doi:10.20944/preprints202201.0239.v1

Three Types of Neutrosophic Alliances based on Connectedness and (Strong) Edges

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Abstract

New setting is introduced to study the alliances. Alliances are about a set of vertices which are applied into the setting of neutrosophic graphs. Neighborhood has the key role to define these notions. Also, neighborhood is defined based on the edges, strong edges and some edges which are coming from connectedness. These three types of edges get a framework as neighborhood and after that, too close vertices have key role to define offensive alliance, defensive alliance, t-offensive alliance, and t-defensive alliance based on three types of edges, common edges, strong edges and some edges which are coming from connectedness. The structure of set is studied and general results are obtained. Also, some classes of neutrosophic graphs containing complete, empty, path, cycle, bipartite, t-partite, star and wheel are investigated in the terms of set, minimal set, number, and neutrosophic number. In this study, there's an open way to extend these results into the family of these classes of neutrosophic graphs. The family of neutrosophic graphs aren't study but it seems that analogous results are determined. There's a question. How can be related to each other, two sets partitioning the vertex set of a graph? The ideas of neighborhood and neighbors based on different edges illustrate open way to get results. A set is alliance when two sets partitioning vertex set have uniform structure. All members of set have different amount of neighbors in the set and out of set. It leads us to the notion of offensive and defensive. New ideas, offensive alliance, defensive alliance, t-offensive alliance, t-defensive alliance, strong offensive alliance, strong defensive alliance, strong t-offensive alliance, strong t-defensive alliance, connected offensive alliance, connected defensive alliance, connected t-offensive alliance, and connected t-defensive alliance are introduced. Two numbers concerning cardinality and neutrosophic cardinality of alliances are introduced. A set is alliance when its complement make a relation in the terms of neighborhood. Different edges make different neighborhoods. Three types of edges are applied to define three styles of neighborhoods. General edges, strong edges and connected edges are used where connected edges are the edges arising from connectedness amid two endpoints of the edges. These notions are applied into neutrosophic graphs as individuals and family of them. Independent set as an alliance is a special set which has no neighbor inside and it implies some drawbacks for this notions. Finding special sets which are well-known, is an open way to purse this study. Special set which its members have only one neighbor inside, characterize the connected components where the cardinality of its complement is the number of connected components. Some problems are proposed to pursue this study. Basic familiarities with graph theory and neutrosophic graph theory are proposed for this article.

Keywords: Alliance, Offensive Alliance, Defensive Alliance
AMS Subject Classification: 05C17, 05C22, 05E45

1 Background

Fuzzy set in **Ref.** [15], neutrosophic set in **Ref.** [2], related definitions of other sets in **Refs.** [2, 13, 14], graphs and new notions on them in **Refs.** [5–11], neutrosophic graphs in **Ref.** [3], studies on neutrosophic graphs in **Ref.** [1], relevant definitions of other graphs based on fuzzy graphs in **Ref.** [12], related definitions of other graphs based on neutrosophic graphs in **Ref.** [4], are proposed.

In this section, I use two subsections to illustrate a perspective about the background of this study.

1.1 Motivation and Contributions

In this study, there's an idea which could be considered as a motivation.

Question 1.1. *Is it possible to use mixed versions of ideas concerning "alliance", "offensive" and "defensive" to define some notions which are applied to neutrosophic graphs?*

It's motivation to find notions to use in any classes of neutrosophic graphs. Real-world applications about time table and scheduling are another thoughts which lead to be considered as motivation. Connections amid two vertices have key roles to assign alliances, defensive alliances and offensive alliances. Thus they're used to define new ideas which conclude to the structure alliances, defensive alliances and offensive alliances. The concept of having general edge inspires me to study the behavior of general, strong edges and connected edge in the way that, three types of numbers and set, e.g., alliances, defensive alliances and offensive alliances are the cases of study in the settings of individuals and in settings of families. Also, there are some extensions into alliances, t-defensive alliances and t-offensive alliances.

The framework of this study is as follows. In the beginning, I introduced basic definitions to clarify about preliminaries. In subsection "Preliminaries", new notions of (strong/connected)alliances, (strong/connected)t-defensive alliances and (strong/connected)t-offensive alliances are applied to set of vertices of neutrosophic graphs as individuals. In section "In the Setting of Set", specific sets have the key role in this way. Classes of neutrosophic graphs are studied in the terms of different sets in section "Classes of Neutrosophic Graphs" as individuals. In the section "In the Setting of Number", usages of general numbers have key role in this study as individuals. In section "Classes of Neutrosophic Graphs", both numbers have applied into individuals. And as a concluding result, there's one statement about the family of neutrosophic graphs in this section. In section "Applications in Time Table and Scheduling", some applications are posed for alliances concerning time table and scheduling when the suspicions are about choosing some subjects. In section "Open Problems", some problems and questions for further studies are proposed. In section "Conclusion and Closing Remarks", gentle discussion about results and applications are featured. In section "Conclusion and Closing Remarks", a brief overview concerning advantages and limitations of this study alongside conclusions are formed.

1.2 Preliminaries

Definition 1.2. (Graph).

$G = (V, E)$ is called a **graph** if V is a set of objects and E is a subset of $V \times V$ (E is a set of 2-subsets of V) where V is called **vertex set** and E is called **edge set**. Every two vertices have been corresponded to at most one edge.

Definition 1.3. (Neutrosophic Graph).

$NTG = (V, E, \sigma = (\sigma_1, \sigma_2, \sigma_3), \mu = (\mu_1, \mu_2, \mu_3))$ is called a **neutrosophic graph** if it's graph, $\sigma_i : V \to [0, 1],\ \mu_i : E \to [0, 1]$, and for every $v_i v_j \in E$,

$$\mu(v_i v_j) \leq \sigma(v_i) \wedge \sigma(v_j).$$

(i) : σ is called **neutrosophic vertex set**.

(ii) : μ is called **neutrosophic edge set**.

(iii) : $|V|$ is called **order** of NTG and it's denoted by $\mathcal{O}(NTG)$.

(iv) : $\Sigma_{v \in V} \sigma(v)$ is called **neutrosophic order** of NTG and it's denoted by $\mathcal{O}_n(NTG)$.

(v) : $|E|$ is called **size** of NTG and it's denoted by $\mathcal{S}(NTG)$.

(vi) : $\Sigma_{e \in E} \mu(e)$ is called **neutrosophic size** of NTG and it's denoted by $\mathcal{S}_n(NTG)$.

Definition 1.4. Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) : a sequence of vertices $P : x_0, x_1, \cdots, x_n$ is called a **path** where $x_i x_{i+1} \in E,\ i = 0, 1, \cdots, n-1$;

(ii) : **strength** of path $P : x_0, x_1, \cdots, x_n$ is $\bigwedge_{i=0,\cdots,n-1} \mu(x_i x_{i+1})$;

(iii) : **connectedness** amid vertices x_0 and x_n is

$$\mu^{\infty}(x, y) = \bigwedge_{P : x_0, x_1, \cdots, x_n} \bigwedge_{i=0,\cdots,n-1} \mu(x_i x_{i+1}).$$

(iv) : a sequence of vertices $P : x_0, x_1, \cdots, x_n$ is called a **path** where $x_i x_{i+1} \in E,\ i = 0, 1, \cdots, n-1$ and there are two edges xy and uv such that $\mu(xy) = \mu(uv) = \bigwedge_{i=0,1,\cdots,n-1} \mu(v_i v_{i+1})$;

(v) : it's a **t-partite** where V is partitioned to t parts, $V_1, V_2, \cdots, V_t$ and the edge xy implies $x \in V_i$ and $y \in V_j$ where $i \neq j$. If it's complete, then it's denoted by $K_{\sigma_1, \sigma_2, \cdots, \sigma_t}$ where σ_i is σ on V_i instead V which mean $x \notin V_i$ induces $\sigma_i(x) = 0$.

(v) : an t-partite is **complete bipartite** If $t = 2$, and it's denoted by K_{σ_1, σ_2}.

(vi) : a complete bipartite is **star** if $|V_1| = 1$, and it's denoted by S_{1, σ_2}.

(vii) : a vertex in V is **center** if the vertex joins to all vertices of a cycle. Then it's **wheel** and it's denoted by W_{1, σ_2}.

$(viii)$: it's a **complete** where $\forall uv \in V,\ \mu(uv) = \sigma(u) \wedge \sigma(v)$.

(ix) : it's a **strong** where $\forall uv \in E,\ \mu(uv) = \sigma(u) \wedge \sigma(v)$.

Based on different edges, it's possible to define different neighbors as follows.

Definition 1.5. (Different Neighbors).

Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Suppose $x \in V$.

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$(i):\ N(x)=\{y\in V \mid xy\in E\}$;

$(ii):\ N_s(x)=\{y\in N(x) \mid \mu(xy)=\sigma(x)\wedge\sigma(y)\}$;

$(iii):\ N_c(x)=\{y\in N(x) \mid \mu(xy)=\mu^{\infty}(x,y)\}$.

Definition 1.6. Let $NTG:(V,E,\sigma,\mu)$ be a neutrosophic graph. A set S is called

$(i):$ **offensive alliance** if $\forall a\in S,\ |N(a)\cap S|>|N(a)\cap(V\setminus S)|$;

$(ii):$ **defensive alliance** if $\forall a\in S,\ |N(a)\cap S|<|N(a)\cap(V\setminus S)|$;

$(iii):$ **t-offensive alliance** if $\forall a\in S,\ |(N(a)\cap S)-(N(a)\cap(V\setminus S))|>t$;

$(iv):$ **t-defensive alliance** if $\forall a\in S,\ |(N(a)\cap S)-(N(a)\cap(V\setminus S))|<t$.

Definition 1.7. Let $NTG:(V,E,\sigma,\mu)$ be a neutrosophic graph. A set S is called

$(i):$ **strong offensive alliance** if $\forall a\in S,\ |N_s(a)\cap S|>|N_s(a)\cap(V\setminus S)|$;

$(ii):$ **strong defensive alliance** if $\forall a\in S,\ |N_s(a)\cap S|<|N_s(a)\cap(V\setminus S)|$;

$(iii):$ **strong t-offensive alliance** if $\forall a\in S,\ |(N_s(a)\cap S)-(N_s(a)\cap(V\setminus S))|>t$;

$(iv):$ **strong t-defensive alliance** if $\forall a\in S,\ |(N_s(a)\cap S)-(N_s(a)\cap(V\setminus S))|<t$.

Definition 1.8. Let $NTG:(V,E,\sigma,\mu)$ be a neutrosophic graph. A set S is called

$(i):$ **connected offensive alliance** if $\forall a\in S,\ |N_c(a)\cap S|>|N_c(a)\cap(V\setminus S)|$;

$(ii):$ **connected defensive alliance** if $\forall a\in S,\ |N_c(a)\cap S|<|N_c(a)\cap(V\setminus S)|$;

$(iii):$ **connected t-offensive alliance** if $\forall a\in S,\ |(N_c(a)\cap S)-(N_c(a)\cap(V\setminus S))|>t$;

$(iv):$ **connected t-defensive alliance** if $\forall a\in S,\ |(N_c(a)\cap S)-(N_c(a)\cap(V\setminus S))|<t$.

Definition 1.9. Let $NTG:(V,E,\sigma,\mu)$ be a neutrosophic graph. Then

$(i):$ **number** of NTG is $\bigwedge_{S \text{ is alliance}}|S|$;

$(ii):$ **neutrosophic number** of NTG is $\bigwedge_{S \text{ is alliance}}\Sigma_{s\in S}\sigma(s)$.

2 In the Setting of Set

Proposition 2.1. *Let $NTG:(V,E,\sigma,\mu)$ be a neutrosophic graph. Then V is*

$(i):$ *offensive alliance;*

$(ii):$ *strong offensive alliance;*

$(iii):$ *connected offensive alliance;*

$(iv):$ *δ-offensive alliance;*

$(v):$ *strong δ-offensive alliance;*

$(vi):$ *connected δ-offensive alliance.*

Proof. Suppose $NTG:(V,E,\sigma,\mu)$ is a neutrosophic graph. Consider V. All members of V have at least one neighbor inside the set more than neighbor out of set. Thus,

(i). V is offensive alliance since the following statements are equivalent.

$$\forall a \in S,\ |N(a) \cap S| > |N(a) \cap (V \setminus S)| \equiv$$
$$\forall a \in V,\ |N(a) \cap V| > |N(a) \cap (V \setminus V)| \equiv$$
$$\forall a \in V,\ |N(a) \cap V| > |N(a) \cap \emptyset| \equiv$$
$$\forall a \in V,\ |N(a) \cap V| > |\emptyset| \equiv$$
$$\forall a \in V,\ |N(a) \cap V| > 0 \equiv$$
$$\forall a \in V,\ \delta > 0.$$

(ii). V is strong offensive alliance since the following statements are equivalent.

$$\forall a \in S,\ |N_s(a) \cap S| > |N_s(a) \cap (V \setminus S)| \equiv$$
$$\forall a \in V,\ |N_s(a) \cap V| > |N_s(a) \cap (V \setminus V)| \equiv$$
$$\forall a \in V,\ |N_s(a) \cap V| > |N_s(a) \cap \emptyset| \equiv$$
$$\forall a \in V,\ |N_s(a) \cap V| > |\emptyset| \equiv$$
$$\forall a \in V,\ |N_s(a) \cap V| > 0 \equiv$$
$$\forall a \in V,\ \delta > 0.$$

(iii). V is connected offensive alliance since the following statements are equivalent.

$$\forall a \in S,\ |N_c(a) \cap S| > |N_c(a) \cap (V \setminus S)| \equiv$$
$$\forall a \in V,\ |N_c(a) \cap V| > |N_c(a) \cap (V \setminus V)| \equiv$$
$$\forall a \in V,\ |N_c(a) \cap V| > |N_c(a) \cap \emptyset| \equiv$$
$$\forall a \in V,\ |N_c(a) \cap V| > |\emptyset| \equiv$$
$$\forall a \in V,\ |N_c(a) \cap V| > 0 \equiv$$
$$\forall a \in V,\ \delta > 0.$$

(iv). V is offensive alliance since the following statements are equivalent.

$$\forall a \in S,\ |(N(a) \cap S) - (N(a) \cap (V \setminus S))| > \delta \equiv$$
$$\forall a \in V,\ |(N(a) \cap V) - (N(a) \cap (V \setminus V))| > \delta \equiv$$
$$\forall a \in V,\ |(N(a) \cap V) - (N(a) \cap (\emptyset))| > \delta \equiv$$
$$\forall a \in V,\ |(N(a) \cap V) - (\emptyset)| > \delta \equiv$$
$$\forall a \in V,\ |(N(a) \cap V)| > \delta.$$

(v). V is strong offensive alliance since the following statements are equivalent.

$$\forall a \in S,\ |(N_s(a) \cap S) - (N_s(a) \cap (V \setminus S))| > \delta \equiv$$
$$\forall a \in V,\ |(N_s(a) \cap V) - (N_s(a) \cap (V \setminus V))| > \delta \equiv$$
$$\forall a \in V,\ |(N_s(a) \cap V) - (N_s(a) \cap (\emptyset))| > \delta \equiv$$
$$\forall a \in V,\ |(N_s(a) \cap V) - (\emptyset)| > \delta \equiv$$
$$\forall a \in V,\ |(N_s(a) \cap V)| > \delta.$$

(vi). V is connected offensive alliance since the following statements are equivalent.

$$\forall a \in S,\ |(N_c(a) \cap S) - (N_c(a) \cap (V \setminus S))| > \delta \equiv$$
$$\forall a \in V,\ |(N_c(a) \cap V) - (N_c(a) \cap (V \setminus V))| > \delta \equiv$$
$$\forall a \in V,\ |(N_c(a) \cap V) - (N_c(a) \cap (\emptyset))| > \delta \equiv$$
$$\forall a \in V,\ |(N_c(a) \cap V) - (\emptyset)| > \delta \equiv$$
$$\forall a \in V,\ |(N_c(a) \cap V)| > \delta.$$

□

Proposition 2.2. *Let* $NTG : (V, E, \sigma, \mu)$ *be a neutrosophic graph. Then* $\emptyset$ *is*

(i) : *defensive alliance;*

(ii) : *strong defensive alliance;*

(iii) : *connected defensive alliance;*

(iv) : *δ-defensive alliance;*

(v) : *strong δ-defensive alliance;*

(vi) : *connected δ-defensive alliance.*

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Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a neutrosophic graph. Consider $\emptyset$. All members of $\emptyset$ have no neighbor inside the set less than neighbor out of set. Thus,

(i). $\emptyset$ is defensive alliance since the following statements are equivalent.

$$\forall a \in S,\ |N(a) \cap S| < |N(a) \cap (V \setminus S)| \equiv$$
$$\forall a \in \emptyset,\ |N(a) \cap \emptyset| < |N(a) \cap (V \setminus \emptyset)| \equiv$$
$$\forall a \in \emptyset,\ |\emptyset| < |N(a) \cap (V \setminus \emptyset)| \equiv$$
$$\forall a \in \emptyset,\ 0 < |N(a) \cap V| \equiv$$
$$\forall a \in \emptyset,\ 0 < |N(a) \cap V| \equiv$$
$$\forall a \in V,\ \delta > 0.$$

(ii). $\emptyset$ is strong defensive alliance since the following statements are equivalent.

$$\forall a \in S,\ |N_s(a) \cap S| < |N_s(a) \cap (V \setminus S)| \equiv$$
$$\forall a \in \emptyset,\ |N_s(a) \cap \emptyset| < |N_s(a) \cap (V \setminus \emptyset)| \equiv$$
$$\forall a \in \emptyset,\ |\emptyset| < |N_s(a) \cap (V \setminus \emptyset)| \equiv$$
$$\forall a \in \emptyset,\ 0 < |N_s(a) \cap V| \equiv$$
$$\forall a \in \emptyset,\ 0 < |N_s(a) \cap V| \equiv$$
$$\forall a \in V,\ \delta > 0.$$

(iii). $\emptyset$ is connected defensive alliance since the following statements are equivalent.

$$\forall a \in S,\ |N_c(a) \cap S| < |N_c(a) \cap (V \setminus S)| \equiv$$
$$\forall a \in \emptyset,\ |N_c(a) \cap \emptyset| < |N_c(a) \cap (V \setminus \emptyset)| \equiv$$
$$\forall a \in \emptyset,\ |\emptyset| < |N_c(a) \cap (V \setminus \emptyset)| \equiv$$
$$\forall a \in \emptyset,\ 0 < |N_c(a) \cap V| \equiv$$
$$\forall a \in \emptyset,\ 0 < |N_c(a) \cap V| \equiv$$
$$\forall a \in V,\ \delta > 0.$$

(iv). $\emptyset$ is defensive alliance since the following statements are equivalent.

$$\forall a \in S,\ |(N(a) \cap S) - (N(a) \cap (V \setminus S))| < \delta \equiv$$
$$\forall a \in \emptyset,\ |(N(a) \cap \emptyset) - (N(a) \cap (V \setminus \emptyset))| < \delta \equiv$$
$$\forall a \in \emptyset,\ |(N(a) \cap \emptyset) - (N(a) \cap (V))| < \delta \equiv$$
$$\forall a \in \emptyset,\ |\emptyset| < \delta \equiv$$
$$\forall a \in V,\ 0 < \delta.$$

(v). $\emptyset$ is strong defensive alliance since the following statements are equivalent.

$$\forall a \in S,\ |(N_s(a) \cap S) - (N_s(a) \cap (V \setminus S))| < \delta \equiv$$
$$\forall a \in \emptyset,\ |(N_s(a) \cap \emptyset) - (N_s(a) \cap (V \setminus \emptyset))| < \delta \equiv$$
$$\forall a \in \emptyset,\ |(N_s(a) \cap \emptyset) - (N_s(a) \cap (V))| < \delta \equiv$$
$$\forall a \in \emptyset,\ |\emptyset| < \delta \equiv$$
$$\forall a \in V,\ 0 < \delta.$$

(vi). $\emptyset$ is connected defensive alliance since the following statements are equivalent.

$$\forall a \in S,\ |(N_c(a) \cap S) - (N_c(a) \cap (V \setminus S))| < \delta \equiv$$
$$\forall a \in \emptyset,\ |(N_c(a) \cap \emptyset) - (N_c(a) \cap (V \setminus \emptyset))| < \delta \equiv$$
$$\forall a \in \emptyset,\ |(N_c(a) \cap \emptyset) - (N_c(a) \cap (V))| < \delta \equiv$$
$$\forall a \in \emptyset,\ |\emptyset| < \delta \equiv$$
$$\forall a \in V,\ 0 < \delta.$$

□

Proposition 2.3. *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then an independent set is*

(i) : *defensive alliance;*

(ii) : *strong defensive alliance;*

(iii) : *connected defensive alliance;*

(iv) : *δ-defensive alliance;*

(v) : *strong δ-defensive alliance;*

(vi) : *connected δ-defensive alliance.*

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a neutrosophic graph. Consider $\emptyset$. All members of $\emptyset$ have no neighbor inside the set less than neighbor out of set. Thus,

(i). An independent set is defensive alliance since the following statements are equivalent.

$$\forall a \in S,\ |N(a) \cap S| < |N(a) \cap (V \setminus S)| \equiv$$
$$\forall a \in S,\ |N(a) \cap S| < |N(a) \cap (V \setminus S)| \equiv$$
$$\forall a \in S,\ |\emptyset| < |N(a) \cap (V \setminus S)| \equiv$$
$$\forall a \in S,\ 0 < |N(a) \cap V| \equiv$$
$$\forall a \in S,\ 0 < |N(a)| \equiv$$
$$\forall a \in V,\ \delta > 0.$$

(ii). An independent set is strong defensive alliance since the following statements are equivalent.

$$\forall a \in S,\ |N_s(a) \cap S| < |N_s(a) \cap (V \setminus S)| \equiv$$
$$\forall a \in S,\ |N_s(a) \cap S| < |N_s(a) \cap (V \setminus S)| \equiv$$
$$\forall a \in S,\ |\emptyset| < |N_s(a) \cap (V \setminus S)| \equiv$$
$$\forall a \in S,\ 0 < |N_s(a) \cap V| \equiv$$
$$\forall a \in S,\ 0 < |N_s(a)| \equiv$$
$$\forall a \in V,\ \delta > 0.$$

(iii). An independent set is connected defensive alliance since the following statements are equivalent.

$$\forall a \in S,\ |N_c(a) \cap S| < |N_c(a) \cap (V \setminus S)| \equiv$$
$$\forall a \in S,\ |N_c(a) \cap S| < |N_c(a) \cap (V \setminus S)| \equiv$$
$$\forall a \in S,\ |\emptyset| < |N_c(a) \cap (V \setminus S)| \equiv$$
$$\forall a \in S,\ 0 < |N_c(a) \cap V| \equiv$$
$$\forall a \in S,\ 0 < |N_c(a)| \equiv$$
$$\forall a \in V,\ \delta > 0.$$

(iv). An independent set is defensive alliance since the following statements are equivalent.

$$\forall a \in S,\ |(N(a) \cap S) - (N(a) \cap (V \setminus S))| < \delta \equiv$$
$$\forall a \in S,\ |(N(a) \cap S) - (N(a) \cap (V \setminus S))| < \delta \equiv$$
$$\forall a \in S,\ |(N(a) \cap S) - (N(a) \cap (V))| < \delta \equiv$$
$$\forall a \in S,\ |\emptyset| < \delta \equiv$$
$$\forall a \in V,\ 0 < \delta.$$

(v). An independent set is strong defensive alliance since the following statements are equivalent.

$$\forall a \in S,\ |(N_s(a) \cap S) - (N_s(a) \cap (V \setminus S))| < \delta \equiv$$
$$\forall a \in S,\ |(N_s(a) \cap S) - (N_s(a) \cap (V \setminus S))| < \delta \equiv$$
$$\forall a \in S,\ |(N_s(a) \cap S) - (N_s(a) \cap (V))| < \delta \equiv$$
$$\forall a \in S,\ |\emptyset| < \delta \equiv$$
$$\forall a \in V,\ 0 < \delta.$$

(vi). An independent set is connected defensive alliance since the following statements are equivalent.

$$\forall a \in S,\ |(N_c(a) \cap S) - (N_c(a) \cap (V \setminus S))| < \delta \equiv$$
$$\forall a \in S,\ |(N_c(a) \cap S) - (N_c(a) \cap (V \setminus S))| < \delta \equiv$$
$$\forall a \in S,\ |(N_c(a) \cap S) - (N_c(a) \cap (V))| < \delta \equiv$$
$$\forall a \in S,\ |\emptyset| < \delta \equiv$$
$$\forall a \in V,\ 0 < \delta.$$

□

3 Classes of Neutrosophic Graphs

Proposition 3.1. *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph which is cycle/path/wheel. Then V is minimal*

(i) : *offensive alliance;*

(ii) : *strong offensive alliance;*

(iii) : *connected offensive alliance;*

(iv) : *$\mathcal{O}(NTG)$-offensive alliance;*

(v) : *strong $\mathcal{O}(NTG)$-offensive alliance;*

(vi) : *connected $\mathcal{O}(NTG)$-offensive alliance.*

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a neutrosophic graph which is cycle/path//wheel.

(i). Consider one vertex is out of S which is alliance. This vertex has one neighbor in S, i.e, Suppose $x \in V \setminus S$ such that $y, z \in N(x)$. By it's cycle, $|N(x)| = |N(y)| = |N(z)| = 2$. Thus

$$\forall a \in S,\ |N(a) \cap S| < |N(a) \cap (V \setminus S)| \equiv$$
$$\forall a \in S,\ |N(a) \cap S| < |N(a) \cap (V \setminus S)| \equiv$$
$$\exists y \in V \setminus \{x\},\ |N(y) \cap S| < |N(y) \cap (V \setminus (V \setminus \{x\}))| \equiv$$
$$\exists y \in V \setminus \{x\},\ |N(y) \cap S| < |N(y) \cap \{x\})| \equiv$$
$$\exists y \in V \setminus \{x\},\ |\{z\}| < |\{x\})| \equiv$$
$$\exists y \in S,\ 1 < 1.$$

Thus it's contradiction. It implies every $V \setminus \{x\}$ isn't offensive alliance in a given cycle. Consider one vertex is out of S which is alliance. This vertex has one neighbor in S, i.e, Suppose $x \in V \setminus S$ such that $y, z \in N(x)$. By it's path, $|N(x)| = |N(y)| = |N(z)| = 2$. Thus

$$\forall a \in S,\ |N(a) \cap S| < |N(a) \cap (V \setminus S)| \equiv$$
$$\forall a \in S,\ |N(a) \cap S| < |N(a) \cap (V \setminus S)| \equiv$$
$$\exists y \in V \setminus \{x\},\ |N(y) \cap S| < |N(y) \cap (V \setminus (V \setminus \{x\}))| \equiv$$
$$\exists y \in V \setminus \{x\},\ |N(y) \cap S| < |N(y) \cap \{x\})| \equiv$$
$$\exists y \in V \setminus \{x\},\ |\{z\}| < |\{x\})| \equiv$$
$$\exists y \in S,\ 1 < 1.$$

Thus it's contradiction. It implies every $V \setminus \{x\}$ isn't offensive alliance in a given path. Consider one vertex is out of S which is alliance. This vertex has one neighbor in S, i.e, Suppose $x \in V \setminus S$ such that $y, z \in N(x)$. By it's wheel, $|N(x)| = |N(y)| = |N(z)| = 2$. Thus

$$\forall a \in S,\ |N(a) \cap S| < |N(a) \cap (V \setminus S)| \equiv$$
$$\forall a \in S,\ |N(a) \cap S| < |N(a) \cap (V \setminus S)| \equiv$$
$$\exists y \in V \setminus \{x\},\ |N(y) \cap S| < |N(y) \cap (V \setminus (V \setminus \{x\}))| \equiv$$
$$\exists y \in V \setminus \{x\},\ |N(y) \cap S| < |N(y) \cap \{x\})| \equiv$$
$$\exists y \in V \setminus \{x\},\ |\{z\}| < |\{x\})| \equiv$$

$$\exists y \in S,\ 1 < 1.$$

Thus it's contradiction. It implies every $V \setminus \{x\}$ isn't offensive alliance in a given wheel.

(ii), (iii) are obvious by (i).

(iv). By (i), $|V|$ is minimal and it's offensive alliance. Thus it's $|V|$-offensive alliance.

$(v), (vi)$ are obvious by (iv). □

Proposition 3.2. *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph which is cycle/path/wheel. Then V is only*

(i) : *offensive alliance;*

(ii) : *strong offensive alliance;*

(iii) : *connected offensive alliance;*

(iv) : *$\mathcal{O}(NTG)$-offensive alliance;*

(v) : *strong $\mathcal{O}(NTG)$-offensive alliance;*

(vi) : *connected $\mathcal{O}(NTG)$-offensive alliance.*

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a neutrosophic graph which is cycle/path//wheel.

(i). Consider one vertex is out of S which is alliance. This vertex has one neighbor in S, i.e, Suppose $x \in V \setminus S$ such that $y, z \in N(x)$. By it's cycle, $|N(x)| = |N(y)| = |N(z)| = 2$. Thus

$$\forall a \in S,\ |N(a) \cap S| < |N(a) \cap (V \setminus S)| \equiv$$
$$\forall a \in S,\ |N(a) \cap S| < |N(a) \cap (V \setminus S)| \equiv$$
$$\exists y \in V \setminus \{x\},\ |N(y) \cap S| < |N(y) \cap (V \setminus (V \setminus \{x\}))| \equiv$$
$$\exists y \in V \setminus \{x\},\ |N(y) \cap S| < |N(y) \cap \{x\})| \equiv$$
$$\exists y \in V \setminus \{x\},\ |\{z\}| < |\{x\})| \equiv$$
$$\exists y \in S,\ 1 < 1.$$

Thus it's contradiction. It implies every $V \setminus \{x\}$ isn't offensive alliance in a given cycle. Consider one vertex is out of S which is alliance. This vertex has one neighbor in S, i.e, Suppose $x \in V \setminus S$ such that $y, z \in N(x)$. By it's path, $|N(x)| = |N(y)| = |N(z)| = 2$. Thus

$$\forall a \in S,\ |N(a) \cap S| < |N(a) \cap (V \setminus S)| \equiv$$
$$\forall a \in S,\ |N(a) \cap S| < |N(a) \cap (V \setminus S)| \equiv$$
$$\exists y \in V \setminus \{x\},\ |N(y) \cap S| < |N(y) \cap (V \setminus (V \setminus \{x\}))| \equiv$$
$$\exists y \in V \setminus \{x\},\ |N(y) \cap S| < |N(y) \cap \{x\})| \equiv$$
$$\exists y \in V \setminus \{x\},\ |\{z\}| < |\{x\})| \equiv$$
$$\exists y \in S,\ 1 < 1.$$

Thus it's contradiction. It implies every $V \setminus \{x\}$ isn't offensive alliance in a given path. Consider one vertex is out of S which is alliance. This vertex has one neighbor in S, i.e, Suppose $x \in V \setminus S$ such that $y, z \in N(x)$. By it's wheel, $|N(x)| = |N(y)| = |N(z)| = 2$. Thus

$$\forall a \in S,\ |N(a) \cap S| < |N(a) \cap (V \setminus S)| \equiv$$
$$\forall a \in S,\ |N(a) \cap S| < |N(a) \cap (V \setminus S)| \equiv$$
$$\exists y \in V \setminus \{x\},\ |N(y) \cap S| < |N(y) \cap (V \setminus (V \setminus \{x\}))| \equiv$$
$$\exists y \in V \setminus \{x\},\ |N(y) \cap S| < |N(y) \cap \{x\})| \equiv$$
$$\exists y \in V \setminus \{x\},\ |\{z\}| < |\{x\})| \equiv$$
$$\exists y \in S,\ 1 < 1.$$

Thus it's contradiction. It implies every $V \setminus \{x\}$ isn't offensive alliance in a given wheel.

(ii), (iii) are obvious by (i).

(iv). By (i), V is minimal and it's offensive alliance. Thus it's $\mathcal{O}(NTG)$-offensive alliance.

$(v), (vi)$ are obvious by (iv). □

Proposition 3.3. *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph which is star/complete bipartite/complete t-partite. Then center and n half +1 vertices is minimal*

(i) : *offensive alliance;*

(ii) : *strong offensive alliance;*

(iii) : *connected offensive alliance;*

(iv) : $\frac{\mathcal{O}(NTG)}{2} + 1$*-offensive alliance;*

(v) : *strong* $\frac{\mathcal{O}(NTG)}{2} + 1$*-offensive alliance;*

(vi) : *connected* $\frac{\mathcal{O}(NTG)}{2} + 1$*-offensive alliance.*

Proof. (i). Consider n half +1 vertices are out of S which is alliance. This vertex has n half neighbor in S. If the vertex is non-center, then

$$\forall a \in S, \ |N(a) \cap S| > |N(a) \cap (V \setminus S)| \equiv$$
$$\forall a \in S, \ 1 > 0.$$

If the vertex is center, then

$$\forall a \in S, \ |N(a) \cap S| > |N(a) \cap (V \setminus S)| \equiv$$
$$\forall a \in S, \ \frac{n}{2} > \frac{n}{2} - 1.$$

Thus it's proved. It implies every S is offensive alliance in a given star.

Consider n half +1 vertices are out of S which is alliance. This vertex has n half neighbor in S.

$$\forall a \in S, \ |N(a) \cap S| > |N(a) \cap (V \setminus S)| \equiv$$
$$\forall a \in S, \ \frac{n}{2} > \frac{n}{2} - 1.$$

Thus it's proved. It implies every S is offensive alliance in a given complete bipartite which isn't a star.

Consider n half +1 vertices are out of S which is alliance and they are chosen from different parts, equally or almost equally as possible. This vertex has n half neighbor in S.

$$\forall a \in S, \ |N(a) \cap S| > |N(a) \cap (V \setminus S)| \equiv$$
$$\forall a \in S, \ \frac{n}{2} > \frac{n}{2} - 1.$$

Thus it's proved. It implies every S is offensive alliance in a given complete t-partite which isn't neither a star nor complete bipartite.

(ii), (iii) are obvious by (i).

(iv). By (i), $\{x_i\}_{i=1}^{\frac{\mathcal{O}(NTG)}{2}+1}$ is minimal and it's offensive alliance. Thus it's $\frac{\mathcal{O}(NTG)}{2} + 1$-offensive alliance.

$(v), (vi)$ are obvious by (iv). □

Proposition 3.4. *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph which is star/complete bipartite/complete t-partite. Then center and n half +1 vertices is only*

(i) : *offensive alliance;*

(ii) : *strong offensive alliance;*

(iii) : *connected offensive alliance;*

$(iv):$ *δ-offensive alliance;*

$(v):$ *strong δ-offensive alliance;*

$(vi):$ *connected δ-offensive alliance.*

Proof. (i). Consider n half $+1$ vertices are out of S which is alliance. This vertex has n half neighbor in S. If the vertex is non-center, then

$$\forall a \in S,\ |N(a) \cap S| > |N(a) \cap (V \setminus S)| \equiv$$
$$\forall a \in S,\ 1 > 0.$$

If the vertex is center, then

$$\forall a \in S,\ |N(a) \cap S| > |N(a) \cap (V \setminus S)| \equiv$$
$$\forall a \in S,\ \frac{n}{2} > \frac{n}{2} - 1.$$

Thus it's proved. It implies every S is offensive alliance in a given star.

Consider n half $+1$ vertices are out of S which is alliance. This vertex has n half neighbor in S.

$$\forall a \in S,\ |N(a) \cap S| > |N(a) \cap (V \setminus S)| \equiv$$
$$\forall a \in S,\ \frac{n}{2} > \frac{n}{2} - 1.$$

Thus it's proved. It implies every S is offensive alliance in a given complete bipartite which isn't a star.

Consider n half $+1$ vertices are out of S which is alliance and they are chosen from different parts, equally or almost equally as possible. This vertex has n half neighbor in S.

$$\forall a \in S,\ |N(a) \cap S| > |N(a) \cap (V \setminus S)| \equiv$$
$$\forall a \in S,\ \frac{n}{2} > \frac{n}{2} - 1.$$

Thus it's proved. It implies every S is offensive alliance in a given complete t-partite which isn't neither a star nor complete bipartite.

(ii), (iii) are obvious by (i).

(iv). By (i), $\{x_i\}_{i=1}^{\frac{\mathcal{O}(NTG)}{2}+1}$ is minimal and it's offensive alliance. Thus it's $\frac{\mathcal{O}(NTG)}{2} + 1$-offensive alliance.

$(v), (vi)$ are obvious by (iv). □

4 In the Setting of Number

Proposition 4.1. *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. The number of connected component is $|V - S|$ if there's a set which is*

$(i):$ *offensive alliance;*

$(ii):$ *strong offensive alliance;*

$(iii):$ *connected offensive alliance;*

$(iv):$ *1-offensive alliance;*

$(v):$ *strong 1-offensive alliance;*

$(vi):$ *connected 1-offensive alliance.*

Proof. (i). Consider some vertices are out of S which is alliance. This vertex has n half neighbor in S but no vertex out of S. Thus

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$$\forall a \in S,\ |N(a) \cap S| > |N(a) \cap (V \setminus S)| \equiv$$
$$\forall a \in S,\ 1 > 0.$$

Thus it's proved. It implies every S is offensive alliance and number of connected component is $|V - S|$.

Consider some vertices are out of S which is alliance. This vertex has n half neighbor in S but no vertex out of S. Thus

$$\forall a \in S,\ |N(a) \cap S| > |N(a) \cap (V \setminus S)| \equiv$$
$$\forall a \in S,\ 1 > 0.$$

Thus it's proved. It implies every S is offensive alliance and number of connected component is $|V - S|$.

Consider some vertices are out of S which is alliance. This vertex has n half neighbor in S but no vertex out of S. Thus

$$\forall a \in S,\ |N(a) \cap S| > |N(a) \cap (V \setminus S)| \equiv$$
$$\forall a \in S,\ 1 > 0.$$

Thus it's proved. It implies every S is offensive alliance and number of connected component is $|V - S|$.

(ii), (iii) are obvious by (i).

(iv). By (i), $\{x\}$ is minimal and it's offensive alliance. Thus it's 1-offensive alliance.

$(v), (vi)$ are obvious by (iv). □

Proposition 4.2. *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then the number is at most $\mathcal{O}(NTG)$ and the neutrosophic number is at most $\mathcal{O}_n(NTG)$.*

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a neutrosophic graph. Consider V. All members of V have at least one neighbor inside the set more than neighbor out of set. Thus,

V is offensive alliance since the following statements are equivalent.

$$\forall a \in S,\ |N(a) \cap S| > |N(a) \cap (V \setminus S)| \equiv$$
$$\forall a \in V,\ |N(a) \cap V| > |N(a) \cap (V \setminus V)| \equiv$$
$$\forall a \in V,\ |N(a) \cap V| > |N(a) \cap \emptyset| \equiv$$
$$\forall a \in V,\ |N(a) \cap V| > |\emptyset| \equiv$$
$$\forall a \in V,\ |N(a) \cap V| > 0 \equiv$$
$$\forall a \in V,\ \delta > 0.$$

V is strong offensive alliance since the following statements are equivalent.

$$\forall a \in S,\ |N_s(a) \cap S| > |N_s(a) \cap (V \setminus S)| \equiv$$
$$\forall a \in V,\ |N_s(a) \cap V| > |N_s(a) \cap (V \setminus V)| \equiv$$
$$\forall a \in V,\ |N_s(a) \cap V| > |N_s(a) \cap \emptyset| \equiv$$
$$\forall a \in V,\ |N_s(a) \cap V| > |\emptyset| \equiv$$
$$\forall a \in V,\ |N_s(a) \cap V| > 0 \equiv$$
$$\forall a \in V,\ \delta > 0.$$

V is connected offensive alliance since the following statements are equivalent.

$$\forall a \in S,\ |N_c(a) \cap S| > |N_c(a) \cap (V \setminus S)| \equiv$$
$$\forall a \in V,\ |N_c(a) \cap V| > |N_c(a) \cap (V \setminus V)| \equiv$$
$$\forall a \in V,\ |N_c(a) \cap V| > |N_c(a) \cap \emptyset| \equiv$$
$$\forall a \in V,\ |N_c(a) \cap V| > |\emptyset| \equiv$$
$$\forall a \in V,\ |N_c(a) \cap V| > 0 \equiv$$
$$\forall a \in V,\ \delta > 0.$$

V is offensive alliance since the following statements are equivalent.

$$\forall a \in S,\ |(N(a) \cap S) - (N(a) \cap (V \setminus S))| > \delta \equiv$$
$$\forall a \in V,\ |(N(a) \cap V) - (N(a) \cap (V \setminus V))| > \delta \equiv$$
$$\forall a \in V,\ |(N(a) \cap V) - (N(a) \cap (\emptyset))| > \delta \equiv$$

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$$\forall a \in V,\ |(N(a)\cap V) - (\emptyset)| > \delta \equiv$$
$$\forall a \in V,\ |(N(a)\cap V)| > \delta.$$

V is strong offensive alliance since the following statements are equivalent.

$$\forall a \in S,\ |(N_s(a)\cap S) - (N_s(a)\cap (V\setminus S))| > \delta \equiv$$
$$\forall a \in V,\ |(N_s(a)\cap V) - (N_s(a)\cap (V\setminus V))| > \delta \equiv$$
$$\forall a \in V,\ |(N_s(a)\cap V) - (N_s(a)\cap (\emptyset))| > \delta \equiv$$
$$\forall a \in V,\ |(N_s(a)\cap V) - (\emptyset)| > \delta \equiv$$
$$\forall a \in V,\ |(N_s(a)\cap V)| > \delta.$$

V is connected offensive alliance since the following statements are equivalent.

$$\forall a \in S,\ |(N_c(a)\cap S) - (N_c(a)\cap (V\setminus S))| > \delta \equiv$$
$$\forall a \in V,\ |(N_c(a)\cap V) - (N_c(a)\cap (V\setminus V))| > \delta \equiv$$
$$\forall a \in V,\ |(N_c(a)\cap V) - (N_c(a)\cap (\emptyset))| > \delta \equiv$$
$$\forall a \in V,\ |(N_c(a)\cap V) - (\emptyset)| > \delta \equiv$$
$$\forall a \in V,\ |(N_c(a)\cap V)| > \delta.$$

Thus V is alliance and V is the biggest set in NTG. Then the number is at most $\mathcal{O}(NTG)$ and the neutrosophic number is at most $\mathcal{O}_n(NTG)$. □

Proposition 4.3. *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph which is complete. The number is $\frac{\mathcal{O}(NTG)}{2} + 1$ and the neutrosophic number is $\min \Sigma_{v\in\{v_1,v_2,\cdots,v_t\}_{t>\frac{\mathcal{O}(NTG)}{2}}\subseteq V}\sigma(v)$, in the setting of*

(i) : *offensive alliance;*

(ii) : *strong offensive alliance;*

(iii) : *connected offensive alliance;*

(iv) : *$(\frac{\mathcal{O}(NTG)}{2} + 1)$-offensive alliance;*

(v) : *strong $(\frac{\mathcal{O}(NTG)}{2} + 1)$-offensive alliance;*

(vi) : *connected $(\frac{\mathcal{O}(NTG)}{2} + 1)$-offensive alliance.*

Proof. (i). Consider n half $+1$ vertices are out of S which is alliance. This vertex has n half neighbor in S.

$$\forall a \in S,\ |N(a)\cap S| > |N(a)\cap (V\setminus S)| \equiv$$
$$\forall a \in S,\ \frac{n}{2} > \frac{n}{2} - 1.$$

Thus it's proved. It implies every S is offensive alliance in a given complete graph. Thus the number is $\frac{\mathcal{O}(NTG)}{2} + 1$ and the neutrosophic number is $\min \Sigma_{v\in\{v_1,v_2,\cdots,v_t\}_{t>\frac{\mathcal{O}(NTG)}{2}}\subseteq V}\sigma(v)$, in the setting of offensive alliance.

(ii). Consider n half $+1$ vertices are out of S which is alliance. This vertex has n half neighbor in S.

$$\forall a \in S,\ |N_s(a)\cap S| > |N_s(a)\cap (V\setminus S)| \equiv$$
$$\forall a \in S,\ \frac{n}{2} > \frac{n}{2} - 1.$$

Thus it's proved. It implies every S is strong offensive alliance in a given complete graph. Thus the number is $\frac{\mathcal{O}(NTG)}{2} + 1$ and the neutrosophic number is $\min \Sigma_{v\in\{v_1,v_2,\cdots,v_t\}_{t>\frac{\mathcal{O}(NTG)}{2}}\subseteq V}\sigma(v)$, in the setting of strong offensive alliance.

(iii). Consider n half $+1$ vertices are out of S which is alliance. This vertex has n half neighbor in S.

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$$\forall a \in S,\ |N_c(a) \cap S| > |N_c(a) \cap (V \setminus S)| \equiv$$
$$\forall a \in S,\ \frac{n}{2} > \frac{n}{2} - 1.$$

Thus it's proved. It implies every S is connected offensive alliance in a given complete graph. Thus the number is $\frac{\mathcal{O}(NTG)}{2} + 1$ and the neutrosophic number is $\min \Sigma_{v \in \{v_1, v_2, \cdots, v_t\}_{t > \frac{\mathcal{O}(NTG)}{2}} \subseteq V} \sigma(v)$, in the setting of connected offensive alliance.

(iv). Consider n half $+1$ vertices are out of S which is alliance. This vertex has n half neighbor in S.

$$\forall a \in S,\ |N(a) \cap S| > |N(a) \cap (V \setminus S)| \equiv$$
$$\forall a \in S,\ \frac{n}{2} > \frac{n}{2} - 1.$$

Thus it's proved. It implies every S is $(\frac{\mathcal{O}(NTG)}{2} + 1)$-offensive alliance in a given complete graph. Thus the number is $\frac{\mathcal{O}(NTG)}{2} + 1$ and the neutrosophic number is $\min \Sigma_{v \in \{v_1, v_2, \cdots, v_t\}_{t > \frac{\mathcal{O}(NTG)}{2}} \subseteq V} \sigma(v)$, in the setting of $(\frac{\mathcal{O}(NTG)}{2} + 1)$-offensive alliance.

(v). Consider n half $+1$ vertices are out of S which is alliance. This vertex has n half neighbor in S.

$$\forall a \in S,\ |N_s(a) \cap S| > |N_s(a) \cap (V \setminus S)| \equiv$$
$$\forall a \in S,\ \frac{n}{2} > \frac{n}{2} - 1.$$

Thus it's proved. It implies every S is strong $(\frac{\mathcal{O}(NTG)}{2} + 1)$-offensive alliance in a given complete graph. Thus the number is $\frac{\mathcal{O}(NTG)}{2} + 1$ and the neutrosophic number is $\min \Sigma_{v \in \{v_1, v_2, \cdots, v_t\}_{t > \frac{\mathcal{O}(NTG)}{2}} \subseteq V} \sigma(v)$, in the setting of strong $(\frac{\mathcal{O}(NTG)}{2} + 1)$-offensive alliance.

(vi). Consider n half $+1$ vertices are out of S which is alliance. This vertex has n half neighbor in S.

$$\forall a \in S,\ |N_c(a) \cap S| > |N_c(a) \cap (V \setminus S)| \equiv$$
$$\forall a \in S,\ \frac{n}{2} > \frac{n}{2} - 1.$$

Thus it's proved. It implies every S is connected $(\frac{\mathcal{O}(NTG)}{2} + 1)$-offensive alliance in a given complete graph. Thus the number is $\frac{\mathcal{O}(NTG)}{2} + 1$ and the neutrosophic number is $\min \Sigma_{v \in \{v_1, v_2, \cdots, v_t\}_{t > \frac{\mathcal{O}(NTG)}{2}} \subseteq V} \sigma(v)$, in the setting of connected $(\frac{\mathcal{O}(NTG)}{2} + 1)$-offensive alliance. □

Proposition 4.4. *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph which is $\emptyset$. The number is 0 and the neutrosophic number is 0, for an independent set in the setting of*

(i) : *offensive alliance;*

(ii) : *strong offensive alliance;*

(iii) : *connected offensive alliance;*

(iv) : 0-*offensive alliance;*

(v) : *strong* 0-*offensive alliance;*

(vi) : *connected* 0-*offensive alliance.*

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a neutrosophic graph. Consider $\emptyset$. All members of $\emptyset$ have no neighbor inside the set less than neighbor out of set. Thus,

(i). $\emptyset$ is defensive alliance since the following statements are equivalent.

$$\forall a \in S,\ |N(a) \cap S| < |N(a) \cap (V \setminus S)| \equiv$$
$$\forall a \in \emptyset,\ |N(a) \cap \emptyset| < |N(a) \cap (V \setminus \emptyset)| \equiv$$
$$\forall a \in \emptyset,\ |\emptyset| < |N(a) \cap (V \setminus \emptyset)| \equiv$$
$$\forall a \in \emptyset,\ 0 < |N(a) \cap V| \equiv$$
$$\forall a \in \emptyset,\ 0 < |N(a) \cap V| \equiv$$
$$\forall a \in V,\ \delta > 0.$$

The number is 0 and the neutrosophic number is 0, for an independent set in the setting of offensive alliance.

(ii). $\emptyset$ is strong defensive alliance since the following statements are equivalent.

$$\forall a \in S,\ |N_s(a) \cap S| < |N_s(a) \cap (V \setminus S)| \equiv$$
$$\forall a \in \emptyset,\ |N_s(a) \cap \emptyset| < |N_s(a) \cap (V \setminus \emptyset)| \equiv$$
$$\forall a \in \emptyset,\ |\emptyset| < |N_s(a) \cap (V \setminus \emptyset)| \equiv$$
$$\forall a \in \emptyset,\ 0 < |N_s(a) \cap V| \equiv$$
$$\forall a \in \emptyset,\ 0 < |N_s(a) \cap V| \equiv$$
$$\forall a \in V,\ \delta > 0.$$

The number is 0 and the neutrosophic number is 0, for an independent set in the setting of strong offensive alliance.

(iii). $\emptyset$ is connected defensive alliance since the following statements are equivalent.

$$\forall a \in S,\ |N_c(a) \cap S| < |N_c(a) \cap (V \setminus S)| \equiv$$
$$\forall a \in \emptyset,\ |N_c(a) \cap \emptyset| < |N_c(a) \cap (V \setminus \emptyset)| \equiv$$
$$\forall a \in \emptyset,\ |\emptyset| < |N_c(a) \cap (V \setminus \emptyset)| \equiv$$
$$\forall a \in \emptyset,\ 0 < |N_c(a) \cap V| \equiv$$
$$\forall a \in \emptyset,\ 0 < |N_c(a) \cap V| \equiv$$
$$\forall a \in V,\ \delta > 0.$$

The number is 0 and the neutrosophic number is 0, for an independent set in the setting of connected offensive alliance.

(iv). $\emptyset$ is defensive alliance since the following statements are equivalent.

$$\forall a \in S,\ |(N(a) \cap S) - (N(a) \cap (V \setminus S))| < \delta \equiv$$
$$\forall a \in \emptyset,\ |(N(a) \cap \emptyset) - (N(a) \cap (V \setminus \emptyset))| < \delta \equiv$$
$$\forall a \in \emptyset,\ |(N(a) \cap \emptyset) - (N(a) \cap (V))| < \delta \equiv$$
$$\forall a \in \emptyset,\ |\emptyset| < \delta \equiv$$
$$\forall a \in V,\ 0 < \delta.$$

The number is 0 and the neutrosophic number is 0, for an independent set in the setting of 0-offensive alliance.

(v). $\emptyset$ is strong defensive alliance since the following statements are equivalent.

$$\forall a \in S,\ |(N_s(a) \cap S) - (N_s(a) \cap (V \setminus S))| < \delta \equiv$$
$$\forall a \in \emptyset,\ |(N_s(a) \cap \emptyset) - (N_s(a) \cap (V \setminus \emptyset))| < \delta \equiv$$
$$\forall a \in \emptyset,\ |(N_s(a) \cap \emptyset) - (N_s(a) \cap (V))| < \delta \equiv$$
$$\forall a \in \emptyset,\ |\emptyset| < \delta \equiv$$
$$\forall a \in V,\ 0 < \delta.$$

The number is 0 and the neutrosophic number is 0, for an independent set in the setting of strong 0-offensive alliance.

(vi). $\emptyset$ is connected defensive alliance since the following statements are equivalent.

$$\forall a \in S,\ |(N_c(a) \cap S) - (N_c(a) \cap (V \setminus S))| < \delta \equiv$$
$$\forall a \in \emptyset,\ |(N_c(a) \cap \emptyset) - (N_c(a) \cap (V \setminus \emptyset))| < \delta \equiv$$
$$\forall a \in \emptyset,\ |(N_c(a) \cap \emptyset) - (N_c(a) \cap (V))| < \delta \equiv$$
$$\forall a \in \emptyset,\ |\emptyset| < \delta \equiv$$
$$\forall a \in V,\ 0 < \delta.$$

The number is 0 and the neutrosophic number is 0, for an independent set in the setting of connected 0-offensive alliance. □

Proposition 4.5. *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph which is complete. Then there's no independent set.*

5 Classes of Neutrosophic Graphs

Proposition 5.1. *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph which is cycle/path/wheel. The number is $\mathcal{O}(NTG)$ and the neutrosophic number is $\mathcal{O}_n(NTG)$, in the setting of*

(i) : *offensive alliance;*

(ii) : *strong offensive alliance;*

(iii) : *connected offensive alliance;*

(iv) : *$\mathcal{O}(NTG)$-offensive alliance;*

(v) : *strong $\mathcal{O}(NTG)$-offensive alliance;*

(vi) : *connected $\mathcal{O}(NTG)$-offensive alliance.*

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a neutrosophic graph which is cycle/path/wheel.
(i). Consider one vertex is out of S which is alliance. This vertex has one neighbor in S, i.e, Suppose $x \in V \setminus S$ such that $y, z \in N(x)$. By it's cycle, $|N(x)| = |N(y)| = |N(z)| = 2$. Thus

$$\forall a \in S,\ |N(a) \cap S| < |N(a) \cap (V \setminus S)| \equiv$$
$$\forall a \in S,\ |N(a) \cap S| < |N(a) \cap (V \setminus S)| \equiv$$
$$\exists y \in V \setminus \{x\},\ |N(y) \cap S| < |N(y) \cap (V \setminus (V \setminus \{x\}))| \equiv$$
$$\exists y \in V \setminus \{x\},\ |N(y) \cap S| < |N(y) \cap \{x\})| \equiv$$
$$\exists y \in V \setminus \{x\},\ |\{z\}| < |\{x\})| \equiv$$
$$\exists y \in S,\ 1 < 1.$$

Thus it's contradiction. It implies every $V \setminus \{x\}$ isn't offensive alliance in a given cycle. Consider one vertex is out of S which is alliance. This vertex has one neighbor in S, i.e, Suppose $x \in V \setminus S$ such that $y, z \in N(x)$. By it's path, $|N(x)| = |N(y)| = |N(z)| = 2$. Thus

$$\forall a \in S,\ |N(a) \cap S| < |N(a) \cap (V \setminus S)| \equiv$$
$$\forall a \in S,\ |N(a) \cap S| < |N(a) \cap (V \setminus S)| \equiv$$
$$\exists y \in V \setminus \{x\},\ |N(y) \cap S| < |N(y) \cap (V \setminus (V \setminus \{x\}))| \equiv$$
$$\exists y \in V \setminus \{x\},\ |N(y) \cap S| < |N(y) \cap \{x\})| \equiv$$
$$\exists y \in V \setminus \{x\},\ |\{z\}| < |\{x\})| \equiv$$
$$\exists y \in S,\ 1 < 1.$$

Thus it's contradiction. It implies every $V \setminus \{x\}$ isn't offensive alliance in a given path. Consider one vertex is out of S which is alliance. This vertex has one neighbor in S, i.e, Suppose $x \in V \setminus S$ such that $y, z \in N(x)$. By it's wheel, $|N(x)| = |N(y)| = |N(z)| = 2$. Thus

$$\forall a \in S,\ |N(a) \cap S| < |N(a) \cap (V \setminus S)| \equiv$$
$$\forall a \in S,\ |N(a) \cap S| < |N(a) \cap (V \setminus S)| \equiv$$
$$\exists y \in V \setminus \{x\},\ |N(y) \cap S| < |N(y) \cap (V \setminus (V \setminus \{x\}))| \equiv$$
$$\exists y \in V \setminus \{x\},\ |N(y) \cap S| < |N(y) \cap \{x\})| \equiv$$
$$\exists y \in V \setminus \{x\},\ |\{z\}| < |\{x\})| \equiv$$
$$\exists y \in S,\ 1 < 1.$$

Thus it's contradiction. It implies every $V \setminus \{x\}$ isn't offensive alliance in a given wheel.
(ii), (iii) are obvious by (i).

(iv). By (i), V is minimal and it's offensive alliance. Thus it's $\mathcal{O}(NTG)$-offensive alliance.

$(v), (vi)$ are obvious by (iv).

Thus the number is $\mathcal{O}(NTG)$ and the neutrosophic number is $\mathcal{O}_n(NTG)$, in the setting of all types of alliance. □

Proposition 5.2. *Let* $NTG : (V, E, \sigma, \mu)$ *be a neutrosophic graph which is star/complete bipartite/complete t-partite. The number is* $\frac{\mathcal{O}(NTG)}{2} + 1$ *and the neutrosophic number is* $\min \Sigma_{v \in \{v_1, v_2, \cdots, v_t\}_{t > \frac{\mathcal{O}(NTG)}{2}} \subseteq V} \sigma(v)$*, in the setting of*

(i) : *offensive alliance;*

(ii) : *strong offensive alliance;*

(iii) : *connected offensive alliance;*

(iv) : $(\frac{\mathcal{O}(NTG)}{2} + 1)$*-offensive alliance;*

(v) : *strong* $(\frac{\mathcal{O}(NTG)}{2} + 1)$*-offensive alliance;*

(vi) : *connected* $(\frac{\mathcal{O}(NTG)}{2} + 1)$*-offensive alliance.*

Proof. (i). Consider n half $+1$ vertices are out of S which is alliance. This vertex has n half neighbor in S. If the vertex is non-center, then

$$\forall a \in S,\ |N(a) \cap S| > |N(a) \cap (V \setminus S)| \equiv$$
$$\forall a \in S,\ 1 > 0.$$

If the vertex is center, then

$$\forall a \in S,\ |N(a) \cap S| > |N(a) \cap (V \setminus S)| \equiv$$
$$\forall a \in S,\ \tfrac{n}{2} > \tfrac{n}{2} - 1.$$

Thus it's proved. It implies every S is offensive alliance in a given star.

Consider n half $+1$ vertices are out of S which is alliance. This vertex has n half neighbor in S.

$$\forall a \in S,\ |N(a) \cap S| > |N(a) \cap (V \setminus S)| \equiv$$
$$\forall a \in S,\ \tfrac{n}{2} > \tfrac{n}{2} - 1.$$

Thus it's proved. It implies every S is offensive alliance in a given complete bipartite which isn't a star.

Consider n half $+1$ vertices are out of S which is alliance and they are chosen from different parts, equally or almost equally as possible. This vertex has n half neighbor in S.

$$\forall a \in S,\ |N(a) \cap S| > |N(a) \cap (V \setminus S)| \equiv$$
$$\forall a \in S,\ \tfrac{n}{2} > \tfrac{n}{2} - 1.$$

Thus it's proved. It implies every S is offensive alliance in a given complete t-partite which isn't neither a star nor complete bipartite.

(ii), (iii) are obvious by (i).

(iv). By (i), $\{x_i\}_{i=1}^{\frac{\mathcal{O}(NTG)}{2}+1}$ is minimal and it's offensive alliance. Thus it's $\frac{\mathcal{O}(NTG)}{2} + 1$-offensive alliance.

$(v), (vi)$ are obvious by (iv).

Thus the number is $\frac{\mathcal{O}(NTG)}{2} + 1$ and the neutrosophic number is $\min \Sigma_{v \in \{v_1, v_2, \cdots, v_t\}_{t > \frac{\mathcal{O}(NTG)}{2}} \subseteq V} \sigma(v)$, in the setting of all alliances. □

Proposition 5.3. *Let $\mathcal{G}$ be a family of $NTGs : (V, E, \sigma, \mu)$ neutrosophic graphs which are from one-type class which the result is obtained for individual. Then results also hold for family $\mathcal{G}$ of these specific classes of neutrosophic graphs.*

Proof. There are neither conditions nor restrictions on the vertices. Thus the result on individual is extended to the result on family. □

6 Applications in Time Table and Scheduling

Designing the programs to achieve some goals is general approach to apply on some issues to function properly. Separation has key role in the context of this style. Separating the duration of work which are consecutive, is the matter and it has important to avoid mixing up.

Step 1. (Definition) Time table is an approach to get some attributes to do the work fast and proper. The style of scheduling implies special attention to the tasks which are consecutive.

Step 2. (Issue) Scheduling of program has faced with difficulties to differ amid consecutive section. Beyond that, sometimes sections are not the same.

Step 3. (Model) The situation is designed as a model. The model uses data to assign every section and to assign to relation amid section, three numbers belong unit interval to state indeterminacy, possibilities and determinacy. There's one restriction in that, the numbers amid two sections are at least the number of the relation amid them. Table (1), clarifies about the assigned numbers to these situation.

Table 1. Scheduling concerns its Subjects and its Connections as a neutrosophic graphs and its alliances in a Model.

Sections of NTG	n_1	$n_2 \cdots$	n_9
Values	$(0.99, 0.98, 0.55)$	$(0.74, 0.64, 0.46) \cdots$	$(0.99, 0.98, 0.55)$
Connections of NTG	E_1	E_2	E_3
Values	$(0.01, 0.01, 0.01)$	$(0.01, 0.01, 0.01)$	$(0.01, 0.01, 0.01)$

Step 4. (Solution) The neutrosophic graphs and its alliances as model, propose to use different types of sets. If the configuration makes complete, the set is different. Also, it holds for other types such that star, wheel, path, and cycle.

7 Open Problems

14 notions concerning alliances are defined in neutrosophic graphs. Thus,

Question 7.1. *Is it possible to use other types neighborhood arising from different types of edges to define new alliances?*

Question 7.2. *Are existed some connections amid different types of alliances in neutrosophic graphs?*

Question 7.3. *Is it possible to construct some classes of which have "nice" behavior?*

Question 7.4. *Which mathematical notions do make an independent study to apply these types in neutrosophic graphs?*

 doi:10.20944/preprints202201.0239.v1

Problem 7.5. *Which parameters are related to this parameter?*

Problem 7.6. *Which approaches do work to construct applications to create independent study?*

Problem 7.7. *Which approaches do work to construct definitions which use all definitions and the relations amid them instead of separate definitions to create independent study?*

8 Conclusion and Closing Remarks

This study uses mixed combinations of different types of definitions concerning alliances to study neutrosophic graphs. The connections of vertices which are clarified by general edges differ them from each other and put them in different categories to represent a set which is called. Further studies could be about changes in the settings to compare this notion amid different settings of neutrosophic graphs theory. One way is finding some relations amid all definitions of notions to make sensible definitions. In Table (2), some limitations and advantages of this study are pointed out.

Table 2. A Brief Overview about Advantages and Limitations of this study

Advantages	Limitations
1. Defining Alliances	1. Specific Results
2. Defining Strong Alliances	
3. Defining Connected Alliances	2. Specific Connections
4. Applying on Individuals	
5. Applying on Family	3. Connections of Results

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