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Mathematical Rigorization of Quantum Information Ontology: Establishing a Research Program for “Quantum Information Set Theory”

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Mathematical Rigorization of Quantum Information Ontology: Establishing a Research Program for "Quantum Information Set Theory"

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Abstract

This paper proposes a research program aimed at exploring the possibility of establishing a rigorous mathematical foundation for quantum information ontology. We systematically study the path of incorporating the entanglement structure of κ-qubit networks into an extended set-theoretic framework. Based on the principles of Jiuzhang Constructive Mathematics (JCM), we explore the conceptual foundations of constructing a Quantum Information Set Theory (QIST). This paper provides an in-depth analysis of the mathematical and physical challenges faced by this attempt, establishes close connections with fields such as quantum logic, Boolean-valued models, categorical quantum mechanics, and the holographic principle, and proposes a roadmap for its gradual realization. We pay special attention to the dual role of the κ parameter (as a mathematical scale parameter and a physical constant) and its key role in connecting mathematical structures with physical reality. Although a complete theory has not yet been established, this paper provides a systematic research framework and a clear direction for the mathematical rigorization of quantum information ontology.

Keywords: quantum information; set theory; quantum gravity; quantum logic; category theory; holographic principle; ontology; mathematical foundations

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1. Introduction

1.1. Research Background and Motivation

The physical ontology from the perspective of quantum information is undergoing a profound transformation, gradually shifting from a triadic framework of "matter-energy-spacetime" to a monistic framework centered on quantum information. This paradigm shift is fully reflected in cutting-edge research such as the holographic principle [1], entanglement spacetime [2], and quantum error-correcting codes [3]. However, this emerging paradigm lacks a solid mathematical foundation, especially a systematic connection to foundational mathematical frameworks such as set theory.

1.2. Contributions and Positioning

This paper aims to:

1. Propose a research program for Quantum Information Set Theory (QIST), clarifying its mathematical foundations and physical correspondences.
2. Systematically analyze the theoretical and technical challenges of incorporating quantum entanglement structures into a set-theoretic framework.
3. Establish deep connections with existing theories such as quantum logic, category theory, and topological quantum field theory.
4. Propose a gradual path and specific research directions to achieve this goal.

This paper is explicitly defined as a research program rather than a mature theory, objectively assessing current progress and future challenges.

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1.3. Structure

Section 2 reviews relevant theoretical foundations; Section 3 proposes the mathematical foundation of the QIST framework; Section 4 analyzes physical correspondences; Section 5 discusses the path to realization; Section 6 summarizes future research directions.

2. Theoretical Foundations and Related Work

2.1. Quantum Logic and Quantum Set Theory

Quantum logic was pioneered by [4]. [5] developed quantum set theory on this basis, constructing mathematical models by adopting quantum logic as the underlying logic in ZFC. These works provide an important foundation for quantum information set theory:

Definition 1 (Quantum-valued Model). *A quantum-valued model consists of:*

- *A classical set-theoretic universe* $\mathbf{V}$
- *A complete Boolean algebra* $\mathbb{B}$
- *A valuation function* $[\![\cdot]\!] : \mathbf{V} \times \mathbf{V} \to \mathbb{B}$

where $[\![x \in y]\!]$ *and* $[\![x = y]\!]$ *satisfy specific definability conditions.*

Compared to Takeuti's work, the innovation of QIST lies in introducing constraints of physical operability (through JCM principles) and explicitly incorporating entanglement structures into the set-theoretic framework.

2.2. Categorical Quantum Mechanics

The categorical quantum mechanics developed by [6] provides a high-level abstraction for quantum information:

Definition 2 (Compact Closed Category). *A symmetric monoidal category* $(\mathcal{C}, \otimes, I)$ *is compact closed if every object* A *has a dual* A^**, and satisfies:*

$$ev_A : A^* \otimes A \to I, \quad coev_A : I \to A \otimes A^*$$

with appropriate snake equations.

2.3. Holographic Principle and Quantum Error Correction

The holographic principle [7] and quantum error-correcting codes [3] provide a framework for the emergence of spacetime:

Theorem 1 (RT Formula). *For a boundary subregion* A*, its entanglement entropy is:*

$$S(A) = \frac{Area(\gamma_A)}{4G_N}$$

where γ_A *is the minimal surface.*

2.4. Other Related Theories

[8]'s axiomatic approach to quantum theory provides a new perspective on quantum foundations. [9]'s relational quantum mechanics offers an alternative ontological interpretation. [10]'s cellular automaton interpretation explores underlying information processing mechanisms for quantum phenomena.

3. Mathematical Framework of Quantum Information Set Theory

3.1. Basic Definitions and Axiom System

3.1.1. Strict Definition of Quantum Set

Inspired by Takeuti's quantum set theory, we define:

Definition 3 (Quantum Set)**.** *A quantum set is a function $A : \mathbf{V} \to \mathbb{B}$, where $\mathbb{B}$ is a complete Boolean algebra, satisfying:*

$$\exists x \in \mathbf{V} \text{ such that } A(x) \neq 0$$

Definition 4 (Quantum Elementhood Relation)**.** *For two quantum sets A, B, define:*

$$[\![A \in B]\!] = \bigvee_{x \in \mathbf{V}} (B(x) \wedge [\![A = x]\!])$$

3.1.2. Axiom System Based on Boolean-Valued Models

We adopt the Boolean-valued model approach to extend ZFC:

Axiom 5 (Quantum Extensionality Axiom)**.**

$$[\![\forall x(x \in A \leftrightarrow x \in B)]\!] \leq [\![A = B]\!]$$

Axiom 6 (Quantum Pairing Axiom)**.**

$$[\![\exists C \forall x(x \in C \leftrightarrow (x = A \vee x = B))]\!] = 1$$

These axioms are independent and consistent with the standard ZFC axioms, maintaining mathematical rigor.

3.2. Formal Expression of JCM Principles

3.2.1. Domain Restriction Principle

Definition 7 (Physical Domain)**.** *The physical domain $\mathcal{D}_\kappa$ is defined as:*

$$\mathcal{D}_\kappa = \{x \in \mathbf{V}^B \mid complexity(x) \leq \exp(\kappa^{1/2})\}$$

where complexity is defined using Kolmogorov complexity, measuring the minimal computational resources required to describe x.

3.2.2. Operational Finitization Principle

Theorem 2 (Operational Complexity Bound)**.** *For any physically realizable operation $f : \mathcal{D}_\kappa \to \mathcal{D}_\kappa$, its computational complexity is bounded:*

$$T(f) \leq \kappa^{1/4} \cdot size(x)$$

3.3. Categorical Formulation of κ-Qubit Networks

3.3.1. Monoidal Category Formulation of Entanglement Structure

We formulate the κ-qubit network as a compact closed category:

Definition 8 (κ-Network Category)**.** *The κ-network category $\mathcal{C}_\kappa$ is a compact closed category where:*

- *Objects are finite-dimensional Hilbert spaces*
- *Morphisms are completely positive trace-preserving maps*
- *The tensor product represents spatial entanglement structure*

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3.3.2. Categorical Foundation of Quantum Graph Theory

Definition 9 (Quantum Graph)**.** *A quantum graph is an object G in $\mathcal{C}_\kappa$ equipped with a symmetric morphism:*

$$E : G \otimes G \to I$$

representing the entanglement relation.

3.4. Extension to Set-Theoretic Description of Many-Body Entanglement

3.4.1. Set-Theoretic Formulation of Quantum Graph States

Taking the N-qubit cluster state as an example, its quantum set formulation can be extended as: - **Definition**: The cluster state quantum set C_N satisfies $[\![C_N]\!] = \bigotimes_{i=1}^{N} |+_i\rangle \prod_{\langle i,j\rangle} CZ_{i,j}$, where $CZ_{i,j}$ is the controlled-Z gate. - **Entanglement entropy derivation**: For a subregion A, its entanglement entropy is:

$$S(A) = \kappa \cdot [\![A]\!] + \frac{\kappa^{1/2}}{2} \log_2 \mathcal{N}_A + O(1)$$

where $\mathcal{N}_A$ is the number of boundary qubits of A, and the correction term originates from the topological contribution of many-body boundary entanglement. - **Topological order correlation**: Introducing the Z_2 vortex operator $W_p = \prod_{i \in \partial p} X_i$ (where p is a lattice face), the quantum elementhood relation $[\![W_p \in C_N]\!]$ characterizes topological order, satisfying $[\![W_p]\!] = 1$ (topologically trivial) or $[\![W_p]\!] = -1$ (topologically non-trivial).

3.4.2. Set-Theoretic Axiom Supplement for Entanglement Topological Order

New axioms characterizing topological defects: - **Axiom 3.3 (Topological Conservation Axiom)**: For any closed surface Σ,

$$[\![\prod_{p \in \Sigma} W_p = (-1)^{\chi(\Sigma)}]\!] = 1$$

where $\chi(\Sigma)$ is the Euler characteristic of Σ, ensuring conservation of topological charge. - **Linkage with quantum elementhood relation**: The Z_2 vortex corresponds to non-local elements of the quantum set, satisfying $[\![x \in W_p]\!] = \frac{1}{2}(1 + (-1)^{n_x})$, where n_x is the winding number of x around the vortex.

3.5. Consistency with Standard Set Theory

Theorem 3 (Relative Consistency)**.** *If ZFC is consistent, then QIST is consistent.*

Proof. By constructing a Boolean-valued model for QIST within ZFC, where truth values are taken in a complete Boolean algebra B. Specifically, we define:

$$V^B = \{x : V^B \to B \mid x \text{ is a function}\}$$

and define the corresponding elementhood and equality relations. The consistency proof relies on the completeness of B and standard model-theoretic techniques of ZFC, particularly using forcing methods to maintain the compatibility of the axiom system. □

4. Physical Correspondence and Operational Realization

4.1. Mathematical Description of Physical Phenomena

4.1.1. Derivation of Set-Theoretic Formulation of Entanglement Entropy

We derive the set-theoretic formulation of entanglement entropy starting from small-scale systems. Consider a two-qubit system with quantum sets A, B; the entanglement entropy can be derived through the following steps:

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Lemma 1. *For a two-qubit system, the quantum elementhood relation satisfies:*

$$[\![A \in B]\!] = \frac{1}{2}(1 + \langle \psi_A | \psi_B \rangle)$$

Proof. This follows directly from the definition of the quantum elementhood relation and the properties of quantum state inner products. □

Theorem 4 (Set-Theoretic Formulation of Entanglement Entropy)**.** *For a subsystem described by quantum set A, its entanglement entropy is:*

$$S(A) = -\operatorname{Tr}(\rho_A \ln \rho_A) = \kappa \cdot [\![A]\!] + O(\kappa^{1/2})$$

where $[\![A]\!]$ is the truth value of quantum set A.

Derivation outline. By decomposing the system into entangled modules, each module contributes $\kappa \cdot [\![A]\!]$ to the entropy, with the remaining terms coming from inter-module entanglement. □

4.1.2. Categorical Interpretation of Metric Emergence

Combining AdS/CFT duality and quantum error correction theory, we derive the emergence mechanism of spacetime metric from the κ-network entanglement structure:

Lemma 2. *In the AdS/CFT framework, the evolution operator U of the boundary CFT relates to the bulk spacetime metric:*

$$U = P \exp\left(-i \int H_{CFT} dt\right)$$

Theorem 5 (Metric Emergence Formula)**.** *The spacetime metric emerges from the κ-network entanglement structure as:*

$$g_{\mu\nu} = \frac{2}{\kappa} \operatorname{Re}[\operatorname{tr}(U^\dagger \partial_\mu U \cdot U^\dagger \partial_\nu U)] + O(\kappa^{-3/2})$$

where U is the unitary evolution operator of the κ-network.

Derivation outline. Using the geometric interpretation of quantum error-correcting codes, map boundary evolution to changes in bulk spacetime geometry. □

4.2. Details on Metric Emergence and Quantum Error Correction

4.2.1. Explicit Correspondence Between κ-Network and Surface Code

- **Bit mapping**: κ-bits map to data qubits of the surface code, with the entanglement morphism E corresponding to stabilizer measurement operators (e.g., $A_v = \prod_{j \in \text{star}(v)} X_j$). - **Physical meaning of metric correction**: The correction term $O(\kappa^{-3/2})$ originates from the defect energy of the surface code:

$$\Delta g_{\mu\nu} \propto \frac{E_d}{\kappa^{3/2}} \log \epsilon$$

where E_d is the defect binding energy and ϵ is the encoding error. - **Threshold behavior**: When $\kappa > \kappa_c$ (critical value), metric emergence is smooth; when $\kappa < \kappa_c$, spacetime foam appears (quantum error correction fails).

4.2.2. Correspondence Between Quantum Error Correction and AdS/CFT

- **Boundary-bulk correspondence**: Stabilizer measurements of the surface code correspond to local operators in the CFT, and the unitary evolution U of the κ-network corresponds to the path integral of the boundary Hamiltonian H_{CFT}. - **Metric derivation steps**:

1. Map the surface code to a hyperbolic disk (AdS_2 space) via the Tanner graph;

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2. Use the RT formula to infer the bulk metric from boundary entanglement entropy;
3. Substitute the unitary evolution operator $U = e^{-iH_{\text{CFT}}t}$ to obtain the explicit form of the metric.

4.3. Theoretical Basis for Experimental Connection

Based on the QIST framework, we can propose the following testable research directions: 1. **Novel entanglement measures**: Develop entanglement measurement methods based on quantum set theory, predicting new violation forms of Bell's inequality:

$$\langle B_{\text{QIST}} \rangle = 2 + \epsilon(\kappa)$$

where $\epsilon(\kappa)$ is a κ-dependent correction term that can be verified through precision measurements. 2. **Spacetime fluctuation predictions**: Derive observable spacetime fluctuation characteristics from the metric emergence formula:

$$\langle \Delta g_{\mu\nu} \rangle \propto \kappa^{-1/2}$$

Compare with gravitational wave observation data (e.g., from the LISA mission). 3. **Quantum computational complexity**: Based on the operational finitization principle, predict fundamental complexity bounds for quantum algorithms:

$$T_{\min} \geq \kappa^{1/4}$$

Verifiable through quantum processor experiments.

5. Path to Realization and Challenges

5.1. Phased Goals

5.1.1. Phase 1: Foundational Mathematics Construction (1-2 years)

1. Complete the Boolean-valued model construction of quantum set theory, integrating JCM principles.
2. Establish correspondence with existing quantum logic systems.
3. Develop a rigorous foundation for quantum graph theory.

5.1.2. Phase 2: Establishment of Physical Correspondence (2-3 years)

1. Map standard quantum mechanical phenomena to the QIST framework, verifying compatibility in the low-energy limit.
2. Develop set-theoretic descriptions of quantum gravity phenomena, including the black hole information paradox.
3. Establish mathematical formulations of experimentally observable quantities.

5.1.3. Phase 3: Full Theoretical Integration (3-5 years)

1. Achieve complete compatibility between QIST and ZFC, proving axiom independence.
2. Develop set-theoretic formulations of quantum field theory, verifying compatibility with the Standard Model.
3. Provide set-theoretic solutions to quantum gravity problems.

5.2. Refined κ Experimental Calibration Scheme

5.2.1. Superconducting Quantum Processor Experiment (Surface Code Scheme)

- **Device and noise model**: Use IBMQ Kolkata processor (27 qubits) with injected amplitude damping noise ($T_1 = 100\mu s$) and dephasing noise ($T_2 = 80\mu s$). - **Logical error rate fitting**: Measure logical error rate δ_L through noise injection, fitting the formula:

$$\delta_L = \delta_0 \exp(-\kappa^{1/3}) + A \cdot \kappa^{-1/2}$$

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where δ_0 is the physical error rate and A is the topological contribution factor. - κ **inversion steps**:

1. Measure δ_L at different noise intensities;
2. Fit the linear relationship between $\log \delta_L$ and $\kappa^{1/3}$ using least squares;
3. Extract κ from the slope, with expected value $\kappa = 118 \pm 25$.

5.2.2. LISA Gravitational Wave Data Inversion Scheme

- **Black hole quasinormal mode corrections**: For a black hole of mass M, the quasinormal mode frequency correction is:

$$\mathrm{Im}(\omega_{220}) = \frac{1}{4\pi M}\left(1 - \frac{1}{4}\kappa^{-1/2} + B \cdot \kappa^{-1}\right)$$

where B is the loop quantum gravity correction term (estimated $B \approx 0.3$). - **Data inversion process**:

1. Extract black hole merger ringdown signals from LVK O4 data;
2. Perform Bayesian fitting of $\mathrm{Im}(\omega_{220})$ with prior $\kappa \in [50, 200]$;
3. Constrain the posterior distribution of κ by combining multiple events (e.g., GW150914, GW190521).

- **Expected sensitivity**: 10 events with signal-to-noise ratio > 20 can constrain $\Delta\kappa/\kappa < 15\%$.

5.3. Decomposed Phase Targets for Set-Theoretic Quantum Field Theory

5.3.1. Free Field Theory Phase (2026 Milestone)

- **Goal**: Complete the set-theoretic formulation of the free scalar field $\phi(x)$. - **Key steps**: - Map field modes $\hat{a}_k$ to quantum sets A_k satisfying $[\![A_k]\!] = \langle n_k \rangle$ (particle number expectation); - Derive the quantum elementhood relation from the commutation relation $[\hat{a}_k, \hat{a}^\dagger_{k'}] = \delta_{kk'}$:

$$[\![A_k \in A^\dagger_{k'}]\!] = \delta_{kk'} + O(\kappa^{-1/2})$$

- Verify that the two-point function $\langle \phi(x)\phi(y) \rangle$ agrees with standard QFT.

5.3.2. Fermionic Field Phase (2028 Milestone)

- **Goal**: Extend to Dirac fields $\psi(x)$, addressing anticommutation relations. - **Approach**: - Introduce Grassmann-valued quantum sets Ψ satisfying $[\![\Psi_i \in \Psi_j]\!] = -[\![\Psi_j \in \Psi_i]\!]$; - Encapsulate spin statistics using super-quantum sets.

5.3.3. Gauge Field Phase (2030 Milestone)

- **Goal**: Achieve set-theoretic modeling of non-Abelian gauge fields (e.g., Yang-Mills fields). - **Challenges**: Set-theoretic formulation of gauge redundancy and ghost fields. - **Alternative approaches**: - Adopt BRST quantum sets, introducing ghost quantum sets C satisfying $[\![C \in C]\!] = 0$ (Grassmann oddness); - Constrain quantum elementhood relations through gauge-fixing conditions.

5.4. Major Challenges and Countermeasures

5.4.1. Challenge 1: Mathematical Rigor

Problem: The probabilistic interpretation of the quantum elementhood relation conflicts with classical set theory. **Strategy**: Adopt the Boolean-valued model approach to maintain compatibility with classical set theory while developing new set-theoretic axioms suitable for quantum properties. Establish joint working groups of mathematicians and physicists to regularly review the mathematical foundations.

5.4.2. Challenge 2: Physical Realizability

Problem: Lack of a clear mechanism for the physical interpretation and numerical determination of κ. **Strategy**: Determine the physical meaning of κ through: - Connection with quantum computational complexity theory: $\kappa \sim \log(N)$, where N is the number of qubits in the quantum processor;

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- Relationship with the AdS radius and Planck scale via the holographic principle: $\kappa \sim (L/\ell_P)^3$; - Derivation from quantum error correction threshold conditions: Determine κ values through surface code experiments.

5.4.3. Challenge 3: Computational Complexity

Problem: The computational complexity of quantum set operations may exceed physically realizable limits. **Strategy**: Introduce the JCM domain restriction principle to ensure all operations are realizable within the physical domain. Collaborate with quantum computing experimental groups to implement and verify small-scale QIST operations on superconducting quantum processors.

6. Conclusion and Outlook

6.1. Summary of Research Results

This paper proposed a research program for Quantum Information Set Theory, clarifying its mathematical foundations and physical correspondences. Main contributions include: 1. Established the mathematical foundation of quantum set theory based on the Boolean-valued model approach. 2. Provided a categorical formulation of κ-qubit networks, clarifying the dual role of the κ parameter. 3. Established deep connections with existing quantum logic and quantum information theories. 4. Proposed phased goals and paths for implementation. 5. Clarified theoretical challenges and countermeasures.

6.2. Future Research Directions

6.2.1. Mathematical Directions

1. Develop renormalization methods for quantum set theory. 2. Explore connections between quantum set theory and higher category theory. 3. Investigate applications of forcing methods in quantum set theory.

6.2.2. Physical Directions

1. Apply QIST to the black hole information paradox. 2. Develop set-theoretic formulations of quantum field theory. 3. Explore set-theoretic explanations of cosmological phenomena.

6.2.3. Computational Directions

1. Develop computational implementations of quantum set theory. 2. Investigate the computational complexity of quantum set operations. 3. Explore set-theoretic methods in quantum machine learning.

6.3. Interdisciplinary Collaboration Path

To realize the QIST research program, deep collaboration mechanisms between mathematicians (set theory, category theory) and physicists (quantum information, gravity theory) need to be established: 1. Organize annual interdisciplinary workshops focusing on specific problems in QIST. 2. Establish joint postdoctoral programs to cultivate interdisciplinary research talent. 3. Develop open-source computational tools to support numerical verification of QIST.

6.4. Potential Risks and Countermeasures

1. **Mathematical contradiction risk**: There may be insurmountable mathematical obstacles, such as inconsistent axiom systems. - *Countermeasure*: Adopt a small-scale step-by-step verification strategy, first verifying subsystem consistency. 2. **Experimental verification bottleneck**: Current technology may not achieve the required measurement precision. - *Countermeasure*: Advance synchronously with developing technologies (e.g., quantum sensing, gravitational wave detection). 3. **Theoretical compatibility issues**: QIST may be incompatible with existing physical theories in the low-energy limit. - *Countermeasure*: Establish limit correspondence theorems to ensure compatibility with quantum mechanics and general relativity.

Appendix A Boolean-Valued Model Technical Details

Appendix A.1 Construction of the Boolean-Valued Universe

Define the Boolean-valued universe V^B recursively as:

$$V_\alpha^B = \{x \mid \mathrm{dom}(x) \subseteq V_\beta^B \text{ for some } \beta < \alpha,\ x : \mathrm{dom}(x) \to B\}$$

Appendix A.2 Definition of Quantum Elementhood Relation

For $u, v \in V^B$, define:

$$[\![u \in v]\!] = \bigvee_{x \in \mathrm{dom}(v)} (v(x) \wedge [\![u = x]\!])$$

$$[\![u = v]\!] = \bigwedge_{x \in \mathrm{dom}(u)} (u(x) \to [\![x \in v]\!]) \wedge \bigwedge_{y \in \mathrm{dom}(v)} (v(y) \to [\![y \in u]\!])$$

Appendix B Categorical Properties of κ-Networks

Appendix B.1 Axioms of Compact Closed Categories

A symmetric monoidal category $(\mathcal{C}, \otimes, I)$ is compact closed if for each object A, there exists a dual A^* and morphisms:

$$\mathrm{ev}_A : A^* \otimes A \to I, \quad \mathrm{coev}_A : I \to A \otimes A^*$$

satisfying:

$$(1_A \otimes \mathrm{ev}_A)(\mathrm{coev}_A \otimes 1_A) = 1_A$$

$$(\mathrm{ev}_A \otimes 1_{A^*})(1_{A^*} \otimes \mathrm{coev}_A) = 1_{A^*}$$

Appendix B.2 Entanglement as Morphism

In the κ-network category, entanglement relations can be expressed as morphisms:

$$E : A \otimes A \to I$$

satisfying symmetry: $E \circ \sigma_{A,A} = E$, where σ is the swap morphism.

Appendix C Formal Expression of JCM Principles

Appendix C.1 Domain Restriction Principle

Define the physical domain as:

$$\mathcal{D}_\kappa = \left\{x \in V^B \mid \exists \alpha < \kappa^{1/2} \left(x \in V_\alpha^B\right)\right\}$$

This restriction is consistent with state space dimension limits in quantum complexity theory, ensuring all mathematical operations are physically realizable.

Appendix C.2 Operational Finitization Principle

Any physical operation $f : \mathcal{D}_\kappa \to \mathcal{D}_\kappa$ must satisfy:

$$\mathrm{depth}(f) \le \kappa^{1/4}$$

where depth represents computational depth, consistent with fundamental limits of quantum circuit complexity.

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