple undirected graph. Triples $(T_E(e), I_E(e), F_E(e))$ attached to each $e \in E$ are precisely the edge grades of a neutrosophic edge graph, with the same admissibility condition $0 \leq T_E + I_E + F_E \leq 3$. $\square$

2.7. Neutrosophic Vertex SuperHyperGraph

A Neutrosophic Vertex SuperHyperGraph equips each n-level supervertex with neutrosophic truth, indeterminacy, and falsity degrees, while superedges connect supervertices across layers, capturing hierarchical uncertainty patterns.

Definition 20 (Neutrosophic Vertex SuperHyperGraph). Let $\text{SHG}^{(n)} = (V, E)$ be an n -SuperHyperGraph. A Neutrosophic Vertex SuperHyperGraph on (V, E) is a tuple

$$\text{NVSHG}^{(n)} = (V, E; T_V, I_V, F_V; T_{\text{inc}}, I_{\text{inc}}, F_{\text{inc}}),$$

where

- $T_V, I_V, F_V : V \rightarrow [0, 1]$ assign to each supervertex $v \in V$ its truth-, indeterminacy-, and falsity-memberships, with

$$0 \leq T_V(v) + I_V(v) + F_V(v) \leq 3 \quad (\forall v \in V).$$

- $T_{\text{inc}}, I_{\text{inc}}, F_{\text{inc}} : E \times V \rightarrow [0, 1]$ are neutrosophic incidence maps (membership of v in e) such that, for all $e \in E$ and $v \in V$,

$$v \notin e \implies T_{\text{inc}}(e, v) = I_{\text{inc}}(e, v) = F_{\text{inc}}(e, v) = 0,$$

and the (componentwise) vertex-domination constraints hold:

$$T_{\text{inc}}(e, v) \leq T_V(v), \quad I_{\text{inc}}(e, v) \leq I_V(v), \quad F_{\text{inc}}(e, v) \leq F_V(v).$$

A canonical choice is $T_{inc}(e, v) = T_V(v)\mathbf{1}_{\{v \in e\}}$ and similarly for I and F .

Example 13 (Neutrosophic Vertex SuperHyperGraph (real-life, emergency response, $n = 2$)). Scenario. Three agencies coordinate emergency response: EMS (E), Fire (F), and Police (P). Take the base set $V_0 = \{E, F, P\}$ and form $\mathcal{P}^2(V_0) = \mathcal{P}(\mathcal{P}(V_0))$. Choose the supervertex set

$$V = \{v_E = \{\{E\}\}, v_F = \{\{F\}\}, v_{EF} = \{\{E\}, \{F\}\}\} \subseteq \mathcal{P}^2(V_0),$$

and superedges (multi-team “operations”)

$$e_{med} = \{v_E, v_{EF}\}, \quad e_{fire} = \{v_F, v_{EF}\}, \quad E = \{e_{med}, e_{fire}\} \subseteq \mathcal{P}^*(V).$$

Vertex neutrosophic grades (T : suitability, I : uncertainty, F : unsuitability) are

$$\begin{aligned} \langle T_V(v_E), I_V(v_E), F_V(v_E) \rangle &= (0.75, 0.15, 0.10), \\ \langle T_V(v_F), I_V(v_F), F_V(v_F) \rangle &= (0.65, 0.20, 0.15), \\ \langle T_V(v_{EF}), I_V(v_{EF}), F_V(v_{EF}) \rangle &= (0.90, 0.06, 0.04), \end{aligned}$$

(all sums ≤ 3). Incidence triples for each operation (only listed pairs are nonzero; all others vanish by support) are

$$\begin{aligned} \langle T_{inc}(e_{med}, v_E), I_{inc}(e_{med}, v_E), F_{inc}(e_{med}, v_E) \rangle &= (0.70, 0.15, 0.10), \\ \langle T_{inc}(e_{med}, v_{EF}), I_{inc}(e_{med}, v_{EF}), F_{inc}(e_{med}, v_{EF}) \rangle &= (0.85, 0.05, 0.04), \\ \langle T_{inc}(e_{fire}, v_F), I_{inc}(e_{fire}, v_F), F_{inc}(e_{fire}, v_F) \rangle &= (0.60, 0.20, 0.15), \\ \langle T_{inc}(e_{fire}, v_{EF}), I_{inc}(e_{fire}, v_{EF}), F_{inc}(e_{fire}, v_{EF}) \rangle &= (0.80, 0.06, 0.04). \end{aligned}$$

These satisfy the vertex-domination constraints $T_{inc}(e, v) \leq T_V(v)$, $I_{inc}(e, v) \leq I_V(v)$, $F_{inc}(e, v) \leq F_V(v)$ componentwise. Interpretation: the joint team v_{EF} has high suitability for both operations (large T_{inc} , small I_{inc}, F_{inc}), while the single-agency teams contribute with their own (more uncertain) profiles.

Theorem 5 (Vertex model generalizes both NV-hypergraphs and superhypergraphs). Let $NVSHG^{(n)}$ be as in Definition 20.

- (a) (Reduction to Neutrosophic Vertex HyperGraph) For $n = 1$, any $NVSHG^{(1)}$ is exactly a Neutrosophic Vertex HyperGraph on the hypergraph (V, E) : supervertices are ordinary vertices and superedges are ordinary hyperedges. Conversely, any Neutrosophic Vertex HyperGraph arises as an $NVSHG^{(1)}$.
- (b) (Reduction to crisp n -SuperHyperGraph) Every n -SuperHyperGraph $SHG^{(n)} = (V, E)$ embeds faithfully into $NVSHG^{(n)}$ by setting, for all $v \in V, e \in E$,

$$T_V(v) = 1, \quad I_V(v) = 0, \quad F_V(v) = 0, \quad T_{inc}(e, v) = \mathbf{1}_{\{v \in e\}}, \quad I_{inc}(e, v) = 0, \quad F_{inc}(e, v) = 0.$$

Forgetting the neutrosophic labels recovers (V, E) .

Proof. (a) When $n = 1$ we have $V \subseteq \mathcal{P}(V_0)$ and $E \subseteq \mathcal{P}^*(V)$, i.e. (V, E) is an ordinary hypergraph. The data (T_V, I_V, F_V) (on vertices) and $(T_{inc}, I_{inc}, F_{inc})$ (on incidences) together with the support and domination constraints are precisely the neutrosophic vertex hypergraph structure; hence the notions coincide.

(b) The assigned triples satisfy $T_V + I_V + F_V = 1 \leq 3$ and, for each (e, v) , $v \notin e \Rightarrow (T_{inc}, I_{inc}, F_{inc}) = (0, 0, 0)$, while $v \in e$ gives $T_{inc}(e, v) = 1 \leq T_V(v) = 1$ and $I_{inc} = F_{inc} = 0 \leq I_V(v), F_V(v)$. Thus all constraints in Definition 20 hold. Discarding the labels returns exactly the underlying $SHG^{(n)}$, proving a faithful embedding. $\square$

2.8. Neutrosophic Edge SuperHyperGraph

A Neutrosophic Edge SuperHyperGraph assigns neutrosophic truth, indeterminacy, and falsity degrees to n -level superedges, with crisp supervertices, capturing uncertain multilayer connectivity across powerset hierarchies effectively.

Definition 21 (Neutrosophic Edge SuperHyperGraph). Let $\text{SHG}^{(n)} = (V, E)$ be an n -SuperHyperGraph. A Neutrosophic Edge SuperHyperGraph on (V, E) is a tuple

$$\text{NESHG}^{(n)} = (V, E; T_E, I_E, F_E),$$

where $T_E, I_E, F_E : E \rightarrow [0, 1]$ attach to each superedge $e \in E$ its neutrosophic truth-, indeterminacy-, and falsity-memberships, with

$$0 \leq T_E(e) + I_E(e) + F_E(e) \leq 3 \quad (\forall e \in E).$$

(If vertex labels T_V, I_V, F_V are also given, one may optionally impose compatibility bounds $T_E(e) \leq \min_{v \in e} T_V(v)$, $I_E(e) \geq \max_{v \in e} I_V(v)$, $F_E(e) \geq \max_{v \in e} F_V(v)$; these are not required in the basic definition.)

Example 14 (Neutrosophic Edge SuperHyperGraph (real-life, urban logistics, $n = 2$)). Scenario. A city coordinates freight transfers across hub-of-hubs. Let $V_0 = \{A, B, C\}$ denote physical hubs. In $\mathcal{P}^2(V_0)$ take the supervertices

$$V = \{v_A = \{\{A\}\}, v_{AB} = \{\{A\}, \{B\}\}, v_{BC} = \{\{B\}, \{C\}\}\} \subseteq \mathcal{P}^2(V_0).$$

Define three superedges (multi-hub corridors)

$$e_1 = \{v_A, v_{AB}\}, \quad e_2 = \{v_{AB}, v_{BC}\}, \quad e_3 = \{v_A, v_{BC}\}, \quad E = \{e_1, e_2, e_3\}.$$

Assign neutrosophic edge grades (T : on-time reliability, I : operational uncertainty, F : disruption risk):

$$\begin{aligned} \langle T_E(e_1), I_E(e_1), F_E(e_1) \rangle &= (0.88, 0.08, 0.04), \\ \langle T_E(e_2), I_E(e_2), F_E(e_2) \rangle &= (0.62, 0.25, 0.13), \\ \langle T_E(e_3), I_E(e_3), F_E(e_3) \rangle &= (0.55, 0.30, 0.15), \end{aligned}$$

each summing to ≤ 3 . Here vertices need not carry neutrosophic labels—the uncertainty is encoded at the corridor (superedge) level. Interpretation: the corridor e_1 is highly reliable (large T_E), e_2 is moderately reliable with congestion-driven uncertainty (larger I_E), and e_3 faces higher disruption risk (larger F_E), e.g., due to roadworks or weather.

Theorem 6 (Edge model generalizes both NE-hypergraphs and superhypergraphs). Let $\text{NESHG}^{(n)}$ be as in Definition 21.

- (a) (Reduction to Neutrosophic Edge HyperGraph) For $n = 1$, any $\text{NESHG}^{(1)}$ is exactly a Neutrosophic Edge HyperGraph on the hypergraph (V, E) : superedges become ordinary hyperedges with neutrosophic edge labels (T_E, I_E, F_E) . Conversely, any Neutrosophic Edge HyperGraph arises as an $\text{NESHG}^{(1)}$.
- (b) (Reduction to crisp n -SuperHyperGraph) Every n -SuperHyperGraph $\text{SHG}^{(n)} = (V, E)$ embeds faithfully into $\text{NESHG}^{(n)}$ by setting, for all $e \in E$,

$$T_E(e) = 1, \quad I_E(e) = 0, \quad F_E(e) = 0.$$

Forgetting the neutrosophic labels recovers (V, E) .

Proof. (a) For $n = 1$ we again have an ordinary hypergraph (V, E) . Attaching to each hyperedge $e \in E$ the triple $(T_E(e), I_E(e), F_E(e)) \in [0, 1]^3$ with $T_E + I_E + F_E \leq 3$ is precisely the definition of a neutrosophic edge hypergraph; hence the notions coincide.

(b) The constant assignments satisfy $T_E + I_E + F_E = 1 \leq 3$ for every $e \in E$, and they do not alter the incidence structure. Discarding the labels recovers the original SHG⁽ⁿ⁾, establishing a faithful embedding. $\square$

3. Conclusions

This paper investigated the Plithogenic Graph, the Plithogenic Vertex Graph, and the Plithogenic Edge Graph. In addition, it examined the Intuitionistic Fuzzy Vertex Graph and Edge Graph, the Neutrosophic Vertex HyperGraph and Neutrosophic Edge HyperGraph, and the Neutrosophic Vertex SuperHyperGraph and Neutrosophic Edge SuperHyperGraph. In future work, we plan to explore extensions based on Directed Graphs[88,89], Bidirected Graphs[90–92], Line Graphs[93,94], and Directed HyperGraphs[95,96], aiming to further generalize the proposed framework.

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