Brief Report

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Keywords: Inhomogeneous wave equation; Weber's electrodynamics; vacuum polarization; telegraphy equation; charge conservation

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# Inhomogeneous Wave Equation and Weber's Electrodynamics

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**Abstract**

It is well known that the inhomogeneous electric wave equation can be derived from Maxwell's equations. On the other hand, it has been a common view that Weber's electrodynamics—an alternative to Maxwell–Lorentz electromagnetism—is unable to produce a wave equation capable of explaining energy radiation. Recently, by introducing the concept of a polarizable vacuum, both longitudinal and transverse wave equations have been derived from Weber's electrodynamics. However, these derivations did not incorporate source terms. In this paper, we attempt to include source terms, thereby deriving an inhomogeneous wave equation from Weber's electrodynamics. Remarkably, this inhomogeneous wave equation takes the same form as the one derived from Maxwell's equations. Although the equation itself permits longitudinal wave propagation, the requirement of charge conservation restricts the solution in vacuum to exclude longitudinal waves. Nevertheless, longitudinal waves can exist in the case of conduction currents (i.e., the movement of free charges) in electric wires.

**Keywords:** inhomogeneous wave equation; Weber's electrodynamics; vacuum polarization; telegraphy equation; charge conservation

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## Introduction

Maxwell's equations were used to derive the inhomogeneous wave equation, which can be used to explain the phenomenon of electric radiation first discovered by Hertz [1]. The Maxwell equations are shown below:

$$\nabla \cdot \boldsymbol{E} = \frac{\rho}{\varepsilon_0} \ (Gauss's\ law)$$

$$\nabla \cdot \boldsymbol{B} = 0 \ (Gauss's\ law\ for\ magnetism)$$

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t} \ (Faraday's\ law)$$

$$\nabla \times \boldsymbol{B} = \mu_0 \boldsymbol{J} + \frac{1}{c^2}\frac{\partial \boldsymbol{E}}{\partial t} \ (Ampère - Maxwell's\ law) \ (1)$$

where $\boldsymbol{E}$ is the electric field vector, $\boldsymbol{B}$ is the magnetic field vector, $\boldsymbol{J}$ is the current density vector, $\rho$ is the electric charge density, $\varepsilon_0$ is the vacuum permittivity, $\mu_0$ is the vacuum permeability, $c$ is the speed of light in vacuum.

Taking the curl of Faraday's law and substituting Ampère–Maxwell's law, we get:

$$\nabla \times \nabla \times \boldsymbol{E} = -\frac{\partial (\nabla \times \boldsymbol{B})}{\partial t} = -\mu_0 \frac{\partial \boldsymbol{J}}{\partial t} - \frac{1}{c^2}\frac{\partial^2 \boldsymbol{E}}{\partial t^2} \ (2)$$

Using the vector identity $\nabla(\nabla \cdot \boldsymbol{E}) = \nabla \times \nabla \times \boldsymbol{E} + \nabla^2 \boldsymbol{E}$, we obtain:

$$\nabla(\nabla \cdot \boldsymbol{E}) - \nabla^2 \boldsymbol{E} = -\mu_0 \frac{\partial \boldsymbol{J}}{\partial t} - \frac{1}{c^2}\frac{\partial^2 \boldsymbol{E}}{\partial t^2} \ (3)$$

Substituting Gauss's law and rearranging terms yields:

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$$\nabla^2 \boldsymbol{E} - \frac{1}{c^2}\frac{\partial^2 \boldsymbol{E}}{\partial t^2} = \frac{1}{\varepsilon_0}\nabla\rho + \mu_0 \frac{\partial \boldsymbol{J}}{\partial t} \ (4)$$

The above equation is the well-known inhomogeneous wave equation for the electric field in vacuum. For electric signal propagation in wires, Heaviside derived the telegraph equation based on Maxwell's equations [2]. For the case of a lossless, dual-conductor transmission wire system, the equation is:

$$\frac{\partial^2 I}{\partial x^2} - LC\frac{\partial^2 I}{\partial t^2} = 0 \ (5)$$

where $I$ is the current in the wire, $L$ is the distributed inductance, and $C$ is the capacitance between the two conductors.

It is worth mentioning that the telegraph equation was first derived using Weber's electrodynamics by Kirchhoff and Weber independently [3]. Later, it was extended to the case of a planar conductor by Assis [4].

Maxwell's equations use the field concept, which is essential for explaining electromagnetic wave propagation in vacuum [5]. According to this framework, electromagnetic fields are self-propagating through the vacuum. In contrast, Weber's electrodynamics focuses on forces between charged particles and, as originally formulated, cannot easily predict wave or force propagation in vacuum, where there are no explicit charges [6].

This limitation to Weber's electrodynamics can be changed if utilizing the concept of vacuum polarization [7,8]. The presence of positive–negative charge pairs in vacuum provides a medium for the propagation of Weber's force, and hence the electric field (defined as the force per unit test charge). These charge pairs can undergo oscillatory displacements, closely resembling Maxwell's concept of displacement current [9], though in this case with a clearer and more physical interpretation. By analyzing the balance of Weber's force, one can derive both longitudinal [10] and transverse wave propagations [11], corresponding to irrotational and rotational charge displacements, respectively.

However, both of these derivations lack source terms and thus cannot be directly applied to study radiation or related phenomena. In this paper, we incorporate source terms and derive an inhomogeneous wave equation. We then analyze its solution under the constraint of charge continuity equation (charge conservation). We show that this constraint prohibits longitudinal wave propagation in vacuum or in media without free charges. Nevertheless, longitudinal wave propagation of free charges can readily occur inside wires, similar to the behavior described by the telegraph equation.

## Irrotational and Rotational Currents

We postulate that space is filled with polarizable positive–negative charge pairs and contains some freely movable charges. The charge pairs can only oscillate relative to each other, whereas the free charges can travel long distances. These charge movements constitute a total current density $\boldsymbol{J}_{total}$, which includes the displacement current density $\boldsymbol{J}_D$ from oscillating charges and the conduction current densit $\boldsymbol{J}_s$ from free charges (sources):

$$\boldsymbol{J}_{total} = \boldsymbol{J}_D + \boldsymbol{J}_s \ (6)$$

Both current densities can be decomposed into irrotational ($\overline{\boldsymbol{J}_D}$, $\overline{\boldsymbol{J}_s}$) and rotational components ($\widetilde{\boldsymbol{J}_D}$, $\widetilde{\boldsymbol{J}_s}$), according to the Helmholtz decomposition:

$$\boldsymbol{J}_D = \overline{\boldsymbol{J}_D} + \widetilde{\boldsymbol{J}_D} \ \nabla \times \overline{\boldsymbol{J}_D} = 0 \ \nabla \cdot \widetilde{\boldsymbol{J}_D} = 0$$
$$\boldsymbol{J}_s = \overline{\boldsymbol{J}_s} + \widetilde{\boldsymbol{J}_s} \ \nabla \times \overline{\boldsymbol{J}_s} = 0 \ \nabla \cdot \widetilde{\boldsymbol{J}_s} = 0 \ (7)$$

Since current densities are due to the motion of charges, we can express them as:

$$\boldsymbol{J}_D = \rho_D' \left(\frac{\partial \boldsymbol{D}_+}{\partial t} - \frac{\partial \boldsymbol{D}_-}{\partial t}\right) = 2\rho_D' \frac{\partial \boldsymbol{D}}{\partial t}$$
$$\boldsymbol{J}_s = \rho_s' \frac{\partial \boldsymbol{D}_s}{\partial t} \ (8)$$

Here, $\rho'_D$ is the density of oscillating charges in the charge pairs, $\rho'_s$ is the density of free charges, $\boldsymbol{D}_+$ and $\boldsymbol{D}_-$ are the displacements of the positive and negative charges in the pairs, respectively, and $\boldsymbol{D}_s$ is the displacement of the free charge. By definition, $\boldsymbol{D}_+ = -\boldsymbol{D}_- = \boldsymbol{D}$. We approximate the Lagrangian derivative by the Eulerian derivative (e.g., $\frac{d\boldsymbol{D}}{dt} \approx \frac{\partial \boldsymbol{D}}{\partial t}$) under the assumption of low charge velocity. This approximation is applied throughout the derivation in this paper.

The irrotational and rotational components of the current densities are associated with the irrotational displacements ($\overline{\boldsymbol{D}}$, $\overline{\boldsymbol{D}_s}$) and the rotational displacements ($\widetilde{\boldsymbol{D}}$, $\widetilde{\boldsymbol{D}_s}$), respectively:

$$\overline{\boldsymbol{J}_D} = 2\rho'_D \frac{\partial \overline{\boldsymbol{D}}}{\partial t} \quad \nabla \times \overline{\boldsymbol{D}} = 0$$
$$\widetilde{\boldsymbol{J}_D} = 2\rho'_D \frac{\partial \widetilde{\boldsymbol{D}}}{\partial t} \quad \nabla \cdot \widetilde{\boldsymbol{D}} = 0$$
$$\overline{\boldsymbol{J}_s} = \rho'_s \frac{\partial \overline{\boldsymbol{D}_s}}{\partial t} \quad \nabla \times \overline{\boldsymbol{D}_s} = 0$$
$$\widetilde{\boldsymbol{J}_s} = \rho'_s \frac{\partial \widetilde{\boldsymbol{D}_s}}{\partial t} \quad \nabla \cdot \widetilde{\boldsymbol{D}_s} = 0 \; (9)$$

## Inhomogeneous Wave Equation for Irrotational Field

In the case where both the current density and the charge displacement are irrotational, a force equilibrium equation was derived in a previous paper [10] (Equation (20) in that paper):

$$\nabla \rho + \frac{1}{c^2} \rho \boldsymbol{a} = 0 \; (10)$$

where $\rho$ is the density of positive or negative charge, and $\boldsymbol{a}$ is the acceleration vector of the corresponding charge. This force equilibrium equation was obtained under the assumption that the velocity of charges is much lower than the speed of light. In the previous work, only oscillating charge pairs were considered, without any freely moving charges. Here, we extend the model by including free charges, while assuming the force balance equation still holds:

$$\nabla(2\rho'_D + \rho'_s) + \frac{1}{c^2}(2\rho'_D \boldsymbol{a}_D + \rho'_s \boldsymbol{a}_s) = 0 \; (11)$$

where $\boldsymbol{a}_D$ is the acceleration vector of the oscillating charges and $\boldsymbol{a}_s$ is the acceleration vector of the free charges. The factor of 2 for the oscillating charge density arises because each charge pair consists of both a positive and a negative charge. After the cancellation of positive and negative charge densities, the gradient of the net charge density remains the same. Therefore, Equation (11) can be rewritten as:

$$\nabla(2\rho_D + \rho_s) + \frac{1}{c^2}(2\rho'_D \boldsymbol{a}_D + \rho'_s \boldsymbol{a}_s) = 0 \; (12)$$

Here, $\rho_D$ represents the net density of oscillating charges after cancellation, and $\rho_s$ is the net density of free charges.

These charge densities are related to the divergence of charge displacements:

$$\rho_D = -\rho'_D \nabla \cdot \overline{\boldsymbol{D}}$$
$$\rho_s = \rho_0 - \rho'_s \nabla \cdot \overline{\boldsymbol{D}_s} \; (13)$$

Here, $\rho_0$ is the net density of free charges when charge displacement is zero. Substituting these into the force balance Equation (12), and approximating the Lagrangian derivatives with Eulerian derivatives ($\boldsymbol{a}_D = \frac{d^2\overline{\boldsymbol{D}}}{dt^2} \approx \frac{\partial^2 \overline{\boldsymbol{D}}}{\partial t^2}$ and $\rho'_s \boldsymbol{a}_s = \frac{d\overline{\boldsymbol{J}_s}}{dt} \approx \frac{\partial \overline{\boldsymbol{J}_s}}{\partial t}$), we obtain:

$$-2\rho'_D \nabla(\nabla \cdot \overline{\boldsymbol{D}}) + \nabla \rho_s + \frac{2\rho'_D}{c^2} \frac{\partial^2 \overline{\boldsymbol{D}}}{\partial t^2} + \frac{1}{c^2} \frac{\partial \overline{\boldsymbol{J}_s}}{\partial t} = 0 \; (14)$$

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Using the vector identity $\nabla(\nabla\cdot\overline{\boldsymbol{D}}) = \nabla\times\nabla\times\overline{\boldsymbol{D}} + \nabla^2\overline{\boldsymbol{D}}$, and assuming a linear relationship between the irrotational electric field $\overline{\boldsymbol{E}}$ and the irrotational displacement $\overline{\boldsymbol{D}}$,

$$\overline{\boldsymbol{D}} = \frac{\varepsilon}{2\rho_D'}\overline{\boldsymbol{E}} \; (15)$$

we arrive at the inhomogeneous wave equation for the irrotational electric field:

$$\nabla^2\overline{\boldsymbol{E}} - \frac{1}{c^2}\frac{\partial^2\overline{\boldsymbol{E}}}{\partial t^2} = \frac{1}{\varepsilon}\nabla\rho_s + \mu\frac{\partial\overline{\boldsymbol{J}_s}}{\partial t} \; (16)$$

## Inhomogeneous Wave Equation for Rotational Field

In a previous paper on deriving the transverse wave from Weber's electrodynamics [11], the following two equations (Equations 15 and 22 in that paper) were obtained by considering only rotational fields. In that derivation, the Lagrangian derivative was approximated by the Eulerian derivative under the assumption of low charge velocity. These equations are similar to the standard forms of Ampère's law and Faraday's law. The key difference is that only rotational fields are considered here, whereas the standard laws do not explicitly require this restriction:

$$\nabla\times\boldsymbol{B} = \mu\tilde{\boldsymbol{J}} \; (Amp\grave{e}re's\ law)$$

$$\nabla\times\tilde{\boldsymbol{E}} = -\frac{\partial\boldsymbol{B}}{\partial t} \; (Faraday's\ law) \; (17)$$

In the previous work [10], only the current due to oscillating charges was considered. Here, we also include the current from freely moving charges:

$$\nabla\times\boldsymbol{B} = \mu\left(\widetilde{\boldsymbol{J}_D} + \widetilde{\boldsymbol{J}_s}\right) \; (18)$$

Applying the curl operator to Faraday's law (Equation (17)) and substituting in Ampère's law (Equation (18)), we obtain:

$$\nabla\times\nabla\times\tilde{\boldsymbol{E}} = -\frac{\partial(\nabla\times\boldsymbol{B})}{\partial t} = -\mu\frac{\partial\widetilde{\boldsymbol{J}_D}}{\partial t} - \mu\frac{\partial\widetilde{\boldsymbol{J}_s}}{\partial t} = -2\mu\rho_D'\frac{\partial^2\tilde{\boldsymbol{D}}}{\partial t^2} - \mu\frac{\partial\widetilde{\boldsymbol{J}_s}}{\partial t} \; (19)$$

Using the vector identity $\nabla\left(\nabla\cdot\tilde{\boldsymbol{E}}\right) = \nabla\times\nabla\times\tilde{\boldsymbol{E}} + \nabla^2\tilde{\boldsymbol{E}}$, and the linear relationship between the rotational electric field $\tilde{\boldsymbol{E}}$ and displacement $\tilde{\boldsymbol{D}}$,

$$\tilde{\boldsymbol{D}} = \frac{\varepsilon}{2\rho_D'}\tilde{\boldsymbol{E}} \; (20)$$

we can rewrite Equation (19) as:

$$\nabla\left(\nabla\cdot\tilde{\boldsymbol{E}}\right) - \nabla^2\tilde{\boldsymbol{E}} = -\mu\varepsilon\frac{\partial^2\tilde{\boldsymbol{E}}}{\partial t^2} - \mu\frac{\partial\widetilde{\boldsymbol{J}_s}}{\partial t} \; (21)$$

Since the divergence of the rotational electric field is zero $\left(\nabla\cdot\tilde{\boldsymbol{E}} = 0\right)$, we obtain the inhomogeneous wave equation for the rotational electric field:

$$\nabla^2\tilde{\boldsymbol{E}} - \frac{1}{c^2}\frac{\partial^2\tilde{\boldsymbol{E}}}{\partial t^2} = \mu\frac{\partial\widetilde{\boldsymbol{J}_s}}{\partial t} \; (22)$$

## Inhomogeneous wave equation for combined field

Now we have obtained the inhomogeneous wave equations for both the irrotational and rotational components of the electric field. By summing Equations (16) and (22), and using Equation (7) along with the assertion that the total electric field is composed of irrotational and rotational parts, $\boldsymbol{E} = \overline{\boldsymbol{E}} + \tilde{\boldsymbol{E}}$, we arrive at:

$$\nabla^2\boldsymbol{E} - \frac{1}{c^2}\frac{\partial^2\boldsymbol{E}}{\partial t^2} = \frac{1}{\varepsilon}\nabla\rho_s + \mu\frac{\partial\boldsymbol{J}_s}{\partial t} \; (23)$$

This wave equation has the same form as the inhomogeneous wave equation derived from Maxwell's equations (Equation (4)). However, there are important underlying differences.

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The derivation based on Maxwell's equations does not rely on any explicit velocity assumptions. In contrast, the derivation presented here assumes that charge velocities are much lower than the speed of light, allowing the approximation of the Lagrangian derivative by the Eulerian derivative and enabling the application of the force balance equation.

Furthermore, Maxwell's derivation explicitly includes Gauss's law for electricity, which imposes the condition of zero divergence for the electric field in the absence of free charges—thereby excluding longitudinal wave propagation in vacuum [1]. In contrast, our derivation does not impose this constraint, and the resulting wave equation does not inherently rule out a nonzero divergence of the electric field in regions without free charge. As a result, our equation permits the possibility of longitudinal wave propagation.

In the next section, we will show that longitudinal waves do not propagate in a vacuum due to the requirement of charge conservation—i.e., the absence of a source for longitudinal waves.

## Non-Existence of Longitudinal Solutions in Vacuum

We examine whether longitudinal solutions can exist by analyzing the inhomogeneous wave equation. First, consider the decomposition of the electric field using a scalar potential $\varphi$ and a vector potential $\boldsymbol{A}$:

$$\boldsymbol{E} = -\nabla\varphi - \frac{\partial \boldsymbol{A}}{\partial t} \quad (24)$$

Substituting this expression into the wave Equation (23), we obtain:

$$\nabla^2(-\nabla\varphi) - \frac{1}{c^2}\frac{\partial^2(-\nabla\varphi)}{\partial t^2} + \nabla^2\left(-\frac{\partial \boldsymbol{A}}{\partial t}\right) - \frac{1}{c^2}\frac{\partial^2\left(-\frac{\partial \boldsymbol{A}}{\partial t}\right)}{\partial t^2} = \frac{1}{\varepsilon}\nabla\rho_s + \mu\frac{\partial \boldsymbol{J}_s}{\partial t} \quad (25)$$

This equation can be satisfied if the scalar and vector potentials satisfy the following two equations respectively:

$$\frac{1}{c^2}\frac{\partial^2\varphi}{\partial t^2} - \nabla^2\varphi = \frac{\rho_s}{\varepsilon}$$
$$\frac{1}{c^2}\frac{\partial^2\boldsymbol{A}}{\partial t^2} - \nabla^2\boldsymbol{A} = \mu\boldsymbol{J}_s \quad (26)$$

There exist infinitely many combinations of scalar and vector potentials that satisfy Equations (24) and (25). Without a rigorous proof, we assume that there is always at least one combination that satisfies Equation (26).

Now apply the time derivative to the first equation in (26) and the divergence to the second:

$$\frac{1}{c^2}\frac{\partial^2\left(\frac{1}{c^2}\frac{\partial\varphi}{\partial t}\right)}{\partial t^2} - \nabla^2\left(\frac{1}{c^2}\frac{\partial\varphi}{\partial t}\right) = \frac{1}{\varepsilon}\left(\frac{1}{c^2}\frac{\partial\rho_s}{\partial t}\right)$$
$$\frac{1}{c^2}\frac{\partial^2(\nabla\cdot\boldsymbol{A})}{\partial t^2} - \nabla^2(\nabla\cdot\boldsymbol{A}) = \mu(\nabla\cdot\boldsymbol{J}_s) \quad (27)$$

Adding these two results:

$$\frac{1}{c^2}\frac{\partial^2\left(\frac{1}{c^2}\frac{\partial\varphi}{\partial t} + \nabla\cdot\boldsymbol{A}\right)}{\partial t^2} - \nabla^2\left(\frac{1}{c^2}\frac{\partial\varphi}{\partial t} + \nabla\cdot\boldsymbol{A}\right) = \mu\left(\frac{\partial\rho_s}{\partial t} + \nabla\cdot\boldsymbol{J}_s\right) \quad (28)$$

By the continuity equation for electric charge, the right-hand side equals zero. Assuming the potentials vanish at infinity, we obtain:

$$\frac{1}{c^2}\frac{\partial\varphi}{\partial t} + \nabla\cdot\boldsymbol{A} = 0 \quad (29)$$

This is the Lorenz gauge condition, which arises naturally from the requirement of charge conservation.

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Next, consider the right-hand side of Ampère–Maxwell's law, which we denote as $\mathbf{\Gamma}$:

$$\mathbf{\Gamma} = \mu \boldsymbol{J}_s + \frac{1}{c^2}\frac{\partial \boldsymbol{E}}{\partial t} \; (30)$$

According to Ampère–Maxwell's law, $\nabla \cdot \mathbf{\Gamma} = 0$. However, in this paper, we begin with Weber's force law and do not assume Ampère–Maxwell's law a priori. Let's derive the properties of $\mathbf{\Gamma}$ from our theoretical approach.

Using the electric field expression from Equation (24), Equation (30) becomes:

$$\mathbf{\Gamma} = \mu \boldsymbol{J}_s - \frac{1}{c^2}\frac{\partial(\nabla\varphi)}{\partial t} - \frac{1}{c^2}\frac{\partial^2 \boldsymbol{A}}{\partial t^2} \; (31)$$

Rewriting:

$$\mathbf{\Gamma} = \mu \boldsymbol{J}_s - \frac{1}{c^2}\frac{\partial(\nabla\varphi)}{\partial t} + \nabla^2 \boldsymbol{A} - \nabla^2 \boldsymbol{A} - \frac{1}{c^2}\frac{\partial^2 \boldsymbol{A}}{\partial t^2} \; (32)$$

Using the vector potential Equation (26), we substitute and simplify:

$$\mathbf{\Gamma} = -\nabla^2 \boldsymbol{A} - \frac{1}{c^2}\frac{\partial(\nabla\varphi)}{\partial t} \; (33)$$

Using the vector identity $\nabla(\nabla \cdot \boldsymbol{A}) = \nabla \times \nabla \times \boldsymbol{A} + \nabla^2 \boldsymbol{A}$, we find:

$$\mathbf{\Gamma} = \nabla \times \nabla \times \boldsymbol{A} - \nabla\left(\nabla \cdot \boldsymbol{A} + \frac{1}{c^2}\frac{\partial \varphi}{\partial t}\right) \; (34)$$

Due to the Lorenz gauge condition (Equation (29)), this simplifies to:

$$\mathbf{\Gamma} = \nabla \times \nabla \times \boldsymbol{A} \; (35)$$

Thus,

$$\nabla \cdot \mathbf{\Gamma} = \nabla \cdot \left(\mu \boldsymbol{J}_s + \frac{1}{c^2}\frac{\partial \boldsymbol{E}}{\partial t}\right) = 0 \; (36)$$

This leads to:

$$\mu \frac{\partial(\rho_s' \nabla \cdot \boldsymbol{D}_s)}{\partial t} + \frac{1}{c^2}\frac{\partial(\nabla \cdot \boldsymbol{E})}{\partial t} = 0 \; (37)$$

From the charge density expression (Equation (13)), we get:

$$\frac{\partial(\rho_0 - \rho_s + \varepsilon \nabla \cdot \boldsymbol{E})}{\partial t} = 0 \; (38)$$

Therefore:

$$\rho_0 - \rho_s + \varepsilon \nabla \cdot \boldsymbol{E} = constant \; (39)$$

Which implies:

$$\nabla \cdot \boldsymbol{E} = \frac{\rho_s}{\varepsilon} + \frac{(constant - \rho_0)}{\varepsilon} \; (40)$$

This closely resembles Gauss's law. If the constant equals $\rho_0$, it becomes identical. It follows that the divergence of the electric field is either constant or zero over time in regions without free charge. Therefore, longitudinal wave propagation of the electric field cannot exist in vacuum.

From this derivation, we conclude that the absence of longitudinal waves is not due to the "infinite stiffness" of the medium, but rather arises naturally from the requirement of charge conservation, which places strict constraints on allowable source/field configurations.

## Existence of Longitudinal Solutions in a Wire

Although longitudinal waves cannot exist in vacuum, they can exist in the context of free charge motion—such as in a conducting wire. For example, electric signals that travel along a wire propagate as longitudinal waves, described by the telegrapher's equation [3]. This can be derived from Equation (23) by setting the left-hand side to zero. Here, we assume the wire has no resistivity, implying the electric field inside the wire is zero:

$$\frac{1}{\varepsilon}\nabla\rho_s + \mu\frac{\partial \boldsymbol{J}_s}{\partial t} = 0 \; (41)$$

Substituting Equations (8) and (13) into Equation (41), we obtain:

$$\frac{1}{\varepsilon}\nabla(\rho_0 - \rho_s'\nabla\cdot\overline{\boldsymbol{D}_s}) + \mu\frac{\partial\left(\rho_s'\frac{\partial \boldsymbol{D}_s}{\partial t}\right)}{\partial t} = 0 \; (42)$$

Inside a one dimensional wire, there is no rotational displacement of free charge, $\overline{\boldsymbol{D}_s} = \boldsymbol{D}_s$ and $\nabla\times\boldsymbol{D}_s = 0$. Using the vector identity $\nabla(\nabla\cdot\boldsymbol{D}_s) = \nabla\times\nabla\times\boldsymbol{D}_s + \nabla^2\boldsymbol{D}_s$, we can simplify the equation as:

$$\nabla^2\boldsymbol{D}_s - \frac{1}{c^2}\frac{\partial^2\boldsymbol{D}_s}{\partial t^2} = \frac{1}{\rho_s'}\nabla(\rho_0) \; (43)$$

This represents longitudinal wave propagation of free charge motion. The same wave equation can also be expressed in terms of current density, using the relationship between current density and free charge motion (Equation (8)):

$$\nabla^2\boldsymbol{J}_s - \frac{1}{c^2}\frac{\partial^2\boldsymbol{J}_s}{\partial t^2} = 0 \; (44)$$

## Discussion

Although we obtained an inhomogeneous wave equation that is identical in form to the one derived from Maxwell's equations, the starting points of the two derivations are fundamentally different. In this paper, we began solely with Weber's force law, whereas the classical approach relies on Faraday's law, the Maxwell–Ampère law, and Gauss's law. Previous work has shown that Maxwell's equations can be derived from Weber's force [12], and the results of this paper provide further support for that claim.

In the derivation based on Maxwell's equations, the applicable domain of the wave equations is often left vague. It is generally assumed that they are valid across all classical physics scenarios, including relativistic regimes [1]. However, in the present method, it is clear that the derivation is an approximation valid only in the low-velocity regime. This raises the question: could Maxwell's equations themselves also be approximations that hold primarily in low-velocity cases?

Although the derived inhomogeneous wave equation does not explicitly prohibit the existence of longitudinal electric field waves, we still find that such waves cannot exist in a vacuum. This is due to the requirement of charge conservation—the continuity of source charges. Extended electrodynamics has been proposed as a modification of Maxwell's theory to allow for longitudinal waves [13]. However, the outcome of this paper suggests that such extensions may be unnecessary.

On the other hand, longitudinal waves can be derived in the case of free charge movement. This result is consistent with the telegrapher's equations [3] and the nature of signal propagation in wires, where free electrons are present. The derivation using Weber's force is more direct than the traditional approach based on Maxwell's equations, which requires first determining the distributed inductance and capacitance—quantities that are not always easy to calculate. Furthermore, the traditional method only yields the telegrapher's equation for dual-conductor wire systems, as the capacitance of a single-conductor wire is not well defined [2].

There have been several past attempts to extend Weber's force to account for wave propagation in a vacuum. One such approach is to assume retarded force propagation [14]. Another treats Weber's force as a special case derivable from Maxwell's equations under specific assumptions, forming the so-called Maxwell–Weber electrodynamics [15]. In contrast, our method is more straightforward; it relies solely on the postulate of a polarizable vacuum, an idea that is physically intuitive.

In this paper, we treat wave propagation as occurring through a medium. Historically, such an idea has been challenged by the argument that a medium would need to be infinitely rigid to suppress longitudinal waves, and infinitely non-viscous to allow matter to move through it freely [16]. However, we have shown here that the assumption of an infinitely rigid medium is not required.

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We have derived the wave equation for the electric field. As shown earlier, the electric field derived from Weber's force contains six components [17]. It remains unclear how each of these components propagates individually. This question is left for future investigation.

## Conclusions

We have successfully derived the inhomogeneous electric wave equation from Weber's electrodynamics. This wave equation has the same form as the one derived from Maxwell's equations. Throughout the derivation, we also confirmed the consistency of Maxwell's equations with Weber's electrodynamics and the postulate of a polarizable vacuum.

Although the wave equation itself permits the possibility of longitudinal electric waves, the requirement of charge conservation (i.e., charge continuity) eliminates the existence of such waves in a vacuum. In contrast, longitudinal wave propagation can occur in the case of free charge movement inside a conductor, such as in a wire.

Our derivation is based on low-velocity approximations. This raises an important question: Do Maxwell's equations—and the associated inhomogeneous wave equation—apply only within the low-velocity regime? In traditional derivations, the displacement current is introduced primarily for mathematical consistency. In this work, we provided a physical interpretation of the displacement current. This may suggest that we are uncovering a more fundamental layer of physics underlying Maxwell's equations.

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