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A Relationship between the Subatomic Particles' Mass and the Square of the Magnetic Flux Quantum Provides a Novel Significance of the Mass and the Wave Function

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Mass and the Square of the Magnetic Flux Quantum Provides a Novel Significance of the Mass and the Wave Function

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Abstract: The motivation for investigating the issues presented in this article stemmed from a discovery that resulted from using the magnetic flux quantum that combine the Planck's constant and the Elementary charge, and combining them in the mathematical expression of Coulomb's law. It led to a new mathematical relationship between the combined expressions. The new relationship yields a novel theoretical finding that indicates that the mass of the electron is associated with the magnitude of the square of the magnetic flux quantum which makes up the particle. It reveils also a novel significance of the vacuum permittivity constant (in SI units), which also relies on a result from a different prespective demonstrated in this article through an analogy to the kinetic theory of gases. It shows that the vacuum permittivity constant is associated with the Bohr radius and it is about sixth of it. Using the concept of the nucleus motion around the center of mass shared with the electron in the Hydrogen atom, along with defineing the orbital angular momentum of the proton at the trajectory around the center of mass, resulted in finding the velocity of the proton at this trajectory. This velocity divided by the speed of light in vaccum, yields a new physical constant which fulfill a similar role like the fine structure constant in the atom domain. The new constant and the fine structure constant are combined together along with the square of the magnetic flux quantum in most of the equations that yield results for the proton and neutron masses and their radii. Another aspect that is presented in a briefly way demonstrates the connection between the square of the magnetic flux quantum through the Bohr radius that provides a novel significance of the wave function in the atom. The theoretical results are in full accordance with experimental results published by NIST CODATA 2018 that I've used, validating the results.

Keywords: A novel significance of the mass; A novel significance of the wave function

Introduction.

The theory presented in this article deal with the magnitudes of Universal Constants in physics. The theory presents the use of the combination of the magnetic flux quantum constant with using Universal Constants from different fields (Eelectromagnetic, Gravitation and Nuclear) and finding new relations through them that have not yet been reflected in the knowledge available today. The formalism developed in this article introduces a relationship between the masses of the electron, proton, and neutron to the square of the magnetic flux quantum. This relationship is unknown to science today. The method used today to calculate the proton and neutron masses theoretically is based (there are some other methods as well) on the quantum Chromodynamics theory of binding energy, which combines the kinetic energy of the quarks and the energy of the gluons within these particles. The theory presented here calculates the electron, proton, and neutron masses in a new straightforward nearly identical formulas, whose their main component is the square of the magnetic flux quantum; the only difference between them is their Compton wavelength component responsible for their different masses. Another formalism yields the proton's and the neutron's radii in a newly approach. It involves the using of Natural Units constants such as Stoney and Planck's masses and the universal Gravitational Constant, which yield a new constant unfamiliar to science, in

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which the strong coupling constant in QCD is seemingly its derivative. With using this new constant, we obtain the proton and the neutron radii (this theory suggest that the proton charge radius is only a part of the actual proton radius), and through them we calculate their Compton wavelength constants which are used to calculate their masses later. The theory also presents a novel way to describe the Planck's Mass and Length, and the Gravitation Constant thru them.

Method of analysis.

Using parameters like the universal constants from different fields (electromagnetic, gravitation and nuclear) and combine them in equations in order to find a new relations between them that have not yet been reflected in the knowledge available today. For instance analyzing an established equation, finding connecting factors between its parts that can define new equations by using them. The subjects in the article are presented in such order that each new topic is based on the development of its predecessor that explains where it stems from. The article presents methods of analyzing traditional physics concepts to extract embedded information. In every step the findings are checked and matched with the highly important tool such as the data provided from experiments by NIST CODATA 2018 publication. The technique uses a simple mathematical means that is practical to obtain results.

Results and discussion.

1. Relationships between the electron mass and the square of the magnetic flux quantum and the Bohr radius and the vacuum permittivity.

The magnetic flux quantum Φ_0 [1] is defined according to the following

$$e = h/2\Phi_0 \rightarrow e^2 = h^2/4\Phi_0^2 \quad \textbf{(1)}$$

Where e is the elementary charge of an electron and h is Planck's constant. The electrostatic force acting on the electron at the Bohr level is

$$e^2/4\pi a_0^2\varepsilon_0 = m_e v_e^2/a_0$$

$$e^2 = m_e v_e^2 4\pi a_0 \varepsilon_0 \quad \textbf{(2)}$$

Where a_0 is the Bohr radius, ε_0 is the vacuum permittivity, m_e is the electron mass, and v_e is the electron velocity at the Bohr radius. The electron's angular momentum at the Bohr radius

$$\frac{h}{2\pi} = m_e v_e a_0 \;\rightarrow\; h = 2\pi m_e v_e a_0 \quad \textbf{(3)}$$

By substituting Eq. (3) in Eq. (1) (the squared term) $e^2 = h^2/4\Phi_0^2$, we obtain

$$e^2 = m_e v_e 2\pi a_0 \times m_e v_e 2\pi a_0/4\Phi_0^2 \quad (4)$$

We can then rewrite Eq. (4) as follows:

$$e^2 = m_e v_e^2 4\pi a_0 \times (\pi a_0 m_e/4\Phi_0^2) \quad \textbf{(5)}$$

By multiplying both the numerator and denominator of Eq. (5) by ε_0, we have

$$e^2 = m_e v_e^2 4\pi a_0 \varepsilon_0 (\pi a_0 m_e/4\varepsilon_0 \Phi_0^2) \quad \textbf{(6)}$$

According to Eq. (2), the expression in parentheses in Eq. (6) should equal unity; thus, we obtain

$$(\pi a_0 m_e/4\varepsilon_0 \Phi_0^2) = 1 \quad \textbf{(7)}$$

We can then substitute the following values published by the National Institute of Standards and Technology Committee on Data for Science and Technology in 2018 (NIST CODATA 2018) [2] in Eq. (7) (SI units):

$a_0 = 5.291772109 \times 10^{-11}$ m

$\varepsilon_0 = 8.8541878128 \times 10^{-12}$ A^2 s^4 kg^{-1}m^{-3}

$\Phi_0 = 2.067833848 \times 10^{-15}$ kg m^2 s^{-2} A^{-1}

$\Phi_0^2 = 4.275936823 \times 10^{-30}$ kg^2 m^4 s^{-4} A^{-2}

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$m_e = 9.1093837015 \times 10^{-31}\,\text{kg}$

These substitutions give us the following relationship:

$$\frac{\pi}{4} \times \frac{5.291\,772\,109 \times 10^{-11}\ \text{m}}{8.854\,187\,812\,8 \times 10^{-12}\ \text{A}^2\text{s}^4\text{kg}^{-1}\text{m}^{-3}} \times \frac{9.109\,383\,701\,5 \times 10^{-31}\ \text{kg}}{4.275\,936\,823 \times 10^{-30}\ \text{kg}^2\text{m}^4\text{s}^{-4}\text{A}^{-2}} = 1 \quad \textbf{(8)}$$

We can consider the multiplication of the vacuum permittivity and the square of the magnetic flux in the denominator of Eq. (8) and multiply their units:

$$(\text{A}^2\text{s}^4\text{kg}^{-1}\text{m}^{-3})(\text{kg}^2\text{m}^4\text{s}^{-4}\text{A}^{-2}) = \text{kg m} \quad \textbf{(9)}$$

We consider this reduction further in the next section.

2. Analysis of Equation (8).

a. After reducing the units in the denominator of Eq. (8), we obtained units corresponding to those in the numerator in Eq. (9). The result in Eq. (9) implies two options: Either Φ_0^2 has units of mass $[\text{kg}]$ or units of length $[\text{m}]$ or, vice versa, ε_0 has units of mass $[\text{kg}]$ or has units of length $[\text{m}]$.

b. The orders of magnitude in Eq. (8) indicate the scale of magnitude of these parameters**.** The Bohr radius scale is 10^{-11}m, and the vacuum permittivity scale is 10^{-12}F m^{-1}. The scale of the electron mass is 10^{-31}kg, and the scale of the square of the magnetic flux quantum is $10^{-30}[\text{Wb}]^2$. From this consideration and the two options presented in (a), it is clearly nonsensical for ε_0 to have units of mass or Φ_0^2 to have units of length. No particle mass on the scale of 10^{-12}kg or a length on the scale of 10^{-30}m exist in the atom.

c. From these considerations, we can conclude with a high degree of certainty that the Bohr radius and the vacuum permittivity are the same entity:

$$\varepsilon_0 \equiv a_0 \quad \textbf{(10)}$$

The electron mass and square of magnetic flux quantum are the same entity:

$$m_e \equiv \Phi_0^2 \quad \textbf{(11)}$$

d. The last conclusion is not as strange as it may seem, as the square of the magnetic flux quantum Φ_0^2 appears in the context of magnetic energy in a current loop and according to Albert Einstein's special theory of relativity, energy is equivalent to mass.

* See also section #8 further on for the meaning of the vacuum permittivity ε_0!

3. Using the conclusions from Sections 2(c) and 2(d) for the electron mass and vacuum permittivity.

By rearranging Eq. (7) for the electron mass calculation and substituting the values of $\varepsilon_0, a_0, \Phi_0^2$ from NIST CODATA 2018, we obtain

$$m_e = \frac{4}{\pi} \times \frac{\varepsilon_0}{a_0} \times \Phi_0^2 = \frac{4}{\pi} \times \frac{8.8541878128 \times 10^{-12}\text{m}}{5.291772109 \times 10^{-11}\text{m}} \times 4.275936823 \times 10^{-30}\text{kg} = 9.109383697 \times 10^{-31}\text{kg} \quad \textbf{(12)}$$

This value compares well with the NIST value of $m_e = 9.1093837015 \times 10^{-31}\text{kg}$

We can obtain another expression for m_e from Eq. (3) with the electron Compton wavelength $\lambda_e\ (v_e = c\alpha)$

$$\lambda_e = h / m_e c = \alpha 2\pi a_0 \quad \textbf{(13)}$$

Where c is the speed of light in vacuum and α is the fine structure constant.

Here, we can rearrange Eq. (12) as $a_0 = 4\varepsilon_0\Phi_0^2\pi^{-1}m_e^{-1}$ and substitute this term in Eq. (13) for another expression of m_e:

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$$m_e = \left(\frac{8\ \alpha\ \varepsilon_0}{\lambda_e} \right) \Phi_0^2 \tag{14}$$

To calculate the value of the vacuum permittivity ε_0, we rewrite Eq. (7) and substitute the values of Φ_0^2, m_e, a_0 from NIST CODATA 2018, which yields

$$\varepsilon_0 = \frac{\pi}{4} \times \frac{m_e}{\Phi_0^2} \times a_0 = \frac{\pi}{4} \times \frac{9.1093837015 \times 10^{-31}\,\mathrm{kg}}{4.275936823 \times 10^{-30}\,\mathrm{kg}} \times 5.291772109 \times 10^{-11}\,\mathrm{m} \tag{15}$$

$$= 8.854187816 \times 10^{-12}\,\mathrm{m}$$

This value compares well with NIST value of $\varepsilon_0 = 8.8541878128 \times 10^{-12}\ \mathrm{A^2\,s^4\,kg^{-1}m^{-3}}$

4. Using the conclusions from Sections 2(c) and 2(d) for elementary charge e and the Planck constant h.

In this subsection, we derive a new expression for the elementary charge e with $m_e = 4\varepsilon_0 \Phi_0^2 \pi^{-1} a_0^{-1}$ from Eq. (12) and with the fine structure constant α and the speed of light in vacuum c.

Substitute m_e in Eq. (2) with $v_e^2 = c^2 \alpha^2$ to obtain an expression of e:

$$e^2 = m_e c^2 \alpha^2 4\pi\varepsilon_0 a_0 = (4\varepsilon_0 \Phi_0^2 \pi^{-1} a_0^{-1})(c^2 \alpha^2 4\pi\varepsilon_0 a_0)$$

$$e = 4c\alpha\varepsilon_0 \Phi_0 \tag{16}$$

In Eq. (16), we substitute the values of ε_0, Φ_0, the speed of light in vacuum c, and the fine structure constant α from NIST CODATA 2018:

$c = 2.99792458 \times 10^8\ \mathrm{m\,s^{-1}}$ and $\alpha = 7.2973525684 \times 10^{-3}$, this yields

$$e = 1.602176633 \times 10^{-19}\,\mathrm{C} \tag{17}$$

This value compares well with the NIST CODATA value of $e = 1.602176634 \times 10^{-19}\,\mathrm{C}$.

We can substitute $c^2 \varepsilon_0 = \mu_0^{-1}$ (where μ_0 is the vacuum permeability) in Eq.(16) to obtain another expression for the elementary charge e:

$$e = 4\alpha\Phi_0 (\varepsilon_0 / \mu_0)^{1/2} \tag{18}$$

Substituting the expression of e from Eq. (16) in Eq. (1) for a new h expression

$$h = 8c\alpha\varepsilon_0 \Phi_0^2 \tag{19}$$

Applying the values of $\varepsilon_0, \Phi_0, c, \alpha$ from NIST CODATA 2018 in Eq. (19), gives

$$h = 6.626070146 \times 10^{-34}\,\mathrm{kg\,m^2 s^{-1}}$$

This value compares well with the NIST value $h = 6.626070146 \times 10^{-34}\,\mathrm{kg\,m^2 s^{-1}}$

Moreover, we obtain another expression by utilizing $c^2 \varepsilon_0 = \mu_0^{-1}$:

$$h = 8\alpha(\varepsilon_0 / \mu_0)^{1/2} \Phi_0^2 \tag{20}$$

5. Radius of the proton (hydrogen nucleus).

For further development, it is necessary to find the proton's radius. According to the Physicist Arthur Beiser on the nucleus motion in his book 'Concepts of Modern Physics' [3], the nucleus of a hydrogen atom (proton) revolves around the center of mass shared with the electron. The rotation of both the electron and nucleus arises from considerations of momentum conservation in an isolated system and is taken into account by a computational correction called the reduced mass of the electron. The notion of reduced mass played an important part in the discovery of the deuterium and also corrects a small but definite discrepancy between the predicted wavelengths of the spectral lines of hydrogen and the measured ones. The center of mass is very close to the axis of the nucleus because of its larger mass; thus, we can assume that the trajectory depicted by the nucleus while revolving around the center of mass lies at a distance almost equivalent to the nucleus radius. We will denote this radius as the proton radius, validated in the final result. As a side note, this radius is not equivalent to the proton's charge radius; however, there is a connection between these two parameters, which will be clarified in Section 5b. To find the proton's radius, we will use nown formulas generated for the natural units of the Stoney [4] and Planck [5] scales.

We will start with the Stoney scale, from which we move to the Planck scale.

The Stoney length l_s in natural units is

$$l_s = \sqrt{\frac{e^2 G}{4\pi\varepsilon_0 c^4}} \tag{21}$$

Here G is the gravitational constant. The Stoney mass m_s in the natural units is

$$m_s = \sqrt{\frac{e^2}{4\pi\varepsilon_0 G}} \tag{22}$$

Or rewrite Eq. (22) for the gravitational constant G:

$$G = \frac{e^2}{4\pi\varepsilon_0 m_s^2} \tag{23}$$

By substituting the relation $e^2 = 2hc\alpha\varepsilon_0$ from the relation introduced by the Physicist Arnold Sommerfeld $\alpha = e^2 / 2hc\varepsilon_0$ [6], in Eq. (23), we obtain

$$G = \frac{2hc\alpha\varepsilon_0}{4\pi\varepsilon_0 m_s^2} \tag{24}$$

The orbital angular momentum of the proton at the trajectory around the center of mass should be expressed by the reduced Planck constant. The proton's velocity at this trajectory is denoted here as v_p. An initial estimation of the velocity v_p yields approximately one fifth of the speed of light in vacuum. Therefore, It is necessary to add a relativistic element $(1 - v_p^2/c^2)^{1/2}$ with $v_p = c\beta$.

$$h = m_p c\beta 2\pi r_p \sqrt{1-\beta^2} \tag{25}$$

Where m_p is the proton mass, β is the ratio of v_p to c, and r_p is the proton radius. By substituting the expression of h from Eq. (25) in Eq. (24) and reducing the expression

$$G = \frac{m_p c^2 \alpha\beta\, r_p \sqrt{1-\beta^2}}{m_s^2} \tag{26}$$

The β is similar to the fine structure constant α also known as the electromagnetic coupling constant, and it appears in the electron's velocity expression at the Bohr radius as $v_e = c\alpha$. We can divide Eq. (21) by Eq. (22) ($4\pi\varepsilon_0$ is reduced, the elementary charge e is partially reduced):

$$\frac{l_s}{m_s} = \sqrt{\frac{e^2}{e^2} \times \frac{G^2}{c^4}} = \frac{e}{e} \times \frac{G}{c^2} \tag{27}$$

Then rearrange Eq. (27) to obtain an expression for G:

$$G = \frac{e}{e} \times \frac{l_s}{m_s} c^2 \tag{28}$$

By setting the expressions in Eq. (28) and Eq. (26) equal to each other, here

$$\frac{e}{e} \times \frac{l_s}{m_s} c^2 = \frac{m_p\, c^2 \alpha\beta\, r_p \sqrt{1-\beta^2}}{m_s^2} \tag{29}$$

Dividing both sides of Eq. (29) by (e/e), multiply by m_s^2, reduce and rearrange, gives

$$m_s \times l_s \times c^2 = \left[\left(\frac{1}{e} m_p \beta\right)\left(\frac{\alpha\, r_p \sqrt{1-\beta^2}}{\frac{1}{e}}\right)\right] \times c^2 \tag{30}$$

Eq. (30) presents a similarity between the right and left flanks (mass component and length component). The expression is split into two parts on the right-hand side of the equation because it contains the solutions corresponding to actual experimental results in the final analysis. The following expressions from Eq. (30), are proposed for the Stoney units.

New expression of Stoney mass m_s: $m_s = \frac{1}{e} m_p \beta$

New expression of Stoney length l_s: $l_s = \left(\frac{\alpha\, r_p \sqrt{1-\beta^2}}{\frac{1}{e}}\right)$

Note that the $\left(\frac{1}{e}\right)$ expression in Eq. (30), represents a dimensionless number, for instance, the number of charged particles in one Coulomb $[\mathrm{C}]$, here

$$\frac{1}{e} = \frac{1\cancel{C}}{1.602176634 \times 10^{-19}\cancel{C}} = 6.2415090744 \times 10^{18} \tag{31}$$

This number, as a multiplier, creates a quantity of charged particles (in our case, the number of protons contained within the Stoney mass, which corresponds to a quintillion protons) or, as a divisor, creates the smallest length (in our case, a contracted radius of the proton within the Stoney mass under internal attraction forces, which corresponds to a quintillionth of the proton radius that represents the Stoney length). This expression is displayed in the following equations as in Eq. (31) to indicate that this value is dimensionless. The gravitational constant G with the propsed Stoney units in Eq. (28), is

$$G = \frac{\left(\frac{\alpha\, r_p \sqrt{1-\beta^2}}{\frac{1}{e}}\right)}{\left(\frac{1}{e} m_p \beta\right)} \times c^2 \tag{32}$$

We can then set Eq. (23) and Eq. (32) equal to each other and substitute the square of the Stoney mass $m_s^2 = \left(1/e\, m_p \beta\right)^2$ in the denominator of Eq. (23)

$$\frac{e^2}{4\pi\varepsilon_0 \left(\frac{1}{e} m_p \beta\right)^2} = \frac{\alpha\, r_p \sqrt{1-\beta^2}}{\frac{1}{e}\left(\frac{1}{e} m_p \beta\right)} \times c^2 \tag{33}$$

Multiplying both sides of Eq. (33) by $4\pi\varepsilon_0 m_p \beta$, reducing, and rearranging, gives

$$e^2 = 2\left[m_p c \beta 2\pi r_p \sqrt{1-\beta^2}\right] c\alpha\varepsilon_0 \tag{34}$$

The expression of Eq. (34) shows the equivalence of $e^2 = 2hc\alpha\varepsilon_0$, where the right-hand side (in brackets) contains the expression of the Planck constant h with the proton parameters introduced in Eq. (25). This result confirms the choice of the proposed solutions for the Stoney units of mass and length from Eq. (30). Although this option was based on a logical consideration, there are additional combinations that could be chosen that yield incorrect results. We multiply the numerator and denominator of Eq. (32) by $\alpha^{-1/2}$ to obtain the gravitational constant at Planck's scale

$$G=\frac{\left(\dfrac{\alpha^{1/2}\, r_p\sqrt{1-\beta^2}}{\dfrac{1}{e}}\right)}{\dfrac{1}{\alpha^{1/2}}\left(\dfrac{1}{e}m_p\beta\right)}\times c^2 \tag{35}$$

Note: The difference between the Stoney and Planck units arises from the need to multiply Planck units by the square root of the fine structure constant $\alpha^{1/2}$, it gives

New expression of Planck mass m_{pl}: $m_{\mathrm{pl}}=\dfrac{1}{\alpha^{1/2}}\left(\dfrac{1}{e}m_p\beta\right)$

New expression of Planck length l_{pl}: $l_{\mathrm{pl}}=\left(\dfrac{\alpha^{1/2}\, r_p\sqrt{1-\beta^2}}{\dfrac{1}{e}}\right)$

By using the Planck mass in natural units and the new expression of Planck mass, we can derive the expression and value of β . The Planck mass defined by natural units, is

$$m_{\mathrm{pl}}=\sqrt{\frac{hc}{2\pi G}} \tag{36}$$

The new expression for the Planck mass from Eq. (35) is

$$m_{\mathrm{pl}}=\frac{1}{\alpha^{1/2}}\left(\frac{1}{e}m_p\beta\right) \tag{37}$$

Setting Eq. (36) and Eq. (37) as equal:

$$\sqrt{\frac{hc}{2\pi G}}=\frac{1}{\alpha^{1/2}}\left(\frac{1}{e}m_p\beta\right) \tag{38}$$

Then rearrange Eq. (38) to obtain an expression for β:

$$\beta=\sqrt{\frac{hc\alpha}{2\pi G}}\times\frac{1}{1/e\times m_p} \tag{39}$$

In Eq. (39), we substitute the values of h, c, α, m_p and the following values from NIST CODATA 2018: $1/e=6.2415090744\times10^{18}$ and $G=6.6743\times10^{-11}\mathrm{m}^3\,\mathrm{kg}^{-1}\mathrm{s}^{-2}$

We obtain $\beta=0.178090537$

As a side note, the relationship of β to nuclear research is through the strong coupling constant in QCD; $\alpha_s=2/3\,\beta\rightarrow\alpha_s=2/3\times0.178090537=0.118727$.

This value compares well with the value obtained experimentally [7], [8], $\alpha_s\approx0.1187$

Using the Planck length from natural units and the new expression for the Planck length, we can derive the expression and value of the proton radius r_p .

$$l_{\mathrm{pl}}=\sqrt{\frac{hG}{2\pi c^3}} \tag{40}$$

The new expression for the Planck length from Eq. (35) is

$$l_{\mathrm{pl}}=\left(\frac{\alpha^{1/2}\, r_p\sqrt{1-\beta^2}}{\dfrac{1}{e}}\right) \tag{41}$$

By setting the expressions in Eq. (40) and Eq. (41) as equal, we obtain

$$\sqrt{\frac{hG}{2\pi c^3}} = \left(\frac{\alpha^{1/2} r_p \sqrt{1-\beta^2}}{\frac{1}{e}} \right) \tag{42}$$

We then rearrange Eq. (42) for the proton radius r_p:

$$r_p = \sqrt{\frac{hG}{2\pi c^3 \alpha}} \times \frac{1}{e} \times \frac{1}{\sqrt{1-\beta^2}} \tag{43}$$

Substituting $\beta = 0.178090537$ and the values of h, G, c, α and $(1/e)$ from NIST CODATA 2018 in Eq. (43), It yields

$$r_p = 1.200094665 \times 10^{-15}\,\text{m}$$

This value compares well with the value obtained experimentally [9], explained next:

a. The proton radius obtained in Eq. (43) complies with the experimental formulation that assumes a spherical nucleus with radius expressed by the Fermi equation for the nuclear radius R_n: $R_\text{n} = R_0 \text{A}^{1/3}$, where R_0 is from experimental results $R_0 = 1.2 \times 10^{-15}\,\text{m}$ and A is the atomic number. For Hydrogen A=1 and $R = 1.2 \times 10^{-15}\,\text{m}$.

* The example for the proton charge radius in the following paragraph is presented without overall proof, which requires a separate article.

b. The proton charge radius r_{pcr} represents the maximum distance from the proton axis that the electron or muon reaches in their penetration to the proton due to interactions with Up quarks. This radius is $r_{pcr} = 4\beta \times r_p (1-\beta^2)^{1/2}$

Substituting the following in the expression for r_{pcr}; with $r_p = 1.200094665 \times 10^{-15}\,\text{m}$ and $\beta = 0.178090537$, we obtain $r_{pcr} = 8.4123564 \times 10^{-16}\,\text{m}$.

The proton's Compton wavelength from Eq. (25) is

$$\lambda_p = \frac{h}{m_p c} = 2\pi\beta r_p \sqrt{1-\beta^2} \tag{44}$$

Substituting the values of β and r_p in Eq. (44), it yields

$$\lambda_p = 2\pi \times 0.178090537 \times 1.200094665 \times 10^{-15}\,\text{m} \times 0.984014106$$
$$= 1.3214098539 \times 10^{-15}\,\text{m}$$

This values compares well with the NIST value of $\lambda_p = 1.3214098539 \times 10^{-15}\,\text{m}$

The last result shows that β combined with the proton radius r_p obtained from Eq. (43) and used in Eq. (44) is entirely consistent with the value of the proton's Compton wavelength λ_p from NIST CODATA 2018, confirming the validity of our approach. To obtain the gravitational constant G, we utilize Eq. (32) and substitute the m_p, c, α from NIST CODATA 2018 and also $1/e = 6.2415090744 \times 10^{18}$, β and r_p:

$$G = \frac{\left(\frac{r_p \sqrt{1-\beta^2}}{6.2415090744 \times 10^{18}} \right) \alpha}{\left(6.2415090744 \times 10^{18} \times m_p \right) \beta} \times c^2 \tag{45}$$

It yields $G = 6.6743 \times 10^{-11}\,\text{m}^3\text{kg}^{-1}\text{s}^{-2}$

Which compares well with the NIST CODATA value of $G = 6.6743 \times 10^{-11}\,\text{m}^3\text{kg}^{-1}\text{s}^{-2}$

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6. Radius of the neutron.

The ratio between the proton mass and neutron mass is the same as the ratio between the neutron Compton wavelength and proton Compton wavelength. The values of $m_p, m_n, \lambda_n, \lambda_p$ are substituted from the NIST CODATA 2018 in the following ratios:

$$\frac{m_p}{m_n} = \frac{1.67262192369 \times 10^{-27}\,\text{Kg}}{1.67492749804 \times 10^{-27}\,\text{Kg}} = 0.998623478$$

$$\frac{\lambda_n}{\lambda_p} = \frac{1.31959090581 \times 10^{-15}\,\text{m}}{1.32140985539 \times 10^{-15}\,\text{m}} = 0.998623478$$

This result indicates that the ratio is also appropriate for the ratio between the neutron and proton radii.

$$\frac{m_p}{m_n} = \frac{r_n}{r_p} \rightarrow r_n = \frac{m_p}{m_n} \times r_p \qquad \textbf{(46)}$$

By substituting the values of m_p, m_n from NIST CODATA 2018 and the radius r_p from Eq. 43 in Eq. (46) for the neutron radius, we obtain

$$r_n = \frac{1.67262192369 \times 10^{-27}\,\text{kg}}{1.67492749804 \times 10^{-27}\,\text{kg}} \times 1.200094667 \times 10^{-15}\,\text{m} = 1.19844271 \times 10^{-15}\,\text{m}$$

The proton and neutron are almost identical in size, and the β constant is found to be related to both radii. Consequently, $v_n = v_p = c\beta$. It is validated by the neutron Compton wavelength λ_n with β and r_n, as follows

$$\lambda_n = \frac{h}{m_n c} = 2\pi\beta r_n \sqrt{1-\beta^2} \qquad \textbf{(47)}$$

Substituting the values of β and r_n in Eq. (47) for λ_n, we obtain

$$\lambda_n = 2\pi \times 0.178090537 \times 1.19844271 \times 10^{-15}\,\text{m} \times 0.984014105 = 1.31959090471 \times 10^{-15}\,\text{m}$$

This value matches well with the NIST CODATA value of

$$\lambda_n = 1.31959090581 \times 10^{-15}\,\text{m}$$

The result obtained by combining β with the neutron radius r_n given by Eq. (46) and substituting in Eq. (47) is entirely consistent with the NIST CODATA 2018 value of the neutron Compton wavelength λ_n, confirming our approach

7. Additional expressions for the proton and neutron masses and radii.

We divide Eq. (3) (with $(v_e = c\alpha)$ by Eq. (25) as follows:

$$\frac{m_e \alpha 2\pi a_0}{m_p \beta 2\pi r_p \sqrt{1-\beta^2}} = 1 \qquad \textbf{(48)}$$

By rearranging Eq. (48) and solving for the proton mass, we obtain

$$m_p = \frac{m_e \alpha a_0}{\beta \times r_p \sqrt{1-\beta^2}} \qquad \textbf{(49)}$$

Substituting $m_e = 4\varepsilon_0 \Phi_0^2 \pi^{-1} a_0^{-1}$ from Eq. (12) in Eq. (49) yields the proton m_p:

$$m_p = \frac{4}{\pi} \times \frac{\alpha}{\beta} \times \frac{\varepsilon_0}{r_p \sqrt{1-\beta^2}} \times \Phi_0^2 \qquad \textbf{(50)}$$

Substituting $\lambda_e = \alpha 2\pi a_0$ and $\lambda_p = \beta 2\pi r_p \sqrt{1-\beta^2}$ in Eq. (48) yields

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$$\frac{m_e}{m_p} = \frac{\lambda_p}{\lambda_e} \quad \text{Or} \rightarrow m_p = \frac{m_e \lambda_e}{\lambda_p} \tag{51}$$

Substitute $\lambda_e = \alpha 2\pi a_0$ and m_e from Eq. (12) in Eq. (51) for m_p, it gives

$$m_p = \left(\frac{8\ \alpha\ \varepsilon_0}{\lambda_p} \right) \Phi_0^2 \tag{52}$$

Substituting the values $\alpha, \varepsilon_0, \lambda_p, \Phi_0^2$ from NIST CODATA in Eq. (52) for m_p, it gives

$$m_p = \frac{8 \times 7.297352569 \times 10^{-3} \times 8.8541878128 \times 10^{-12}\,\mathrm{m}}{1.321409\,85539 \times 10^{-15}\,\mathrm{m}} \times 4.27593682 \times 10^{-30}\,\mathrm{kg} \tag{53}$$
$$= 1.6726219217 \times 10^{-27}\,\mathrm{kg}$$

This value matches well with the NIST value of $m_p = 1.67262192369 \times 10^{-27}\,\mathrm{kg}$

Rearrange Eq. (47) to obtain the orbital angular momentum of the neutron:

$$\frac{h}{2\pi} = m_n c \beta r_n \sqrt{1-\beta^2} \text{ Or } h = m_n c \beta 2\pi r_n \sqrt{1-\beta^2} \tag{54}$$

We then divide Eq. (3) by Eq. (54), as follows:

$$\frac{m_e c \alpha 2\pi a_0}{m_n c \beta\, 2\pi r_n \sqrt{1-\beta^2}} = 1 \tag{55}$$

Rearranging Eq. (55) for the neutron mass, gives

$$m_n = \frac{m_e \alpha a_0}{\beta\, r_n \sqrt{1-\beta^2}} \tag{56}$$

Substituting m_e from Eq. (12) in Eq. (56) yields an expression for m_n mass:

$$m_n = \frac{4}{\pi} \times \frac{\alpha}{\beta} \times \frac{\varepsilon_0}{r_n \sqrt{1-\beta^2}} \times \Phi_0^2 \tag{57}$$

Substituting $\lambda_e = \alpha 2\pi a_0$ from Eq. (13) and the neutron $\lambda_n = \beta 2\pi r_n \sqrt{1-\beta^2}$ from Eq. (47) in Eq. (55), gives

$$\frac{m_e}{m_n} = \frac{\lambda_n}{\lambda_e} \quad \text{Or} \rightarrow m_n = \frac{m_e \lambda_e}{\lambda_n} \tag{58}$$

We substitute $\lambda_e = \alpha 2\pi a_0$ and $m_e = 4\varepsilon_0 \Phi_0^2 \pi^{-1} a_0^{-1}$ from Eq. (12) in Eq. (58) for another expression of m_n, it gives

$$m_n = \left(\frac{8\ \alpha\ \varepsilon_0}{\lambda_n} \right) \Phi_0^2 \tag{59}$$

Now, substitue the values of $\alpha, \varepsilon_0, \lambda_n, \Phi_0^2$ from NIST CODATA 2018 in Eq. (59), it gives

$$m_n = \frac{8 \times 7.2973525693 \times 10^{-3} \times 8.8541878128 \times 10^{-12}\,\mathrm{m}}{1.319\,59090581 \times 10^{-15}\,\mathrm{m}} \times 4.275936823 \times 10^{-30}\,\mathrm{kg} \tag{60}$$
$$= 1.6749274973 \times 10^{-27}\,\mathrm{kg}$$

This value compares well with the NIST value $m_n = 1.67492749804 \times 10^{-27}\,\mathrm{kg}$.

The ratio of proton mass to electron mass is obtained from Eq. (49). After rearranging and substituting the NIST CODATA 2018 values for α, a_0 and β and r_p, it gives

$$m_p / m_e = \alpha / \beta \times a_0 / r_p \sqrt{1-\beta^2} = 1836.152675 \tag{61}$$

This ratio can also be obtained from Eq. (51). This value matches well with the NIST ratio of

$$m_p/m_e = 1836.1526734$$

The ratio of neutron mass to electron mass is obtained from Eq. (56). After rearranging and substituting the NIST CODATA 2018 values for α, a_0 and β and r_n, it gives

$$m_n/m_e = \alpha/\beta \times a_0/r_n \sqrt{1-\beta^2} = 1838.683661 \quad \textbf{(62)}$$

This ratio can also be obtained from Eq. (58). This value matches well with the NIST ratio of

$$m_n/m_e = 1838.683661$$

8. The meaning of the permittivity of vacuum constant ε_0 from a different aspect.

The electron in the Hydrogen atom revolves around the center of mass shared with the proton as was described in section #5. Let's assume now that the virtual photons, responsible for the force acting between them, collide with the virtual shell that represents the supposed locations of the electron at the Bohr level (The absorption and emission process). What if it is possible to describe the virtual photons' rapid motion within the 'shell space' in terms of molecules in an ideal gas?

The centripetal force F_c that holds the electron at Bohr radius a_0 level [10] , is

$$F_c = \frac{m_e v_e^2}{a_0} \quad \textbf{(63)}$$

And the electric force F_e that holds the electron at Bohr radius a_0 level, is

$$F_e = \frac{e^2}{4\pi\varepsilon_0 a_0^2} \quad \textbf{(64)}$$

The condition for a dynamically stable at Bohr radius a_0 level, is

$$F_c = F_e$$

$$\frac{m_e v_e^2}{a_0} = \frac{e^2}{4\pi\varepsilon_0 a_0^2} \quad \textbf{(65)}$$

Multiplying both sides of Eq. (65) by $\left(\frac{a_0}{2}\right)$, it gives

$$\frac{m_e v_e^2}{2} = \frac{e^2}{8\pi\varepsilon_0 a_0} \quad \textbf{(66)}$$

We obtain at the left side of Eq. (66) the kinetic energy E_k of the electron at a_0 level, as follows

$$E_k = \frac{m_e v_e^2}{2} \quad \textbf{(67)}$$

Where m_e is the electron mass, and v_e is the electron velocity at a_0 level.

The potential energy of the electron that equals to the kinetic energy E_k at the right side of Eq. (66), is

$$E_p = \frac{e^2}{8\pi\varepsilon_0 a_0} \quad \textbf{(68)}$$

Where e is the elementary charge of the electron, and ε_0 is the vcuum permittivity constant.

The average kinetic energy E_k of the virtual photons exchanged between the electron and the proton that support the movement of the electron in its trajectory at the Bohr level (as an approximation of molecules in motion in Ideal gas), is

$$PV = \frac{2}{3} E_k \quad \textbf{(69)}$$

Where V is the volume contained within the spherical shell, and P is the pressure that the virtual photons impose on the spherical shell.
Substituting E_k from Eq. (67) in Eq. (69), as flollows

$$PV = \frac{2}{3}\left[\frac{1}{2}m_e v_e^2\right] \quad \textbf{(70)}$$

The pressure P is defined as the force acting on a given surface. If the spherical shell surface is $\mathrm{A} = 4\pi a_0^2$, and a_0 is the Bohr radius, the pressure is

$$P = \frac{F}{4\pi a_0^2} \quad \textbf{(71)}$$

Where F is the total force imposed by the virtual photons on the inner surface of the spherical shell. Substituting Eq. (71) into Eq. (70) and rearrange, gives

$$FV = \frac{2}{3}\left(\frac{1}{2}m_e v_e^2\right)4\pi a_0^2 \rightarrow FV = 2\left[\frac{m_e v_e^2 4\pi a_0^2}{6}\right] \quad \textbf{(72)}$$

The same mathematical development can be applied for the potential energy, as

$$PV = \frac{2}{3}E_p \quad \textbf{(73)}$$

Substituting Eq. (68) (from the equality of $E_k = E_p$ at Eq. (66)) and Eq. (71) in Eq. (73) and rearrange , it gives

$$FV = \frac{2}{3}\left(\frac{e^2}{8\pi\varepsilon_0 a_0}\right)4\pi a_e^2 \rightarrow FV = e^2 \times 2\left(\frac{a_0}{6\varepsilon_0}\right) \quad \textbf{(74)}$$

Setting Eq. (74) as equal to Eq. (72) with reducing the integer 2 from both sides, gives

$$e^2\left(\frac{a_e}{6\varepsilon_0}\right) = \frac{m_e v_e^2 4\pi a_0^2}{6} \quad \textbf{(75)}$$

Set the expression in parentheses at the left side of Eq. (75) as equal to unity

$$\left(\frac{a_e}{6\varepsilon_0}\right) = 1 \quad \text{or} \quad \frac{a_0}{6} = \varepsilon_0 \quad \textbf{(76)}$$

And then substituting the last expression obtained in Eq. (76) for ε_0 in the right side of Eq. (75) ,we obtain the electrostatic force acting on the electron at the Bohr level from Eq. (2) at section #1, as follows

$$e^2 = m_e v_e^2 4\pi a_0 \varepsilon_0 \quad \textbf{(77)}$$

Note, that Eq. (76) shows the relation between the Bohr radius a_0 and the vcuum permittivity ε_0 constant that corresponds with the conclussion from the analysis of Equation (8) at section #2.

9. Developing the new expressions proposed for the Stoney mass and length from a different aspect.

Using the term of $e^2 = h^2/4\Phi_0^2$ from Eq. (1) and substitute the terms of $h = 2\pi m_e v_e a_0$ (with $v_e = c\alpha$) from Eq. (3) and $h = m_p c\beta 2\pi r_p\sqrt{1-\beta^2}$ from Eq. (25) in it, as follows

$$e^2 = \frac{m_e c\alpha 2\pi a_0 \times m_p c\beta 2\pi r_p\sqrt{1-\beta^2}}{4\Phi_0^2} \quad \textbf{(78)}$$

Substituting $a_0 = 4\varepsilon_0\Phi_0^2\pi^{-1}m_e^{-1}$ from Eq. (7) in Eq. (78), and after reducing and rearranging, it gives

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$$e^2 = \frac{m_e c \alpha 2\pi \left(4\varepsilon_0 \Phi_0^2 \pi^{-1} m_e^{-1}\right) \times m_p c \beta 2\pi r_p \sqrt{1-\beta^2}}{4\Phi_0^2} \rightarrow e^2 = m_p c^2 \alpha \beta 4\pi \varepsilon_0 r_p \sqrt{1-\beta^2} \quad \textbf{(79)}$$

Rearranging Eq. (79), as follows

$$\frac{e^2}{4\pi\varepsilon_0} = \left(m_p \beta\right) \alpha r_p \sqrt{1-\beta^2} c^2 \quad \textbf{(80)}$$

Dividing both sides of Eq. (80) by the square of the Stoney mass term $m_s^2 = \left(\frac{1}{e} m_p \beta\right)^2$ (proposed at Eq. (30)), after reducing and rearrenging, it gives

$$\frac{e^2}{4\pi\varepsilon_0 \left(\frac{1}{e} m_p \beta\right)^2} = \frac{\alpha r_p \sqrt{1-\beta^2}}{\frac{1}{e}\left(\frac{1}{e} m_p \beta\right)} \times c^2 = G \quad \textbf{(81)}$$

We obtain in Eq. (81) the equality of the Gravitational Constant G acording to Eq. (33).

10. The squared values of the magnetic flux quantum used in the wave function, yield solutions which depict the flow pattern of the magnetic flux surrounding electrons at a given energy level.

Using the Normalized Wave Function of the 'Hydrogen-like atom' equation [11]

$$R_{nl}(r) = \sqrt{\left(\frac{2Z}{na_0}\right)^3 \frac{(n-1-l)!}{2n[(n+l)!]^3}} \left(\frac{2Zr}{na_0}\right)^l e^{-zr/na_0} L_{n+l}^{2l+1}\left(\frac{2Zr}{na_0}\right) \quad \textbf{(82)}$$

Substituting $a_0 = 4\varepsilon_0 \Phi_0^2 \pi^{-1} m_e^{-1}$ from Eq. (7) in Eq. (82) and rearranging, it gives

$$R_{nl}(r) = \left(\frac{\pi}{2\varepsilon_0} \times \frac{Zm_e}{n\Phi_0^2}\right)^{3/2} \left(\frac{(n-1-l)!}{2n[(n+l)!]^3}\right)^{1/2}$$
$$\times \left(\frac{\pi r}{2\varepsilon_0} \times \frac{Zm_e}{n\Phi_0^2}\right)^l e^{-zr/na_0} L_{n+l}^{2l+1}\left(\frac{\pi r}{2\varepsilon_0} \times \frac{Zm_e}{n\Phi_0^2}\right) \quad \textbf{(83)}$$

Now writing the third expression in parentheses at Eq. (83), as follows

$$\left(\frac{\pi r}{2\varepsilon_0} \times \frac{Zm_e}{n\Phi_0^2}\right)^l \quad \textbf{(84)}$$

Using it as an example that provides the value of the derivative r at the electron wave function of the 'Hydrogen-like Atom' equation by rearranging it as

$$r = \frac{2\varepsilon_0}{\pi Z m_e} \times n\Phi_0^2 \quad \text{When} \quad l = 0 \rightarrow \left(\frac{\pi r}{2\varepsilon_0} \times \frac{Zm_e}{n\Phi_0^2}\right)^0 \quad \textbf{(85)}$$

Conclusions.

a. The conclusions from Eq. (85) for the electron wave function of 'Hydrogen like Atom' $R_{nl}(r)$ that it is mainly a function of the changing value of the square of the magnetic flux quantum $n\Phi_0^2$ and the number of electrons Zm_e at the relevant level (Please notice that the number of electrons in a neutral atom is equal to the number of protons in the nucleus of the atom presented as the atomic number Z).

b. The electron wave function in a 'Hydrogen-like atom' depends on the radius r of the atomic level expressed by several Bohr radii, and since the Bohr radius is a function of the square of the magnetic flux quantum , the electron wave function describes the magnitude of the flow pattern of the magnetic flux that surrounds the electrons at the energy levels in the atom.

c. The mass of the electron and other subatomic particles is related to the magnitude of the square of the magnetic flux quantum which makes up the particles. This relationship results in a novel expression of universal constants. The formalism developed in this paper yields the radii of the proton and the neutron from theory.

d. The Gravitational constant is identified based on Newton's law of universal gravitation. The new formula for the Gravitational constant developed in this paper contains elements from the atomic domain (proton's mass and radius) presented by the new proposed Stoney and Planck units, which represent the quantum reality environment; in this way they demonstrate the integration of the quantum and gravity levels.

e. I raise here (as an educated guess) a possibility from obserbing the equations obtained in this article, like for the elementary electric charge in Eq. (16) and Planck's constant in Eq (19), and especially in the equation for the mass of the electron at Eq. (14), that the product ($\alpha\varepsilon_0$) that appears in them and in some other equations, describes the size of the electron radius r_e:

$$r_e = \alpha\varepsilon_0 = 6.461213018\times10^{-14}\,\text{m}$$

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