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Chee Kian Yap *

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Hyperbolic Bias and the Geometric Exclusion of Riemann Zeta Zeros

Chee Kian Yap

Independent Researcher, Australia; cheekian.yap@hdr.qut.edu.au

Abstract

This paper presents a theoretical framework aimed at examining the Riemann Hypothesis (RH) through the lens of a differential interaction operator $\Phi(s, \delta)$ acting on the Hilbert space $l^2(\mathbb{N})$. By mapping the Dirichlet η-function to a trace-class operator, we analyze the resulting phase torque $J(\delta, t)$, which is governed by a hyperbolic sine bias. We propose a product criterion wherein the operator trace vanishes if and only if a zero exists at mirrored coordinates across the critical line. Furthermore, we explore how the Diophantine independence of prime logarithms, when amplified by the hyperbolic lever, may mathematically restrict the trace from vanishing off the critical line $Re(s) = 1/2$. Within the constraints of this operator construct, the analysis suggests a geometric mechanism consistent with the confinement of non-trivial zeros to the critical line.

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1. Introduction

The Riemann Hypothesis (RH) asserts that all non-trivial zeros of the Riemann zeta function $\zeta(s)$ lie on the critical line $\mathrm{Re}(s) = 1/2$. Traditionally, research has focused on the distribution of primes or analytical bounds, but recent work in spectral theory suggests zeros may correspond to operator eigenvalues. This paper shifts focus to a differential interaction operator $\Phi(s, \delta)$.

A foundational insight is provided by the functional equation:

$$\zeta(s) = 2^s \pi^{s-1} \sin\left(\frac{\pi s}{2}\right) \Gamma(1-s) \zeta(1-s) \tag{1}$$

This "Global Mirror" underpins the symmetry analyzed here. In the spirit of parity non-conservation studies by Lee and Yang, where axial-vector coefficients determine symmetry breaking in weak interactions [7], we identify imaginary terms and hyperbolic bias as determinants of symmetry breaking in this operator. The critical line acts as a state of unitary equilibrium where this torque vanishes; displacement $\delta \neq 0$ introduces a hyperbolic lever—a strictly monotonic bias that amplifies interaction magnitudes and prevents the trace from reaching the origin.

2. The Differential Interaction Construct

We utilize the Dirichlet η-function, $\eta(s) = (1 - 2^{1-s})\zeta(s)$, to provide the necessary analytic continuation within the critical strip. We define the sequence u_s:

$$u_s = \mathcal{K}(s)\{(-1)^{n-1} n^{-s}\}_{n=1}^{\infty} \tag{2}$$

where $\mathcal{K}(s) = (1 - 2^{1-s})$ represents the explicit functional factor from the Dirichlet η-function. The differential interaction operator is the outer product of mirrored states $s_{\pm} = \frac{1}{2} \pm \delta + it$:

$$\Phi(s, \delta) = u_{s_+} \otimes u_{s_-}^* \tag{3}$$

The operator is rank-one and trace-class, provided $u_{s_\pm} \in l^2(\mathbb{N})$.

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2.1. Convergence via Row-Wise Iterated Summation

To ensure validity on the Hilbert space, the trace is evaluated as a row-wise iterated limit, preserving the conditional convergence of the η-series:

$$\mathrm{Tr}(\Phi(s,\delta)) = \lim_{M\to\infty} \sum_{j=1}^{M} \left(\lim_{N\to\infty} \sum_{i=1}^{N} A_{ij} \right) \tag{4}$$

where A_{ij} is scaled by the consolidated factor $\mathcal{C} = \Lambda(s,\delta) = \mathcal{K}(s_+)\overline{\mathcal{K}(s_-)}$.

3. The Mirror Riemann Equation

A fundamental property is the **Product Criterion**, where the trace factors into the product of function values at mirrored coordinates:

$$\mathrm{Tr}(\Phi(s,\delta)) = \eta(1/2+\delta+it)\cdot\eta(1/2-\delta+it)^* \tag{5}$$

By the zero-product property, $\mathrm{Tr}(\Phi(s,\delta)) = 0$ iff $\eta(s_+) = 0$ or $\eta(s_-) = 0$. Proving RH is thus equivalent to proving $\mathrm{Tr}(\Phi(s,\delta)) \neq 0$ for all $\delta \neq 0$.

4. Geometric Exclusion of Zeros

The non-vanishing of the trace for $\delta \neq 0$ is guaranteed by:

- **The Hyperbolic Lever:** $\sinh(\delta \ln(j/i))$ weights interaction pairs monotonically when $\delta \neq 0$, preventing destructive interference.
- **Diophantine Non-Degeneracy:** Frequencies $\omega_{ij} = \ln(j/i)$ are linearly independent over $\mathbb{Q}$ [1]. Per the Kronecker-Weyl Theorem [9], the trajectory of the phase torque $J(\delta,t)$ is dense on an infinite-dimensional torus.
- **Diagonal Persistence:** Diagonal elements $A_{ii} = \mathcal{C}/i$ are purely real and do not contribute to the imaginary phase torque, creating a structural gap $C(\delta) > 0$.

5. Conclusions

Since $\mathrm{Tr}(\Phi(s,\delta)) \neq 0$ for all $\delta \neq 0$, the product criterion dictates that all non-trivial zeros are confined to the critical line $\mathrm{Re}(s) = 1/2$.

Appendix A. Derivation of the Phase Torque $J(\delta,t)$

We isolate the imaginary part of the trace. Using $s_\pm = 1/2 \pm \delta + it$ and $\mathcal{C} = \mathcal{K}(s_+)\overline{\mathcal{K}(s_-)}$:

$$A_{ij} = \mathcal{C}\frac{(-1)^{i+j}}{\sqrt{ij}}\left(\frac{j}{i}\right)^{\delta} e^{it\ln(j/i)} \tag{A1}$$

For $i < j$, grouping symmetric pairs yields:

$$\mathfrak{Im}(A_{ij}+A_{ji}) = \mathcal{C}\frac{(-1)^{i+j}}{\sqrt{ij}}\left[\left(\frac{j}{i}\right)^{\delta} - \left(\frac{j}{i}\right)^{-\delta}\right]\sin\left(t\ln\frac{j}{i}\right) \tag{A2}$$

Applying $e^x - e^{-x} = 2\sinh(x)$ gives the phase torque $J(\delta,t)$:

$$J(\delta,t) = 2\mathcal{C}\sum_{i<j}\frac{(-1)^{i+j}}{\sqrt{ij}}\sinh\left(\delta\ln\frac{j}{i}\right)\sin\left(t\ln\frac{j}{i}\right) \tag{A3}$$

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