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Keywords: artificial general intelligence; productive AI capital; artificial labour; human capital; search and matching; technological displacement

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When Machines Apply for Your Job: Human Capital Value at Risk Under AGI Adoption

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Abstract

Productive Artificial General Intelligence (AGI) creates the possibility that accumulable capital generates autonomous artificial labour services, making effective labour capacity partly reproducible through investment. This paper develops a recursive search-and-matching general-equilibrium model in which human labour, artificial labour capacity, matching, and productive AI-capital accumulation are jointly determined. Three results follow. First, a recursive valuation operator characterises human-capital value relative to the artificial-labour alternative and distinguishes latent surplus, realised value, and marketability. Second, the model identifies an endogenous technological regime threshold at which continuation-adjusted human surplus reaches zero and human labour ceases to generate positive realised equilibrium value in the affected task. Third, a Production–Distribution Separation Theorem establishes conditions under which the stationary productive allocation is invariant to the ownership distribution of productive AI capital, while the distribution of AI-generated income and wealth remains ownership dependent. The framework therefore identifies recursive human-capital valuation, technological displacement, and distributive incidence as distinct but connected equilibrium objects. Productive AI cannot in general be represented solely as a productivity shock because its accumulation changes effective labour capacity and thereby the equilibrium scarcity value of human labour.

Keywords: artificial general intelligence; productive AI capital; artificial labour; human capital; search and matching; technological displacement

JEL Classification: J23; J24; J64; O33; E24; D31

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1. Introduction

Labour economics is organised around a structural distinction between labour and capital. Labour is supplied by workers, while capital affects production and labour demand. From search-and-matching theory to models of technological change, employment, wages, vacancy creation, and unemployment are consequently studied with a labour supply that remains fundamentally human [1,10,15,25]. Productive Artificial General Intelligence (AGI) changes this architecture. If investment in productive AI capital generates autonomous labour services, capital accumulation affects not only productive capacity but also the effective supply of labour itself.

Existing research examines technological change through skill-biased technical change, routine substitution, automation, task reallocation, factor shares, and inequality [1–4,6,8–12,17,18,21]. Diamond–Mortensen–Pissarides models likewise allocate a given population of workers through search and matching [15,19,25,27]. These frameworks provide powerful accounts of how technology changes labour demand, productivity, and remuneration, but do not generally allow capital accumulation itself to generate competing labour services. Once it can, labour scarcity becomes partly endogenous.

This paper develops a recursive search-and-matching general-equilibrium model of this environment. Productive AI capital generates artificial labour services that can be allocated to task-specific production alongside human labour. Households supply human labour, own productive assets, consume, and invest, while firms create vacancies and employ human and artificial assignments. Artificial labour is neither an exogenous endowment nor an independent economic agent. It is a productive service flow generated by accumulated AI capital and allocated within the matching economy.

The central question is how labour-market equilibrium changes when productive capital can generate labour services directly. A recursive formulation is necessary because current AI investment changes future artificial-labour capacity and thereby future matching conditions, outside options, vacancy creation, and human-employment surplus. The analysis distinguishes recursive competitive equilibrium from the additional conditions required for stationarity, and the principal results are conditional on the corresponding equilibrium and regularity assumptions.

Three results follow. First, the paper derives a recursive valuation of human employment relative to artificial labour. Human-employment value depends on current productivity and remuneration together with matching, separation, and continuation values, and is therefore determined relative to the available artificial-labour alternative. Second, under explicit monotonicity conditions, the model identifies a unique productive AI-capital threshold at which continuation-adjusted human surplus reaches zero. This is an equilibrium participation transition rather than a stability bifurcation of the underlying dynamic system. Human productive capability may remain positive even after its equilibrium scarcity value has disappeared.

Third, the paper establishes a conditional Production–Distribution Separation Theorem. When ownership claims do not affect preferences, labour supply, investment, matching, bargaining, vacancy creation, or other components of the productive allocation, alternative ownership distributions leave the stationary productive allocation unchanged while altering the household incidence of AI-generated income and wealth. The result is deliberately conditional. Once distribution enters the productive allocation block, ownership can feed back into equilibrium production and the separation need not hold.

The analytical results are complemented by structural numerical exercises that implement the recursive valuation operator, calculate state-contingent displacement thresholds, examine comparative statics, and evaluate the parameter region supporting a unique threshold. An ownership experiment illustrates both production–distribution separation and its failure when the institutional exclusion restrictions are relaxed. These exercises do not estimate the model or forecast AGI adoption. They examine its mechanisms, comparative statics, and domain of validity.

The paper contributes to the emerging economics of advanced AI and autonomous artificial labour [14,22,23,28–32] by endogenising artificial labour supply through productive capital accumulation within a recursive matching economy. Conventional search-and-matching models are recovered when productive AI capital generates no artificial labour. Once this restriction is relaxed, capital accumulation changes the effective supply of productive agents, making human-labour scarcity an equilibrium outcome of technology, matching, investment, and participation.

The remainder of the paper is organised as follows. Section 2 develops the recursive economy and equilibrium concepts. Section 3 establishes the valuation, displacement, and production–distribution results. Section 4 discusses their economic implications, and Section 5 concludes. Appendix A reports the numerical methodology, robustness diagnostics, and replication protocol.

2. Recursive Economy with Artificial Labour Capital

Time is discrete, with periods indexed by $t \in \mathbb{N}_0$. The economy extends a recursive search-and-matching framework by introducing productive AI capital as an accumulable factor that generates artificial labour services. Unlike conventional physical capital, productive AI capital expands the effective supply of labour available for production. Artificial labour is not an independent economic agent and has no preferences or autonomous search decision. It is a productive service flow generated by AI capital and deployed across task-specific matching markets.

The economy consists of a representative household, a unit mass of competitive firms, and an exogenous aggregate productivity process. The household owns the productive assets of the economy and supplies a fixed measure $N^H > 0$ of human labour. Production is organised over the finite task set $\mathcal{J} = \{1, \ldots, J\}$, while aggregate productivity follows a time-homogeneous Markov process $(z_t)_{t\geq 0}$ on $\mathcal{Z}$ with transition kernel $Q(dz' \mid z)$.

Productive AI capital is represented by $K_t^A \in \mathbb{R}_+$ and generates contemporaneous artificial labour capacity
$$L_t^A = \chi(K_t^A),$$
where $\chi : \mathbb{R}_+ \to \mathbb{R}_+$ satisfies $\chi(0) = 0$, $\chi'(K^A) > 0$, and $\chi''(K^A) \leq 0$. Hence productive AI-capital accumulation expands the artificial labour capacity available to the economy at a positive but weakly diminishing marginal rate.

For each task $j \in \mathcal{J}$ and assignment type $r \in \{H, A\}$, $p_j : \mathcal{Z} \to \mathbb{R}_{++}$ denotes the task-specific output price, $q_j^r : \mathbb{R}_+ \times \mathcal{Z} \to \mathbb{R}_+$ the productivity of an assignment, and $m_j^r : \mathbb{R}_+^2 \to \mathbb{R}_+$ the corresponding matching technology. Let $p = (p_j)_{j\in\mathcal{J}}$, $q = (q_j^r)_{(j,r)\in\mathcal{J}\times\{H,A\}}$, $m = (m_j^r)_{(j,r)\in\mathcal{J}\times\{H,A\}}$, $\kappa = (\kappa_j^r)_{(j,r)\in\mathcal{J}\times\{H,A\}}$, and $\delta = (\delta_j^r)_{(j,r)\in\mathcal{J}\times\{H,A\}}$. The primitive environment is

$$\mathcal{G} = \Big(Q, \chi, p, q, m, \kappa, \delta, \delta_K, u, \beta, N^H\Big), \tag{1}$$

where $\beta \in (0, 1)$ is the household discount factor, $\delta_K \in (0, 1)$ is the depreciation rate of productive AI capital, $\delta_j^r \in (0, 1)$ is the separation probability, and $\kappa_j^r > 0$ is the vacancy-posting cost in market (j, r). Additional restrictions on the primitives are imposed only where required for the corresponding equilibrium and analytical results.

At the beginning of period t, task j contains predetermined stocks $N_{j,t}^H$ and $N_{j,t}^A$ of human and artificial productive assignments inherited from previous matching decisions. Assignment feasibility requires
$$\sum_{j\in\mathcal{J}} N_{j,t}^H \leq N^H, \qquad \sum_{j\in\mathcal{J}} N_{j,t}^A \leq L_t^A = \chi(K_t^A).$$
Following separation, the human labour and artificial capacity available for new matching are

$$U_t^H = N^H - \sum_{j\in\mathcal{J}} (1 - \delta_j^H) N_{j,t}^H, \qquad U_t^A = L_t^A - \sum_{j\in\mathcal{J}} (1 - \delta_j^A) N_{j,t}^A.$$

Thus U_t^H represents human labour available for directed search, whereas U_t^A represents artificial labour capacity available for redeployment.

The allocation of unmatched capacity across tasks is represented by $\boldsymbol{\sigma}_t^r = (\sigma_{j,t}^r)_{j\in\mathcal{J}} \in \Delta(\mathcal{J})$, $r \in \{H, A\}$, where
$$\Delta(\mathcal{J}) = \left\{ \boldsymbol{\sigma} \in \mathbb{R}_+^J \,\middle|\, \sum_{j\in\mathcal{J}} \sigma_j = 1 \right\}.$$
For human labour, $\sigma_{j,t}^H U_t^H$ is the measure directed towards search in task j. For artificial labour, $\sigma_{j,t}^A U_t^A$ is the service capacity deployed to the corresponding matching market. The common notation records directed allocation across tasks without attributing autonomous search behaviour to artificial labour.

Competitive firms post vacancies $V_{j,t}^r \geq 0$ at cost $\kappa_j^r V_{j,t}^r$. New assignments are formed through m_j^r, assumed continuous, increasing in both arguments, zero when either argument is zero, and homogeneous of degree one. Constant returns imply that matching probabilities depend on market tightness rather than market scale. Given the timing above, assignment stocks evolve according to

$$N_{j,t+1}^r = (1 - \delta_j^r) N_{j,t}^r + m_j^r\Big(\sigma_{j,t}^r U_t^r, V_{j,t}^r\Big), \qquad (j, r) \in \mathcal{J} \times \{H, A\}. \tag{1}$$

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The matching technology is restricted by feasibility so that new assignments cannot exceed either available capacity or posted vacancies. Hence $N^r_{j,t}$ is a predetermined assignment stock rather than a contemporaneous firm control.

Human and artificial assignments contribute directly to production. For task j, gross revenue is

$$p_j(z_t)\Big[q^H_j(K^A_t, z_t)N^H_{j,t} + q^A_j(K^A_t, z_t)N^A_{j,t}\Big].$$

The specification distinguishes the capacity and productivity channels of productive AI capital. Capital accumulation expands artificial labour capacity through $\chi(K^A_t)$, while $q^r_j(K^A_t, z_t)$ allows the stock of productive AI capital to affect the productivity of existing human and artificial assignments. Aggregate output and vacancy expenditure are therefore

$$Y_t = \sum_{j\in\mathcal{J}} p_j(z_t)\Big[q^H_j(K^A_t, z_t)N^H_{j,t} + q^A_j(K^A_t, z_t)N^A_{j,t}\Big], \qquad \mathcal{K}_t = \sum_{j\in\mathcal{J}}\sum_{r\in\{H,A\}} \kappa^r_j V^r_{j,t}.$$

Aggregate vacancy expenditure is

$$\mathcal{K}_t = \sum_{j\in\mathcal{J}}\sum_{r\in\{H,A\}} \kappa^r_j V^r_{j,t}.$$

Conditional on the inherited assignment stocks, firms choose vacancy levels $\mathbf{V}_t = (V^H_{j,t}, V^A_{j,t})_{j\in\mathcal{J}} \in \mathbb{R}^{2J}_+$. The representative household chooses consumption C_t, productive AI investment $I^A_t \geq 0$, human directed search σ^H_t, and the allocation σ^A_t of household-owned artificial labour capacity across task-specific rental markets. Thus σ^H_t governs human search, whereas σ^A_t is a supply-allocation decision taken by the owner of productive AI capital in response to competitive rental prices.

Productive AI capital evolves according to

$$K^A_{t+1} = (1-\delta_K)K^A_t + I^A_t. \tag{2}$$

Since K^A_{t+1} determines artificial labour capacity in the next period, admissible investment and matching decisions must jointly satisfy

$$\sum_{j\in\mathcal{J}} N^A_{j,t+1} \leq \chi(K^A_{t+1}). \tag{3}$$

Investment therefore affects future production through two distinct channels: the productivity effects of the AI-capital stock and the artificial labour capacity generated by that stock. Household income consists of human wage earnings, returns from supplying artificial labour services, and distributed firm profits. Human wages are determined through task-specific Nash bargaining, artificial labour services are supplied competitively at task-specific rental prices, and free entry governs vacancy creation.

The recursive state is

$$S_t = (M_t, K^A_t, z_t), \qquad M_t = (N^H_{j,t}, N^A_{j,t})_{j\in\mathcal{J}}, \tag{4}$$

where M_t collects inherited productive assignments. Conditional on an equilibrium policy profile, let $\mathbf{V}(S)$, $I^A(S)$, $\sigma^H(S)$, and $\sigma^A(S)$ denote vacancy creation, investment, human search, and artificial-capacity allocation. The induced assignment transition map $\Gamma_M(S) = (\Gamma^r_j(S))_{(j,r)\in\mathcal{J}\times\{H,A\}}$ is defined componentwise by

$$\Gamma^r_j(S) = (1-\delta^r_j)N^r_j + m^r_j\Big(\sigma^r_j(S)U^r(S), V^r_j(S)\Big), \tag{5}$$

while productive AI capital evolves according to

$$\Gamma_K(S) = (1-\delta_K)K^A + I^A(S). \tag{6}$$

Hence assignments and productive AI capital evolve endogenously conditional on equilibrium policies, whereas aggregate productivity evolves exogenously.

The corresponding state transition is represented by

$$\mathcal{P}(dS' \mid S) = \delta_{\Gamma_M(S)}(dM')\, \delta_{\Gamma_K(S)}(dK^{A\prime})\, Q(dz' \mid z), \tag{7}$$

where δ_x denotes the Dirac measure concentrated at x. Equation (7) defines the state transition induced by a given equilibrium policy profile and does not itself establish existence or uniqueness of recursive competitive equilibrium.

The endogenous price system consists of human wages $\mathbf{w}_t^H = (w_{j,t}^H)_{j\in\mathcal{J}}$ and artificial-labour rental prices $\mathbf{r}_t^A = (r_{j,t}^A)_{j\in\mathcal{J}}$, together with the primitive task-output prices $\mathbf{p}(z_t)$. Human wages divide match surplus through task-specific Nash bargaining, whereas rental prices allocate household-owned artificial labour capacity competitively across tasks. The assignment stocks $N_{j,t}^r$ are predetermined state variables generated by previous matching and separation rather than contemporaneous Walrasian factor supplies. Market consistency requires productive use of each assignment type to coincide with its available assignment stock, while current vacancy creation and matching determine the assignments inherited by the next state.

Finally, aggregate output is allocated between consumption, productive AI investment, and vacancy creation. Aggregate feasibility requires

$$Y_t = C_t + I_t^A + \mathcal{K}_t. \tag{8}$$

Because the representative household owns productive AI capital and the competitive corporate sector, human wage payments, artificial-labour rental payments, and distributed profits are internal transfers under aggregation. Financial claims are therefore in zero net supply and, under the maintained representative-household structure and absence of additional financial frictions, do not constitute an independent aggregate state variable.

Equations (1)–(8), together with household optimisation, firms' vacancy-posting decisions, task-specific Nash bargaining, competitive allocation of artificial labour services, free entry, market consistency, and rational expectations, define the equilibrium conditions of the economy. Conditional on an equilibrium policy profile, these conditions induce allocations, prices, and state transitions from the primitive environment $\mathcal{G}$.

The resulting recursive structure separates endogenous state transitions from exogenous uncertainty. Productive assignments evolve through matching and separation, productive AI capital through investment and depreciation, and aggregate productivity through the exogenous Markov process. Consequently, $S_t = (M_t, K_t^A, z_t)$ contains the payoff-relevant predetermined information required for the recursive formulation developed below.

2.1. Recursive Optimisation

Let $S = (M, K^A, z) \in \mathcal{S}$ denote the aggregate state. To represent the recursive value of production without attributing aggregate state control to an individual competitive firm, let $F(S)$ denote the value of the consolidated productive sector. Conditional on prices and household allocation policies, its recursive representation is

$$F(S) = \max_{\mathbf{V}\in\mathcal{V}(S)} \left\{ \Pi(S,\mathbf{V}) + \beta\mathbb{E}\left[F(S') \mid S\right] \right\}, \tag{9}$$

where $\mathbf{V} = (V_j^H, V_j^A)_{j\in\mathcal{J}}$, the feasible correspondence $\mathcal{V}(S)$ incorporates matching feasibility, and the aggregate assignment transition is

$$N_j^{r\prime} = (1-\delta_j^r)N_j^r + m_j^r\left(\sigma_j^r(S)U^r(S), V_j^r\right), \qquad (j,r) \in \mathcal{J}\times\{H,A\}, \tag{10}$$

with

$$U^r(S) = L^r(S) - \sum_{j\in\mathcal{J}}(1-\delta_j^r)N_j^r, \qquad L^H(S) = N^H, \qquad L^A(S) = \chi(K^A). \tag{11}$$

Here $\sigma_j^H(S)U^H(S)$ is human labour directed towards task j, whereas $\sigma_j^A(S)U^A(S)$ is household-owned artificial labour capacity allocated to that task. Current productive-sector profits are

$$\Pi(S,\mathbf{V}) = \sum_{j\in\mathcal{J}}\left[p_j(z)\left(q_j^H(K^A,z)N_j^H + q_j^A(K^A,z)N_j^A\right) - w_j^H N_j^H - r_j^A N_j^A - \sum_{r\in\{H,A\}}\kappa_j^r V_j^r\right]. \tag{12}$$

Equation (9) is an aggregate recursive representation of the productive sector rather than the optimisation problem of an atomistic firm. Individual vacancy creation is decentralised through the free-entry conditions specified below.

The representative household owns productive AI capital and the corporate sector and chooses $(C, I^A, \boldsymbol{\sigma}^H, \boldsymbol{\sigma}^A)$ from the state-dependent feasible correspondence $\mathcal{A}(S)$. Human $\boldsymbol{\sigma}^H$ governs directed search, while $\boldsymbol{\sigma}^A$ allocates household-owned artificial labour capacity across task-specific rental markets, taking $\mathbf{r}^A$ as given. The household value function satisfies

$$H(S) = \max_{(C,I^A,\boldsymbol{\sigma}^H,\boldsymbol{\sigma}^A)\in\mathcal{A}(S)}\left\{u(C) + \beta\mathbb{E}\left[H(S') \mid S\right]\right\}, \tag{13}$$

subject to

$$C + I^A \le \sum_{j\in\mathcal{J}} w_j^H N_j^H + \sum_{j\in\mathcal{J}} r_j^A N_j^A + \Pi(S,\mathbf{V}), \tag{14}$$

the capital transition

$$K^{A\prime} = (1-\delta_K)K^A + I^A, \tag{15}$$

the assignment transition (10), and the artificial-capacity constraint

$$\sum_{j\in\mathcal{J}} N_j^{A\prime} \le \chi(K^{A\prime}). \tag{16}$$

Thus $\mathcal{A}(S)$ incorporates the non-negativity, simplex, budget, transition, and capacity restrictions required for feasibility.

For an interior investment choice at which the household value function is differentiable and the artificial-capacity constraint is locally slack, the first-order condition with respect to I_t^A is

$$u'(C_t) = \beta\mathbb{E}_t[H_{K^A}(S_{t+1})]. \tag{17}$$

The continuation derivative $H_{K^A}(S_{t+1})$ incorporates all channels through which productive AI capital affects future household value, including artificial labour capacity, assignment opportunities, productivity, rental returns, and subsequent state transitions. If the capacity constraint binds, its multiplier enters the corresponding Kuhn–Tucker condition. Under the maintained regularity conditions, optimal investment additionally satisfies

$$\lim_{T\to\infty}\beta^T\mathbb{E}\left[u'(C_T)K_{T+1}^A\right] = 0. \tag{18}$$

Since the representative household owns productive AI capital and the competitive corporate sector, aggregate financial claims are internal and in zero net supply. Conditional on this ownership structure and the absence of additional financial frictions, financial wealth is not an independent aggregate state variable. The payoff-relevant predetermined state is therefore

$$S = (M, K^A, z), \tag{19}$$

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which records inherited productive assignments, productive AI capital, and aggregate productivity and is sufficient for the recursive equilibrium formulation below.

2.2. Recursive Competitive Equilibrium

The recursive allocation is decentralised through human wages, competitive rental prices for artificial labour services, and vacancy creation. Human wages are determined within each task through Nash bargaining. Given worker bargaining weight $\eta_j \in (0,1)$, the equilibrium wage satisfies

$$w_j^H(S) \in \arg\max_w \Big[\mathcal{S}_j^H(S;w)\Big]^{\eta_j} \Big[J_j^H(S;w)\Big]^{1-\eta_j}, \qquad j \in \mathcal{J}, \tag{20}$$

where $\mathcal{S}_j^H(S;w)$ and $J_j^H(S;w)$ denote the worker and firm surpluses associated with a human assignment in task j.

Artificial labour services are supplied competitively by the owner of productive AI capital. Since task output is linear in artificial assignments, the competitive rental price equals marginal revenue product,

$$r_j^A(S) = p_j(z)\, q_j^A(K^A, z), \qquad j \in \mathcal{J}. \tag{21}$$

Accordingly, $\boldsymbol{\sigma}^A(S)$ allocates available artificial labour capacity across task-specific rental markets rather than representing an autonomous search decision by artificial labour.

For vacancy creation, let $J_j^r(S)$ denote the private value to a firm of a productive assignment of type r in task j. This private match value is distinct from the consolidated productive-sector value $F(S)$ in (9). It satisfies the recursive relation

$$J_j^r(S) = \pi_j^r(S) + \beta(1-\delta_j^r)\mathbb{E}\Big[J_j^r(S') \mid S\Big], \qquad (j,r) \in \mathcal{J} \times \{H, A\}, \tag{22}$$

where the current net return from an existing assignment is

$$\pi_j^H(S) = p_j(z) q_j^H(K^A, z) - w_j^H(S), \qquad \pi_j^A(S) = p_j(z) q_j^A(K^A, z) - r_j^A(S). \tag{23}$$

The recursion in (22) values an assignment already in production. A match formed through vacancy posting at state S enters production in the next period and therefore has current value $\beta\mathbb{E}[J_j^r(S') \mid S]$.

For $V_j^r(S) > 0$, define the vacancy-filling probability by

$$\phi_j^r(S) = \frac{m_j^r\Big(\sigma_j^r(S) U^r(S), V_j^r(S)\Big)}{V_j^r(S)}. \tag{24}$$

Free entry then requires

$$0 \le V_j^r(S) \;\perp\; \kappa_j^r - \beta\,\phi_j^r(S)\mathbb{E}\Big[J_j^r(S') \mid S\Big] \ge 0, \qquad (j,r) \in \mathcal{J} \times \{H, A\}. \tag{25}$$

Thus an active vacancy has zero expected value, while no vacancy is created when its expected return lies below its posting cost.

The representative household chooses $\boldsymbol{\sigma}^H(S)$ and $\boldsymbol{\sigma}^A(S)$ jointly with consumption and productive AI investment. The first directs available human labour across task-specific search markets. The second allocates household-owned artificial labour capacity across competitive rental markets, taking $\mathbf{r}^A(S)$ as given. These allocations affect matching probabilities and the endogenous evolution of productive assignments.

Equilibrium feasibility requires

$$\sum_{j \in \mathcal{J}} N_j^H \le N^H, \qquad \sum_{j \in \mathcal{J}} N_j^A \le \chi(K^A), \tag{26}$$

together with the aggregate resource constraint (8). Intertemporal consistency of artificial assignments additionally requires

$$\sum_{j \in \mathcal{J}} N_j^{A\prime} \leq \chi(K^{A\prime}). \tag{27}$$

Since the representative household owns productive AI capital and the competitive corporate sector, wages, artificial-labour rental payments, and distributed profits are internal transfers under aggregation.

Given an equilibrium policy profile, assignments and productive AI capital evolve according to

$$M' = \Gamma_M(S), \qquad K^{A\prime} = \Gamma_K(S), \tag{28}$$

while aggregate productivity follows $Q(dz' \mid z)$. The induced equilibrium transition kernel is

$$P^E(dS' \mid S) = \delta_{\Gamma_M(S)}(dM')\, \delta_{\Gamma_K(S)}(dK^{A\prime})\, Q(dz' \mid z). \tag{29}$$

Definition 1 (Recursive Competitive Equilibrium)**.** *A recursive competitive equilibrium is a collection*

$$\mathcal{E} = \left(F, H, \mathbf{J}, \mathbf{V}, C, I^A, \boldsymbol{\sigma}^H, \boldsymbol{\sigma}^A, \mathbf{w}^H, \mathbf{r}^A, P^E\right) \tag{30}$$

such that, for every feasible state $S \in \mathcal{S}$,

1. *the consolidated productive-sector value satisfies (9), while private assignment values satisfy (22);*
2. *the representative household solves (13) subject to its budget, transition, and capacity constraints;*
3. *human wages satisfy (20), artificial labour services satisfy (21), and vacancy creation satisfies (25);*
4. *assignment and artificial-capacity feasibility satisfy (26) and (27), while aggregate goods-market feasibility satisfies (8);*
5. *household decisions, private vacancy decisions, matching outcomes, prices, and aggregate policies are mutually consistent; and*
6. *expectations are rational, so the perceived aggregate law of motion coincides with the induced transition kernel (29).*

Definition 1 identifies recursive competitive equilibrium as a fixed point in optimisation, prices, matching outcomes, aggregate policies, and the perceived law of motion. It does not by itself assert existence or uniqueness. The analytical results below are stated conditional on a recursive competitive equilibrium and, where stationarity is required, on a stationary recursive competitive equilibrium satisfying the additional conditions imposed for the relevant result.

2.3. Stationary Recursive Equilibrium

A recursive competitive equilibrium is stationary when its value functions, prices, allocations, and policy rules are time invariant and depend on calendar time only through the current state. Stationarity of equilibrium policies is distinct from stationarity of the induced state distribution, which additionally requires an invariant probability measure.

Definition 2 (Stationary Recursive Competitive Equilibrium)**.** *A stationary recursive competitive equilibrium is a recursive competitive equilibrium*

$$\mathcal{E}^* = \left(F^*, H^*, \mathbf{J}^*, \Pi^*, P^{E*}\right), \tag{31}$$

whose equilibrium policy profile Π^* and induced transition kernel P^{E*} are time invariant, such that

1. *F^*, H^*, and $\mathbf{J}^*$ satisfy the stationary productive-sector, household, and private assignment-value recursions and generate stationary vacancy, consumption, investment, human-search, and artificial-capacity allocation policies;*

2. *human wages, artificial-labour rental prices, and vacancy creation satisfy the stationary Nash bargaining, competitive pricing, and free-entry conditions;*
3. *productive assignments and productive AI capital evolve according to*

$$M' = \Gamma^*_M(S), \qquad K^{A\prime} = \Gamma^*_K(S), \tag{32}$$

while aggregate productivity evolves according to $Q(dz' \mid z)$;

4. *assignment, artificial-capacity, and aggregate resource feasibility hold at every feasible state;*
5. *household decisions, private vacancy decisions, matching outcomes, prices, and aggregate policies are mutually consistent, and expectations are rational under the induced stationary transition kernel*

$$P^{E*}(dS' \mid S) = \delta_{\Gamma^*_M(S)}(dM')\, \delta_{\Gamma^*_K(S)}(dK^{A\prime})\, Q(dz' \mid z). \tag{33}$$

If P^{E*} admits an invariant probability measure $\mu^* \in \mathcal{P}(\mathcal{S})$, then

$$\mu^*(B) = \int_{\mathcal{S}} P^{E*}(B \mid S)\, \mu^*(dS), \qquad B \in \mathcal{B}(\mathcal{S}), \tag{34}$$

where $\mathcal{B}(\mathcal{S})$ denotes the Borel σ-algebra on the state space. The measure μ^* is then a stationary distribution of the recursive economy. If the invariant measure is unique, it is the unique stationary distribution associated with P^{E*}.

The distinction between stationary equilibrium and an invariant state distribution is maintained throughout the analysis. A stationary recursive competitive equilibrium requires time-invariant equilibrium rules, whereas existence and uniqueness of an invariant measure are separate properties of the induced Markov kernel. The theoretical results below are therefore stated conditional on a stationary recursive competitive equilibrium and, only where a stationary distribution is required, on the existence of an invariant measure satisfying (34).

3. Analysis and Results

The recursive economy developed above makes effective labour supply endogenous through the accumulation of productive AI capital. Productive AI capital generates artificial labour capacity according to $L^A = \chi(K^A)$, whereas the aggregate human labour endowment remains fixed. Human employment values, matching probabilities, and vacancy incentives therefore depend on the joint equilibrium evolution of human assignments, artificial labour capacity, and productive AI capital.

The analysis develops three results. First, it characterises the recursive surplus and marketability of human employment. Second, it identifies conditions under which accumulation of productive AI capital drives human employment surplus to its participation boundary. Third, it examines conditions under which productive allocation is invariant to the distribution of ownership claims on productive AI capital. All results are conditional on a recursive competitive equilibrium as defined in Definition 1 and, where required, on a stationary recursive competitive equilibrium satisfying the additional conditions stated below.

3.1. Dynamic Valuation and Marketability of Human Capital

Fix a task $j \in \mathcal{J}$. Let $B(\mathcal{S})$ denote the Banach space of bounded measurable functions on $\mathcal{S}$, equipped with the supremum norm, and define the equilibrium Markov operator $\mathcal{P} : B(\mathcal{S}) \to B(\mathcal{S})$ by

$$(\mathcal{P}h)(S) = \int_{\mathcal{S}} h(S') P^E(dS' \mid S) = \mathbb{E}[h(S') \mid S]. \tag{35}$$

Let $W_j^E(S)$ denote the value to a human worker currently assigned to task j, and let $W^U(S)$ denote the value of being available for search. Let $\omega^U(S)$ denote the current flow value associated with non-employment. An existing human assignment satisfies

$$W_j^E(S) = w_j^H(S) + \beta\mathcal{P}\Big[(1-\delta_j^H)W_j^E + \delta_j^H W^U\Big](S). \tag{36}$$

An available human worker chooses a task according to $\sigma^H(S)$. Conditional on being directed to task j, define the job-finding probability

$$f_j^H(S) = \frac{m_j^H\Big(\sigma_j^H(S)U^H(S), V_j^H(S)\Big)}{\sigma_j^H(S)U^H(S)}, \tag{37}$$

whenever $\sigma_j^H(S)U^H(S) > 0$, with $f_j^H(S) = 0$ when the corresponding search pool is empty. The value conditional on search in task j is therefore

$$W_j^U(S) = \omega^U(S) + \beta\mathcal{P}\Big[f_j^H W_j^E + (1-f_j^H)W_j^U\Big](S). \tag{38}$$

Define the human worker's match surplus by

$$H_j(S) = W_j^E(S) - W_j^U(S). \tag{39}$$

Subtracting (38) from (36) gives

$$H_j(S) = w_j^H(S) - \omega^U(S) + \beta\Big(1-\delta_j^H - f_j^H(S)\Big)(\mathcal{P}H_j)(S). \tag{40}$$

Thus the continuation value of a human match depends on both separation and the option value of returning to search.

Let $J_j^H(S)$ denote the private firm value of an existing human assignment introduced in (22). Its recursion is

$$J_j^H(S) = p_j(z)q_j^H(K^A, z) - w_j^H(S) + \beta(1-\delta_j^H)(\mathcal{P}J_j^H)(S). \tag{41}$$

Productive AI capital enters this valuation through the state $S = (M, K^A, z)$, the productivity function $q_j^H(K^A, z)$, equilibrium prices and matching conditions, and the transition kernel P^E. No independent utility, bargaining problem, or participation condition is assigned to artificial labour.

Define total human match surplus by

$$\Sigma_j(S) = H_j(S) + J_j^H(S). \tag{42}$$

Whenever $\Sigma_j(S) > 0$, Nash bargaining implies

$$H_j(S) = \eta_j\Sigma_j(S), \qquad J_j^H(S) = (1-\eta_j)\Sigma_j(S). \tag{43}$$

Adding (40) and (41), and using (43), yields

$$\Sigma_j(S) = b_j(S) + \beta a_j(S)(\mathcal{P}\Sigma_j)(S), \tag{44}$$

where

$$b_j(S) = p_j(z)q_j^H(K^A, z) - \omega^U(S) \tag{45}$$

and

$$a_j(S) = 1 - \delta_j^H - \eta_j f_j^H(S). \tag{46}$$

The coefficient $a_j(S)$ captures the effective persistence of bilateral human-match surplus after accounting for separation and the worker's option value of rematching.

For the valuation results below, define the linear operator

$$(\mathcal{T}_j h)(S) = b_j(S) + \beta a_j(S)(\mathcal{P}h)(S). \tag{47}$$

If $b_j \in \mathcal{B}(\mathcal{S})$ and

$$\beta \sup_{S\in\mathcal{S}} |a_j(S)| < 1, \tag{48}$$

then $\mathcal{T}_j$ is a contraction on $\mathcal{B}(\mathcal{S})$. Hence the latent surplus equation

$$\widehat{\Sigma}_j = \mathcal{T}_j \widehat{\Sigma}_j \tag{49}$$

admits a unique bounded solution,

$$\widehat{\Sigma}_j = (I - \beta \mathcal{A}_j \mathcal{P})^{-1} b_j, \qquad (\mathcal{A}_j h)(S) = a_j(S)h(S). \tag{50}$$

The latent object $\widehat{\Sigma}_j$ records the recursive valuation before imposition of the participation boundary. Realised equilibrium surplus is therefore

$$\Sigma_j(S) = [\widehat{\Sigma}_j(S)]_+, \qquad [x]_+ = \max\{x, 0\}, \tag{51}$$

with realised surplus shares

$$H_j(S) = \eta_j [\widehat{\Sigma}_j(S)]_+, \qquad J_j^H(S) = (1 - \eta_j)[\widehat{\Sigma}_j(S)]_+. \tag{52}$$

Finally, define the equilibrium marketability of human capital in task j by

$$\mathcal{M}_j^H(S) = f_j^H(S) H_j(S). \tag{53}$$

Marketability combines the probability that available human labour forms a productive assignment with the equilibrium surplus accruing to the worker conditional on that assignment. Productive AI capital can therefore reduce human-capital marketability through changes in matching probabilities, human productivity, equilibrium continuation values, or the induced law of motion, without requiring artificial labour to be modelled as an independent economic agent.

Theorem 1 (Dynamic Human-Capital Valuation and Displacement). *Fix $j \in \mathcal{J}$ and suppose that $b_j \in \mathcal{B}(\mathcal{S})$, $a_j \in \mathcal{B}(\mathcal{S})$, and*

$$\beta \sup_{S\in\mathcal{S}} |a_j(S)| < 1. \tag{54}$$

Let $\mathcal{A}_j$ denote the multiplication operator

$$(\mathcal{A}_j h)(S) = a_j(S)h(S),$$

and define the resolvent

$$\mathcal{R}_j = (I - \beta \mathcal{A}_j \mathcal{P})^{-1}. \tag{55}$$

Then the following statements hold.

(i) Recursive valuation. *The latent human-match surplus equation*

$$\widehat{\Sigma}_j = b_j + \beta \mathcal{A}_j \mathcal{P} \widehat{\Sigma}_j$$

admits a unique bounded solution given by

$$\widehat{\Sigma}_j = \mathcal{R}_j b_j = \sum_{n=0}^{\infty} \beta^n (\mathcal{A}_j \mathcal{P})^n b_j. \tag{56}$$

Consequently, realised worker surplus and human-capital marketability satisfy

$$H_j(S) = \eta_j\big[(\mathcal{R}_j b_j)(S)\big]_+, \qquad \mathcal{M}_j^H(S) = \eta_j f_j^H(S)\big[(\mathcal{R}_j b_j)(S)\big]_+. \tag{57}$$

Hence

$$H_j(S) > 0 \iff (\mathcal{R}_j b_j)(S) > 0, \tag{58}$$

and

$$\mathcal{M}_j^H(S) > 0 \iff f_j^H(S) > 0 \text{ and } (\mathcal{R}_j b_j)(S) > 0. \tag{59}$$

(ii) Comparative displacement. *Consider two equilibrium environments with the same operator $\mathcal{A}_j\mathcal{P}$ but flow-surplus functions b_j and $\widetilde{b}_j = b_j - d_j$, where $d_j \in \mathcal{B}(\mathcal{S})$ satisfies $d_j \geq 0$. If $\mathcal{A}_j\mathcal{P}$ is a positive operator, then*

$$\widetilde{\widehat{\Sigma}}_j = \widehat{\Sigma}_j - \mathcal{R}_j d_j \leq \widehat{\Sigma}_j. \tag{60}$$

The inequality is strict at S_0 whenever

$$\big[(\mathcal{A}_j\mathcal{P})^n d_j\big](S_0) > 0 \tag{61}$$

for some $n \in \mathbb{N}_0$. Hence a reduction in current or expected future human flow surplus weakly reduces the recursive value of human employment and, where the inequality crosses the participation boundary, eliminates realised human surplus.

(iii) AI-capital displacement threshold. *Suppose the equilibrium objects can be indexed by productive AI capital K^A, and write*

$$\widehat{\Sigma}_j(S;K^A) = \Big[\mathcal{R}_j(K^A) b_j(K^A)\Big](S).$$

Assume, on an interval $\mathcal{K} \subseteq \mathbb{R}_+$, that for every $S \in \mathcal{S}$:

(a) $K^A \mapsto \widehat{\Sigma}_j(S;K^A)$ *is continuous;*
(b) $K^A \mapsto \widehat{\Sigma}_j(S;K^A)$ *is strictly decreasing;*
(c) *there exist $K_L^A, K_H^A \in \mathcal{K}$, with $K_L^A < K_H^A$, such that*

$$\widehat{\Sigma}_j(S;K_L^A) > 0, \qquad \widehat{\Sigma}_j(S;K_H^A) \leq 0. \tag{62}$$

Then there exists a unique threshold $K_j^\dagger(S) \in (K_L^A, K_H^A]$ satisfying

$$\widehat{\Sigma}_j\Big(S;K_j^\dagger(S)\Big) = 0, \tag{63}$$

and

$$H_j(S;K^A) > 0 \iff K^A < K_j^\dagger(S), \tag{64}$$

for $K^A \in \mathcal{K}$. Human-capital marketability satisfies

$$\mathcal{M}_j^H(S;K^A) > 0 \iff f_j^H(S;K^A) > 0 \text{ and } K^A < K_j^\dagger(S). \tag{65}$$

Corollary 1 (Global Human-Capital Survival and Displacement). *Suppose the conditions of Theorem 1(iii) hold for every $S \in \mathcal{S}$, and let $K_j^\dagger(S)$ denote the corresponding state-dependent displacement threshold. Define*

$$\underline{K}_j^\dagger = \inf_{S\in\mathcal{S}} K_j^\dagger(S), \qquad \overline{K}_j^\dagger = \sup_{S\in\mathcal{S}} K_j^\dagger(S). \tag{66}$$

Then, for every admissible stock of productive AI capital K^A, the following statements hold.

(i) Global survival. *If*

$$K^A < \underline{K}_j^\dagger, \tag{67}$$

then

$$H_j(S;K^A) > 0 \qquad \textit{for every } S \in \mathcal{S}. \tag{68}$$

Moreover,

$$\mathcal{M}_j^H(S;K^A) > 0 \iff f_j^H(S;K^A) > 0. \tag{69}$$

(ii) State-dependent displacement. *If*

$$\underline{K}_j^\dagger \le K^A < \overline{K}_j^\dagger, \tag{70}$$

define the human-survival and displacement regions by

$$\mathcal{S}_j^+(K^A) = \Big\{S \in \mathcal{S} : K^A < K_j^\dagger(S)\Big\}, \tag{71}$$

$$\mathcal{S}_j^0(K^A) = \Big\{S \in \mathcal{S} : K^A \ge K_j^\dagger(S)\Big\}. \tag{72}$$

These sets partition the state space,

$$\mathcal{S} = \mathcal{S}_j^+(K^A) \cup \mathcal{S}_j^0(K^A), \qquad \mathcal{S}_j^+(K^A) \cap \mathcal{S}_j^0(K^A) = \varnothing, \tag{73}$$

and human surplus satisfies

$$H_j(S;K^A) > 0 \iff S \in \mathcal{S}_j^+(K^A), \tag{74}$$

whereas

$$H_j(S;K^A) = 0 \iff S \in \mathcal{S}_j^0(K^A). \tag{75}$$

Human-capital marketability is positive precisely when the state supports both positive human surplus and a positive probability of matching,

$$\mathcal{M}_j^H(S;K^A) > 0 \iff S \in \mathcal{S}_j^+(K^A) \textit{ and } f_j^H(S;K^A) > 0. \tag{76}$$

(iii) Global displacement. *If*

$$K^A \ge \overline{K}_j^\dagger, \tag{77}$$

then

$$H_j(S;K^A) = \mathcal{M}_j^H(S;K^A) = 0 \qquad \textit{for every } S \in \mathcal{S}. \tag{78}$$

Thus $\underline{K}_j^\dagger$ is the boundary below which positive human surplus is guaranteed throughout the state space, while $\overline{K}_j^\dagger$ is the boundary at or above which positive human surplus is absent throughout the state space. Between these boundaries, the viability and marketability of human employment are state dependent.

Theorem 1 and Corollary 1 establish a hierarchy of equilibrium displacement boundaries generated by productive AI capital. Theorem 1 identifies the state-specific threshold $K_j^\dagger(S)$ at which human employment in task j ceases to generate positive equilibrium surplus. Building on this result, Corollary 1 characterises the corresponding boundaries across the state space. The lower threshold $\underline{K}_j^\dagger$ identifies the region below which positive human surplus is sustained in every state, whereas the upper threshold $\overline{K}_j^\dagger$ identifies the region at or above which positive human surplus is absent in every state. Between these thresholds, human-employment viability is state contingent.

Importantly, the displacement concept in Theorem 1 and Corollary 1 concerns the equilibrium value and marketability of human employment rather than the instantaneous disappearance of existing human assignments. Since N_j^H is a predetermined assignment stock, existing human matches may persist after the surplus associated with human employment reaches its participation boundary and subsequently decline through the assignment law of motion. The thresholds therefore identify the loss of positive equilibrium surplus, not the contemporaneous condition $N_j^H = 0$.

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The state dependence established by Theorem 1 follows from the recursive valuation

$$\widehat{\Sigma}_j(S;K^A) = \Big[\mathcal{R}_j(K^A)b_j(K^A)\Big](S),$$

which incorporates the equilibrium channels operating through b_j, f_j^H, $\mathcal{A}_j$, and P^E. Human productivity, matching conditions, vacancy creation, equilibrium prices, continuation values, and assignment dynamics can therefore shift the state-specific threshold $K_j^{\dagger}(S)$. Corollary 1 then translates this state dependence into the global bounds $\underline{K}_j^{\dagger}$ and $\overline{K}_j^{\dagger}$. Productive AI capital alone therefore does not determine human-employment viability; the displacement boundary depends jointly on productive AI capacity and the equilibrium state in which that capacity operates.

3.2. *Technological Regime Transition and Human-Capital Scarcity*

The preceding results establish state-contingent thresholds for the equilibrium value of human employment. We now examine whether an analogous boundary arises along a stationary equilibrium path indexed by the stock of productive AI capital. The relevant question is whether increasing K^A can move the stationary economy from a regime in which human employment generates positive equilibrium surplus to one in which the human participation constraint binds.

For each admissible K^A, let $S^*(K^A)$ denote the stationary equilibrium state, or a selected stationary equilibrium state when the stationary equilibrium correspondence is not singleton-valued. For task $j \in \mathcal{J}$, define the stationary latent human-surplus function by

$$\Gamma_j(K^A) = \widehat{\Sigma}_j\Big(S^*(K^A);K^A\Big) = \Big[\mathcal{R}_j(K^A)b_j(K^A)\Big]\Big(S^*(K^A)\Big), \tag{79}$$

where

$$\mathcal{R}_j(K^A) = \Big[I - \beta\mathcal{A}_j(K^A)\mathcal{P}(K^A)\Big]^{-1}. \tag{80}$$

Thus $\Gamma_j(K^A)$ incorporates both the direct dependence of current human flow surplus on productive AI capital and the indirect equilibrium effects operating through matching, vacancy creation, prices, continuation values, and the stationary transition law.

Realised stationary human surplus and its Nash shares are

$$\Sigma_j^*(K^A) = [\Gamma_j(K^A)]_+, \qquad H_j^*(K^A) = \eta_j[\Gamma_j(K^A)]_+, \qquad J_j^{H*}(K^A) = (1-\eta_j)[\Gamma_j(K^A)]_+. \tag{81}$$

The corresponding marketability of human capital is

$$\mathcal{M}_j^{H*}(K^A) = f_j^{H*}(K^A)H_j^*(K^A). \tag{82}$$

Define the human-participation indicator

$$\mathbb{I}_j^H(K^A) = \mathbf{1}\Big\{\Gamma_j(K^A) > 0\Big\}, \tag{83}$$

and the associated stationary regime map

$$\mathfrak{R}_j(K^A) = \begin{cases} \mathsf{H}, & \Gamma_j(K^A) > 0, \\ \mathsf{N}, & \Gamma_j(K^A) \le 0, \end{cases} \tag{84}$$

where H denotes positive human participation surplus and N denotes the regime in which the human participation boundary binds. The regime map is discrete even when the underlying surplus function is continuous. A technological regime bifurcation therefore refers here to a change in this equilibrium regime classification induced by a zero crossing of Γ_j.

Definition 3 (Technological Regime Transition). *For task $j \in \mathcal{J}$, a productive AI-capital stock $\widehat{K}_j^A > 0$ is a technological regime bifurcation point if*

$$\Gamma_j(\widehat{K}_j^A) = 0 \tag{85}$$

and there exists $\varepsilon > 0$ such that

$$\Gamma_j(K^A) > 0 \quad \text{for } K^A \in (\widehat{K}_j^A - \varepsilon, \widehat{K}_j^A), \tag{86}$$

whereas

$$\Gamma_j(K^A) < 0 \quad \text{for } K^A \in (\widehat{K}_j^A, \widehat{K}_j^A + \varepsilon). \tag{87}$$

Equivalently, the discrete regime map changes locally from H *to* N *as productive AI capital crosses $\widehat{K}_j^A$ from below.*

Assumption 1 (Stationary Surplus Monotonicity). *For every task $j \in \mathcal{J}$, the selected stationary equilibrium surplus function $\Gamma_j : \mathbb{R}_+ \to \mathbb{R}$ is continuously differentiable and satisfies*

$$\Gamma_j(0) > 0, \qquad \Gamma_j'(K^A) < 0 \quad \text{for all } K^A \geq 0, \qquad \lim_{K^A \to \infty} \Gamma_j(K^A) < 0. \tag{88}$$

Thus the stationary recursive value of human employment is initially positive, declines strictly as productive AI capital increases, and eventually crosses the human participation boundary.

Theorem 2 (Stationary Human-Capital Regime Threshold). *Suppose Assumption 1 holds. Then, for every $j \in \mathcal{J}$, there exists a unique $\widehat{K}_j^A \in (0, \infty)$ satisfying*

$$\Gamma_j(\widehat{K}_j^A) = 0. \tag{89}$$

Moreover,

$$\Gamma_j(K^A) \begin{cases} > 0, & K^A < \widehat{K}_j^A, \\ = 0, & K^A = \widehat{K}_j^A, \\ < 0, & K^A > \widehat{K}_j^A, \end{cases} \tag{90}$$

and therefore

$$\mathfrak{R}_j(K^A) = \begin{cases} \mathsf{H}, & K^A < \widehat{K}_j^A, \\ \mathsf{N}, & K^A \geq \widehat{K}_j^A. \end{cases} \tag{91}$$

Hence $\widehat{K}_j^A$ is the unique technological regime transformation point in the sense of Definition 3.

For $K^A < \widehat{K}_j^A$,

$$\Gamma_j(K^A) > 0 \iff \Sigma_j^*(K^A) > 0 \iff H_j^*(K^A) > 0 \iff J_j^{H*}(K^A) > 0. \tag{92}$$

For $K^A \geq \widehat{K}_j^A$,

$$\Sigma_j^*(K^A) = H_j^*(K^A) = J_j^{H*}(K^A) = 0. \tag{93}$$

Finally,

$$\mathcal{M}_j^{H*}(K^A) > 0 \iff K^A < \widehat{K}_j^A \text{ and } f_j^{H*}(K^A) > 0. \tag{94}$$

Thus $\widehat{K}_j^A$ is the unique stationary-equilibrium threshold at which task-j human employment ceases to generate positive match surplus. The theorem concerns the equilibrium viability of human employment and does not imply the instantaneous disappearance of inherited human assignments.

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Corollary 2 (Regularity of Equilibrium Human-Capital Values)**.** *Suppose Assumption 1 holds. Then the realised stationary human-match surplus and its Nash shares,*

$$\Sigma_j^*(K^A) = [\Gamma_j(K^A)]_+, \qquad H_j^*(K^A) = \eta_j[\Gamma_j(K^A)]_+, \qquad J_j^{H*}(K^A) = (1-\eta_j)[\Gamma_j(K^A)]_+,$$

are continuous on $\mathbb{R}_+$ and continuously differentiable on

$$[0, \widehat{K}_j^A) \cup (\widehat{K}_j^A, \infty).$$

For $K^A \neq \widehat{K}_j^A$,

$$\frac{d\Sigma_j^*}{dK^A} = \begin{cases} \Gamma_j'(K^A), & K^A < \widehat{K}_j^A, \\ 0, & K^A > \widehat{K}_j^A, \end{cases} \tag{95}$$

and therefore

$$\frac{dH_j^*}{dK^A} = \eta_j \frac{d\Sigma_j^*}{dK^A}, \qquad \frac{dJ_j^{H*}}{dK^A} = (1-\eta_j)\frac{d\Sigma_j^*}{dK^A}. \tag{96}$$

At the threshold, the one-sided derivatives of realised surplus satisfy

$$D_-\Sigma_j^*(\widehat{K}_j^A) = \Gamma_j'(\widehat{K}_j^A) < 0, \qquad D_+\Sigma_j^*(\widehat{K}_j^A) = 0. \tag{97}$$

Accordingly,

$$D_-H_j^*(\widehat{K}_j^A) = \eta_j\Gamma_j'(\widehat{K}_j^A), \qquad D_+H_j^*(\widehat{K}_j^A) = 0, \tag{98}$$

$$D_-J_j^{H*}(\widehat{K}_j^A) = (1-\eta_j)\Gamma_j'(\widehat{K}_j^A), \qquad D_+J_j^{H*}(\widehat{K}_j^A) = 0. \tag{99}$$

Hence

$$\Sigma_j^*,\ H_j^*,\ J_j^{H*} \in C^0(\mathbb{R}_+)$$

but none is differentiable at $\widehat{K}_j^A$. The stationary participation boundary therefore generates a kink in realised human-capital valuation even though the underlying latent surplus function Γ_j remains continuously differentiable.

Theorem 2 identifies $\widehat{K}_j^A$ as the stationary equilibrium boundary at which the continuation-adjusted surplus of human employment in task j reaches zero. Its significance is economic rather than physical. Crossing the threshold changes the participation regime by eliminating positive realised surplus from a human assignment; it does not require the contemporaneous disappearance of the inherited assignment stock N_j^H. Existing human matches may remain in the state vector and subsequently decline through separation and the endogenous assignment transition. The theorem therefore characterises the loss of human labour's equilibrium scarcity value rather than instantaneous technological replacement.

The threshold is also endogenous to the recursive equilibrium rather than a fixed measure of technological capability. Through (79), productive AI accumulation affects human-employment surplus through the equilibrium configuration of productivity, matching, vacancy creation, prices, continuation values, and state transitions. Accordingly, $\widehat{K}_j^A$ identifies the point at which these forces jointly exhaust the positive surplus supporting human participation in task j.

Corollary 2 reveals the corresponding structure of equilibrium valuation. Although the latent surplus $\Gamma_j(K^A)$ remains smooth through $\widehat{K}_j^A$, realised surplus satisfies

$$\Sigma_j^*(K^A) = [\Gamma_j(K^A)]_+.$$

Its marginal response to productive AI capital is therefore $\Gamma_j'(K^A) < 0$ below the threshold and zero above it. The resulting kink is generated by the binding participation constraint, not by a discontinuity

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in technology or latent valuation. Productive AI may continue to reduce the latent relative value of human employment beyond the threshold, but this deterioration is no longer transmitted into realised surplus because equilibrium participation truncates that surplus at zero. The theorem and corollary therefore distinguish technological capability, latent human-employment value, and realised equilibrium participation as analytically separate components of the displacement mechanism.

3.3. Institutional Separation of Production and Distribution

The preceding analysis determines the productive allocation generated by the recursive competitive equilibrium. We now consider an institutional extension in which claims on productive AI-capital income may be distributed across households without changing the technologies, market structure, or aggregate resource constraints of the baseline economy. The purpose is to identify the conditions under which the ownership of productive AI capital affects the distribution of equilibrium income without altering the productive allocation itself.

Let

$$\Omega = (\omega_1, \ldots, \omega_N) \in \Delta^{N-1}, \qquad \omega_i \geq 0, \qquad \sum_{i=1}^{N} \omega_i = 1, \tag{100}$$

denote the distribution of claims on aggregate productive AI-capital income across N households. The baseline representative-household economy is recovered as the special case in which these claims are consolidated within a single household. The extension introduced here changes only the incidence of AI-capital income and does not introduce household-specific production, matching, investment, or labour-supply decisions.

For an ownership regime Ω, let $\mathcal{E}^*_\Omega$ denote the stationary productive allocation and price system, and let $\mathcal{D}^*_\Omega$ denote the associated distribution of household income and wealth claims. Write the stationary allocation system as

$$\mathcal{F}^A(\mathcal{E}^*; \mathcal{G}) = 0, \tag{101}$$

where $\mathcal{G}$ denotes the primitive environment defined in (1). Conditional on the resulting stationary equilibrium, the distributional system is

$$\mathcal{D}^*_\Omega = \mathcal{F}^D(\mathcal{E}^*, \Omega; \mathcal{G}). \tag{102}$$

The allocation system (101) jointly determines the stationary productive allocation, equilibrium prices, matching outcomes, and AI-capital accumulation. The distribution system (102) maps this equilibrium together with the ownership regime Ω into the corresponding household distributions of income and wealth. Institutional separation therefore requires ownership to affect distributional outcomes through $\mathcal{F}^D$ without entering the equilibrium conditions governing the productive allocation in $\mathcal{F}^A$.

Assumption 2 (Ownership Neutrality)**.** *For the institutional extension considered in this subsection, suppose the following conditions hold.*

- *(i)* *Productive AI services are allocated through competitive rental markets. Firms condition their production and vacancy decisions on equilibrium prices and productive characteristics rather than on the identity of AI-capital owners.*
- *(ii)* *Ownership shares affect only the household incidence of claims on AI-generated income and do not alter aggregate preferences, human labour supply, directed search, matching, bargaining, vacancy creation, or aggregate resource feasibility.*
- *(iii)* *Household-specific AI-generated income does not enter worker outside options, bargaining surpluses, matching technologies, firm payoffs, or any other condition determining human or artificial productive assignments.*
- *(iv)* *Aggregate productive AI investment I^A and the law of motion for K^A depend only on aggregate equilibrium objects and are invariant to the distribution of ownership claims.*

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(v) *Redistribution of ownership claims introduces no taxes, transaction costs, borrowing constraints, portfolio restrictions, incomplete-market effects, or other financial frictions through which household wealth can affect the productive allocation.*

(vi) *For every admissible primitive environment $\mathcal{G}$, the stationary allocation system (101) admits a unique solution.*

Lemma 1 (Ownership Invariance of the Stationary Productive Allocation)**.** *Suppose Assumption 2 holds. For any two ownership regimes $\Omega, \widetilde{\Omega} \in \Delta^{N-1}$ associated with the same primitive environment $\mathcal{G}$, the stationary productive allocation satisfies*

$$\mathcal{E}^*_{\Omega} = \mathcal{E}^*_{\widetilde{\Omega}} = \mathcal{E}^*(\mathcal{G}). \tag{103}$$

Hence the stationary productive allocation and equilibrium price system are invariant to the distribution of claims on productive AI-capital income.

The associated household distributions need not be invariant. In particular, if productive AI capital generates strictly positive distributable income and $\Omega \neq \widetilde{\Omega}$ assigns different income claims to at least one household, then

$$\mathcal{D}^*_{\Omega} \neq \mathcal{D}^*_{\widetilde{\Omega}}. \tag{104}$$

Lemma 1 establishes the exclusion result underlying the institutional analysis. Under Assumption 2, redistributing claims on productive AI-capital income leaves the stationary productive allocation, matching outcomes, vacancy decisions, productive AI-capital stock, and equilibrium prices unchanged, while potentially altering the household distribution of income and wealth.

The result is conditional rather than a general neutrality proposition. Ownership can affect the productive equilibrium whenever redistribution changes labour supply, search, bargaining positions, investment, aggregate demand, or financial constraints, or introduces taxation, transaction costs, incomplete markets, or other ownership-dependent frictions. The lemma therefore isolates the institutional restrictions under which productive allocation and distributional incidence can be analysed separately.

Theorem 3 (Production–Distribution Separation)**.** *Suppose Assumption 2 holds. For any two ownership regimes $\Omega, \widetilde{\Omega} \in \Delta^{N-1}$ defined over the same primitive environment $\mathcal{G}$, the stationary productive allocation is ownership invariant,*

$$\mathcal{E}^*_{\Omega} = \mathcal{E}^*_{\widetilde{\Omega}} = \mathcal{E}^*(\mathcal{G}). \tag{105}$$

Hence every equilibrium object determined exclusively by the allocation block is identical across ownership regimes. This includes aggregate output, productive assignments, matching probabilities, vacancy creation, equilibrium prices, productive AI capital, human-match surplus, and technological regime thresholds generated by the stationary allocation system.

Define aggregate distributable income generated by artificial labour services as

$$R^{A,*} = \sum_{j \in \mathcal{J}} r_j^{A,*} N_j^{A,*}. \tag{106}$$

Because $\mathbf{r}^{A,*}$ and $\mathbf{N}^{A,*}$ are determined by $\mathcal{E}^*(\mathcal{G})$, $R^{A,*}$ is ownership invariant. Under ownership regime $\Omega = (\omega_1, \ldots, \omega_N)$, household i receives

$$y^{A,*}_{i,\Omega} = \omega_i R^{A,*}, \tag{107}$$

so that

$$\sum_{i=1}^{N} y^{A,*}_{i,\Omega} = R^{A,*} \tag{108}$$

for every $\Omega \in \Delta^{N-1}$, while

$$y^{A,*}_{i,\Omega} - y^{A,*}_{i,\widetilde{\Omega}} = (\omega_i - \widetilde{\omega}_i) R^{A,*}. \tag{109}$$

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Therefore, if $R^{A,*} > 0$ *and* $\Omega \neq \widetilde{\Omega}$, *there exists at least one household* i *for which*

$$y^{A,*}_{i,\Omega} \neq y^{A,*}_{i,\widetilde{\Omega}}. \tag{110}$$

The stationary productive allocation and the household incidence of AI-generated income are therefore determined by distinct equilibrium mechanisms. The primitive environment $\mathcal{G}$ *determines* $\mathcal{E}^*(\mathcal{G})$ *and hence the aggregate income* $R^{A,*}$ *generated by artificial labour services, whereas the ownership regime* Ω *determines how that income is distributed across households. Ownership can consequently alter distributive incidence without altering the productive equilibrium that generates the distributable income.*

Corollary 3 (Ownership Neutrality and Distributional Incidence)**.** *Suppose the conditions of Theorem 3 hold, and let*

$$d\Omega = (d\omega_1, \dots, d\omega_N) \in \mathbb{R}^N$$

be a feasible redistribution of ownership claims satisfying

$$\sum_{i=1}^{N} d\omega_i = 0, \qquad \Omega + d\Omega \in \Delta^{N-1}. \tag{111}$$

Then the following statements hold.

(i) *The stationary productive allocation is invariant,*

$$\mathcal{E}^*_{\Omega+d\Omega} = \mathcal{E}^*_{\Omega} = \mathcal{E}^*(\mathcal{G}). \tag{112}$$

Consequently, every equilibrium object determined exclusively by the allocation block, including aggregate output, productive assignments, matching probabilities, vacancy creation, equilibrium prices, productive AI capital, human-match surplus, and the technological regime thresholds characterised in Theorem 2, is unchanged.

(ii) *Household i's claim on AI-generated income changes according to*

$$y^{A,*}_{i,\Omega+d\Omega} - y^{A,*}_{i,\Omega} = d\omega_i R^{A,*}. \tag{113}$$

Hence, whenever $R^{A,*} > 0$, *every household satisfying* $d\omega_i \neq 0$ *experiences a non-zero change in its AI-generated income claim.*

(iii) *Aggregate AI-generated income is invariant,*

$$\sum_{i=1}^{N} \left(y^{A,*}_{i,\Omega+d\Omega} - y^{A,*}_{i,\Omega} \right) = 0. \tag{114}$$

Ownership redistribution therefore changes the household incidence of the fixed equilibrium flow $R^{A,*}$ *without changing its aggregate magnitude.*

(iv) *Suppose, additionally, that stationary household wealth claims satisfy*

$$a^*_i = \mathcal{A}_i\left(y^{A,*}_i ; \mathcal{G} \right), \tag{115}$$

where $\mathcal{A}_i(\cdot;\mathcal{G})$ *is injective. If* $R^{A,*} > 0$, *then*

$$d\omega_i \neq 0 \quad \Longrightarrow \quad a^*_{i,\Omega+d\Omega} \neq a^*_{i,\Omega}. \tag{116}$$

Hence every non-trivial feasible ownership redistribution changes at least one household's stationary wealth claim under the maintained wealth mapping.

Ownership is therefore neutral with respect to the stationary productive allocation but non-neutral with respect to the household incidence of positive AI-generated income. Its implications for stationary wealth follow whenever wealth claims respond injectively to the corresponding AI-income claims.

Theorem 3 and Corollary 3 establish the institutional dimension of the recursive economy. Theorem 3 shows that, under Assumption 2, the stationary productive allocation is invariant to the distribution of ownership claims. Productive AI accumulation, matching, vacancy creation, productive assignments, equilibrium prices, and human-capital valuation are determined within the allocation block, while ownership determines the household incidence of the AI-generated income $R^{A,*}$. Corollary 3 sharpens this result by showing that a feasible redistribution of ownership claims redistributes a fixed aggregate flow of AI-generated income without changing the productive allocation that generates it.

The institutional result complements the displacement mechanism established in Theorem 2. Productive AI accumulation determines whether human labour continues to generate positive equilibrium surplus, while ownership determines who receives the income generated by artificial labour services conditional on the resulting allocation. Technological displacement and distributive incidence are therefore analytically distinct but economically connected. An economy may undergo a substantial change in the scarcity value of human labour without the distribution of AI-generated income being uniquely determined by that technological transition.

This separation remains conditional on Assumption 2. If ownership affects consumption, investment, labour supply, directed search, bargaining, taxation, financing conditions, or another object entering the allocation system, then Ω enters the allocation operator $\mathcal{F}^A$ and distribution can feed back into production. Theorem 3 and Corollary 3 therefore identify the institutional boundary between economies in which ownership changes only the incidence of AI-generated income and those in which ownership also becomes part of the determination of the productive equilibrium.

4. Discussion

The analysis identifies a structural change in labour-market equilibrium when productive AI capital generates reproducible artificial labour services. In standard Diamond–Mortensen–Pissarides environments, human labour is an exogenous productive endowment allocated through search and matching [15,19,25,27]. In the present economy, the human endowment remains fixed, but effective labour capacity becomes partly reproducible because accumulation of productive AI capital K^A expands artificial labour capacity according to $L^A = \chi(K^A)$. Capital accumulation consequently affects not only productive capacity but also the composition of labour services available for productive assignment.

This mechanism differs from the principal margins studied in the literature on skill-biased technological change, occupational restructuring, and task substitution. Those approaches examine how technology changes relative labour demand, task content, or returns to skills [1–3,8–10,17]. The present model introduces an additional equilibrium margin. Productive capital generates artificial labour capacity that can be deployed across task-specific markets. Accumulation therefore changes the effective supply of productive labour services itself. Human-capital valuation must consequently incorporate not only current human productivity, but also matching conditions, continuation values, vacancy creation, and the equilibrium evolution of productive AI capital.

Theorem 1 formalises this mechanism through the recursive valuation of human assignments. Human surplus is not determined by contemporaneous productivity alone. It depends on the discounted sequence of future states through the resolvent operator

$$\widehat{\Sigma}_j = \left(I - \beta \mathcal{Q}_j \mathcal{P}\right)^{-1} g_j. \tag{117}$$

Realised worker value is

$$H_j(S) = \eta_j \left[\widehat{\Sigma}_j(S)\right]_+, \tag{118}$$

while human-capital marketability additionally requires a positive job-finding probability. This distinction is economically important. A human assignment may possess positive latent value while having limited marketability because matching opportunities are scarce. Conversely, once continuation-adjusted surplus reaches the participation boundary, realised human value disappears even though the underlying latent valuation remains well defined.

The finite-state numerical implementation verifies this recursive valuation mechanism. Under the baseline specification, $\rho(A) = 0.72582 < 1$ and $\|A\|_\infty = 0.74784 < 1$, while $\mathrm{cond}(I - A) = 2.34607$. The associated Neumann representation converges rapidly to the direct resolvent solution. Human worker values across the Low, Normal, and High states are approximately 0.656, 0.803, and 0.960, while marketability is approximately 0.144, 0.241, and 0.365. The same human endowment can therefore possess different equilibrium values across states because productivity, matching, and continuation conditions differ. These calculations are structural illustrations of the analytical mechanism rather than estimates of an empirical economy.

The result complements task-based theories of automation. Existing models distinguish displacement, productivity, new-task, and reinstatement effects [2,3], while empirical research documents heterogeneous employment, wage, and occupational responses to automation and AI [4,7,13]. The mechanism developed here concerns a different equilibrium object. Once artificial labour services are generated through accumulable productive capital, the scarcity value of human employment becomes endogenous to the stock and deployment of that capital. The limiting question is therefore not only which tasks can be automated, but whether human employment continues to generate positive recursive surplus as reproducible artificial labour capacity expands [28,30].

The state-contingent displacement analysis makes this distinction explicit. The numerical thresholds

$$a^\dagger = (0.31288,\ 0.40151,\ 0.50286) \tag{119}$$

for the Low, Normal, and High states show that improvements in the artificial-labour alternative need not affect human value uniformly across the state space. Human-capital resilience is itself state dependent because the recursive valuation incorporates the future equilibrium environments reachable from the current state. Technological capability alone is therefore insufficient to determine whether human capital remains valuable.

Theorem 2 extends this valuation result from a state-contingent comparative-static experiment to a stationary technological transition generated by productive AI accumulation. Under Assumption 1, the continuation-adjusted stationary surplus $\Gamma_j(K^A)$ is continuous, strictly decreasing, initially positive, and eventually negative. These conditions generate a unique productive AI-capital threshold $\widehat{K}_j^A$ at which the participation boundary binds. In the numerical structural specification,

$$\widehat{K}^A = 1.847953. \tag{120}$$

This threshold is not an exogenously imposed automation frontier. It is an equilibrium object generated by the interaction between human productivity, the artificial-labour alternative, matching, continuation values, and the accumulation of productive AI capacity.

The economic interpretation of this threshold requires care. Crossing $\widehat{K}_j^A$ does not imply that inherited human assignments vanish instantaneously. Productive assignments are predetermined components of the recursive state and evolve according to their transition law. Human workers already assigned to a task may therefore remain employed temporarily after the participation boundary has been reached. What disappears at $\widehat{K}_j^A$ is positive realised human match surplus. The threshold is consequently a transition in the equilibrium scarcity value of human employment rather than an instantaneous technological deletion of the existing human workforce.

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Corollary 2 further establishes that this regime transition is generated by the participation constraint rather than by an instability of the recursive system. The latent-surplus function $\Gamma_j(K^A)$ remains smooth at the threshold, whereas realised surplus satisfies

$$\Sigma_j^*(K^A) = \left[\Gamma_j(K^A)\right]_+. \tag{121}$$

The numerical implementation confirms this geometry. The left and right derivatives of latent surplus at the threshold are both approximately -0.76664, while the realised human-value derivative changes from approximately -0.42165 on the left to zero on the right. The kink is therefore produced by the endogenous participation boundary. It is not evidence of non-existence, instability, or discontinuity of the underlying recursive valuation problem.

The local comparative statics identify the economic forces determining the location of this boundary. Under the maintained numerical specification, the elasticity of $\widehat{K}^A$ with respect to human productivity is approximately 3.23648. The corresponding elasticities with respect to the scale of artificial labour and its marginal productive contribution are both approximately -1.73475, while those associated with the artificial-labour outside-option parameter, AI-capital conversion efficiency, and the human outside option are approximately -1.03567, -1, and -0.46605, respectively. Greater human productivity therefore increases the productive AI-capital stock required to reach the participation boundary, whereas changes that strengthen the artificial-labour alternative bring that boundary forward.

The zero local derivatives with respect to β, η, and δ require a narrower interpretation. They arise from the maintained one-state functional form, in which these parameters affect the magnitude or division of surplus but not the zero of its numerator. They should therefore not be interpreted as general comparative-static invariance results of the recursive economy.

The global parameter experiment provides a separate diagnostic of the assumptions required for the threshold theorem. Of 4,000 independently sampled parameter vectors, 2,772, or 69.3%, satisfy the sufficient conditions for a unique theorem-consistent threshold. The remaining draws include cases in which initial surplus is non-positive, terminal surplus remains positive, or the surplus path is non-monotone. These cases do not contradict Theorem 2. They lie outside its maintained assumptions. The exercise therefore shows that displacement is not mechanically implied by the existence or accumulation of productive AI capital. A unique displacement threshold emerges only when the equilibrium surplus function possesses the sign and monotonicity properties imposed by the theorem.

The final analytical result concerns a different dimension of the equilibrium. Advanced AI may affect growth, factor shares, employment, and income distribution [12,22,23], while capital intensity and automation have broader implications for labour shares and inequality [6,11,21,26]. Theorem 3 establishes a benchmark in which the productive and distributive consequences of AI can be separated. Under Assumption 2, redistribution of ownership claims leaves the stationary allocation unchanged but alters the household incidence of AI-generated income whenever aggregate AI-capital income is strictly positive.

The numerical ownership experiment illustrates this separation directly. Equal, concentrated, and single-owner structures generate the same productive AI-capital stock, $K^A = 5$, the same rental price, $r^A = 0.8$, and the same aggregate AI-capital income of 4. Output, human and artificial employment, vacancy creation, market tightness, productive AI capital, rental prices, and the technological threshold are unchanged across ownership regimes. The distribution of AI income, however, changes substantially, with Gini coefficients of approximately 0, 0.527, and 0.990. Ownership is therefore neutral for the allocation block under the theorem's exclusion restrictions but non-neutral for distributive incidence.

The qualification is essential. Theorem 3 is not a general irrelevance theorem for ownership. If ownership affects consumption, investment, labour supply, directed search, bargaining, taxation, borrowing constraints, aggregate demand, or the accumulation of productive AI capital, distribution enters the productive block and allocation invariance need not survive. The numerical demand-feedback counterfactual makes precisely this point. Once aggregate demand depends on the ownership distribution, output differs across ownership regimes and production–distribution separation fails. The theorem should therefore be interpreted as an analytically explicit benchmark identifying the conditions under which technological allocation and institutional incidence can be separated.

The three analytical results consequently identify distinct stages of the same recursive economic mechanism. Theorem 1 determines the recursive value and marketability of human employment. Theorem 2 identifies the productive AI-capital stock at which positive human match surplus reaches its participation boundary. Theorem 3 establishes the conditions under which the ownership of the resulting productive AI income changes distribution without changing the allocation that generates it. Recursive valuation, technological regime transition, and distributive incidence are therefore connected through the same equilibrium system, but they remain analytically distinct objects.

This structure also gives precise economic meaning to the question posed in the title. A machine "applies" for a human job not because it literally participates in a human hiring process, but because productive AI becomes an alternative source of labour services for the same productive assignment. Human capital is at risk when that alternative changes the continuation-adjusted surplus generated by assigning human rather than artificial labour. The relevant boundary is therefore not technological feasibility alone. It is the equilibrium point at which human employment ceases to generate positive recursive surplus.

The broader implication is that productive AI cannot generally be represented as another exogenous productivity shock. When accumulable capital generates artificial labour services, capital accumulation changes effective labour capacity and thereby enters the equilibrium determination of human labour scarcity. The decisive economic question is consequently not only whether a machine can perform a human job, but whether human labour continues to possess positive equilibrium value when machines can compete for the same productive assignment.

5. Conclusions

Productive artificial intelligence changes labour-market equilibrium when accumulable capital generates artificial labour services. Human labour remains an exogenous endowment, but effective labour capacity becomes partly reproducible through productive AI capital. Capital accumulation can therefore alter not only productivity and labour demand, but also the scarcity value of human employment.

This paper develops a recursive search-and-matching equilibrium framework for this environment and establishes three results. First, human-capital value is a recursive surplus determined by productivity, matching conditions and continuation values. Second, under the maintained monotonicity conditions, there exists a productive AI-capital threshold at which positive human match surplus reaches its participation boundary. Third, under ownership neutrality, redistributing claims on AI-generated income changes their household incidence without changing the stationary productive allocation.

The analysis therefore separates human-capital valuation, technological displacement and distributive incidence. Displacement is not imposed through an exogenous automation frontier and technological capability alone does not determine it. It emerges endogenously when the continuation-adjusted surplus from human employment reaches zero. Ownership neutrality is likewise a conditional benchmark. If ownership affects investment, labour supply, search, bargaining or other components of the allocation system, distribution may feed back into production.

These results give precise economic meaning to the question posed in the title. A machine effectively enters the market for a human job when productive AI capital generates artificial labour services that can substitute for human labour within the same productive assignment. The relevant economic boundary is therefore not whether a machine is technologically capable of performing a job. It is whether human employment continues to generate positive recursive equilibrium surplus once artificial labour provides a productive alternative.

Appendix A. Numerical Specification and Reproducibility

Appendix A.1. Purpose and Methodological Status

The numerical analysis provides a computational counterpart to the analytical results developed in the paper. It has four purposes. First, it verifies a finite-state implementation of the recursive human-capital valuation operator in Theorem 1. Second, it illustrates the state-contingent displacement thresholds implied by that valuation problem and the stationary technological transition characterised in Theorem 2. Third, it examines the local sensitivity of the productive-AI threshold and evaluates the maintained sufficient conditions for a unique threshold over a wider parameter domain. Fourth, it illustrates the production–distribution separation established in Theorem 3 and shows, through a counterfactual experiment, why that separation fails once ownership feeds back into the productive block.

The exercises are structural rather than empirical. The model is not estimated or calibrated to a particular economy, and the calculations do not constitute forecasts of the development, adoption, or labour-market consequences of Artificial General Intelligence. Parameter values are selected to provide transparent numerical representations of the analytical mechanisms. The results should therefore be interpreted as computational verification, comparative-static analysis, and assumption diagnostics. None of the analytical theorems depends upon the numerical specification reported below.

Appendix A.2. Finite-State Recursive Valuation

Consider the finite state space

$$\mathcal{S} = \{\text{Low}, \text{Normal}, \text{High}\}, \tag{A1}$$

with Markov transition matrix

$$P = \begin{pmatrix} 0.75 & 0.20 & 0.05 \\ 0.15 & 0.70 & 0.15 \\ 0.05 & 0.20 & 0.75 \end{pmatrix}. \tag{A2}$$

Each row is non-negative and sums to one.

For the numerical implementation of Theorem 1, define

$$q = 1 - \delta - \eta f^H, \tag{A3}$$

where all operations are componentwise, and let

$$A = \beta \, \mathrm{diag}(q) P. \tag{A4}$$

The vector of contemporaneous human productive advantages is

$$g = Y^H - \Lambda^A - b. \tag{A5}$$

The finite-state analogue of the recursive valuation operator is therefore

$$R = (I - A)^{-1}, \tag{A6}$$

and latent human surplus satisfies

$$\widehat{\Sigma} = Rg = (I - A)^{-1} g. \tag{A7}$$

Realised worker value, firm value, and human-capital marketability are computed as

$$H = \eta [\widehat{\Sigma}]_+, \qquad J = (1 - \eta)[\widehat{\Sigma}]_+, \qquad M = f^H \odot H, \tag{A8}$$

where $\odot$ denotes componentwise multiplication and the positive-part operator is applied state by state. Equation (A8) preserves the analytical distinction between latent surplus, realised match value, and marketability. In particular, positive human value and positive human-capital formation are not equivalent unless the relevant job-finding probability is also strictly positive.

Appendix A.3. Numerical Specification

The baseline finite-state parameters are

$$\beta = 0.96, \qquad \eta = 0.55, \tag{A9}$$

with

$$\delta = (0.10, 0.08, 0.06), \qquad f^H = (0.22, 0.30, 0.38), \tag{A10}$$

and

$$Y^H = (1.00, 1.20, 1.45), \qquad \Lambda^A = (0.58, 0.62, 0.69), \qquad b = (0.18, 0.18, 0.18). \tag{A11}$$

These values imply

$$q = 1 - \delta - \eta f^H = (0.779, 0.755, 0.731), \tag{A12}$$

and

$$g = Y^H - \Lambda^A - b = (0.240, 0.400, 0.580). \tag{A13}$$

Table A1 reports the resulting recursive valuation.

Table A1. Baseline recursive valuation of human capital.

State	$q(s)$	$g(s)$	$\widehat{\Sigma}(s)$	$H(s)$	$J(s)$	$M(s)$
Low	0.779	0.240	1.193	0.656	0.537	0.144
Normal	0.755	0.400	1.460	0.803	0.657	0.241
High	0.731	0.580	1.746	0.960	0.785	0.365

The calculations illustrate the state dependence of recursive human-capital valuation. Human worker value increases from approximately 0.656 in the Low state to 0.960 in the High state, while marketability rises from approximately 0.144 to 0.365. These differences arise from the joint effect of contemporaneous productive advantage, matching conditions, and continuation values. They should not be interpreted as estimated differences between empirical labour-market states.

Appendix A.4. Operator Diagnostics and Recursive Convergence

For the baseline specification,

$$\rho(A) = 0.72582, \qquad \|A\|_\infty = 0.74784, \qquad \mathrm{cond}(I - A) = 2.34607. \tag{A14}$$

Thus both

$$\rho(A) < 1 \tag{A15}$$

and the stronger sufficient condition

$$\|A\|_\infty < 1 \tag{A16}$$

hold. The inverse in (A7) is therefore well defined for the baseline finite-state system and is numerically well conditioned.

The direct resolvent solution is compared with the truncated Neumann representation

$$\widehat{\Sigma}_N = \sum_{n=0}^{N} A^n g. \tag{A17}$$

Convergence is measured using

$$e_N = \left\| \widehat{\Sigma}_N - \widehat{\Sigma} \right\|_\infty. \tag{A18}$$

Table A2. Convergence of the Neumann representation.

Truncation order N	Sup-norm error e_N
0	1.16550
1	7.95674×10^{-1}
2	5.57727×10^{-1}
5	2.27013×10^{-1}
10	4.69400×10^{-2}
20	1.91433×10^{-3}
50	1.27930×10^{-7}
100	1.42109×10^{-14}

The approximation converges rapidly to the direct resolvent solution. The error falls from 1.16550 at $N = 0$ to 1.91433×10^{-3} at $N = 20$, 1.27930×10^{-7} at $N = 50$, and approximately machine precision at $N = 100$.

The finite-state calculations also clarify the status of the contraction condition imposed in Theorem 1. Consider a diagnostic specification satisfying

$$\rho(\beta QP) = 0.504 < 1, \qquad \beta\|q\|_\infty = 1.152 > 1. \tag{A19}$$

The resolvent nevertheless exists, and the Neumann approximation has error approximately 1.11×10^{-16} at $N = 100$. Thus the sup-norm condition used in the general theorem is sufficient but not necessary in a finite-dimensional representation. The theorem retains the stronger restriction because it provides an immediate contraction argument on $\mathcal{B}(\mathcal{S})$, whereas the spectral-radius condition can be sharper in the finite-state case.

Appendix A.5. State-Contingent Displacement Thresholds

To implement the state-contingent displacement result in Theorem 1, consider an additive improvement $a \geq 0$ in the artificial-labour alternative. The perturbed flow advantage is

$$g_a = g - a\mathbf{1}, \tag{A20}$$

so that latent surplus becomes

$$\widehat{\Sigma}_a = R(g - a\mathbf{1}) = Rg - aR\mathbf{1}. \tag{A21}$$

For state s, the threshold at which latent human surplus reaches zero is therefore

$$a^\dagger(s) = \frac{(Rg)(s)}{(R\mathbf{1})(s)}. \tag{A22}$$

The resulting thresholds are reported in Table A3.

Table A3. State-contingent human-capital displacement thresholds.

State	Flow advantage $g(s)$	Threshold $a^\dagger(s)$
Low	0.240	0.31288
Normal	0.400	0.40151
High	0.580	0.50286

The Low state reaches the participation boundary first, whereas the High state requires the largest improvement in the artificial-labour alternative. The difference is not determined by contemporaneous

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productivity alone because $a^{\dagger}(s)$ depends upon the entire recursive valuation $R = (I - A)^{-1}$. The thresholds therefore illustrate the state-contingent survival result rather than three independent static productivity comparisons.

Appendix A.6. Stationary Productive-AI Regime Transition

The state-contingent experiment above varies an additive artificial-labour advantage while holding the finite-state recursive environment fixed. The technological bifurcation in Theorem 2 is a different exercise. It examines how stationary equilibrium surplus changes with the stock of productive AI capital itself.

For this purpose, artificial labour capacity is represented by

$$\chi(K^A) = \bar{\chi}\Big(1 - \exp(-\kappa K^A)\Big), \tag{A23}$$

while the human job-finding rate is

$$f^H(K^A) = f_{\min} + (f_0 - f_{\min})\exp(-\xi K^A). \tag{A24}$$

The stationary continuation-adjusted human surplus is

$$\Gamma(K^A) = \frac{Y^H - \lambda_0 - \lambda_1 \chi(K^A) - b}{1 - \beta[1 - \delta - \eta f^H(K^A)]}. \tag{A25}$$

Realised human value is

$$H^*(K^A) = \eta\Big[\Gamma(K^A)\Big]_+. \tag{A26}$$

The structural parameter vector is

$$\beta = 0.96, \qquad \delta = 0.08, \qquad \eta = 0.55, \qquad f_0 = 0.38, \qquad f_{\min} = 0.08, \qquad \xi = 0.18, \tag{A27}$$

and

$$Y^H = 1.25, \qquad b = 0.18, \qquad \lambda_0 = 0.40, \qquad \lambda_1 = 1.05, \qquad \bar{\chi} = 1, \qquad \kappa = 0.55. \tag{A28}$$

The surplus function is evaluated over

$$K^A \in [0, 12], \tag{A29}$$

using a grid of 600 points. The first sign-changing interval is identified, after which Brent's bracketing algorithm is used to solve

$$\Gamma(\widehat{K}^A) = 0. \tag{A30}$$

The resulting threshold is

$$\boxed{\widehat{K}^A = 1.847953}. \tag{A31}$$

This value is the numerical counterpart of the participation threshold in Theorem 2. It identifies the productive AI-capital stock at which continuation-adjusted human surplus reaches zero under the maintained stationary specification. It should not be interpreted as an empirical forecast of an AI-capital stock at which human employment disappears.

The distinction is also dynamically important. The threshold concerns the valuation and participation condition for human assignments. It does not imply that inherited human assignments disappear instantaneously at $K^A = \widehat{K}^A$. Existing assignments remain state variables governed by the transition law of the recursive economy. The numerical experiment therefore identifies the disappearance of positive realised human surplus, not the instantaneous physical elimination of all existing human employment.

Appendix A.7. Regularity at the Participation Boundary

The numerical implementation also illustrates Corollary 2. One-sided finite differences at $\widehat{K}^A$ give

$$D_-\Gamma(\widehat{K}^A) \approx -0.76664, \qquad D_+\Gamma(\widehat{K}^A) \approx -0.76664. \tag{A32}$$

The latent-surplus function is therefore locally smooth at the threshold.

For realised human value,

$$D_-H^*(\widehat{K}^A) \approx -0.42165, \qquad D_+H^*(\widehat{K}^A) \approx 0. \tag{A33}$$

Hence H^* is continuous but not differentiable at the threshold. The non-differentiability arises from the positive-part participation constraint in (A26), not from a discontinuity or instability of the underlying latent-surplus recursion. Numerically, the bifurcation is therefore a kink in realised equilibrium value generated by endogenous participation.

Appendix A.8. Local Comparative Statics of the Technological Threshold

For each parameter θ, the local derivative of $\widehat{K}^A$ is approximated using a centred finite difference with step

$$h_\theta = 10^{-4}\max\{|\theta|, 1\}. \tag{A34}$$

The corresponding local elasticity is

$$\varepsilon_\theta = \frac{\partial \widehat{K}^A}{\partial \theta}\frac{\theta}{\widehat{K}^A}. \tag{A35}$$

Table A4. Local comparative statics of the technological threshold.

Parameter	Baseline	$\partial \widehat{K}^A/\partial\theta$	Elasticity
Y^H	1.25	4.78469	3.23648
$\bar{\chi}$	1.00	−3.20574	−1.73475
λ_1	1.05	−3.05309	−1.73475
λ_0	0.40	−4.78469	−1.03567
κ	0.55	−3.35991	−1.00000
b	0.18	−4.78469	−0.46605
β	0.96	0.00000	0.00000
η	0.55	0.00000	0.00000
δ	0.08	0.00000	0.00000

The threshold is most responsive to human productivity under this specification. Increasing Y^H raises the productive AI-capital stock required to reach the participation boundary. Increases in $\bar{\chi}$, λ_1, λ_0, or κ strengthen the artificial-labour alternative and reduce the threshold.

The zero derivatives for β, η, and δ are not general comparative-static results. They are consequences of (A25). Under the maintained one-state functional form, these parameters enter the denominator of $\Gamma(K^A)$ but not the numerator whose zero determines $\widehat{K}^A$. They can therefore alter the magnitude of continuation-adjusted surplus away from the threshold without moving the threshold itself.

Appendix A.9. Global Parameter Diagnostics

The preceding threshold calculation is conditional on the regularity requirements of Assumption 1. A global parameter experiment is used to examine how frequently the numerical specification satisfies these sufficient conditions over a broader parameter domain.

The experiment contains

$$N_{\mathrm{MC}} = 4000 \tag{A36}$$

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independent parameter vectors generated using

$$\texttt{numpy.random.default\_rng(20260726)}. \tag{A37}$$

Each parameter is drawn independently from the uniform intervals reported in Table A5.

Table A5. Parameter ranges used in the global diagnostic experiment.

Parameter	Lower bound	Upper bound
Y^H	0.90	1.70
b	0.05	0.35
λ_0	0.15	0.75
λ_1	0.35	1.55
$\bar{\chi}$	0.65	1.35
κ	0.18	1.20
η	0.25	0.80
δ	0.03	0.20
β	0.88	0.99

For every draw, $\Gamma(K^A)$ is evaluated over the same numerical domain. A draw is classified as theorem-consistent when the numerical path satisfies all of the following conditions

$$\Gamma(0) > 0, \tag{A38}$$

$$\Gamma(12) < 0, \tag{A39}$$

the evaluated surplus path is strictly decreasing on the numerical grid, and exactly one sign-changing interval is identified.

The resulting diagnostics are reported in Table A6.

Table A6. Diagnostics from the global parameter experiment.

Diagnostic	Count	Share
Unique theorem-consistent threshold	2772	69.3%
No positive initial surplus	39	1.0%
No negative terminal surplus	1189	29.7%
Non-monotone numerical path	953	23.8%

The failure categories are not mutually exclusive. A single parameter vector may violate more than one maintained condition, so the reported percentages should not be added to construct an exhaustive partition.

More importantly,

$$\frac{2772}{4000} = 0.693 \tag{A40}$$

is not an estimated probability of human displacement and is not evidence that an AGI-induced regime transition occurs with probability 69.3%. It records only the share of the specified parameter domain for which the numerical implementation satisfies the sufficient conditions used by Theorem 2. Parameter vectors generating tangencies, multiple crossings, non-monotone surplus paths, or no crossing lie outside the unique-threshold theorem. Their existence therefore identifies the substantive role of Assumption 1 rather than contradicting the theorem.

Appendix A.10. Ownership, Distribution, and Allocation Invariance

The final numerical exercise illustrates Theorem 3. It deliberately holds the productive allocation fixed while varying the distribution of ownership claims on productive AI capital. The experiment contains

$$N_H = 100 \tag{A41}$$

households and aggregate AI-capital income

$$R^A = r^A K^A = 4. \tag{A42}$$

Three ownership structures are considered. The first distributes productive AI capital equally across households. The second uses a concentrated rank-based ownership profile proportional to $i^{-0.85}$. The third assigns all productive AI ownership to a single household.

Table A7. Ownership regimes and AI-capital income.

Ownership regime	Aggregate AI income	Income Gini	K^A	r^A
Equal ownership	4.000	0.000	5.000	0.800
Concentrated ownership	4.000	0.527	5.000	0.800
Single owner	4.000	0.990	5.000	0.800

Output, human employment, artificial employment, vacancy creation, market tightness, productive AI capital, the AI rental price, and the technological threshold are identical across the three ownership regimes. The maximum allocation residual between the equal-ownership and single-owner economies is numerically zero, while the aggregate-income residual is

$$4.44 \times 10^{-16}. \tag{A43}$$

By contrast, the L^1-distance between the corresponding household AI-income distributions is

$$7.92. \tag{A44}$$

The experiment therefore illustrates the exact content of Theorem 3. Conditional on the institutional-separation assumptions, ownership redistributes claims on a fixed flow of AI-capital income without changing the productive allocation generating that income. The large change in distribution is therefore compatible with exact allocation invariance.

Appendix A.11. Failure of Separation Under Distributional Feedback

The ownership-neutrality result depends upon the exclusion restrictions in Assumption 2. To make this dependence explicit, consider the counterfactual specification

$$Y(\Omega) = Y_0[1 - \psi G(\Omega)], \tag{A45}$$

where $G(\Omega)$ denotes the Gini coefficient associated with ownership regime Ω and $\psi > 0$ measures the strength of the distributional feedback.

For every $\psi > 0$, ownership now enters the productive block through aggregate demand, so output differs across ownership regimes. The allocation invariance established in Theorem 3 consequently no longer follows.

Equation (A45) is not proposed as an alternative calibrated model of aggregate demand. Its purpose is diagnostic. It demonstrates that production–distribution separation depends upon ownership remaining excluded from the equations determining production, investment, labour supply, matching, bargaining, vacancy creation, and productive AI-capital accumulation. Once any such channel is introduced, ownership may affect both distribution and allocation.

Appendix A.12. Computational Environment and Replication Protocol

All calculations are implemented in Python using `NumPy`, `SciPy`, `pandas`, and `Matplotlib`. Matrix eigenvalues are computed using `scipy.linalg.eigvals`, and scalar threshold equations are solved using `scipy.optimize.brentq`. Calculations use IEEE-754 double precision.

A successful replication should reproduce, up to the displayed rounding, the operator diagnostics

$$\rho(A) = 0.72582, \qquad \|A\|_\infty = 0.74784, \qquad \mathrm{cond}(I - A) = 2.34607, \tag{A46}$$

the state-contingent thresholds

$$a^\dagger = (0.31288,\ 0.40151,\ 0.50286), \tag{A47}$$

the stationary technological threshold

$$\widehat{K}^A = 1.847953, \tag{A48}$$

the global diagnostic share

$$\frac{N_{\text{valid}}}{N_{\text{MC}}} = \frac{2772}{4000} = 0.693, \tag{A49}$$

and the ownership Gini coefficients

$$(0,\ 0.527,\ 0.990). \tag{A50}$$

The Neumann approximation should satisfy

$$e_{100} \approx 1.42109 \times 10^{-14}, \tag{A51}$$

while the ownership experiment should produce a maximum allocation residual of zero up to numerical precision and an aggregate-income residual of approximately 4.44×10^{-16}.

These replication targets verify the numerical implementation of the paper's analytical mechanisms. They do not constitute empirical moment conditions or calibration targets. Their purpose is to permit independent reproduction of the recursive valuation, displacement-threshold, regularity, comparative-static, and ownership-separation exercises reported in the paper.

Appendix B. Technical Appendix

Appendix B.1. Proof of Theorem 1

Let

$$\mathcal{R}_j = \left(I - \beta \mathcal{A}_j \mathcal{P}\right)^{-1}.$$

Under the contraction condition imposed in Theorem 1, the inverse exists as a bounded linear operator on $\mathcal{B}(\mathcal{S})$ and admits the Neumann-series representation

$$\mathcal{R}_j = \sum_{n=0}^{\infty} \beta^n \left(\mathcal{A}_j \mathcal{P}\right)^n. \tag{A52}$$

Applying this operator to the latent-surplus recursion yields

$$\widehat{\Sigma}_j = \mathcal{R}_j b_j,$$

which proves (56). Equations (57)–(59) then follow from the realised-surplus rule (51) and the Nash surplus-sharing condition (52). For the displacement result, let $d_j \geq 0$ denote the reduction in the current relative surplus of human labour. Linearity of the resolvent gives

$$\mathcal{R}_j(b_j - d_j) = \mathcal{R}_j b_j - \mathcal{R}_j d_j.$$

Moreover,

$$(\mathcal{R}_j d_j)(S_0) = \sum_{n=0}^{\infty} \beta^n \Big[(\mathcal{A}_j \mathcal{P})^n d_j \Big] (S_0).$$

Every term in this series is non-negative. If

$$\Big[(\mathcal{A}_j \mathcal{P})^n d_j \Big] (S_0) > 0$$

for at least one $n \in \mathbb{N}_0$, then

$$(\mathcal{R}_j d_j)(S_0) > 0.$$

Hence the continuation-adjusted latent human surplus is strictly lower at S_0, establishing the strict displacement result in Theorem 1. For the threshold result, define

$$\varphi_j(K^A; S) = \widehat{\Sigma}_j(S; K^A).$$

By the conditions of Theorem 1, $\varphi_j(\cdot; S)$ is continuous and strictly decreasing on $\mathcal{K}$, with

$$\varphi_j(K_L^A; S) > 0, \qquad \varphi_j(K_H^A; S) \leq 0.$$

The intermediate value theorem therefore implies the existence of $K_j^\dagger(S) \in (K_L^A, K_H^A]$ satisfying

$$\varphi_j\Big(K_j^\dagger(S); S\Big) = 0.$$

Strict monotonicity implies uniqueness and yields

$$\varphi_j(K^A; S) > 0 \quad \Longleftrightarrow \quad K^A < K_j^\dagger(S).$$

Applying the positive-part rule gives (64). Since $f_j^H(S; K^A) \geq 0$, multiplication by the equilibrium job-finding probability gives (65). This establishes all claims of Theorem 1.

□

Appendix B.2. Proof of Corollary 1

By Theorem 1(iii),

$$H_j(S; K^A) > 0 \quad \Longleftrightarrow \quad K^A < K_j^\dagger(S). \tag{A53}$$

Suppose first that $K^A < \underline{K}_j^\dagger$. By definition of $\underline{K}_j^\dagger$,

$$K^A < \underline{K}_j^\dagger \leq K_j^\dagger(S) \qquad \text{for every } S \in \mathcal{S}.$$

It follows from (A53) that

$$H_j(S; K^A) > 0 \qquad \text{for every } S \in \mathcal{S}.$$

Since

$$\mathcal{M}_j^H(S; K^A) = f_j^H(S; K^A) H_j(S; K^A),$$

strictly positive human-capital value implies

$$\mathcal{M}_j^H(S; K^A) > 0 \quad \Longleftrightarrow \quad f_j^H(S; K^A) > 0,$$

which establishes (69). Next, suppose

$$\underline{K}_j^\dagger \leq K^A < \overline{K}_j^\dagger.$$

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For each $S \in \mathcal{S}$, exactly one of

$$K^A < K_j^\dagger(S) \qquad \text{and} \qquad K^A \geq K_j^\dagger(S)$$

holds. Hence $\mathcal{S}_j^+(K^A)$ and $\mathcal{S}_j^0(K^A)$ form the partition defined in (73). Applying (A53) state by state gives (74) and (75). Since $f_j^H(S;K^A) \geq 0$, the identity

$$\mathcal{M}_j^H(S;K^A) = f_j^H(S;K^A)H_j(S;K^A)$$

then establishes (76). Finally, suppose $K^A \geq \overline{K}_j^\dagger$. By definition of $\overline{K}_j^\dagger$,

$$K^A \geq \overline{K}_j^\dagger \geq K_j^\dagger(S) \qquad \text{for every } S \in \mathcal{S}.$$

Equation (A53) therefore implies

$$H_j(S;K^A) = 0 \qquad \text{for every } S \in \mathcal{S}.$$

Consequently,

$$\mathcal{M}_j^H(S;K^A) = 0 \qquad \text{for every } S \in \mathcal{S},$$

which establishes the global displacement result.

□

Appendix B.3. Proof of Theorem 2

Assumption 1 implies

$$\Gamma_j(0) > 0 \qquad \text{and} \qquad \lim_{K^A \to \infty} \Gamma_j(K^A) < 0.$$

Hence there exists $\overline{K}_j^A > 0$ such that

$$\Gamma_j(\overline{K}_j^A) < 0.$$

By continuity of Γ_j, the Intermediate Value Theorem therefore implies the existence of $\widehat{K}_j^A \in (0, \overline{K}_j^A)$ satisfying

$$\Gamma_j(\widehat{K}_j^A) = 0.$$

Since

$$\Gamma_j'(K^A) < 0 \qquad \text{for every } K^A \geq 0,$$

the function Γ_j is strictly decreasing. Its zero is therefore unique, and

$$\Gamma_j(K^A)\begin{cases} > 0, & K^A < \widehat{K}_j^A, \\ = 0, & K^A = \widehat{K}_j^A, \\ < 0, & K^A > \widehat{K}_j^A. \end{cases}$$

This establishes (90) and (91). The strict change in sign at $\widehat{K}_j^A$ also satisfies Definition 3. By the realised-surplus rule and Nash surplus sharing,

$$\Sigma_j^*(K^A) = [\Gamma_j(K^A)]_+,$$

$$H_j^*(K^A) = \eta_j \Sigma_j^*(K^A), \qquad J_j^*(K^A) = (1-\eta_j)\Sigma_j^*(K^A),$$

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with $\eta_j \in (0,1)$. Hence

$$\Gamma_j(K^A) > 0 \iff \Sigma_j^*(K^A) > 0 \iff H_j^*(K^A) > 0 \iff J_j^*(K^A) > 0.$$

Combining this equivalence with the unique sign threshold established above proves (92). If $K^A \geq \widehat{K}_j^A$, then $\Gamma_j(K^A) \leq 0$, so the positive-part operator implies

$$\Sigma_j^*(K^A) = H_j^*(K^A) = J_j^*(K^A) = 0,$$

which establishes (93). Finally,

$$M_j^*(K^A) = f_j^{H*}(K^A) H_j^*(K^A),$$

with $f_j^{H*}(K^A) \geq 0$. Therefore

$$M_j^*(K^A) > 0 \iff f_j^{H*}(K^A) > 0 \text{ and } H_j^*(K^A) > 0.$$

Using the preceding equivalence,

$$M_j^*(K^A) > 0 \iff K^A < \widehat{K}_j^A \text{ and } f_j^{H*}(K^A) > 0,$$

which proves (94).

□

Appendix B.4. Proof of Corollary 2

By definition of realised surplus and Theorem 2,

$$\Sigma_j^*(K^A) = \begin{cases} \Gamma_j(K^A), & K^A < \widehat{K}_j^A, \\ 0, & K^A \geq \widehat{K}_j^A. \end{cases} \tag{A54}$$

Since $\Gamma_j \in C^1(\mathbb{R}_+)$ and $\Gamma_j(\widehat{K}_j^A) = 0$, the two branches coincide at $\widehat{K}_j^A$. Hence Σ_j^* is continuous on $\mathbb{R}_+$ and continuously differentiable on

$$[0, \widehat{K}_j^A) \cup (\widehat{K}_j^A, \infty).$$

Differentiating (A54) away from the threshold gives

$$\frac{d\Sigma_j^*}{dK^A} = \begin{cases} \Gamma_j'(K^A), & K^A < \widehat{K}_j^A, \\ 0, & K^A > \widehat{K}_j^A. \end{cases} \tag{A55}$$

At the threshold, the left derivative is

$$D_-\Sigma_j^*(\widehat{K}_j^A) = \Gamma_j'(\widehat{K}_j^A),$$

whereas Σ_j^* is identically zero immediately to the right, so

$$D_+\Sigma_j^*(\widehat{K}_j^A) = 0.$$

Assumption 1 implies

$$\Gamma_j'(\widehat{K}_j^A) < 0.$$

The one-sided derivatives therefore differ, and Σ_j^* is not differentiable at $\widehat{K}_j^A$. Since Nash surplus sharing gives

$$H_j^*(K^A) = \eta_j \Sigma_j^*(K^A), \qquad J_j^*(K^A) = (1-\eta_j)\Sigma_j^*(K^A),$$

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with $\eta_j \in (0,1)$, both H_j^* and J_j^* are continuous on $\mathbb{R}_+$, continuously differentiable away from $\widehat{K}_j^A$, and satisfy

$$D_- H_j^*(\widehat{K}_j^A) = \eta_j \Gamma_j'(\widehat{K}_j^A), \qquad D_+ H_j^*(\widehat{K}_j^A) = 0,$$

and

$$D_- J_j^*(\widehat{K}_j^A) = (1-\eta_j)\Gamma_j'(\widehat{K}_j^A), \qquad D_+ J_j^*(\widehat{K}_j^A) = 0.$$

Because $\eta_j \in (0,1)$ and $\Gamma_j'(\widehat{K}_j^A) < 0$, neither pair of one-sided derivatives coincides. Thus

$$\Sigma_j^*,\ H_j^*,\ J_j^* \in C^0(\mathbb{R}_+) \setminus C^1(\mathbb{R}_+),$$

with each value function exhibiting a kink at the technological threshold $\widehat{K}_j^A$.

□

Appendix B.5. Proof of Lemma 1

Under Assumption 2, the stationary productive allocation under any ownership regime $\Omega \in \Delta^{N-1}$ satisfies

$$\mathcal{F}^A(\mathcal{E}_\Omega^*; \mathcal{G}) = 0. \tag{A56}$$

The ownership vector Ω does not enter the allocation operator $\mathcal{F}^A$. Hence, for any two ownership regimes $\Omega, \widetilde{\Omega} \in \Delta^{N-1}$,

$$\mathcal{F}^A(\mathcal{E}_\Omega^*; \mathcal{G}) = 0, \qquad \mathcal{F}^A\left(\mathcal{E}_{\widetilde{\Omega}}^*; \mathcal{G}\right) = 0. \tag{A57}$$

Both allocations therefore solve the same stationary allocation system under the same primitive environment $\mathcal{G}$. By the uniqueness condition in Assumption 2,

$$\mathcal{E}_\Omega^* = \mathcal{E}_{\widetilde{\Omega}}^* = \mathcal{E}^*(\mathcal{G}), \tag{A58}$$

which proves (103).

The corresponding household distribution under ownership regime Ω is determined by

$$\mathcal{D}_\Omega^* = \mathcal{F}^D(\mathcal{E}^*(\mathcal{G}), \Omega; \mathcal{G}). \tag{A59}$$

Although the productive allocation $\mathcal{E}^*(\mathcal{G})$ is ownership invariant, the ownership vector Ω enters the distribution mapping directly. Hence changes in ownership can alter household-level claims without altering the productive allocation from which those claims arise. In particular, if productive AI generates strictly positive distributable income and $\Omega \neq \widetilde{\Omega}$ assigns different claims to at least one household, then the corresponding household income claims differ. Therefore

$$\mathcal{D}_\Omega^* \neq \mathcal{D}_{\widetilde{\Omega}}^*, \tag{A60}$$

which establishes (104) under the conditions stated in the lemma.

□

Appendix B.6. Proof of Theorem 3

By Lemma 1, the ownership vector does not enter the stationary allocation operator. Hence, under the common primitive environment $\mathcal{G}$,

$$\mathcal{F}^A(\mathcal{E}_\Omega^*; \mathcal{G}) = 0, \qquad \mathcal{F}^A\left(\mathcal{E}_{\widetilde{\Omega}}^*; \mathcal{G}\right) = 0.$$

The uniqueness condition in Assumption 2 therefore implies

$$\mathcal{E}_\Omega^* = \mathcal{E}_{\widetilde{\Omega}}^* = \mathcal{E}^*(\mathcal{G}),$$

which proves (105). Every equilibrium object determined exclusively by the stationary allocation block is consequently ownership invariant. In particular, the equilibrium artificial-labour rental prices $\mathbf{r}^{A,*}$ and productive assignments $\mathbf{N}^{A,*}$ are identical under both ownership regimes. Hence aggregate distributable AI-generated income,

$$R^{A,*} = \sum_{j \in \mathcal{J}} r_j^{A,*} N_j^{A,*},$$

is ownership invariant. Since

$$y_{i,\Omega}^{A,*} = \omega_i R^{A,*},$$

and $\sum_{i=1}^{N} \omega_i = 1$, it follows that

$$\sum_{i=1}^{N} y_{i,\Omega}^{A,*} = R^{A,*},$$

for every $\Omega \in \Delta^{N-1}$. Thus ownership affects neither the productive allocation nor the aggregate magnitude of AI-generated income. For two ownership regimes Ω and $\widetilde{\Omega}$,

$$y_{i,\Omega}^{A,*} - y_{i,\widetilde{\Omega}}^{A,*} = (\omega_i - \widetilde{\omega}_i) R^{A,*},$$

which establishes (109). If $\Omega \neq \widetilde{\Omega}$, then there exists at least one household i such that

$$\omega_i \neq \widetilde{\omega}_i.$$

Whenever $R^{A,*} > 0$, (109) therefore implies

$$y_{i,\Omega}^{A,*} \neq y_{i,\widetilde{\Omega}}^{A,*}.$$

Ownership consequently changes the household incidence of AI-generated income while leaving both its aggregate magnitude and the stationary productive allocation unchanged. This establishes the production–distribution separation claimed in Theorem 3.

□

Appendix B.7. Proof of Corollary 3

By Theorem 3, the stationary productive allocation is invariant to the distribution of ownership claims. Hence, for every feasible redistribution $d\Omega$,

$$\mathcal{E}_{\Omega+d\Omega}^{*} = \mathcal{E}_{\Omega}^{*} = \mathcal{E}^{*}(\mathcal{G}),$$

which proves (112). Every equilibrium object determined exclusively by the allocation block is therefore unchanged. Since aggregate distributable AI-generated income $R^{A,*}$ is determined entirely by the ownership-invariant allocation, household i's income claim under the redistributed ownership vector is

$$y_{i,\Omega+d\Omega}^{A,*} = (\omega_i + d\omega_i) R^{A,*},$$

whereas

$$y_{i,\Omega}^{A,*} = \omega_i R^{A,*}.$$

Subtracting yields

$$y_{i,\Omega+d\Omega}^{A,*} - y_{i,\Omega}^{A,*} = d\omega_i R^{A,*},$$

which proves (113). If $R^{A,*} > 0$, this difference is non-zero if and only if $d\omega_i \neq 0$. Summing across households and using the feasibility condition $\sum_{i=1}^{N} d\omega_i = 0$ gives

$$\sum_{i=1}^{N} \left(y_{i,\Omega+d\Omega}^{A,*} - y_{i,\Omega}^{A,*} \right) = R^{A,*} \sum_{i=1}^{N} d\omega_i = 0,$$

which establishes (114). Finally, suppose stationary household wealth claims satisfy

$$a_i^* = \mathcal{A}_i\left(y_i^{A,*}; \mathcal{G}\right),$$

with $\mathcal{A}_i(\cdot; \mathcal{G})$ injective. If $R^{A,*} > 0$ and $d\omega_i \neq 0$, then

$$y_{i,\Omega+d\Omega}^{A,*} \neq y_{i,\Omega}^{A,*}.$$

Injectivity therefore implies

$$\mathcal{A}_i\left(y_{i,\Omega+d\Omega}^{A,*}; \mathcal{G}\right) \neq \mathcal{A}_i\left(y_{i,\Omega}^{A,*}; \mathcal{G}\right),$$

and hence

$$a_{i,\Omega+d\Omega}^* \neq a_{i,\Omega}^*.$$

Every non-trivial feasible redistribution has $d\omega_i \neq 0$ for at least one household. Thus, whenever $R^{A,*} > 0$, ownership redistribution leaves the stationary productive allocation unchanged while altering at least one household's AI-income claim and, under the maintained injective wealth mapping, its stationary wealth claim.

□

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