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Gravitation as Phase Synchronisation on a Finite Substrate

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Abstract

We derive gravitation in the Finite Ring Cosmology framework from three identifications: mass is the cardinality of a phase-locked cluster of substrate degrees of freedom, distance is decoherence, and time is the shared scale-dilation driving every cell in common. Gravity is then spontaneous synchronisation, and five stated premises yield Newton's law $F = Gm_1m_2/r^2$ with $G = \hbar c/m_P^2$: the inverse square from harmonicity in three derived dimensions, the product m_1m_2 from coherent additivity, universal attraction from the single arrow of the drive, and the equivalence principle as an identity. The relativistic completion gives post-Newtonian $\beta = \gamma = 1$: deflection, Shapiro delay, perihelion, and Lense–Thirring at their observed values. On the theory's exponential branch (the alternative is GR-coincident) the exact static solution is horizonless but operationally black, with a parameter-free shadow 4.6% wider than general relativity predicts (55.7 vs 53.3 μas for Sgr A*, 1σ above current Event Horizon Telescope constraints), a -4.4% ringdown frequency shift, suppressed evaporation, and an area-law entropy recovering the de Sitter entropy $\sim \Omega$ of the substrate. The resolution floor, read as an acceleration, derives the galactic radial-acceleration scale $a_0 = cH_0/2\pi \approx 1.1 \times 10^{-10}\,\mathrm{m\,s^{-2}}$ against the fitted 1.2×10^{-10}. The discrete Fierz–Pauli functional is exhibited and exactly gauge-invariant; its second-order evaluation decides the strong-field branch and carries the remaining coefficients.

Keywords: emergent gravity; phase synchronisation; Kuramoto dynamics; Finite Ring Cosmology; finite fields; equivalence principle; black hole shadow; horizonless compact objects; radial acceleration relation; de Sitter entropy

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1. Introduction

That gravity may be emergent rather than fundamental — a thermodynamic or entropic residue of microscopic degrees of freedom — is a recurring proposal, from Sakharov's induced gravity [1] through Jacobson's derivation of the Einstein equation as an equation of state [2] to Verlinde's entropic force [3]. These accounts share a difficulty: the microscopic substrate is left unspecified, so the derivations consume macroscopic thermodynamic inputs rather than producing them. The present paper takes the opposite route. The substrate is specified completely — a finite arithmetic structure with an explicit observer theory, developed in the Finite Ring Cosmology (FRC) programme [4–7,9] — and gravitation is derived as the synchronisation dynamics of clocks riding that substrate, with the constant G, the post-Newtonian parameters, the strong-field structure, and a galactic-scale acceleration all emerging as computed numbers or priced premises.

The physical picture can be stated in one paragraph. Every massive object is an ensemble of elementary clocks, phase-locked internally — that is what makes it one object — and all clocks tick in a single universal time, the scale-dilation of the substrate, whose largest-scale reading is the Hubble flow. Clocks on a common platform synchronise spontaneously, as coupled metronomes do [10–12]; the relaxation of mutual phase tension between two locked ensembles lowers a free energy whose gradient along their separation is a force. And because, in this framework, distance *is* the decoherence

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between two systems, the dynamics that increases mutual coherence is identically motion toward smaller separation: synchronisation does not cause gravitational attraction, it *is* gravitational attraction.

The paper's results divide into three layers. The Newtonian layer (Section 4): Newton's law exactly, with each of its parts given a provenance — the r^{-2} from harmonicity in three dimensions (themselves derived in the framework, not assumed), the product $m_1 m_2$ from the coherent additivity of locked phases, the universal sign from the single arrow of the drive, $G = \hbar c / m_P^2$ from unit channel capacity — and the equivalence principle as an identity rather than a postulate. The relativistic layer (Section 5): the source is an exact finite dust tensor on the mass shell of a quadratic extension; one further premise (time and distance are two readings of a single correlation capacity) forces the spatial metric bias to equal the temporal one, giving $\gamma = 1$, the full light deflection, and, through multiplicative composition, an exponential metric with $\beta = \gamma = 1$ — all classical tests at their observed values — with the route to the full Einstein dynamics laid out through an exactly constructed gauge structure. The strong-field and observational layer (Sections 6–7): the static problem solved exactly, a horizonless but operationally black object with parameter-free shadow and ringdown signatures a few percent from general relativity, an area-law entropy closing a consistency loop on the substrate's own size, and a derived weak-acceleration scale $a_0 = cH_0/2\pi$ addressing the radial acceleration relation — the one tight empirical law of galactic dynamics that current theory leaves unexplained.

Throughout, the accounting discipline of the FRC programme applies: every statement is explicitly exact, premised, or open. Five premises carry the construction; the single most consequential open item — a discrete stiffness functional on the deformation data of Section 5 — is identified, bounded, and shown to be the common origin of every remaining coefficient.

2. The Substrate: A Primer

This section summarises, self-containedly, the elements of the Finite Ring Cosmology framework that the derivation consumes. The framework is developed in [4–7,9] and applied to the spectral theory of the Riemann zeta function in the companion paper [9], whose physical reading the present paper extends; no familiarity with that corpus is assumed here.

The Carrier and the observer.

The substrate — the *Carrier* — is a finite prime field $\mathbb{F}_\Omega$, with $\Omega \sim 10^{122}$ in Planck units, fixed by the de Sitter entropy of the observable universe. There is no completed infinity: the classical continuum is recovered only as a degenerate idealisation of the finite structure, in the finitist tradition of Lev and Zeilberger [4]. The element 0 (position), the unit 1 (scale), and a multiplicative generator (speed-like) are not absolute marks but *frame data* that an observer assigns; physics is the frame-covariant content, and "no absolute frame" is the framework's relational ontology. An observer — a *Subject* — is a much smaller nested shell $\mathbb{F}_p$, $\Omega \gg p^2$; below the Subject's horizon the Subject and Carrier agree on arithmetic, and all laboratory physics lives in that agreement window.

Time, the drive, and the floor.

Time in the framework is scale-dilation: the multiplicative action $x \mapsto g\,x$ of the frame generator, advancing every substrate cell by one *chronon* per step; its idealised generator is $\hat{H} = -i(x\partial_x + \frac{1}{2})$ [6,9]. The dilation is global: every degree of freedom is driven in common, and the Hubble flow is this drive read at the largest scale. Two derived scales bound every observation. The *coherence horizon* $\sqrt{\Omega} \sim 10^{61}$ Planck lengths is the Hubble radius; and the *resolution floor* $1/\sqrt{\Omega}$ is the finest signal any frame resolves — a quantity established in [9], where it bounds the spectral resolution of arithmetic itself. The floor will appear in this paper four times: as the evaporation cutoff, the preferred-frame residual, the operational-horizon criterion, and the galactic acceleration scale.

Three dimensions from frame freedoms.

The observer's chart of the Carrier is the projective line over $\mathbb{F}_\Omega$ — the Ω field residues closed by the horizon mark — and an oriented frame is a unimodular basis of the underlying plane, so

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the space of frames is a torsor under $\mathrm{SL}_2(\mathbb{F}_\Omega)$ ([9] §9.6). Its three motion cycles are the three frame freedoms: translating the origin (cycle Ω, space), rescaling the unit (cycle $\Omega - 1$, scale-time), and the horizon-carrying boost (cycle $\Omega + 1$). The three dimensions of space are these freedoms, not three copies of the ring; the continuum shadow of the frame space is the 3-sphere, consonant with the spatial sections of de Sitter space. For the present paper the operative consequence is that the local spatial chart is three-dimensional — locally the lattice $\mathbb{Z}^3$ of spacing one Planck cell, with curvature radius $\sqrt{\Omega}$ far beyond any laboratory scale — and that adjacency is the only relational channel the substrate carries.

The quadratic extension and the boost cycle.

The Lorentzian and spinor structure lives one extension up [6,8]: on the quadratic extension K of the shell, with Frobenius involution $z \mapsto z^\sigma$, trace-zero generator η ($\eta^2 = \nu$ a nonsquare), and norm $\mathrm{N}(z) = z\,z^\sigma$. In the basis $\{1, \eta\}$ the norm is the Lorentzian form $a^2 - \nu b^2$, with the nonsquare ν supplying the temporal–spatial signature. The unit-norm circle U is the boost cycle of the framework's Dirac layer; Frobenius acts on it as inversion, supplying conjugation and time reversal.

Mass and the Compton clock.

In the framework's mass reading [9, §9.5], a mass m is a Subject of m degrees of freedom — mass is cardinality — and its rest energy is the share m of the totality, with the Planck mass the locality horizon $m_P = \sqrt{\Omega}$. The object's clock is its Compton frequency, $E = mc^2 = hf$: frequency proportional to cardinality. This reading enters the present paper as Premise 1 and is the hinge of the whole construction: massive objects are literally clocks, and their cardinality is literally their winding rate.

3. Premises

Five identifications carry the construction. Each is stated as a premise; the first three are the physical reading already carried by the FRC corpus, the fourth fixes one magnitude, and the fifth is the single new input of the relativistic layer.

Premise 1 (Mass is locked cardinality)**.** A massive object of mass m is a cluster of m substrate degrees of freedom whose phases are mutually locked. Internal coherence is what makes the cluster one object; its collective phase advances m quanta per chronon — the Compton clock.

Premise 2 (Distance is decoherence)**.** The distance r between two uncorrelated objects is the measure of their decoherence, counted in relational steps. Frame agreement propagates only between adjacent cells, so the decoherence metric coincides with the graph metric of the spatial chart.

Premise 3 (The dilation is the common drive)**.** Every cell is driven by the same chronon step of the global dilation. The Hubble flow is this drive read at the largest scale; it is the platform shared by all clocks, in the precise role the common base plays in the spontaneous synchronisation of metronomes [10–12].

Premise 4 (Unit channel capacity)**.** The coherence channel between adjacent cells carries one phase quantum per chronon: the stiffness of the phase field is $\kappa = 1$ in shell units. This premise fixes the magnitude of G and the location of the saturation scale; the inverse-square form is independent of it.

Premise 5 (Channel unity)**.** Each cell carries one correlation capacity. The clock tick (temporal correlation with the drive) and the decoherence count (spatial correlation with neighbours) are two readings of that single capacity — the scale and translation cycles of one frame torsor — so a bias of the capacity biases both readings.

On these premises the gravitational statement is mechanical: clusters of locked clocks on a common platform synchronise; synchronisation lowers a free energy whose spatial gradient is a force; and since r *is* decoherence (Premise 2), the flow that increases mutual coherence is identically motion toward smaller r.

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4. The Newtonian Layer

4.1. The Coherence Field

Let $\theta_x \in \mathbb{R}$ (linearised from the boost-cycle phase) be the phase at cell $x \in \Lambda \simeq \mathbb{Z}^3$, with the per-chronon update

$$\theta_x \;\longmapsto\; \theta_x + \omega\tau + \kappa\tau \sum_{y\sim x} \sin(\theta_y - \theta_x), \tag{1}$$

the discrete Kuramoto dynamics [10,11] with nearest-neighbour coupling — nearest-neighbour because, by Premise 2, adjacency is the only relational channel the substrate carries. Write $u_x = \theta_x - \omega t$ for the offset from the global drive and linearise about alignment.

Lemma 6 (Gradient flow)**.** *To first order in the offsets, the dynamics (1) is the gradient flow $\dot{u} = -\partial E/\partial u$ of the free energy*

$$E[u] \;=\; \frac{\kappa}{2} \sum_{\langle xy\rangle} (u_x - u_y)^2 \;-\; \sum_i m_i\, u(x_i), \tag{2}$$

where the sum over i runs over the clusters of Premise 1, located at x_i with cardinalities m_i.

Proof. Linearising $\sin(u_y - u_x)$ gives $\dot{u}_x = \kappa(\Delta u)_x$, and $\partial/\partial u_x$ of the quadratic term in (2) is $-\kappa(\Delta u)_x$. The linear term is the cluster coupling, whose coefficient is fixed by the next lemma. □

Lemma 7 (Coherent additivity)**.** *A locked cluster of cardinality m couples to the ambient field with coefficient exactly m. An unlocked ensemble of m independent phases couples with expected magnitude $O(\sqrt{m})$.*

Proof. Each member clock contributes a linear pull on its adjacent ambient cells toward its own phase. For a locked cluster all m pulls are toward the same phase and add as scalars: total coefficient m. For independent phases the pulls are random unit-strength terms on the phase circle; their sum is a random walk of m steps, of typical magnitude $\sqrt{m}$. Coherence converts root-mean-square addition into linear addition. □

Lemma 8 (Stationary bias field)**.** *The minimiser of (2) satisfies $\kappa\, \Delta u = -\sum_i m_i \delta_{x_i}$ and equals*

$$u^*(x) \;=\; \frac{1}{\kappa}\sum_i m_i\, G_\Lambda(x, x_i), \qquad G_{\mathbb{Z}^3}(x,y) \;=\; \frac{1}{4\pi\,|x-y|}\Big(1 + O(|x-y|^{-2})\Big),$$

the discrete harmonic Green's function of the three-dimensional chart, whose Coulomb asymptotics is rigorous lattice potential theory [13,14].

Proof. Stationarity of (2) is the discrete Poisson equation; the Green's function expansion for $\mathbb{Z}^3$ is classical [13,14]. The exponent is forced by the dimension: on a d-dimensional chart $G \sim r^{2-d}$, and $d = 3$ is supplied by the frame-torsor derivation summarised in Section 2, not assumed here. □

4.2. The Inverse Square Law

Theorem 9 (Newtonian limit)**.** *For two clusters of cardinalities m_1, m_2 at decoherence distance r (large in lattice units), the equilibrium free energy carries the interaction term*

$$U(r) \;=\; -\,G\,\frac{m_1 m_2}{r}, \qquad G \;=\; \frac{1}{4\pi\kappa},$$

and the force along the decoherence coordinate is

$$F(r) \;=\; -\frac{dU}{dr} \;=\; -\,G\,\frac{m_1 m_2}{r^2},$$

attractive, with relative corrections $O(r^{-2})$ from the lattice Green's function.

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Proof. Substituting the minimiser of Lemma 8 into (2) gives

$$E[u^*] \;=\; -\frac{1}{2}\sum_{i,j}\frac{m_i m_j}{\kappa}\,G_\Lambda(x_i,x_j) \;=\; (\text{self-energies}) \;-\; \frac{m_1 m_2}{4\pi\kappa\, r}\Big(1+O(r^{-2})\Big),$$

the self-energy terms independent of r. The cross term is $U(r)$; differentiation gives F. The sign is structural: for a field of positive stiffness with *linear* source coupling, the equilibrium energy decreases as like-signed sources approach — the scalar-mediator sign, opposite to electrostatics, where the energy is field-quadratic with no linear reward. Motion toward smaller r is available because, by Premise 2, r is the decoherence measure and the gradient flow of Lemma 6 increases mutual coherence: the relaxation *is* the approach. □

Proposition 10 (Universality of attraction)**.** *All masses are like-signed sources, hence all gravitational interaction is attractive.*

Proof. The sign of a source in (2) is the winding direction of its clocks, and every composed object winds in the one global time direction of the dilation (Premise 3). A repulsive gravitational charge would require a time-reversed clock, which is the Frobenius branch of the boost cycle [8] — the conjugation, not an evolution available to a composed cluster riding the drive. Universal attraction is the universality of the time direction. □

4.3. Three Corollaries

Corollary 11 (Equivalence principle as an identity)**.** *Inertial and gravitational mass are the same cardinality by construction: gravitational mass is the coherent pull of m locked clocks (Lemma 7), inertial mass is the re-phasing cost of the same m locked clocks under a change in the state of motion. Both proportionalities are one coherent additivity applied to one cluster; there is no second parameter to tune.*

Corollary 12 (The bias field is clock rate)**.** *The stationary field u^* is a clock-rate offset: the clock of a bound test mass is its Compton frequency, $E = hf$, so its rate at distance r from a mass m is shifted by the binding, $\Delta f/f = Gm/(rc^2)$ — gravitational time dilation and redshift, derived rather than imposed. Section 5 completes this temporal reading with its spatial partner.*

Corollary 13 (Turnaround against the Hubble drive)**.** *The comoving stretch of the decoherence distance contributes an outward drift of acceleration scale $H^2 r$; net approach requires $Gm/r^2 \gtrsim H^2 r$, i.e. $r \lesssim (Gm/H^2)^{1/3}$: the turnaround radius of structure formation. Bound systems are synchronised clusters decoupled from the flow — the observed non-expansion of bound structures.*

4.4. The Magnitude of G

The form of the law used only Lemmas 6–8; the magnitude uses Premise 4. With $\kappa = 1$ — one phase quantum per cell per chronon — the constant is $G = 1/4\pi$ in shell units. Restoring dimensions with the Planck mass as the locality horizon $m_P = \sqrt{\Omega}$ (Section 2) gives

$$G \;=\; \frac{\hbar c}{m_P^2},$$

the definition of the Planck mass read backwards: G is not a new constant but the statement that the coherence channel has unit capacity. The weakness of gravity is the dilution of one clock's bias across the substrate: the dimensionless coupling of two protons, $(m_p/m_P)^2 \sim 10^{-38}$, is the square of their cardinality fraction — small because Ω is large.

5. The Relativistic Layer

A coherence field read as clock rate alone is the temporal half of a metric: it reproduces the Newtonian limit and the redshift but only half of the observed light deflection, and a scalar coupled to the source's trace, as in Nordström's theory [15], deflects light not at all [16]. The completion has two

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parts — the source must be a symmetric two-index object, and the response must bias space exactly as it biases time — and both are supplied by the framework rather than added to it.

5.1. The Source: The Frobenius-Symmetric Square on the Mass Shell

Proposition 14 (Dust tensor from the quadratic extension)**.** *The state of motion of a cluster is an element* $w = a + b\eta \in K$ *on the Frobenius unit shell*

$$\mathrm{N}(w) \;=\; w\,w^{\sigma} \;=\; a^2 - \nu b^2 \;=\; 1,$$

the unit mass shell of the Lorentzian form of Section 2. The source of the coherence field is the symmetric square

$$T \;=\; m\,(w \otimes w) \;=\; m\begin{pmatrix} a^2 & ab \\ ab & b^2 \end{pmatrix},$$

whose trace under the Frobenius pairing is $m\,\mathrm{N}(w) = m$*: the rest cardinality. Its components are the energy and momentum flux of relativistic dust; at rest* ($a = 1$, $b = 0$) *it reduces to the scalar source of Lemma 6, identified thereby as the rest-frame reading of the tensor source. The source is conserved: cardinality conservation gives the temporal component, torsor equivariance the flux components.*

Proof. The mass-shell identity is the Frobenius norm in the $\{1, \eta\}$ basis; the trace identity is $\mathrm{Tr}_K(w \otimes w) = \mathrm{N}(w)$ under the pairing $\langle z, z' \rangle = z z'^{\sigma}$. Boost covariance is the statement that the boost cycle acts on w by multiplication and on T by the symmetric square of that action, preserving N; the off-diagonal ab is the momentum read by the boosted frame. □

The spatial multiplicity is left implicit, as throughout the corpus: the framework derives the temporal–spatial split from the square class of ν, not from a count of spatial directions [8,9].

5.2. Channel Unity: Space Bends Because Distance Is Correlation

Lemma 15 (The isotropic pair)**.** *To first order in the bias* u*, a cell devoting the fraction* u *of its capacity to a cluster's field presents to ambient processes the line element*

$$ds^2 \;=\; -(1-2u)\,c^2 dt^2 + (1+2u)\,\delta_{ij}\,dx^i dx^j, \qquad u = \frac{Gm}{rc^2},$$

i.e. the spatial bias equals the temporal one: the parameterised-post-Newtonian $\gamma = 1$*.*

Proof. Temporal reading: the cell's correlation with the global drive is reduced by the factor $(1-u)$, so proper time accumulates as $d\tau = (1-u)\,dt$. Spatial reading: the decoherence increment per lattice step is inversely proportional to the remaining capacity, $1/(1-u) \approx 1 + u$, so the proper distance element is $dl = (1+u)\,dx$. Both factors are the one number u by Premise 5: there is no second field to tune. The failure mode of half-deflection theories — time dilates while space stays absolute — is unavailable here, because space is itself a correlation measure and cannot stay absolute while correlation is biased. □

Proposition 16 (Full light deflection)**.** *Correlation propagates at one cell per chronon; in the biased region the effective index is* $n = 1 + 2u$*, and Fermat's principle gives the deflection of a ray of impact parameter* b*:*

$$\alpha \;=\; \int_{-\infty}^{\infty} \big|\nabla_{\perp}(n-1)\big|\,dz \;=\; \frac{4Gm}{c^2 b},$$

the observed value — twice the clock-rate-only result, the factor restored by the spatial half of Lemma 15.

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5.3. Composition: The Exponential Metric and the Classical Tests

Lemma 17 (Multiplicative composition). *Capacity renormalisations of successive cells compose multiplicatively. For an unsaturated configuration the composed factors are* $\exp(\mp U)$ *with* $U = \sum_i Gm_i/(r_i c^2)$ *the superposed potential, giving the exponential isotropic metric*

$$ds^2 = -e^{-2U} c^2 dt^2 + e^{2U} \delta_{ij}\, dx^i dx^j.$$

Validity requires every per-cell factor far from saturation, which Section 6 shows holds down to a microscopic core.

Proposition 18 (All three classical tests). *The exponential isotropic metric has* $g_{00} = -(1 - 2U + 2U^2 + O(U^3))$ *and* $g_{ij} = (1 + 2U + O(U^2))\delta_{ij}$*, hence* $\beta = 1$*,* $\gamma = 1$*: light deflection, Shapiro delay, and perihelion precession all take exactly their observed (general-relativistic) values [16].*

Beyond first post-Newtonian order the exponential metric deviates from Schwarzschild; the classical tests do not resolve the difference, and the regime where it matters is the strong-field regime of Section 6.

5.4. The Route to the Full Field Equations

The field, its gauge structure, and its coupling can be constructed exactly; the dynamics is then pinned by imported uniqueness theorems.

Proposition 19 (The deformation space is the symmetric field). *The symmetric bilinear deformations* $g = g_0 + h$ *of the norm form* $g_0 = \mathrm{diag}(1, -\nu)$ *form a three-parameter space per cell,* $h = (h_{00}, h_{01}, h_{11})$*. A cell-dependent assignment* $x \mapsto h(x)$ *is the gravitational field: the isotropic pair of Lemma 15 is the diagonal slice, and the gravitomagnetic sector is the off-diagonal* h_{01}*, conjugate to the momentum component ab of the source.*

Proposition 20 (Reframings are the gauge). *A reframing is a cell-dependent infinitesimal change of frame data,* $A(x) = 1 + \varepsilon(x)$*,* $\varepsilon \in \mathfrak{sl}_2$*, acting on forms by congruence. A constant* ε *moves* g_0 *inside its congruence orbit and changes no invariant; a varying* $\varepsilon(x)$ *enters every adjacency-local functional only through link differences of* $\xi = g_0\varepsilon$*, so the gauge orbit of the field is the discrete symmetrised gradient* $h_{\mu\nu} \mapsto h_{\mu\nu} + \nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu$*: the linearised diffeomorphism gauge, derived from the relational ontology rather than postulated.*

Proposition 21 (Coupling, conservation, and the imported uniqueness). *The unique frame-invariant linear pairing of field and source is* $h_{\mu\nu} T^{\mu\nu}$*; its gauge invariance requires* $\nabla_\mu T^{\mu\nu} = 0$*, the conservation of Proposition 14. The quadratic, adjacency-local, gauge-invariant stiffness functional of a symmetric two-index field is unique up to normalisation — the Fierz–Pauli functional [17] — and its discrete form can be exhibited at once. With central differences* Δ_μ *on the chart and* $h = \eta^{\mu\nu} h_{\mu\nu}$*,*

$$E_{\mathrm{FP}}[h] = \kappa \sum_x \Big[\tfrac{1}{4} \Delta_\lambda h_{\mu\nu} \Delta^\lambda h^{\mu\nu} - \tfrac{1}{4} \Delta_\lambda h\, \Delta^\lambda h - \tfrac{1}{2} \Delta_\mu h^{\mu\nu} \Delta^\lambda h_{\lambda\nu} + \tfrac{1}{2} \Delta_\mu h^{\mu\nu} \Delta_\nu h \Big], \tag{3}$$

overall normalisation and signature bookkeeping absorbed in κ*. This functional is* exactly *invariant under the discrete gauge orbit of Proposition 20 on the periodic chart: the continuum invariance proof uses only the commutativity of derivatives and their anti-self-adjointness under integration by parts, and central differences carry both properties exactly (*$\Delta_\mu^{\mathsf{T}} = -\Delta_\mu$ *on the periodic chart, with real Fourier symbol* $\sin k_\mu$ *replacing* k_μ *in the continuum identity); one-sided differences, whose adjoint is not their negative, do not, and the invariance fails for them. Both statements are verified to machine precision in the validation suite. Uniqueness within the adjacency-local class is expected from the continuum theorem and remains to be proven on the shell. The requirement that the field couple to its own energy (forced here: the field's energy is itself a cardinality share, and everything carrying cardinality gravitates by Premise 1) then determines the nonlinear completion by Deser's first-order bootstrap [18]. What remains open is therefore no longer the functional itself but its* second-order evaluation*: the self-energy that (3) assigns to its own field, fed back as source. Section 6 shows this evaluation* *is the decision point of the theory's single remaining branch choice, on which the continuum and shell readings part.*

Proposition 22 (No preferred-frame leakage; frame dragging at the observed value)**.** *The framework possesses a global rest frame — the drive — yet the local dynamics inherits no preferred-frame parameters at leading order: source and channel are torsor-equivariant, so boosts act covariantly on all local data, and the drive enters local physics only through the resolution floor, at $O(1/\sqrt{\Omega})$. Hence the preferred-frame parameters obey $\alpha_1, \alpha_2 = O(1/\sqrt{\Omega})$, and with $\gamma = 1$ the gravitomagnetic sector takes its conservative value: the Lense–Thirring precession is the observed $\Omega_{\rm LT} = 2GJ/(c^2r^3)$ up to $O(1/\sqrt{\Omega})$ [16]. The residual is itself a prediction: preferred-frame effects of order 10^{-61}, far below the current 10^{-5} bounds, but the drive's in-principle signature.*

6. The Strong Field

The linearisation of Lemma 6 fails where the bias approaches the channel capacity, and the failure is not a defect but the theory's strong-field content: the capacity bound is exact, and the static problem can be solved against it in full.

Lemma 23 (Capacity bound, exact)**.** *The interaction* (1) *transfers at most κ phase quanta per link per chronon* ($|\sin| \le 1$)*. No static synchronised configuration exists in any region whose maintenance demands more.*

Proposition 24 (Exact static profile)**.** *In shell units, spherically symmetric statics requires the synchronisation flux through every sphere to carry the source: $4\pi r^2 \kappa \sin u'(r) = m$, hence*

$$u'(r) \;=\; \arcsin\Big(\frac{Gm}{r^2}\Big), \qquad u(r) \;=\; \frac{Gm}{r}\Big(1 + \tfrac{1}{30}\big(\tfrac{Gm}{r^2}\big)^2 + \dots\Big),$$

defined exactly on $r \ge r_* = \sqrt{Gm}$ — in physical units $r_* = \sqrt{r_g\,\ell_P}$, the geometric mean of the gravitational radius and the Planck length — and undefined inside: the static synchronised exterior exists down to r_* and no further. Inside r_* phase slips (the depinning of a driven oscillator past its lock-in threshold) replace synchrony: a dynamical slip core, *microscopic on every astrophysical scale (for a solar mass, $r_* \sim 10^{-16}$ m).*

Proof. Gauss's theorem for the discrete dynamics: in a static locked state the net transfer into every closed region vanishes except for the source demand, and a link carries at most κ sin of its phase difference. Monotone radial profiles give the flux law; arcsin requires its argument at most 1, with equality at r_*; the expansion of $u = \int_r^\infty \arcsin(Gm/s^2)\,ds$ is term-by-term. □

Proposition 25 (Horizonless, operationally black)**.** *The clock of a bound test mass is its Compton frequency, and capacity renormalisations compose multiplicatively (Lemma 17), the composition validated down to r_* by the exact profile: the per-cell increment $u'(r)\ell_P$ remains far below saturation at every $r \gg r_*$. Hence*

$$f(r) \;=\; f_\infty\, e^{-u(r)},$$

which vanishes at no finite r: the exact theory is horizonless. *The rate reaches the resolution floor $\Omega^{-1/2}$ at $u = \frac{1}{2}\ln\Omega$, i.e. at*

$$r_f \;=\; \frac{2Gm}{c^2\ln\Omega} \;\approx\; \frac{r_s}{281},$$

inside which no clock is resolvable in any observer frame: operational blackness. Outward correlation export across the surrounding region is redshift-suppressed by the same factor, so the surface is operationally one-way.

Remark 26 (The branch alternative, stated precisely)**.** The two candidate completions agree through first post-Newtonian order ($\beta = \gamma = 1$) and part at second. The continuum bootstrap, taken at face value, yields the Einstein dynamics, whose static vacuum exterior is Schwarzschild — isotropic $g_{00} = 1 - 2U + 2U^2 - \frac{3}{2}U^3 + \dots$, a sharp horizon, general-relativistic strong-field coefficients. The shell's own composition law (Lemma 17) yields the exponential metric, $g_{00} = 1 - 2U + 2U^2 - \frac{4}{3}U^3 + \dots$, horizonless and operationally black. The second-order evaluation of the discrete functional (3) arbitrates: either its self-coupling reproduces the continuum bootstrap, and the strong field is GR-coincident, or the shell composition survives at second order, and the signatures of Section 7 stand. The burden of proof presently lies on the exponential branch, since the continuum uniqueness theorems pull against it; every strong-field number in this paper is accordingly stated as *conditional on the exponential branch* and falsifiable as such, while the Newtonian and post-Newtonian layers, the entropy scaling, and the acceleration floor are branch-independent.

Proposition 27 (Area-law entropy, and the de Sitter closure)**.** *The externally resolvable data of the operationally black region is carried by the channels of its bounding surface, numbering A_f/ℓ_P^2 up to an order-one factor, each running at capacity with one undetermined slip phase: $S = c'_S\, A_f/\ell_P^2$, the area scaling derived, the coefficient open. Relative to the Bekenstein–Hawking value [19,20], $S/S_{\rm BH} = (r_f/r_s)^2 = \ln^{-2}\Omega \approx 1.3\times10^{-5}$. Applied to the cosmological horizon of radius $\sqrt{\Omega}$ Planck lengths, the same area law gives $S\sim\Omega$ up to logarithmic factors: the de Sitter entropy that fixes the Carrier cardinality (Section 2) is recovered by the framework's own horizon law, closing the loop between the assumed substrate size and the derived horizon thermodynamics.*

Remark 28 (Phase-slip radiation)**.** At the threshold the links hover at lock-in, where the residual floor noise drives stochastic slips with characteristic frequency set by the offset gradient — the surface gravity — giving emission at the Hawking scaling $k_BT\sim\hbar\kappa_{\rm sg}/2\pi c$ [20]. On the exponential branch the emission is additionally redshifted by $\Omega^{-1/2}$: luminosity below one quantum per Hubble time. Frozen objects do not evaporate on any relevant timescale.

7. Observational Consequences

The global static solution reads, from the outside in: the post-Newtonian zone $r\gg r_g$, where $\beta=\gamma=1$ and all classical tests hold; the exponential zone $r_*\ll r\lesssim r_g$, where the metric is $-e^{-2u}c^2dt^2+e^{2u}d\mathbf{x}^2$ with the exact potential of Proposition 24 (whose correction to Gm/r at the photon sphere is zero to floor precision for any astrophysical mass); the operational horizon at $r_f=r_s/\ln\Omega$; and the cloaked slip core at r_*. Its observational signatures are parameter-free.

7.1. Strong-Field Signatures Against Current Data

Proposition 29 (Photon sphere and shadow)**.** *Null circular orbits of the exponential zone extremise $b(r) = r\,e^{2u(r)}$: with $u = Gm/(c^2r)$,*

$$r_{\rm ph} = \frac{2Gm}{c^2}\ \textit{exactly},\qquad b_c = \frac{2e\,Gm}{c^2}\approx 5.437\,\frac{Gm}{c^2},$$

against the Schwarzschild $b_c = 3\sqrt{3}\,Gm/c^2\approx 5.196\,Gm/c^2$: the shadow is

$$\delta \;=\; \frac{2e}{3\sqrt{3}}-1 \;=\; +4.63\%$$

wider, independent of mass, distance, and environment.

Sgr A* is the clean arena, its mass-to-distance ratio fixed independently by stellar orbits. With the GRAVITY values ($M = 4.297\times10^6 M_\odot$, $D = 8.277$ kpc, $\theta_g = 5.12\,\mu$as) [21], the predicted shadow diameters are 53.3 μas (general relativity) against 55.7 μas (this work): an excess of 2.5 μas. The Event Horizon Telescope's fractional-deviation constraints, $\delta = -0.04^{+0.09}_{-0.10}$ (Keck priors) and -0.08 ± 0.09 (VLTI priors) [22], place the prediction about one standard deviation high: not excluded, not confirmed, inside the discovery window; next-generation arrays targeting percent-level shadow calibration are decisive. For M87* the corresponding numbers are 39.7 against 41.5 μas, with the caveat that its mass partly derives from the ring itself.

The independent twin rides in gravitational waves: at fixed inspiral-determined mass the eikonal ringdown frequency scales as $1/b_c$, so $f/f_{\rm GR} = 3\sqrt{3}/2e = 0.956$: a -4.4% shift, approachable by stacked events and decisive at Einstein Telescope sensitivity. Late-time echoes, the usual signature of

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horizonless ultracompact objects [23], are by contrast *absent*: the round-trip delay to the floor surface carries the factor $e^{2u(r_f)} = \Omega$, giving $\Delta t \sim (2r_g/c)\,\Omega/\ln^2\Omega \sim 10^{118}$ s — the object mimics a black hole in that channel, and a reported echo detection would falsify this branch. The joint signature — shadow wide by 4.6%, ringdown low by 4.4%, no echoes, no evaporation bursts — has one origin and no tuning. The outcome logic of Remark 26 applies: a null result selects the linear branch, whose strong field is GR-coincident; the predictions are sharp against the exponential branch only. The prediction is stated for slow rotation: the rotating solution is unbuilt, and Kerr spin shifts the general-relativistic mean shadow diameter by up to $\sim 4\%$, making low-spin Sgr A* the preferred target.

7.2. The Weak-Acceleration Floor and the Radial Acceleration Relation

Across rotationally supported galaxies the observed centripetal acceleration follows the baryonic one through a single universal curve with scatter ~ 0.1 dex, transitioning at $a_0 = 1.20 \pm 0.02\,(\text{stat}) \pm 0.24\,(\text{syst}) \times 10^{-10}\,\text{m}\,\text{s}^{-2}$ — the radial acceleration relation [24,25]. The standard cosmology reproduces it only through galaxy-by-galaxy feedback tuning and does not predict the scale; modified-dynamics phenomenology [26] postulates the scale without deriving it; the numerical coincidence $a_0 \approx cH_0/2\pi$ has remained a coincidence [27].

Proposition 30 (The floor acceleration)**.** *The drive decorrelates every link at the Hubble rate: the resolution floor $1/\sqrt{\Omega}$ is, in the synchronisation reading, a phase-noise floor of rate H. A static synchronisation gradient — a gravitational acceleration $g = c^2\nabla u$ — is signal against that floor only if it accumulates at least one phase cycle per Hubble time, i.e. above*

$$a_0 \;=\; \frac{cH_0}{2\pi} \;=\; 1.08 \times 10^{-10}\,\text{m}\,\text{s}^{-2}\;(H_0 = 70)\,, \qquad 1.13 \times 10^{-10}\;(H_0 = 73)\,,$$

against the fitted $1.20 \pm 0.24 \times 10^{-10}$: agreement within ten percent, with no parameter. Equivalently, a_0 is the acceleration whose Unruh temperature matches the de Sitter temperature [27]; in this framework the two statements are one, since both temperatures are readings of the same floor. The 2π-level normalisation has the same status as the strong-field coefficients: owed to the stiffness construction, with the scale cH_0 derived.

Lemma 31 (Flux conservation under noise)**.** *The floor noise does not modify the time-averaged static force law: in a statistically stationary state the mean synchronisation flux through any sphere enclosing a source of cardinality m equals m by Gauss's theorem, slips redistributing the transfer in time but not in mean.*

Remark 32 (The deep regime, constrained and redirected)**.** Lemma 31 is a constraint the analysis itself imposes on any account of the deep regime: the empirical enhancement $g_{\text{eff}} = \sqrt{g_N\,a_0}$ at $g_N < a_0$ — the deep-MOND limit that fits flat rotation curves and the baryonic Tully–Fisher relation [25] — *cannot* arise as a modification of the mean static force, and naive sub-threshold rectification heuristics are thereby excluded. The enhancement, if this framework produces it, must be dynamical: the response of orbits to floor-scale force *fluctuations*, whose amplitude at the threshold is set by a_0 and whose spatial correlations are seeded by the source. No quantitative model of this dynamics yet exists; the radial acceleration relation's observed tightness (0.11 dex) is the figure of merit any candidate must meet, and the bounded next step is fully specified: simulate (1) with drive-noise rate H and a central source, and measure the orbit-averaged effective acceleration as a function of g_N/a_0. The scale derivation of Proposition 30 is independent of this open dynamics and stands on its own.

Two predictions separate this account from both incumbents. From fundamental modified dynamics: a_0 is not a constant of nature but the current noise floor, so it tracks the expansion, $a_0(z) \propto H(z)$ — roughly tripled at $z = 2$ — making high-redshift rotation curves the deciding dataset; the existing high-z samples [28] are precisely the contested arena, and the test cuts both ways. From the standard cosmology: the relation's scale and universality are structural, not emergent from feedback, so its scatter at fixed z has a floor set by frame variance rather than formation history. A third observable discriminates the mechanism itself: a floor set by the global drive, not by the local field, fixes the external-field dependence of the threshold, and the claimed detection of an external-field effect in

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the SPARC sample [29] — if it survives scrutiny — strains the naive floor reading and would force the sub-threshold dynamics to carry the environmental dependence; this is recorded as the sharpest pressure point of the subsection.

7.3. Explicability Dividends

Two items are dissolved rather than computed. The cosmological-constant magnitude: $\Lambda \sim 1/\Omega$ in Planck units is the curvature of the closed chart — the totality datum — not a vacuum energy requiring cancellation across 122 orders; the fine-tuning problem is not solved but unasked. And the large-angle anomaly of the microwave background (the anomalously low large-angle correlation, $p \sim 1\%$ [30]) is the natural signature of the wrapped chart: super-horizon modes are identified, not populated, truncating correlations beyond the horizon scale; turning this consonance into a number requires the framework's primordial spectrum, which does not yet exist and is recorded as open. To these add the dividends of the body: the equivalence principle as an identity (Corollary 11), the weakness of gravity as cardinality dilution (Subsection 4.4), and the absence of a graviton fine-tuning problem — the field is collective synchronisation, not a propagating fundamental quantum.

8. Status: What Is Exact, What Is Premise, What Is Open

Exact. Lemmas 6 and 7 are elementary; Lemma 8 rests on rigorous lattice potential theory [13,14]; Theorem 9 is a substitution. Proposition 14 and the deformation/gauge structure of Propositions 19–20 are finite algebra on the quadratic extension. Lemma 15 and Propositions 16 and 18 are first-order computations from Premise 5, with $\beta = \gamma = 1$ and the classical tests following arithmetically. Lemma 23 is exact at the discrete level; Proposition 24 solves the saturating statics exactly; and the photon sphere, shadow, and ringdown shift are exact consequences of the exponential zone. The dimension $d = 3$ is supplied by the framework's frame-torsor derivation and used as input.

Premises. Five, stated in Section 3. The first three are the physical reading carried by the FRC corpus; the fourth fixes the magnitude of G and the saturation scale; the fifth asserts only that in a framework where distance is correlation there is one correlation, not two. Nothing else is consumed: no field equation, no action principle, and no metric are posited — the potential, the force, the equivalence principle, the redshift, the deflection, the perihelion, the frame dragging, the turnaround radius, the operational horizon, the area law, and the acceleration floor are outputs.

Open. One keystone evaluation and two bounded follow-ups. The keystone: the discrete Fierz–Pauli functional is exhibited in (3) and exactly gauge-invariant, with two items outstanding — its uniqueness within the adjacency-local class on the shell, and above all its *second-order self-coupling evaluation*, which arbitrates the strong-field branch (Remark 26) and owes the remaining normalisations: c'_S, the radiation constant, and the 2π of the acceleration floor. Until that evaluation is carried out, every strong-field signature of Section 7 is conditional on the exponential branch, against which the continuum uniqueness theorems presently weigh; the Newtonian and post-Newtonian results, the entropy scaling, and the acceleration floor are branch-independent. The follow-ups: the sub-threshold dynamics of the weak-acceleration regime — now constrained by Lemma 31 to be a dynamical fluctuation effect, not a static force modification, with the specified lattice experiment as its next step and the radial acceleration relation's 0.11 dex tightness as its figure of merit — and the framework's primordial spectrum, required to turn the large-angle consonance into a number. The rotating strong-field solution is likewise unbuilt.

What the synchronisation reading adds to gravitation is the provenance of each part: the product $m_1 m_2$ from coherent additivity, the r^{-2} from harmonicity in the three frame freedoms, the universal sign from the shared arrow of the dilation, G from the unit capacity of the relational channel, the equality of space and time bias from the unity of that channel, the slip core from its saturation, the operational horizon from its resolution floor — and, at the opposite end of the acceleration scale, the galactic transition from the same floor read as noise. One substrate, one channel, one floor; the constants are its bookkeeping.

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Reproducibility

Every quantitative claim of the paper is independently verified by a numeric validation suite: the exact lattice Green's function and the inverse-square law, the post-Newtonian parameters and classical-test factors, the exact static profile, the photon sphere and shadow, the discrete Fierz–Pauli gauge invariance (with the failure of one-sided differences retained as a control), flux conservation under drive noise, and the acceleration floor against the SPARC value. The suite, with a per-claim mapping to the propositions of the text, is available at https://github.com/gamayos/frc-gravity-numerics.

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