

Review

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Review

# Big Data, Crowdsourcing, and Volunteered Geographic Information Challenge Core Conceptual Neighborhood Graph Assumptions

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## Abstract

The big data revolution transformed how we think of data analytics in many ways. Critical amongst them are the somewhat interconnected ideas of volunteered geographic information, crowdsourcing, and the big data property of variety. The robust literature concerning conceptual neighborhood graphs in two of these cases considers objects whose datatypes are held stable between the relations under consideration. This, however, is a limiting factor in these three application spaces due to the unknown form that data will take. This paper considers two avenues for the conceptual neighborhood graph to take as directions for future research: discretization conceptual neighborhood graphs (changing between corresponding vector and raster spaces) and cartographic generalization conceptual neighborhood graphs (changing the form of the objects in question). This paper provides insights as to what considerations should be considered when embarking upon this idea and demonstrates these concepts applied to prior conceptual neighborhood graphs.

**Keywords:** conceptual neighborhood graph; spatial reasoning; big data; volunteered geographic information; crowdsourcing

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## 1. Introduction

The big data revolution placed an emphasis on a core property of modern data: *variety* [1]. When referring to spatial data, this is particularly pertinent. Spatial data is contained in photographs, text descriptions, spatial databases, and raster files, amongst other forms. While a particular practitioner who collects and curates their own data might be able to keep the variety to a minimum, volunteered geographic information (VGI) [2] and crowdsourcing [3,4] create environments where the data that we collect is not fully within our control, potentially compromising various forms of analytical work. While VGI and crowdsourcing have become increasingly important for solving and/or meeting many societal problems (e.g., [5,6]), the lack of control in data architecture and data sources creates fundamental problems for data analysis [7]. In fact, these variety issues are regarded as the largest of all big data challenges. To tackle these challenges, it is often suggested that universalization of abstraction is the preferable approach [8].

Variety is particularly challenging with spatial data that is presented in some sort of mapped format. Variety can present several problems in lots of geospatial applications, but it is an increased challenge within qualitative spatial and temporal reasoning, which is often a foundational part of the spatial and temporal cognitive process, and thus in our decision making. Qualitative spatial and temporal reasoning bridge spatial and temporal analytical capabilities with aspects common to human cognition of spatial and temporal phenomena [9,10] and thus also human language [11–16].

Qualitative spatial and temporal reasoning also serves as a go-between for querying spatial data via database languages [17] or via drawing mechanisms [18–20]. By focusing on formal qualitative mechanisms such as the 9-intersection matrix [21] and the region connection calculus [22], qualitative spatial and temporal reasoning functions well in cases where cartographic simplification occurs [23], focusing not on the geometry, but rather the topology of the scenario [24,25]. This type of quality control for data is integrated through snapping analyses in modern GIS [26] and in augmented reality applications [27].

While topological mechanisms within these frameworks seem to account for many challenges in searching through spatial and temporal data that can solve some VGI and crowdsourcing paradigms, they are not immune to certain challenges. Four fundamental challenges arise from how spatial and temporal objects are encoded or formalized:

1. The most obvious one is the difference between vectorized objects (such as the shapefile architecture) and discretized objects (such as those originating from a raster). When using the same formalism on the same type of object but switching the embedding space from a vectorized to discretized embedding, we see fundamental shifts in relational diversity, and this can impact reasoning power and data consistency. For example, both RCC-8 and the 9-intersection identify eight planar polygonal relations between objects [21,22], however, in discretized embeddings, this can stay the same by using the standard digital topology endowed by the hyperraster [28,29] or it can be expanded greatly to 16/19 [30] or 62/70 [31] by considering bounding mechanisms such as the digital Jordan curve [32] or the frontier-as-boundary approach [31]. The same is true for temporal intervals: 13 temporal intervals exist in continuous time [33], while for discretized time, that total is 74 [34]. The standard rule is that discretized embeddings have more distinctive symbolic representations than continuous embeddings do, predominantly because of the boundary and its thickness in the discretized embedding space.
2. The next area of difficulty comes from the cartographic generalization of objects to a higher co-dimension with their embedding space, commonly called collapsing [35]. The opposite of this is called expanding. An easy example to conceive of the challenges borne by these types of changes is the modeling of highways or rivers as lines or cities or towns as points, communicating their fundamental purpose in the representation itself. While applications such as Google Maps seamlessly undergo geometric collapse (and expansion) as one dynamically zooms in or out [36,37], when static representations are involved, there is no opportunity to expand; similarly, to collapse would involve the potential of having to encode the static vector image on the fly. In vectorized embeddings, there are numerous relation sets that have been constructed to represent relations between types of objects [21,38–42]. In discretized embeddings, this conceptual space is not fully realized [30,31,34,43]. Furthermore, changing between co-dimensions of objects involves a reformulation of how the topological structure of the space is conceptually utilized, switching from a point-set topological architecture [44] to an algebraic topological architecture [38].
3. Another cartographic generalization challenge through the concept of simplification is the consideration of holes and separations within compound objects [45,46]. It is quite common for geographic objects with exclaves or enclaves to undergo hole-filling or separation-cleansing procedures [47,48]. Relation sets have been considered for such types of objects and their transformations in this regard [41,42,49,50].
4. Apart from discretization and generalization concepts, it is also a common practice for topological relations themselves to be simplified on a linguistic level. The most common example of this is between RCC-8 [22] and RCC-5 [51]. The difference between these relation spaces is that the boundary component is in effect neglected. We see similar legacies of this with the various versions of within from the Clementini operators in modern GIS [17].

The critical point in these cases is that the practice of encoding spatial and temporal information is not always consistent across data sources, and this has a foundational impact on spatial and temporal reasoning applications and the very theories that support them. This creates an important

challenge with respect to stitching together this type of information across disparate sources (the exact scenario presented by VGI, crowdsourcing and big data variety), but also creates challenges for interpreting semantic and/or relational similarity through the concept known as a conceptual neighborhood graph [52].

Conceptual neighborhood graphs model transitions between relations between objects under an allowable set of deformations. Traditionally, these deformations consider topologically homeomorphic outcomes (such as translation, rotation, isotropic scaling, and anisotropic scaling); however, there are applications where various conceptual neighborhood graphs were expanded to account for arbitrary sets of relations [53], multi-granular scenarios [54], and hole and separation changes [50]. Because cartographic generalization and linguistic generalization techniques abound in spatial representation, it is crucial to achieve a framework where all conceptual neighborhood graphs can stitch together into a singular structure. To do that requires an inventory of what was done in the past and what needs to be done going forward. Such an inventory is presented in the discussion section of the manuscript.

The remainder of this paper is structured as follows. Section 2 describes how formal qualitative topological reasoning is conducted, highlighting several base frameworks. Section 3 describes the conceptual neighborhood graph architecture and inventories what types of conceptual neighborhood graphs were constructed for which object relation circumstances. Section 4 proposes an architecture for organizing conceptual neighborhood graphs into a singular framework [55], proposing deformations such as simplification, aggregation, discretization, and linguistic simplification. Section 5 identifies examples of conceptual neighborhood frameworks that cross over various representational divides. Section 6 identifies the parts of the process that were completed and provides the challenge to the qualitative spatial and temporal reasoning community to close the loop and further provides a vision for what such completions would allow for in our data-rich modern world.

## 2. Qualitative Spatial Relation Formalisms and Sets

Qualitative topological reasoning is an attempt to turn formal analytical methods into geometrically agnostic terms, typically mirroring human language. Conceptually, in spatial and temporal terms, qualitative topological reasoning is frequently used to derive spatial or temporal prepositions [14,15,33].

There are several types of formal strategies that are leveraged to create topological relations on a qualitative level. They can broadly be organized into three categories: topological (leading to the 9-intersection family of formalisms), mereotopological (leading to the region connection calculus), and graph-theoretical (leading to partition mappings). In the following subsections we detail each of the three types.

### 2.1. Topological Formalisms

One class of formalisms to define object relations in spatial and temporal settings is based on concepts fundamental to point-set and algebraic topology. These formalisms rely on the classification of points within the embedding space into sets derived from three core concepts: interior, boundary, and exterior.

The 4- and 9-intersection models [21,44] are derived from these three concepts using a point-set topological construction. Definitions 1 through 5 detail the relevant background mathematics.

**Definition 1.** A *topology* on a nonempty set  $X$  is a collection of subsets of  $X$ , called *open sets*, such that:

- a) The empty set  $\emptyset$  and the set  $X$  are open sets;
- b) The union of an arbitrary collection of open sets is also an open set;
- c) The intersection of a finite number of open sets is open.

A subset  $A$  of  $X$  is a **closed set** if and only if its complement  $X \setminus A$  is an open set.

From a practical point of view, Definition 1 creates the foundation for topological reasoning. The definitions of open and closed sets allows us to identify three core concepts: *interior* (Definition 2), *boundary* (Definition 4), and *exterior* (Definition 5). These three concepts are the foundational backbone for the 9-intersection matrix [21].

**Definition 2.** Let  $S$  be a set in topological space  $X$  (Definition 1). The union of all open sets contained fully within  $S$  is called the **interior** of  $S$ , denoted  $S^o$ .

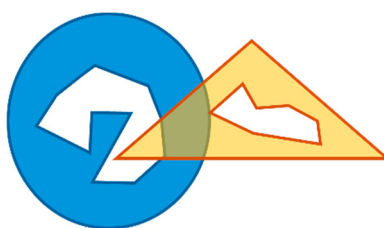
**Definition 3.** Let  $S$  be a set in topological space  $X$  (Definition 1). The intersection of all closed sets that contain  $S$  is called the **closure** of  $S$ , denoted  $\bar{S}$ .

**Definition 4.** Let  $S$  be a set in topological space  $X$  (Definition 1). The set difference between the closure of  $S$  (Definition 3) and the interior of  $S$  (Definition 2) is called the **boundary** of  $S$ , denoted  $\partial S$ .

**Definition 5.** Let  $S$  be a set in topological space  $X$  (Definition 1). The set difference between  $X$  and  $S$  is called the **exterior** of  $S$ , denoted  $S^-$ .

When an object has co-dimension 0 with its embedding space, these definitions work just fine. When an object has relevant components that are no longer co-dimension 0 with the embedding space (e.g., a line in a planar or spherical embedding), these concepts are generalized through algebraic topological constructions. This is accomplished through cell complexes and results in boundaries having dimension  $n - 1$  relative to the object in question. The set minus this boundary then produces the interior while the exterior remains the same [38]. Fundamentally, these structural definitions help to meet intuitive conceptions of a boundary as the furthest extent of objects.

The 4-intersection and 9-intersection matrices are the impetus for a larger family of relationship formalisms including the 9+-intersection [56] and the dimensionally-extended 9-intersection [57]. At their cores is the notion of the topological components of one object being intersected with the topological components of the other object. These models may apply to any objects in a pairwise disjoint manner: no relation between objects can have two distinct signatures if the definitions of interior, boundary, and exterior (or any subcomponents of those) are not changed. These four relation matrix structures are shown in Figure 1 for the same object configuration.



(a)

$$\begin{array}{cc} & B^o \quad \partial B \\ A^o & 1 \quad 1 \\ \partial A & 1 \quad 1 \end{array}$$

(b)

$$\begin{array}{ccccc} & B^o & \partial B^H & \partial B^C & B^{H-} & B^{C-} \\ A^o & 1 & 0 & 1 & 0 & 1 \\ \partial A^H & 1 & 0 & 1 & 0 & 1 \\ \partial A^C & 1 & 1 & 1 & 0 & 1 \\ A^{H-} & 1 & 0 & 1 & 0 & 1 \\ A^{C-} & 1 & 1 & 1 & 1 & 1 \end{array}$$

(d)

$$\begin{array}{ccc} & B^o & \partial B & B^- \\ A^o & 1 & 1 & 1 \\ \partial A & 1 & 1 & 1 \\ A^- & 1 & 1 & 1 \end{array}$$

(c)

$$\begin{array}{ccc} & B^o & \partial B & B^- \\ A^o & 2 & 1 & 2 \\ \partial A & 1 & 0 & 1 \\ A^- & 2 & 1 & 2 \end{array}$$

(e)

**Figure 1.** A spatial scene with (a) object *A* (blue) and object *B* (orange) with four topological intersection model representations: (b) 4-intersection [44], (c) 9-intersection [21], (d) 9+-intersection [56], and (e) dimensionally-extended 9-intersection [57]. Each formalism provides a different set of information about the scene. In (e), the 0 communicates a 0-dimension intersection (i.e., points), whereas in (b-d), 0 would convey no intersection present.

These types of relation formalisms have been used to create many sets of relations in the literature, including in continuous embedding spaces: region-region relations [21,40,44], region-line relations [39], line-line relations in the plane [38], line-line relations in the cycle [58], line-line relations in a linear embedding [33], complex region-region relations on the sphere [41], and compound object relations between any combination of points, lines, or polygons [42]. Discretized relations sets from these models include: region-region relations [29–31,43] and line-line relations in a digital linear embedding [34]. These relation formalisms are the backbone for modern GIS query capabilities [17].

## 2.2. Mereotopological Formalisms

Mereotopological relations are based on the concepts of connection and containment [59]. For region-region relations, these methodologies produce equivalent sets to the 9-intersection (e.g., [21,22] and [40,60]).

Several different types of mereotopological formalisms exist, including [61]:

- Region connection calculus [22]
- Extensional contact algebra [62]
- Normal contact algebra [63,64]
- Local contact algebra [65,66]
- Connected extensional contact algebra [67]

There are several sets of relations that have been derived through the region connection calculus, the most extensively utilized of any of these formalisms:

- RCC-5, categorizing relations in the plane where the boundary is not crucial [51]
- RCC-7, categorizing relations on the sphere where the boundary is not crucial [68]
- RCC-8, categorizing relations in the plane [22]
- RCC\*-9, categorizing relations with lines and regions [69–71]
- RCC-11, categorizing relations on the sphere [60]
- RCC-23, categorizing relations in 3D space [72]

Because the region connection calculus fundamentally focuses on regions, extensions were considered that treat lines and points through the multidimensional region connection calculus [73]. This formalism has identified 36 object relations.

## 2.3. Partition-Mapping Formalisms

A third way of conceptualizing spatial relations is seen through partition-based spaces. Fundamentally discretized, these representations allow us to rely on adjacency properties to consider relations between sets of objects treated as groups. There are few sets of relations derived from this approach, most notably a group of surrounds relations [74] and a basic categorization of region-region relations that are well known in planar embeddings through representations such as MapTree [75] and through collections of nodes in scene networks [76].

## 3. Conceptual Neighborhood Graphs

Conceptual neighborhood graphs represent functional organizations of relation spaces based on allowable deformations [52]. While originally constructed to describe temporal intervals, these types of structures were quickly leveraged to describe relations in all kinds of embedding spaces and object types (e.g., [77–80]).

As a network structure, conceptual neighborhood graphs are mathematical structures that have nodes and edges (Definition 6).

**Definition 6.** Let  $V$  be a set of objects and  $E$  a set of associations between elements of  $V$ . A **graph**, denoted as  $G_{V,E}$ , is a combination of the set of **vertices**  $V$  and the set of **edges**  $E$ .

The nodes in the conceptual neighborhood graph represent relations, while the edges represent a transition from one relation to another via some allowable deformation. As such, conceptual neighborhood graphs are typically described by two concepts: the relation space in question and the allowable deformation(s) in question. In discretized spaces, we could consider a third concept: object extent [78,79], leading to conceptual neighborhood graphs between specific configurations. The organization principle for a conceptual neighborhood graph is simple: an edge exists between nodes if and only if a deformation from a configuration exhibiting one relation forces a transition to another relation without first going through an intermediary relation. Since the reasoning systems underneath these systems are qualitative, we resort typically to reporting salient connections that are independent of the particular objects in question.

There are several conceptual neighborhood graphs in the extant literature, connected inherently to relation sets:

- Translation, isotropic scaling, anisotropic scaling in continuous temporal intervals [52]
- Rotation, translation, isotropic scaling, and anisotropic scaling in continuous region-region relations [40,77]
- Translation, isotropic scaling, and anisotropic scaling for discretized region-region relations [30,78,79]
- Matrix differences for continuous line-region and conversely region-line relations [39]
- Matrix differences for continuous line-line relations [81]
- Matrix differences for arbitrary relation sets [53]
- Integration of RCC-5 and RCC-8 conceptual neighborhood graphs [54]
- Unions and intersections of conceptual neighborhood graphs [82]
- Hole and separation changes to region-region relations [50]
- Translation, isotropic scaling, and anisotropic scaling in discretized temporal intervals [80]
- Translation, isotropic scaling, and anisotropic scaling in discretized lines in a linear embedding [80]

Conceptual neighborhood graphs present several important use cases in a world of spatio-temporal data and a world of geospatial artificial intelligence. For a more rich view of the possibilities, please consult [55].

#### 4. Attempts at Combining Conceptual Neighborhood Graphs

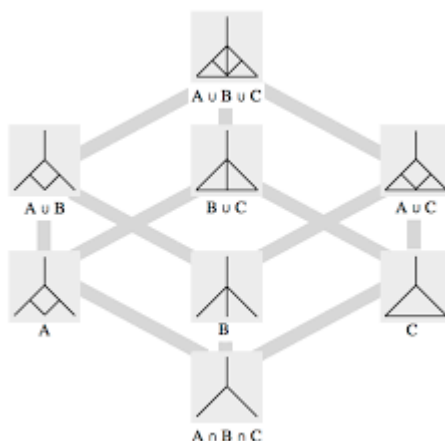
Conceptual neighborhood graphs were combined on a few separate occasions. Each combination of conceptual neighborhood graphs in the past operated on different base principles.

Egenhofer and Al-Taha [77] built conceptual neighborhood graphs from aggregations of paths by default. This is fundamentally essential because relations such as *inside*, *equal*, and *contains* are all fundamentally constrained such as to avoid one another deformationally as *equal* is itself fundamentally restrictive. Only by starting at *equal* can any patterns involving it be adequately assessed. As such, a conceptual neighborhood graph can be described as the union of characteristic paths (Figure 2).



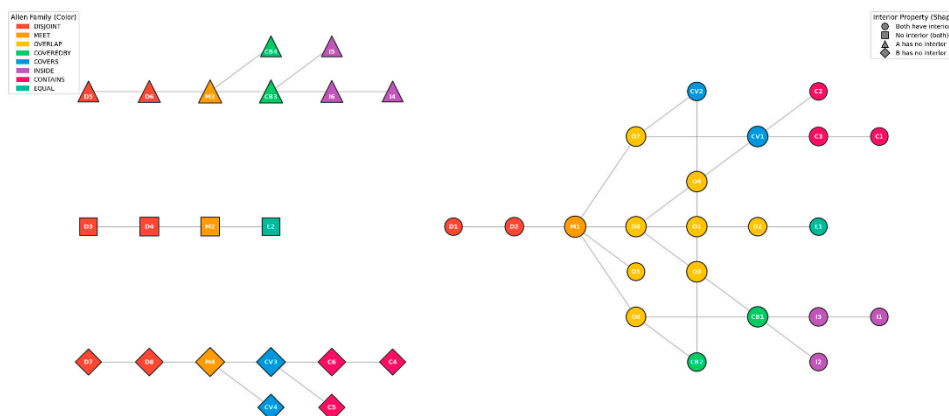
**Figure 2.** Conceptual neighborhood graph generation as a combination of paths. These twelve characteristic paths form the isotropic scaling conceptual neighborhood graph for region-region relations in the plane [77].

Egenhofer [82] took this one step further and instead of considering deformational paths, a proposed concept of conducting unions and intersections of conceptual neighborhood graphs was explored, dubbed the family of conceptual neighborhood graphs. While a single conceptual neighborhood graph for a particular deformation represents a union of paths, the family of conceptual neighborhood graphs attempts to describe changes that are either consistent (intersectional) or that are accessible by one or more of a set of deformations. This approach was considered on the planar region-region relations (Figure 3).



**Figure 3.** The family of conceptual neighborhood graphs on planar region-region relations, where conceptual neighborhood graph A represents the anisotropic scaling neighborhood, conceptual neighborhood graph B represents the translation/rotation neighborhood, and conceptual neighborhood graph C represents the isotropic scaling neighborhood. Conceptual neighborhood graphs on the higher levels of this lattice represent combinations of deformations allowed from the third level. The lowest level represents the characteristic paths that are common to all deformations in the set [82].

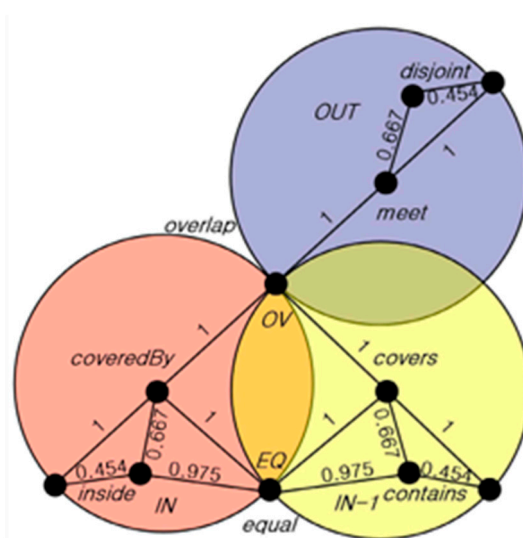
Figures 2 and 3 represent combinations of pathways and conceptual neighborhood graphs that relate relations from the same set to one another. It is possible, however, for relations from a common definition paradigm to be isolated from one another in a conceptual neighborhood graph, thus representing a union of relations that are not achievable from one to another under a deformation type. This is seen in discrete conceptual neighborhood graphs where particular relations may be sufficiently small enough to preclude relations under translation [78–80]. This type of conceptual neighborhood graph can be seen in Figure 4.



**Figure 4.** The conceptual neighborhood graph for translation of discrete temporal intervals [80]. The symbology in this graph is irrelevant in this context and maintained only for continuity with prior literature. This graph is, however, necessarily clustered. One cluster involves only endpoints; one cluster has an interval A with only

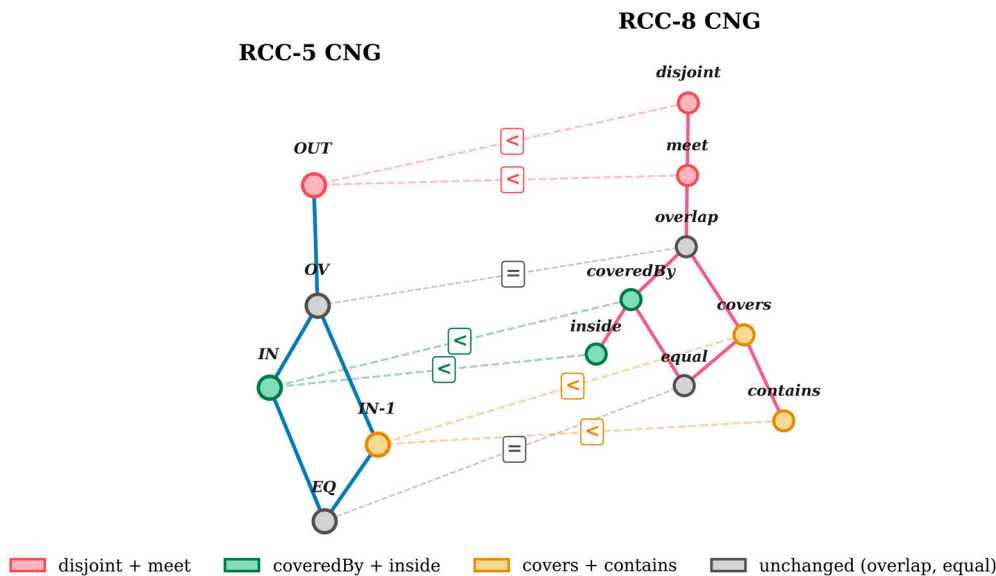
endpoints, while  $B$  has points between them; one cluster represents an interval  $A$  with points between its endpoints, while  $B$  has none; the final cluster represents two intervals each with points between their endpoints. While these subgraphs are isolated from one another, a different deformation (such as isotropic scaling) would connect them if they were considered with the family of conceptual neighborhood graphs approach [82].

The final type of combined conceptual neighborhood graph has to do with linguistic simplification (practically), or more formally a disregard for the boundary (e.g., RCC-5 vs. RCC-8) [54]. At the whole graph level, this was the first merging of conceptual neighborhood graphs. It was practically motivated to place uncertain information in the context of more certain information. As such, it made the choice to isolate the unioned RCC-5 relations (*externally connected; proper part; proper part inverse*) from the relations mutually in both sets (*partial overlap, equal*). As such, it is not a true union of conceptual neighborhood graphs, but rather a direct conceptual aggregation. This conceptual neighborhood graph is shown in Figure 5.

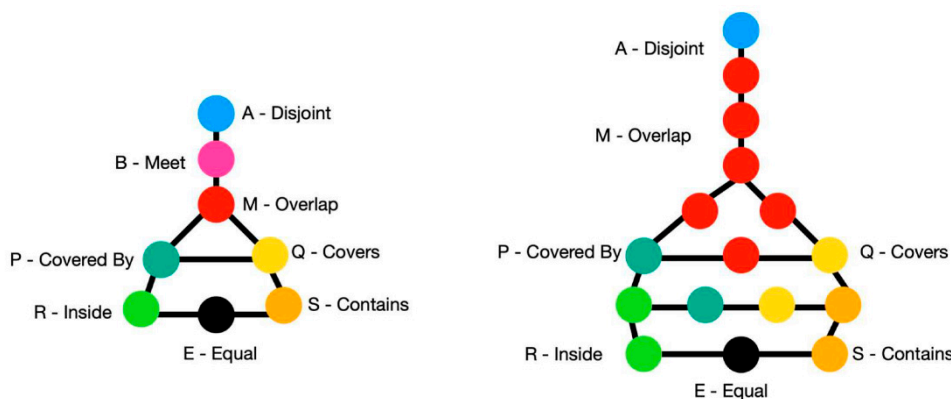


**Figure 5.** The aggregation of the conceptual neighborhood graphs for RCC-5 [51] and RCC-8 [22] with an eye toward treating the RCC-5 relations as uncertain entities [54]. Conceptually, the expectation is that the uncertain relation is more likely to practically relate to the prototype case without boundary contact. This is merely one example of weighting the conceptual neighborhood graph.

Dube and Egenhofer [54], however, started the process of arriving at the graph in Figure 5 from a conceptual framework of aligning the two conceptual neighborhood graphs. Philosophically, that is more important for this work. This process was undertaken by asking which concepts in which graphs were equivalent under that aggregation, visualizing a mapping from RCC-8 to RCC-5 (Figure 6). Hall and Dube [78–80] picked up on this and use it as a litmus test for a discretization neighborhood as in Figure 7.



**Figure 6.** A mapping of concepts in RCC-8 to RCC-5 [54]. This approach represents a conceptual attempt to relate relations from two quasi-distinct sets.



**Figure 7.** The conceptual neighborhood graph of discretized regions under isotropic scaling (right) projected back onto the continuous region-region relations that construct them (left) [79], effectively comparing competing models of boundary definition in the digital plane [29–31,43]. The colors become link points in the graphs. Of note is that *coveredBy* and *covers* are conceptual neighbors as a function of this aggregation and would not be ordinarily in the continuous set. This is due to the nature of discretization not being able to densely transform itself. The set at right did not include *disjointTouch* and therefore did not have the opportunity to model *meet* conceptually in the continuous setting.

### 5. An Integrated Approach to Conceptual Neighborhood Graphs

Given the current architecture of conceptual neighborhood graphs is limited to configurations of identical type, an architecture is needed to facilitate combining them together. The infrastructure to accomplish that task is mathematically an *n*-partite graph.

**Definition 7.** Let  $G_{V,E}$  be a graph on a set of vertices  $V$  such that  $V$  consists of  $n > 1$  jointly exhaustive and pairwise disjoint subsets and  $E$  be a set of edges such that no individual edge  $e$  consists of vertices from the same subset of  $V$ .  $G_{V,E}$  is called an *n-partite graph*.

An *n*-partite graph is the appropriate approach for this task because the types of linking operations in question to aggregate conceptual neighborhood graphs beyond simple unions of deformations [82] and in some instances linguistic simplification [54] result in a connection between

relations that are necessarily in different relational sets. When we collapse or aggregate objects as a cartographic generalization operation (or correspondingly undo those operations), we change the dimension of an object itself, thus leaving the preconditions for our set [35]. When we discretize an embedding space, we are leaving the topological structure inherited from the embedding space [29–31,43].



5.1. Example Discretization Neighborhood: Simple Region-Region Relations to Discretized Region-Region Relations

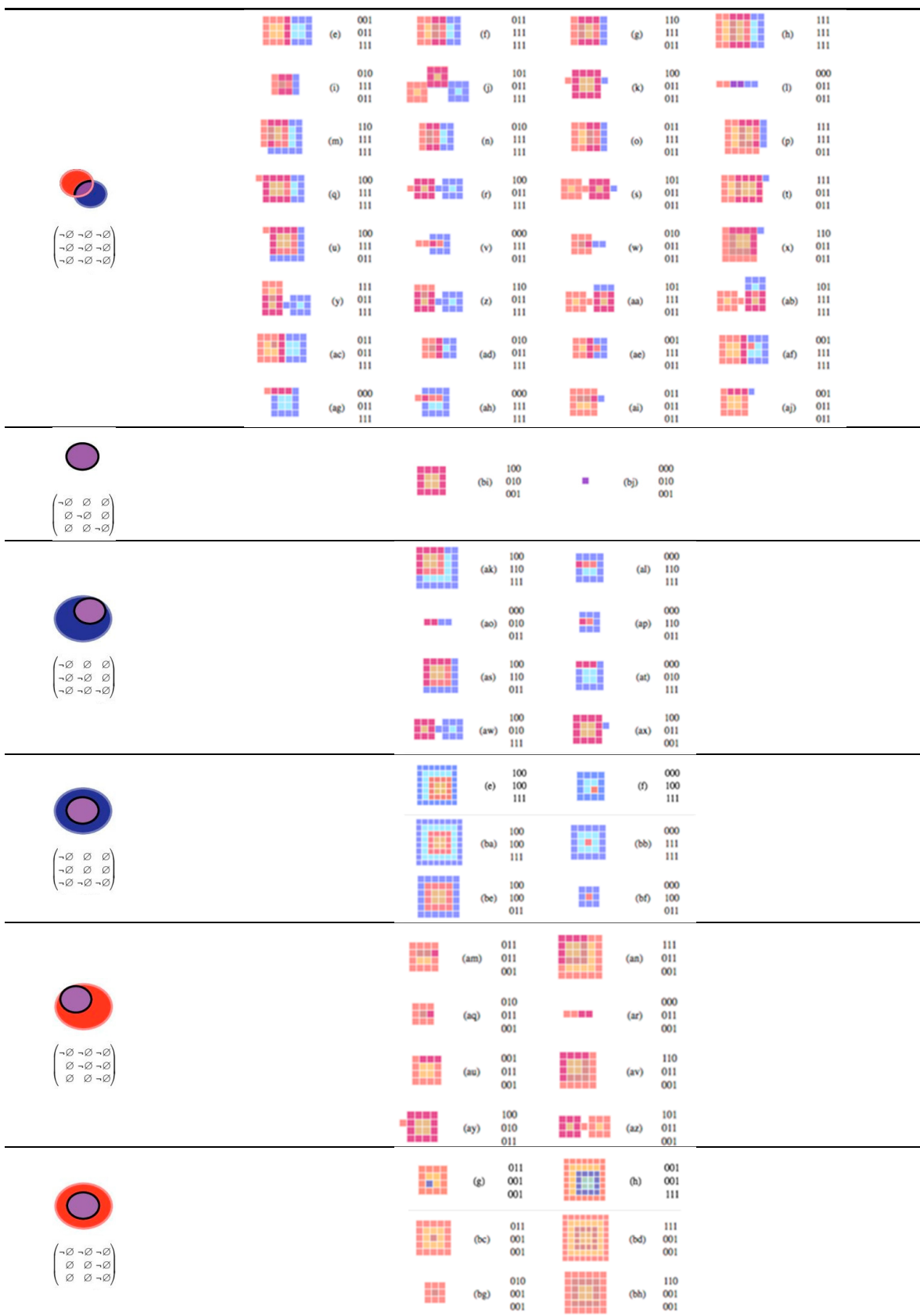
The simple region-region relations are one of the most iconic sets of qualitative relation sets and a common link point between spatial formalisms. [21,22]. Discretized region-region relations exist in two separate forms: a set that has a digital Jordan curve boundary [30,43] and one that has a frontier boundary [31]. Certain scholars advocate for a relation approach that simply considers the discretized object as its continuous counterpart [29]. Both pixel boundary approaches [31,32] lead to considering a linkage of relational concepts between continuous and discretized objects. In the case of vectorizing a region-region relation, this linkage would be analogous to the transformation between the hyperraster model [29] and the frontier-as-boundary relations [31,83]. It may also work similarly when taking a vectorized object and digitizing it, but it could also work in a functional capacity [79], too. In this section, we will consider both approaches.

5.1.1. Hyperraster and Frontier-as-Boundary Relations

The hyperraster model [29] defines a set of relations resulting from a vectorization of a discretized scene that corresponds to the relations from the 9-intersection for the Cartesian plane [21]. Under duality [84], these relations define the corresponding spherical relations from the continuous and digital spheres respectively. Table 1 shows the linkages under this transformation.

**Table 1.** Mapping from region-region relations in the Cartesian plane [21,29] to the region-region relations in the discretized plane [30,31]. Under duality [84], these relations also explain mappings between the continuous and digital spheres [40,43]. Each mapping in this table becomes an edge in a discretization conceptual neighborhood graph.

Region-Region in $\mathbb{R}^2$ [21,29,40]	Region-Region in $\mathbb{Z}^2$ [30,31,43]			
 $\begin{pmatrix} \emptyset & \emptyset & \sim\emptyset \\ \emptyset & \emptyset & \sim\emptyset \\ \sim\emptyset & \sim\emptyset & \sim\emptyset \end{pmatrix}$	 (a) $\begin{matrix} 001 \\ 001 \\ 111 \end{matrix}$	 (b) $\begin{matrix} 000 \\ 001 \\ 011 \end{matrix}$	 (c) $\begin{matrix} 000 \\ 001 \\ 111 \end{matrix}$	 (d) $\begin{matrix} 001 \\ 001 \\ 011 \end{matrix}$
 $\begin{pmatrix} \emptyset & \emptyset & \sim\emptyset \\ \emptyset & \emptyset & \sim\emptyset \\ \sim\emptyset & \sim\emptyset & \sim\emptyset \end{pmatrix}$	 (a) $\begin{matrix} 001 \\ 001 \\ 111 \end{matrix}$	 (b) $\begin{matrix} 000 \\ 001 \\ 011 \end{matrix}$	 (c) $\begin{matrix} 000 \\ 001 \\ 111 \end{matrix}$	 (d) $\begin{matrix} 001 \\ 001 \\ 011 \end{matrix}$


























































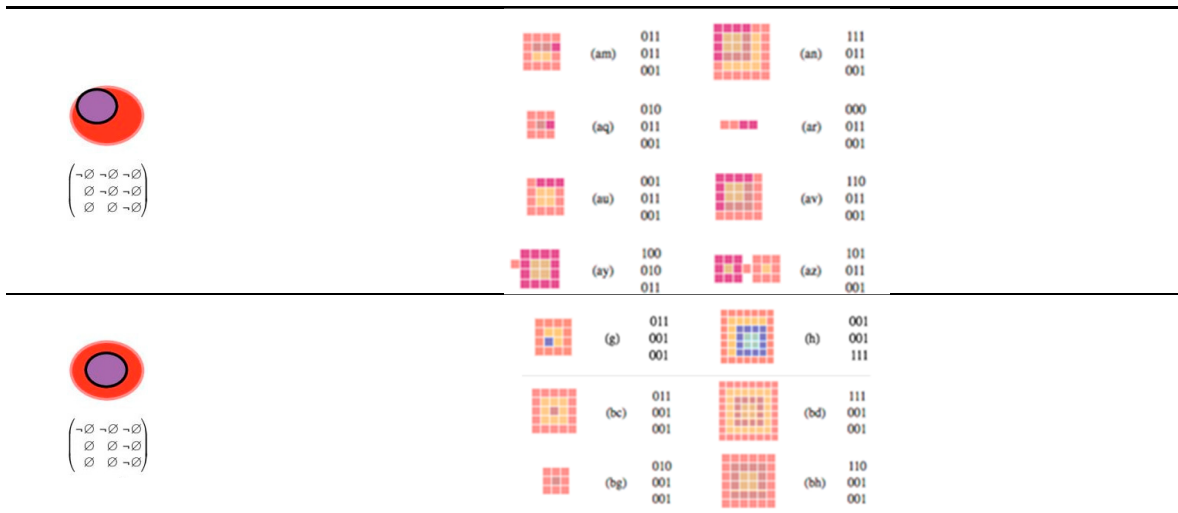
### 5.1.2. Functional Linguistic Mapping Between These Sets

From a simple comparative perspective, these two approaches will not be tremendously different, however, they are conceptually different specifically with what is defined as the *meet* relation. Functionally, a *meet* relation involves sharing only a boundary; as such there are several relations that vectorize to *overlap* where this designation is more appropriate. Similarly, the choice

between where to place the *disjointTouch* relations also has semantic interest. If we proceed solely from boundary sharing, then *disjointTouch* relations go to *disjoint*, and they are replaced with relations that carry the name *meet* in their discretized labeling. This is demonstrated in Table 2.

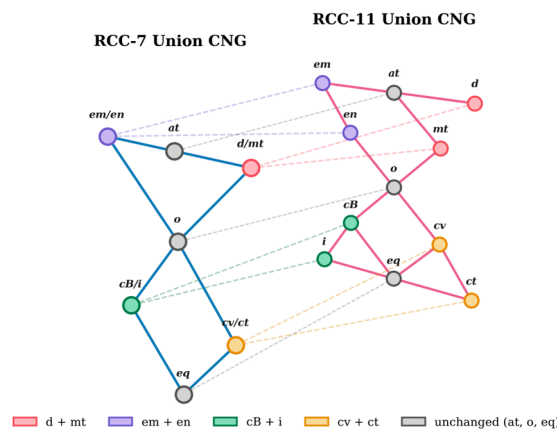
**Table 2.** Mapping from region-region relations in the Cartesian plane [21] to the region-region relations in the discretized plane [30,31] under functional capacities. Under duality [84], these relations also explain mappings between the continuous and digital spheres [40,43]. Each mapping in this table becomes an edge in a discretization conceptual neighborhood graph. Relations in Table 2 are mostly positioned as they are in Table 1, with differences occurring between *disjoint* and *meet* and between *meet* and *overlap*.

Region-Region in $\mathbb{R}^2$ [21,29,40]	Region-Region in $\mathbb{Z}^2$ [30,31,43]			
 $\begin{pmatrix} \emptyset & \emptyset & -\emptyset \\ \emptyset & \emptyset & -\emptyset \\ -\emptyset & -\emptyset & -\emptyset \end{pmatrix}$	 (a) $\begin{matrix} 001 \\ 001 \\ 111 \end{matrix}$	 (b) $\begin{matrix} 000 \\ 001 \\ 011 \end{matrix}$	 (c) $\begin{matrix} 000 \\ 001 \\ 111 \end{matrix}$	 (d) $\begin{matrix} 001 \\ 001 \\ 011 \end{matrix}$
 $\begin{pmatrix} \emptyset & \emptyset & -\emptyset \\ \emptyset & -\emptyset & -\emptyset \\ -\emptyset & -\emptyset & -\emptyset \end{pmatrix}$	 (e) $\begin{matrix} 001 \\ 011 \\ 111 \end{matrix}$	 (ag) $\begin{matrix} 000 \\ 011 \\ 111 \end{matrix}$	 (aj) $\begin{matrix} 001 \\ 011 \\ 011 \end{matrix}$	 (l) $\begin{matrix} 000 \\ 011 \\ 011 \end{matrix}$
 $\begin{pmatrix} -\emptyset & -\emptyset & -\emptyset \\ -\emptyset & -\emptyset & -\emptyset \\ -\emptyset & -\emptyset & -\emptyset \end{pmatrix}$	 (i) $\begin{matrix} 010 \\ 111 \\ 011 \end{matrix}$	 (j) $\begin{matrix} 101 \\ 011 \\ 111 \end{matrix}$	 (k) $\begin{matrix} 100 \\ 011 \\ 011 \end{matrix}$	 (h) $\begin{matrix} 111 \\ 111 \\ 011 \end{matrix}$
	 (m) $\begin{matrix} 110 \\ 111 \\ 111 \end{matrix}$	 (n) $\begin{matrix} 010 \\ 111 \\ 111 \end{matrix}$	 (o) $\begin{matrix} 011 \\ 111 \\ 011 \end{matrix}$	 (p) $\begin{matrix} 111 \\ 111 \\ 011 \end{matrix}$
	 (q) $\begin{matrix} 100 \\ 111 \\ 111 \end{matrix}$	 (r) $\begin{matrix} 100 \\ 011 \\ 111 \end{matrix}$	 (s) $\begin{matrix} 101 \\ 011 \\ 011 \end{matrix}$	 (t) $\begin{matrix} 111 \\ 011 \\ 011 \end{matrix}$
	 (u) $\begin{matrix} 100 \\ 111 \\ 011 \end{matrix}$	 (v) $\begin{matrix} 000 \\ 111 \\ 011 \end{matrix}$	 (w) $\begin{matrix} 010 \\ 011 \\ 011 \end{matrix}$	 (x) $\begin{matrix} 110 \\ 011 \\ 011 \end{matrix}$
	 (y) $\begin{matrix} 111 \\ 011 \\ 111 \end{matrix}$	 (z) $\begin{matrix} 110 \\ 011 \\ 111 \end{matrix}$	 (aa) $\begin{matrix} 101 \\ 111 \\ 011 \end{matrix}$	 (ab) $\begin{matrix} 101 \\ 111 \\ 111 \end{matrix}$
	 (ac) $\begin{matrix} 011 \\ 011 \\ 111 \end{matrix}$	 (ad) $\begin{matrix} 010 \\ 011 \\ 111 \end{matrix}$	 (ae) $\begin{matrix} 001 \\ 111 \\ 011 \end{matrix}$	 (af) $\begin{matrix} 001 \\ 111 \\ 111 \end{matrix}$
		 (ah) $\begin{matrix} 000 \\ 111 \\ 111 \end{matrix}$	 (ai) $\begin{matrix} 011 \\ 011 \\ 011 \end{matrix}$	
 $\begin{pmatrix} -\emptyset & \emptyset & \emptyset \\ \emptyset & -\emptyset & \emptyset \\ \emptyset & \emptyset & -\emptyset \end{pmatrix}$	 (bi) $\begin{matrix} 100 \\ 010 \\ 001 \end{matrix}$	 (bj) $\begin{matrix} 000 \\ 010 \\ 001 \end{matrix}$		
 $\begin{pmatrix} -\emptyset & \emptyset & \emptyset \\ -\emptyset & -\emptyset & \emptyset \\ -\emptyset & -\emptyset & -\emptyset \end{pmatrix}$	 (ak) $\begin{matrix} 100 \\ 110 \\ 111 \end{matrix}$	 (al) $\begin{matrix} 000 \\ 110 \\ 111 \end{matrix}$		
	 (ao) $\begin{matrix} 000 \\ 010 \\ 011 \end{matrix}$	 (ap) $\begin{matrix} 000 \\ 110 \\ 011 \end{matrix}$		
	 (as) $\begin{matrix} 100 \\ 110 \\ 011 \end{matrix}$	 (at) $\begin{matrix} 000 \\ 010 \\ 111 \end{matrix}$		
	 (aw) $\begin{matrix} 100 \\ 010 \\ 111 \end{matrix}$	 (ax) $\begin{matrix} 100 \\ 011 \\ 001 \end{matrix}$		
 $\begin{pmatrix} -\emptyset & \emptyset & \emptyset \\ -\emptyset & \emptyset & \emptyset \\ -\emptyset & -\emptyset & -\emptyset \end{pmatrix}$	 (e) $\begin{matrix} 100 \\ 100 \\ 111 \end{matrix}$	 (f) $\begin{matrix} 000 \\ 100 \\ 111 \end{matrix}$		
	 (ba) $\begin{matrix} 100 \\ 100 \\ 111 \end{matrix}$	 (bb) $\begin{matrix} 000 \\ 111 \\ 111 \end{matrix}$		
	 (be) $\begin{matrix} 100 \\ 100 \\ 011 \end{matrix}$	 (bf) $\begin{matrix} 000 \\ 100 \\ 011 \end{matrix}$		



5.2. Example Linguistic Simplification Neighborhood: RCC-7 and RCC-11

The Region-Connection Calculus and the 9-intersection produce similar relations between simple vectorized regions in both the plane and the sphere. As such, it is common practice in spatial information systems to create operators that have some semantic flexibility to account for whether the boundary is essential to the relation itself, or not. This is manifest in the Region-Connection Calculus by dropping the connection component and focusing on containment between the objects and their complements. Similarly in the 9-intersection, the boundary intersections can be dropped. For the region-region relations on the sphere, this effectively does not impact three of the relations (*overlap*, *attach*, *equal*) and then combines relations that differ only in a boundary-boundary intersection (e.g., *disjoint* and *meet*).



**Figure 8.** Mapping from region-region relations in RCC-11 [60] to region-region relations in RCC-7 [84]. Connections shown in the graphs are the union of the conceptual neighborhood graphs for homeomorphic deformations [82].

5.3. Example Cartographic Generalization Neighborhood: Region-Region to Region-Point Relations in the Cartesian Plane

In a multiscale map, jurisdictions are often generalized to points at a sufficiently zoomed out scale. When this happens, the relationship between jurisdictions changes from a region-region relation to a region-point relation. This changes our relation space from eight relations down to three relations (*inside*, *on the boundary*, *outside*). Effectively this can be managed by the interior and boundary intersections of the region being reduced to a point. Wherever these intersections intersect the other object’s components, an opportunity for a generalization appears, as shown in Table 3. Practically speaking, certain relations are not likely generalizations, but some contextually would

likely hold value. For example, *meet* can generalize to *on the boundary* which would communicate the touching relationship, however, most generalization algorithms would not generalize it that way. If instead the concept is meaningful, it could provide direction to force the generalization to this option. This would be similar for *coveredBy* or *covers* as well.

**Table 3.** Mapping from region-region relations in the Cartesian plane [21] to the region-point relations in the Cartesian plane [42].

Region-Region Relation	Region-Point Relation
<i>Disjoint</i>	<i>Outside</i>
<i>Meet</i>	<i>on the boundary, outside</i>
<i>Overlap</i>	<i>inside, on the boundary, outside</i>
<i>Equal</i>	<i>inside, on the boundary</i>
<i>coveredBy</i>	<i>inside, on the boundary</i>
<i>Inside</i>	<i>Inside</i>
<i>Covers</i>	<i>inside, on the boundary, outside</i>
<i>Contains</i>	<i>inside, on the boundary, outside</i>

## 6. Discussion

Conceptual neighborhood graphs are a crucial mechanism for sorting through the challenges of a data forward world, in particular the notions of variety and verbosity from the big data revolution. There is still much work to be done in these areas that would allow us to fully realize their power, but the increasing role of artificial intelligence in spatial context provides opportunities for this work to bear fruit [55].

Fundamentally, conceptual neighborhood graph approaches very much relate to spatial ontologies [85–90]. Spatial ontologies specifically model types of objects, their properties, the roles that objects or properties function in, and how everything relates. Conceptual neighborhood graphs are a convenient visual ontology that models the *realizable* spatial relations that can occur, concerning two objects because most of the possible 9-intersection configurations are physically impossible. The ability to see the relations and the areas of uncertainty shows that a consistent truth or realm of possibilities can be determined. Formally, an ontology is a resource description framework. Basically, it is an annotated tree-graph composed of objects and the relations between them. It presents a hierarchy of categories for objects, their roles, and relations. It attempts to generalize as much as possible while still restricting the space of what is possible.

An ontology is not true or false, rather, it is the space which dictates whether a statement can be considered true or false. An ontology must be considered consistent for this. Consistent ontologies restrict the possible space of valid statements that conform to the axioms by some series of relations. Conceptual neighborhood graphs model relations as objects; they contain nodes representing some relation. The edges between them are the ways of continuously deforming some relation into another.

An inconsistency indicates a contradiction in the axioms, and any statement applied in the framework is meaningless; they are true and false, simultaneously and ubiquitously. There is no requirement that an ontology be complete, but discrete neighborhood embeddings, it is possible to prove closure [80]. The similarities between various neighborhoods might leverage this when attempting to prove continuous ones. Such proofs may be frivolous, since neighborhoods considered so far are consistent with reality and ignore what is not.

Two popular philosophies within ontologies are the open- and closed-world assumptions. The open- and closed-world assumptions are opposite ways of viewing these frameworks. In the open-world, anything is possible until explicitly restricted, and meaning is built by ruling things out. The closed-world permits only what is explicitly declared. The conceptual neighborhood graphs proposed by [52] are closed-world.

An open-world view might explore how lines may intersect, having an interior-interior overlap without their boundaries overlapping. On a sphere, two disjoint regions may be squeezed to overlap

somewhere in their interiors, as well. The application for this is not immediately apparent, but it is worth noting that just because something is physically impossible in one space or with one pair of objects, it may still be feasible and even meaningful under certain conditions.

Since neighborhoods explore deformations, relating neighborhoods requires varying the representational space, the type of objects, the resolution, or the language depicting the event or scenario. Expanding and connecting the current knowledge base allows us to approach a consistent and complete model. The symmetries and relationships between various neighborhoods can also improve inferencing power, too [52].

Viewing this as an ontological problem allows different degrees of freedom to be explored while all else is held constant. It may not direct exploration by offering convenient next-step suggestions, but it outlines all the possible neighborhoods and assists the exploration by declaring what qualitative changes are allowed. [77] leverages symmetries and visual reasoning to examine how subtle variations in starting configuration may alter the same deformation. Rather than explore how different deformations lead to different relations, they varied the relative placement of objects. Many of the relations make sense, intuitively, but others show that the nearest neighbor with the most resemblance is not always along the traversal. [52] and [55] explore the implications that can be made when time is one of the intersecting objects, since our experience of it is ordered and linear.

While many applications proposed conceptual neighborhood graphs which combine others (e.g., [50,54,79,80,82]), the ground for providing that resource depends upon what already exists for conceptual neighborhood graphs, relation sets, and generalization protocols. Table 4 shows an inventory of where the literature sits in these pursuits.

Object Class	Continuous 1D	Continuous 2D	Discrete 1D	Discrete 2D	Linguistic Simplification
R to R	N/A	Relation set [21,40], CNG [77]	N/A	Relation set [30,31,43], CNG [79]	[51,60]
R to L	N/A	Relation set [39], CNG [39]	N/A		[91]
R to P	N/A	Relation set [42]	N/A	Relation set [31], CNG [79]	
L to L	Relation set [33], CNG [52]	Relation set [38], CNG [81]	Relation set [34], CNG [80]		
L to P	Relation set [42]	Relation set [42]	Relation set [34], CNG [80]		
P to P	Relation set [42]	Relation set [42]	Relation set [34], CNG [80]	Relation set [31], CNG [79]	

Table 4 demonstrates the significant range of missing links, particularly in the discrete realms. There is no adequate definition for discretized lines in a discretized 2D embedding space. Without this definition, relations involving discretized lines are not available to organize. On top of this, the generalization of regions to lines, and lines to points, are beset with difficulties [92]. Similarly, discretizing a continuous object is functionally dependent upon the resolution [93].

It is also of substantive note that a conceptual neighborhood graph only fulfills the role of relating spatial and temporal prepositions to one another [11,14,15]. A similar and relatable problem is the difficulty of matching the objects themselves. While cartographic generalization can create confusions in this regard, it is important to note the foundational concept of place is an additional challenge within this realm. It is not enough to know how relations are combined in this pursuit; we must also know that the objects themselves belong together [94–97]. To solve these relational challenges, both the object challenges and the relational challenges must be resolved. The object challenges of course must be resolved first – we must know that we are referring to the same objects before we consider their relations within various data formats.

The other fundamental challenge is human language itself [12,16]. Linguistic diversity plays a role in text-based spatial data [94,95]. While spatial prepositions and temporal prepositions play prominent roles in many languages, they are not completely universal [14,98]. It is thus critical to consider the complete set of spatial and temporal preposition concepts across all languages – the opposite of the natural semantic metalanguage concept of intersection – and create foundational mappings between those terms. Conceptual neighborhood graphs show substantial potential toward this pursuit as well [14]. It is critical to inventory these prepositions against different types of object representation, similar to the types of qualitative topological relations that are found mirrored by different object domains [42].

Distinguishing between persistent and coincidental neighbors is also important for uncovering an understanding of how relations change in different conditions, too [80]. A coincidental neighbor might occur when one object's resolution is low enough that it does not have a clear boundary and interior region, blurring the ability to distinguish between it being tangent to another object or existing within some region of the other object [80]. If one relation connects coincidental neighbors, it shows that ambiguity lies somewhere in the resolution of one or both objects' size, the deformation occurring between them, or the representation space itself [79].

Though there are missing components to the puzzle, this should not preclude us from striving toward these objectives. The benefit of constructing a unifying conceptual neighborhood graph framework is immense, especially in the worlds of big data, crowdsourcing, and VGI. Each of these and similar phenomena provides an inability control of the data format. Rather than removing our reasoning capabilities, we should be expanding those possibilities. Conceptual neighborhood graphs provide the mechanism by which to do that, functionally creating a ranking function for potential neighboring concepts [55].

Working with an ontology is as much about defining restrictions as it is declaring what exists. [52] shows that understanding general relations can lead to more efficient inferencing capabilities, since knowledge is rarely complete. [52] stresses the need to understand how relations deform into one another rather than consider the less-detailed information in terms of disjunctions and alternatives. The boundaries of a line are disjoint points, yet the boundaries of regional-objects are continuous. This seemingly innocuous difference leads to some interesting properties, like a line being able to intersect with an object without the object ever touching the boundary of the line. If we add dimensions, the object's interior may extend to a plane beyond its boundary, and thus shares properties with the line.

We outline the requisites for an interface where a human can use natural language to define a particular situation or event. Like colloquial dialogues, when more than one category of relation fits the situation, the machine needs ways to disambiguate [52]. Also, it should be able to return a list of possible events and have a ranking mechanism to sort the list based on relative probability [55,79,80].

People are already good at the machine's role. [39] shows that people of various cultures show a large agreement when matching a description to a relation; however, the agreements end when the subjects are asked to write the descriptions. Large language models have the capacity to understand different ways of describing something already, so the bridge to connect language and conceptual models only needs to span an agent's ability to discover uncertainties and attempt to resolve and/or rank them. The ability to trace relations and agreement when describing them is encouraging, as it indicates that descriptive objectivity is possible.

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