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Meaning Computation AI and Semantic Reasoning

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Keywords: semantic reasoning; implicit relations; ontology; axiomatic model; interval relations

Article

Introduction to Meaning Computation AI. Semantic Reasoning

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Abstract: This paper's aim is to extend horizons in symbolic AI. The key to doing so is to establish a scientific basis for knowledge foundations, and specifically for the concept of meaning, and its computation. This led to the creation of the Meaning Computation AI (MCAI). Knowledge structures are at the heart of MCAI. The semantic triple chain, one of such structures, is the subject of this paper. This structure is the core of introducing computational semantic reasoning models across a variety of domains. To efficiently apply knowledge structures to create semantic reasoning applications MCAI contains a development framework consisting of four components. The first component stems from our discovery that knowledge is relational in nature. It can be expressed through implicit relations (which reflect meaning) and semantic relations (which describe how implicit relations act between entities). An algebraical apparatus for relational knowledge, specifically for implicit and semantic relations, represents the first component. Axiomatic models of relational domain ontologies and domain-oriented semantic reasoning models comprise the second and third components. The fourth component contains domain-oriented semantic reasoning engines.

Keywords: semantic reasoning; implicit relations; ontology; axiomatic model; interval relations

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1. Introduction

What is the future of symbolic AI? Is rule-based AI all it can provide? To answer these questions it is vital to have a clear definition of intelligence. General Motors' Head of Research, Charles Kettering, is widely credited with saying "A problem well-stated is a problem half-solved". So, what is the definition of intelligence? The Oxford Dictionary answers this question as follows: intelligence is the ability to acquire and apply knowledge and skills [1]. From such a definition, it may be concluded that knowledge plays a pivotal role in AI, while computational knowledge science holds the key to further AI advancement.

In this regard, the question arises: what kind of mathematical apparatus is suitable for knowledge research? The first objective of this paper is to answer this question. We have shown that knowledge is relational in nature and can be expressed through implicit and semantic relations. The paper introduced an algebraical apparatus for implicit relations.

The essence of knowledge in AI can be defined by two concepts. They are the meaning and its computation. The paper's second aim is to use the algebraical apparatus of implicit relations which reflect meaning to explore these two concepts. Critical role in this process is played by a knowledge structure we call the semantic triple chain.

2. Relational Foundations of Knowledge

2.1. Knowledge, Relations, and Meaning

The closest mathematical notion associated with knowledge is relation. Indeed, when we talk about time, words like *earlier, later, during, and simultaneously* are usually involved. These words represent relations. Or, if we consider knowledge about objects in space, words like *under, over,* and *inside* are involved. These words represent relations too. Relations appear to be fundamentally positioned at the core of knowledge. Therefore, the mathematical apparatus of relations is appropriate for knowledge research.

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Two mathematics departments study relations: set theory and predicate logic. Let's consider the following statements: "Brenda is grandmother of Thomas and Kate", "Mary is grandmother of Peter". These statements represent the grandmother relation. From set theory, this relation can be defined as follows:

$$\text{Grandmother} = \{(\text{Brenda, Thomas}), (\text{Brenda, Kate}), (\text{Mary, Peter})\}. \quad (1)$$

However, from a predicate logic point of view the grandmother relation may be defined as:

$$\text{Grandmother}\ (x,y) \approx \exists z\ [\text{Mother}\ (x, z) \land \text{Parent}\ (z,y)]. \quad (2)$$

Relations defined by set theory will be called explicit relations while relations defined by predicate logic will be called implicit relations.

How can we formalize the meaning of the grandmother's statements above? Can we do it based on explicit relations? Hardly, indeed, let's consider, for example, the *OlderThan* relation between the same people. It can be defined as follows:

$$\text{OlderThan} = \{(\text{ Brenda, Thomas }), (\text{ Brenda, Kate }), (\text{ Mary, Peter })\}. \quad (3)$$

As can be seen from (1) and (3), the same formula defines explicit relations representing different meanings.

Due to its uniqueness, expression (2) better represents the meaning of the grandmother's relation from the perspective of implicit relations. In fact, the predicate formula (2) can only express one meaning. Nonetheless, this definition says nothing about Brenda, Thomas, Kate, Mary, or Peter

To resolve this situation, a new category of relations is needed. Let's look at how to form it. If we link an implicit relation to an explicit relation, provided that the predicate defining the implicit relation is true for each pair of objects in the explicit relation, we will get an entity called a semantic relation [2, 3]. Denoting a semantic relation as $SR_{i,j}$, we can define it as

$$\mathrm{SR}_{i,j} = (\mathrm{P}_i, \mathrm{R}_j), \quad (4)$$

where P_i and R_j are, respectively, implicit and related explicit relations. In our example above, expressions (1) and (2) define two sides of the grandmother semantic relation $SR_{i,j}$, namely, expression (2) defines its meaning, while expression (1) defines its content.

The formal apparatus of implicit relations can be applied to modeling intelligence, such as reasoning in semantic triple chains. The formal apparatus of semantic relations, on the other hand, can be applied to modeling knowledge graphs, including their intelligence. The purpose of this paper is to further research implicit relations, their formal apparatus, and application to modeling reasoning in semantic triple chains.

2.2. *Algebra of Implicit Relations*

There are four algebraic operations on implicit relations: composition, conversion, disjunction, and conjunction, as well as the property of inclusion. Let's consider them. The composition of the implicit relations P_i by P_j is called the implicit relation denoted as

$$\mathrm{P}_k = \mathrm{P}_i \circ \mathrm{P}_j, \quad (5)$$

if and only if the formula

$$\mathrm{P}_k\,(x,y) \approx \exists z\ [\mathrm{P}_i\,(x,z) \land \mathrm{P}_j\,(z,y)] \quad (6)$$

is identically true. E.g., if P_i is the implicit relation "x is the daughter of y" and P_j - "x is the sibling of y", then the implicit relation P_k "x is the niece of y" is the composition of P_i by P_j since it is obvious that the expression (6) in this case is identically true.

The conversion of the implicit relation P_j is called the implicit relation P_i denoted as

$$\mathrm{P}_i = \mathrm{P}_j^{\ -1} \quad (7)$$

if and only if the formula

$$\mathrm{P}_i\,(x,y) \approx \mathrm{P}_j\,(y,x) \quad (8)$$

is identically true. For instance, if P_j is the implicit relation "x is before y", then the implicit relation P_i "x is after y" is the conversion of the implicit relation P_j. If the implicit relation $P_i\,(x,y)$ on the left side of equation (8) is substituted by the definition of the same given in (7), the following property is obtained:

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$$P_j^{-1}(x, y) \approx P_j(y, x). \quad (9)$$

This property signifies that the formula describing the conversion of implicit relation between objects x and y is valid if the same implicit relation is valid between objects y and x.

The implicit relation P_k is called disjunction of implicit relations P_i and P_j and is denoted as $P_k = P_i \vee P_j$, if and only if the formula

$$P_k(x, y) \approx P_i(x, y) \vee P_j(x, y). \quad (10)$$

is identically true. E.g., if P_i is the implicit relation "x is the mother of y" and P_j - "x is the father of y", then the implicit relation P_k "x is the parent of y" is the disjunction of P_i and P_j.

The implicit relation P_k is called the conjunction of implicit relations P_i and P_j and is denoted as $P_k = P_i \wedge P_j$, if and only if the formula

$$P_k(x, y) \approx P_i(x, y) \wedge P_j(x, y). \quad (11)$$

is identically true. E.g., if P_i is the implicit relation "x is the sister of y" and P_j - "x is older than y", then the implicit relation P_k "x is the older sister" is the conjunction of P_i and P_j.

Let's now introduce the inclusion property. We shall define this property as follows: the implicit relation P_i is included in the implicit relation $P_{j,}$ and this property is defined as

$$P_i \subseteq P_j \quad (12)$$

if and only if the formula

$$P_i \rightarrow P_j \quad (13)$$

is identically true. E.g., if P_i is the relation "x is mother of y" then it is included in the relation P_j "x is parent of y".

The algebraic properties of the operations composition and conversion, as well as their interdependence on disjunction and conjunction operations, are characterized through the following correlations, whose validity can be proven:

$$(P_i \circ P_j) \circ P_k = P_i \circ (P_j \circ P_k) \quad (14) \qquad (P_i^{-1})^{-1} = P_i \quad (15)$$

$$(P_i \circ P_j)^{-1} = P_j^{-1} \circ P_i^{-1} \quad (16) \qquad (P_i \vee P_j)^{-1} = P_i^{-1} \vee P_j^{-1} \quad (17)$$

$$(P_i \wedge P_j)^{-1} = P_i^{-1} \wedge P_j^{-1} \quad (18) \qquad (P_i \vee P_j) \circ P_k = (P_i \circ P_k) \vee (P_j \circ P_k) \quad (19)$$

$$(P_i \wedge P_j) \circ P_k \subseteq (P_i \circ P_k) \wedge (P_j \circ P_k) \quad (20)$$

3. Model of Semantic Reasoning using Implicit Relations Algebra

3.1. Algebraic Model of Semantic Reasoning

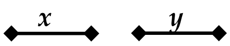
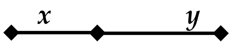
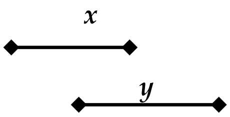
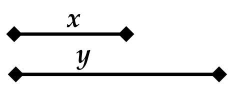
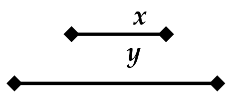
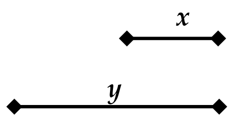
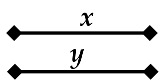
Above we introduced the operation of composition which is vital to modeling semantic reasoning. Indeed, in the 19th century, British mathematician and logician Augustus de Morgan, for the first time, introduced the interpretation of syllogistic reasoning as a composition of relations [4]. In modern times this approach is applied to non-syllogistic reasoning as well. For example, in the early 1980s James F. Allen applied the composition of relations operation to modeling reasoning in the domain of non-metric temporal interval relations [5].

Allen proposed thirteen basic binary relations covering all possible relations between two time intervals. Table 1 below illustrates these relations. For each relation the name, symbol, and symbol for the inverse relation are provided. The meaning of each is portrayed by a diagram relating to the intervals x and y. As an example, the first relation "*x Before y*" means that x ends before y begins, while the second relation "*x Meets y*" means that y begins when x ends. Column 5 in the table illustrates the meaning of these relations in terms of project management.

Each inverse relation in column three can be interpreted as the result of the algebraic conversion of the relation in column one. However, based on expression (9) above, we may conclude that the original and inverse implicit relations represent the same meaning. Indeed, saying that interval a is before interval b is the same as saying that interval b is after interval a. Therefore, we may exclude inverse relations from consideration without losing the completeness of the interval relations representation.

Let's introduce the concept of a semantic triple chain knowledge structure. Figure 1 below shows an example. This knowledge structure tells us that interval *a* is associated with interval *b* via relation S and interval b is associated with interval c via relation D. In other words this semantic triple chain can be described as *aSb*, *bDc*. Figure 1 also shows the unknown implicit relation P_x between intervals *a* and *c*. We call semantic reasoning the process of finding out the unknown implicit relation between entities in the semantic triple chain.

Table 1. List of non-metric interval relations.

Relation Name	Relation Symbol	Symbol for Inverse Relation	Relation Meaning	Interpretation
1	2	3	4	5
x** Before **y	B	Bi		Project ***x*** completed before project ***y*** started
x** Meets **y	M	Mi		Project ***x*** completed at the same time project ***y*** started
x** Overlaps **y	O	Oi		Project ***x*** started before project ***y*** started and completed after project ***y*** started and before project ***y*** completed
x** Starts **y	S	Si		Projects ***x*** and ***y*** started simultaneously and project ***x*** completed before project ***y*** completed
x** During **y	D	Di		Project ***x*** started after project ***y*** started and completed before project ***y*** completed
x** Finished **y	F	Fi		Project ***x*** started after project ***y*** started and completed simultaneously with project ***y***
x** Equals **y	E			Projects ***x*** and ***y*** started and completed simultaneously

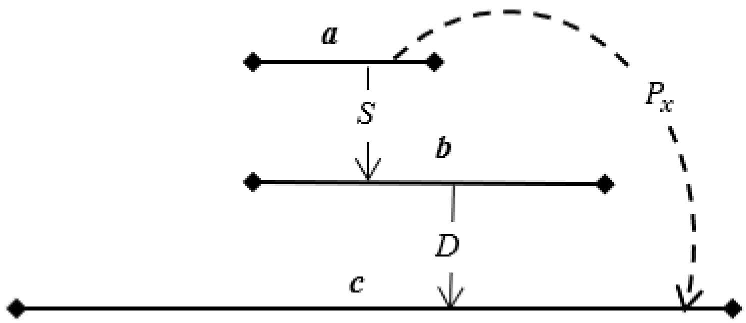


Figure 1. Semantic triple chain knowledge structure.

In terms of algebra of implicit relations semantic reasoning task of finding the value of P_x can be defined as the process of resolving the relational compositional expression as shown below:

$$P_x = S \circ D. \quad (21)$$

Generalizing (21), we arrive at the following algebraic model of semantic reasoning:

$$P_x = \prod_{i=1}^{n} P_i. \quad (22)$$

where P_i represents the implicit relations belonging to the given semantic triple chain $a_1P_1a_2, a_2P_2a_3, \dots, a_{n-1}P_{n-1}a_n$, $a_nP_na_{n+1}$ ($n \geq 2$), and P_x is the unknown relation between the first and the last objects in this chain. We will refer to (22) as the algebraic relational model of the semantic reasoning.

We can see that to obtain the value of the unknown relation P_x in (22), it is necessary to solve the algebraic relational expression on the right side of this formula. In order to tackle this issue, we need to answer two questions: how to represent implicit relations and how to solve compositional relational expressions in different domains?

3.2. *Axiomatic Model of Relational Domain Ontology*

Initially introduced by Aristotle, ontologies are formal models of how we perceive a domain of interest. They provide a precise, logical account of the intended meaning of terms, data structures, and other elements modeling the real world [6]. The term ontology refers to a wide range of formal representations, including taxonomies, hierarchical terminology vocabularies, or detailed logical theories describing a domain [7]. The term ontology was adopted by early AI researchers, who recognized the applicability of the work from mathematical logic and argued that AI researchers could create new ontologies as computational models that enable certain kinds of automated reasoning [8]. In [9], attention was brought to a formal, algebraic approach which identifies an ontology as a pair *(S, A)*. *S* is the vocabulary (or signature) of the ontology and *A* is the set of ontological axioms, which specify the intended interpretation of the vocabulary in a given domain of discourse.

In this paper, we will introduce a similar algebraic approach to axiomatic domain ontology. The peculiarity of the ontology we are considering is that it is a relational ontology since it should represent knowledge about implicit relations. To create an axiomatic ontology, we divide all relations within a domain into two categories: base and derived relations. Base relations are relations without definitions. They are characterized only by their names and properties. Derived relations are relations defined through base relations using implicit relations algebra.

We define the relational domain ontology as an axiomatic model [10], which can be denoted as $\mathcal{RDO}_i$ and defined as

$$\mathcal{RDO}_i = (\Sigma_i, \Delta_i, A_i), \quad (23)$$

where Σ_i is the signature of the axiomatic system. It is denoted as $\Sigma_i = (\Sigma_i^b, \Sigma_i^d)$, where Σ_i^b is the base relations signature component defining the set of base relations names, while Σ_x^d is the derived relations signature component defining the set of derived relations names. Δ_i represents the totality of all derived relations definitions. A_i is the base relations axioms component denoted as $A_i = (A_{i,}^s, A_i^n)$, where $A_{i,}^s$ defines the set of simplification axioms, while A_i^n defines axioms of indeterminacy.

The notion of relational expression in $\mathcal{RDO}_i$ ontology can be defined inductively as follows:

1. If $P \in \Sigma_i^b$ or $P \in \Sigma_i^d$, then P is a relational expression;
2. If P_1 and P_2 are relational expressions, then P_1^{-1}, $P_1 \circ P_2, P_1 \vee P_2$, $P_1 \wedge P_2$, $P_1 \subseteq P_2$ and only they are relational expressions

The relational expression which is a base relation or conversion of a base relation we call an elementary relational expression. We say an expression has been loaded into the signature whenever it has been computed into a base relation, derived relation, or their converts. Otherwise, it is not loaded into the signature. The process of algebraic transformation of an arbitrary relational expression to find out whether it is loaded into the signature or not, we call resolution of this expression. This is, otherwise, the computation of its value.

4. Semantic Reasoning in the Domain of Non-Metric Interval Relations

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4.1. Axiomatic Model of the Non-Metric Interval Relations Ontology

We shall denote the axiomatic model of the non-metric interval relations ontology as $\mathcal{RDO}_{int}$ and define it as

$$\mathcal{RDO}_{int} = (\Sigma_{int}, \Delta_{int}, A_{int}). \quad (24)$$

The signature's base relations component we will define as

$$\Sigma_{int}^{b} = \{M, S, F, E\}, \quad (25)$$

while the signature's derived relations component will be defined as

$$\Sigma_{int}^{d} = \{B, D, O\}. \quad (26)$$

We will also define the derived relations definitions component Δ_{int} by the introduction of the following formulas:

$$B = M \circ M \quad (27)$$

$$B = M \circ F^{-1} \quad (28)$$

$$B = S \circ M \quad (29)$$

$$D = S \circ F \quad (30)$$

$$D = F \circ S \quad (31)$$

$$O = F^{-1} \circ S \quad (32)$$

Below the simplification axioms belonging to component A_{int}^{s}, of axioms A_{int} are shown:

$$M \circ S = M \quad (33)$$

$$M \circ S^{-1} = M \quad (34)$$

$$F^{-1} \circ M = M \quad (35)$$

$$F \circ M = M \quad (36)$$

$$M \circ E = M \quad (37)$$

$$E \circ M = M \quad (38)$$

$$S \circ S = S \quad (39)$$

$$S \circ E = S \quad (40)$$

$$E \circ S = S \quad (41)$$

$$F \circ F = F \quad (42)$$

$$E \circ F = F \quad (43)$$

$$F \circ E = F \quad (44)$$

$$E \circ E = E. \quad (45)$$

Every simplification axiom represents some property of the base relation and is expressed in the form of the transformation of an expression on the left, composed of elementary expressions, into a base relation.

The indeterminacy axioms belonging to component A_{int}^{n}, are defined as follows:

$$M \circ F = O \vee S \vee D \quad (46)$$

$$F^{-1} \circ F = F^{-1} \vee E \vee F \quad (47)$$

$$M \circ M^{-1} = F^{-1} \vee E \vee F \quad (48)$$

$$S^{-1} \circ M = F \vee S^{-1} \vee O \quad (49)$$

$$S \circ F^{-1} = B \vee M \vee O \quad (50)$$

$$S^{-1} \circ S = S^{-1} \vee E \vee S \quad (51)$$

$$S \circ S^{-1} = S^{-1} \vee E \vee S \quad (52)$$

$$M^{-1} \circ M = S^{-1} \vee E \vee S \quad (53)$$

$$F \circ F^{-1} = F^{-1} \vee E \vee F \quad (54)$$

Our ontology has been largely defined by compositions of elementary expressions. Due to this fact, it is possible to represent this ontology using a composition table shown below in Table 2. In this table, rows and columns are denoted by elementary expressions. The composition operation is performed as follows: an elementary expression that represents the table's row is composed by an elementary expression that denotes the column, and the result is given at the intersection of row and column.

One can see that the composition table is filled partially, namely its part divided by its diagonal. The reason is that all other results in the table can be obtained by computation. For example, let's find out an unknown value at the intersection of row "F^{-1}" and column "S^{-1}". We will get:

$$F^{-1} \circ S^{-1} = (S \circ F)^{-1} = D^{-1} \tag{55}$$

Here we have applied the property (16) of conversion of relations composition and relation *D* definition (30).

Table 2. Composition table representing a non-metric interval relations ontology.

	M	S	F	E = E^{-1}	F^{-1}	S^{-1}	M^{-1}
M	B	M	O ∨ S ∨ D	M	B	M	F^{-1} ∨ E ∨ F
S	B	S	D	S	B ∨ M ∨ O	S^{-1} ∨ E ∨ S	
F	M	D	F	F	F^{-1} ∨ E ∨ F		
E = E^{-1}	M	S	F	E			
F^{-1}	M	O	F^{-1} ∨ E ∨ F				
S^{-1}	D^{-1} ∨ F^{-1} ∨ O	S^{-1} ∨ E ∨ S					
M^{-1}	S^{-1} ∨ E ∨ S						

4.2. Ontology-Based Algebraical Model of Semantic Reasoning

As the issue of the non-metric interval relations ontology has been addressed, we are ready to turn our attention to the semantic reasoning process. In (22) we defined this process as the resolution or computation of a compositional expression based on implicit relations which in turn represent the meaning of relations. Therefore semantic reasoning is a process of meaning computation.

Let's now move to development of semantic reasoning model. At the heart of this model are two procedures. The first one is the normalization procedure. It transforms a relational expression into a base-normal form by replacing each derived relation contained within the expression with its definition. Consequently, we will obtain an expression consisting only of elementary expressions. E.g., the base-normal form of the expression $M \circ O$ is the expression $M \circ F^{-1} \circ S$, based on (32). The second procedure aims to simplify the base-normal expression. This is done using ontology simplification axioms and definitions of derived relations. These two steps - normalization and simplification - comprise the essence of the semantic reasoning model.

Now let's return to Figure 1 and consider equation (21) that represents the semantic reasoning task of finding the value of unknown relation P_x. At the normalization stage, using the definition of relation *D* (31) we get:

$$P_x = S \circ D = S \circ F \circ S. \tag{56}$$

At the simplification stage, we can see that in accordance with (30) and (31) respectively, the compositions of the first relation by the second as well as that of the second by the third define derived relation *D*. Here is how this can be resolved:

$$P_x = (S \circ F) \circ S = (F \circ S) \circ S = F \circ (S \circ S) = F \circ S = D. \tag{57}$$

At the beginning of the resolution, we use the grouping rule (14). We shall call it *G-rule*. After that, we swap the relation *D* definition (30) with its definition (31). Such an operation we shall call *Swap-rule*. After that, *G-rule* is used again. Next, we apply the simplification axiom (39). We shall call

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such operation *A-rule*. Then we apply the definition (31) of relation *D*. We shall name such operation *D-rule*. Finally, we conclude that *D* is an unknown relation.

4.3. Checking the Completeness of Non-Metric Relational Domain Ontology

We shall consider that the axiomatic model of relational domain ontology is complete [11] if its axioms and definitions provide resolution of any relational expression. Within the scope of this paper, we are interested only in compositional relational expressions. Therefore, we shall examine the completeness problem regarding this category of expressions. It is evident that any relational expression can be transformed into the base-normal form. Therefore, we should re-define the notion of completeness as follows: the domain ontology is complete if its axioms and definitions provide resolution of any compositional relational expression in the base-normal form.

Now, let's ask ourselves the following question: what are the scenarios where a relational expression in base-normal form cannot be simplified? It may occur when this expression contains the composition of three elementary relational expressions containing the composition of the first expression by the second, as well as that of the second by the third, which define derived relations or their conversions. In order to detect such "difficult" compositions, one may proceed as follows. A table is drawn, whose rows and columns are denoted by definitions of derived relations or their conversions. At row-column intersections, one looks for ternary compositions meeting the above-stated condition and investigates the possibility of simplifying these expressions. If an expression cannot be simplified, its value must be defined axiomatically. Table 3 below represents such an analysis. This table shows that 12 expressions require investigation. However, it may be proved that all of them can be simplified. Therefore, we may conclude that the axiomatic model of non-metric interval relations ontology is complete.

Table 3. Analysis of the completeness of the non-metric interval relations ontology.

	$M \circ M$	$M \circ F^{-1}$	$S \circ M$	$S \circ F$	$F \circ S$	$F^{-1} \circ S$	$S^{-1} \circ F$	$S^{-1} \circ F^{-1}$	$F^{-1} \circ S^{-1}$	$M^{-1} \circ S^{-1}$	$F \circ M^{-1}$	$M^{-1} \circ M^{-1}$
$M \circ M$	$M \circ M \circ M$	$M \circ M \circ F^{-1}$										
$M * F^{-1}$						$M \circ F^{-1} \circ S$			$M \circ F^{-1} \circ S^{-1}$			
$S \circ M$	$S \circ M \circ M$	$S \circ M \circ F^{-1}$										
$S \circ F$					$S \circ F \circ S$							
$F \circ S$			$F \circ S \circ M$	$F \circ S \circ F$								
$F^{-1} * S$			$F^{-1} \circ S \circ M$	$F^{-1} \circ S \circ F$								
$S^{-1} * F$					$S^{-1} \circ F \circ S$							
$S^{-1} * F^{-1}$												
$F^{-1} * S^{-1}$												
$M^{-1} * S^{-1}$												
$F * M^{-1}$												
$M^{-1} * M^{-1}$												

5. Semantic Reasoning in the Domain of Metric Interval Relations

5.1. Introduction to Metric Interval Relations

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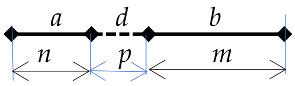
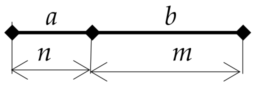
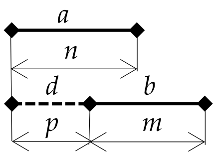
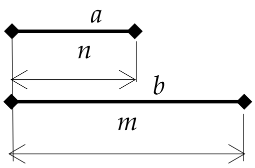
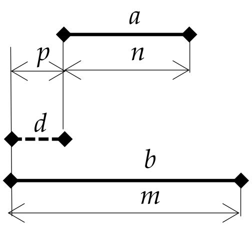
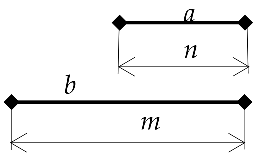
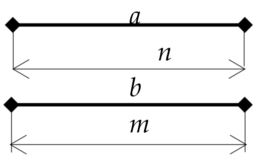
Below in Table 4 the list of metric interval relations is shown. Each interval is denoted by two attributes separated by a dot. The first attribute is the interval name; the second attribute is the interval duration. Solid lines represent real intervals while dotted lines are invisible intervals.

Table 4. List of metric interval relations.

Relation Name	Relation Symbol	Relation Meaning	Notation
1	2	3	4
Interval ***a*** of duration ***n*** *Before* interval ***b*** of duration ***m*** by the duration of ***p***	B		B(a.n, b.m, d.p)
Interval ***a*** of duration ***n*** *Meets* interval ***b*** of duration ***m***	M		M(a.n, b.m)
Interval ***a*** of duration ***n*** *Overlaps* interval ***b*** of duration ***m*** and begins earlier that ***b*** by the duration of ***p***	O		O(a.n, b.m, d.p)
Interval ***a*** of duration ***n*** *Starts* interval ***b*** of duration ***m***	S		S(a.n, b.m), n<m
Interval ***a*** of duration ***n*** *During* interval ***b*** of duration ***m*** and begins later that ***b*** by the duration of ***p***	D		D(a.n, b.m, d.p)
Interval ***a*** of duration ***n*** *Finishes* interval ***b*** of duration ***m***	F		F(a.n, b.m), n<m
Interval ***a*** of duration ***n*** *Equals* interval ***b*** of duration ***m***	E		E(a.n, b.m), n=m

5.2. Axiomatic Model of the Metric Interval Relations Ontology

We shall denote the axiomatic model of the metric interval relations ontology as $\mathcal{RDO}_{mint}$ and define it as

$$\mathcal{RDO}_{\text{mint}} =< \Sigma_{\text{mint}}, \Delta_{\text{mint}}, A_{\text{mint}} >. \tag{59}$$

The model's signature will be denoted $\Sigma_{mint} =< \Sigma^b_{mint}, \Sigma^d_{mint} >$ and defined as

$$\Sigma^b_{mint} = \{ \text{S(a.n, b.m), F(a.n, b.m)} \}, \tag{60}$$

$$\Sigma^d_{mint} = \{ \text{B(a.n, b.m, d.p), M(a.n, b.m), O(a.n, b.m, d.p), D(a.n, b.m, d.p), E(a.n, b.m)} \}. \tag{61}$$

The composition table for the metric interval relations ontology is shown in Table 5. In this table, the derived relations are shown in bold. There are some differences in the definition of non-metric vs metric interval relation ontologies. As opposed to the former case, in the latter case *M* and *E* relations fall into the category of derived relations, not base relations. Table 5 also shows that the metric interval relations ontology does not include indeterminacy axioms.

We will reference relation definitions and properties in the composition table shown in Table 5 using the following format: MIRT.[row number].[column number], where MIRT stands for metric interval relations table. For example, MIRT.2.1 represents the following definition: *D(a.n, b.m, q-n)* = *F(a.n, i.q)* ∘ *S(i.q, b.m)* while MIRT.1.3b represents the definition *M(a.n, b.m)* = *S(a.n, i.q)* ∘ F^{-1}*(i.q,b.m)*, where *q*=*m*+*n*.

Table 5. Metric interval relations ontology composition table

		S(i.q, b.m)	F(i.q, b.m)	F^{-1}(i.q, b.m)	S^{-1}(i.q, b.m)
		1	2	3	4
1	S(a.n, i.q)	S(a.n, b.m)	**D (a.n, b.m, d.(m-q))**	a: **B(a.n, b.m, d.(q-n-m)),** q >m+n *b*: **M(a.n, b.m)**, q = m+n *c*: **O(a.n, b.m, d.(q-m)),** q < m+n	*a*: S^{-1} (a.n, b.m), n > m b: **E(a.n, b.m)**, n = m c: S(a.n, b.m), n < m
2	F(a.n, i.q)	**D(a.n, b.m, d.(q-n))**	F(a.n, b.m)	*a*: F^{-1} (a.n, b.m), n > m *b:* **E(a.n, b.m)**, n = m c: F(a.n, b.m), n < m	
3	F^{-1}(a.n, i.q)	**O(a.n, b.m, d.(n-q))**	*a*: F^{-1} (a.n, b.m), m < n *b:* **E(a.n, b.m),** m = n c: F(a.n, b.m), m > n		
4	S^{-1}(a.n, i.q)	*a*: S^{-1} (a.n, b.m), n > m *b:* **E(a.n, b.m)**, n = m c: S(a.n, b.m), n > m			

5.3. Ontology-Based Algebraical Model of Semantic Reasoning

Let's consider an example of the semantic triple chain shown in Figure 2. One can see that the unknown relation P_x can be defined by the following expression:

$$\mathrm{P_x} = \mathrm{S(a.n, b.m)} \circ \mathrm{D(b.m, c.z, d.p)}. \quad (62)$$

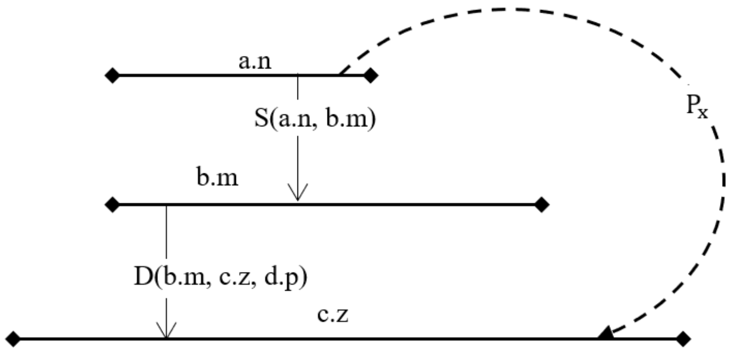


Figure 2. Semantic triple chain based on metric interval relations

Let's compute this expression. At the normalization stage we have to replace the derived relation *D* definition from the composition table with its standard definition in Table 4. Let's consider the composition table definition provided by MIRT.1.2:

$$D\ (a.n,\ b.m,\ d.(m\text{-}q)) = S(a.n,\ i.q) \circ F(i.q,\ b.m) \quad (63)$$

In this particular case we have to reformat the definition of relation *D* (63) to a standard format where relation *D* is denoted as *D(a.n, b.m, d.p)*. On the basis of (63) we will get:

$$p = m\text{-}q \quad (64)$$

From (64) we will get

$$q = m\text{-}p. \quad (65)$$

When we apply (64) and (65) to (63) we will get:

$$D\ (a.n,\ b.m,\ d.p) = S\ (a.n,\ i.(m\text{-}p)) \circ F(i.(m\text{-}p),\ b.m). \quad (66)$$

So, we've transformed the definition of relation *D* provided by (63) into the standardized denotation of this relation (66). To make the terms of (66) compatible with the terms of (62), the following substitutions should be applied: *a/b, n/m, b/c, m/z*. We will obtain the following result:

$$D\ (b.m,\ c.z,\ d.p) = S(b.m,\ i.(z\text{-}p)) \circ F(i.(z\text{-}p),\ c.z). \quad (67)$$

The next step is to replace relation *D(b.m, c.z, d.p)* in (62) with definition (67). We will get:

$$P_x = S(a.n, b.m) \circ D(b.m,\ c.z,\ d.p) = S(a.n, b.m) \circ S(b.m,\ i.(z\text{-}p)) \circ F(i.(z\text{-}p),\ c.z). \quad (68)$$

As a result, we have transformed the initial relational expression (62) into a base-normal form (68). Now let's try to simplify (68). First, we apply *G-rule* by grouping the composition of the first and second elementary expressions in (68). We will get:

$$P_x = (S(a.n, b.m) \circ S(b.m,\ i.(z\text{-}p))) \circ F(i.(z\text{-}p),\ c.z). \quad (69)$$

Next we apply *A-rule* MIRT.1.1 to the grouped expression:

$$P_x = S(a.n, i.(z-p) \circ F(i.(z\text{-}p),\ c.z). \quad (70)$$

Finally, we consider the application of *D-Rule* in accordance with MITD.1.2 which is defined as follows:

$$D\ (a.n,\ b.m,\ d.(m\text{-}q)) = S(a.n,\ i.q) \circ F(i.q,\ b.m) \quad (71)$$

To apply this definition we should reformat the right part of (71) in accordance with the terms of the right part of expression (70). We will get:

$$q = z\text{-}p. \quad (72)$$

Applying (72) to (71) we will get:

$$D\ (a.n,\ b.m,\ d.(m\text{-}(z\text{-}p))) = S(a.n,\ i.(z\text{-}p)) \circ F(i.(z\text{-}p),\ b.m) \quad (73)$$

To make the terms of (73) compatible with the terms of (70), the following substitutions should be applied: b/c, m/z. This will further result in:

$$D\ (a.n,\ c.z,\ d.(z\text{-}(z\text{-}p))) = S(a.n,\ i.(z\text{-}p)) \circ F(i.(z\text{-}p),\ c.z). \quad (74)$$

Finally we obtain that

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$$D\ (a.n,\ c.z,\ d.p) = S(a.n,\ i.(z\text{-}p)) \circ F(i.(z\text{-}p),\ c.z) \qquad (75)$$

So, we come to the following conclusion:

$$P_x = D\ (a.n, c.z, d.p). \qquad (76)$$

Above we considered the application of the following relation expression computation techniques: transforming the source expression into the base-normal form, *G-* , *D-* and *A-Rules*.

Let's now consider how *Swap-Rule* can be applied. For that purpose we will deliberate on the resolution of a semantic reasoning task represented by the semantic triple chain in Figure 3 below.

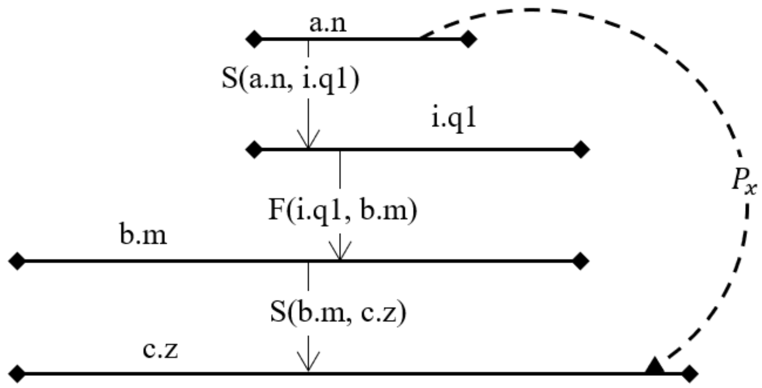

Figure 3. Semantic triple chain illustrating the *Swap-Rule* problem

One can see that the unknown relation P_x can be defined by the following expression:

$$P_x = S(a.n, i.q1) \circ F(i.q1, b.m) \circ S(b.m, c.z). \qquad (77)$$

In (77) composition of the first relation by the second and the second by the third defines the derived relation. This is a classic example of an expression that requires the use of *Swap-Rule*. In this particular case *Swap-Rule* requires to convert the *S∘F* type relational expression defining relation *D* to the *F∘S* type definition of this relation or vice versa.

Let's consider the problem of conversion from *S(a.n, i.q1)* ∘ *F(i.q1, b.m)* which is the *S∘F* type definition to *F∘S* type definition. Figure 4 below shows a graphical representation of such a conversion.

Part *a* in this figure represents the *S∘F* definition of relation *D* while part *b* shows the *F∘S* type definition of the same relation. Our goal is to find conditions where both of these definitions express the same relation. Since the utmost intervals (*a.n* and *b.m*) in both semantic triple chains are the same, to reach this goal the following condition should be met:

$$m - q1 = q2 - n. \qquad (78)$$

From (78) we will get:

$$q2 = m - q1 + n. \qquad (79)$$

As a result of (79), using two ways of presentation of relation D in Figure 4 we arrive at the following formula for conversion of *S∘F* type definition of relation *D* to *F∘S* type definition of this relation:

$$S(a.n,\ i.q1) \circ F(i.q1,\ b.m) = F(a.n,\ i.(m\text{-}q1\text{+}n) \circ S(i.(m\text{-}q1\text{+}n)),\ b.m). \qquad (80)$$

Let's apply this *Swap-Rule* conversion formula. First we should use *G-Rule* by grouping the composition of the first and second elementary expressions in (77). We will get:

$$P_x = (S(a.n, i.q1) \circ F(i.q1, b.m)) \circ S(b.m, c.z) \qquad (81)$$

After the application of *Swap-Rule* (80) in (81) we will get:

$$P_x = F(a.n,\ i.(m\text{-}q1\text{+}n) \circ S(i.(m\text{-}q1\text{+}n),\ b.m) \circ S(b.m,\ c.z) \qquad (82)$$

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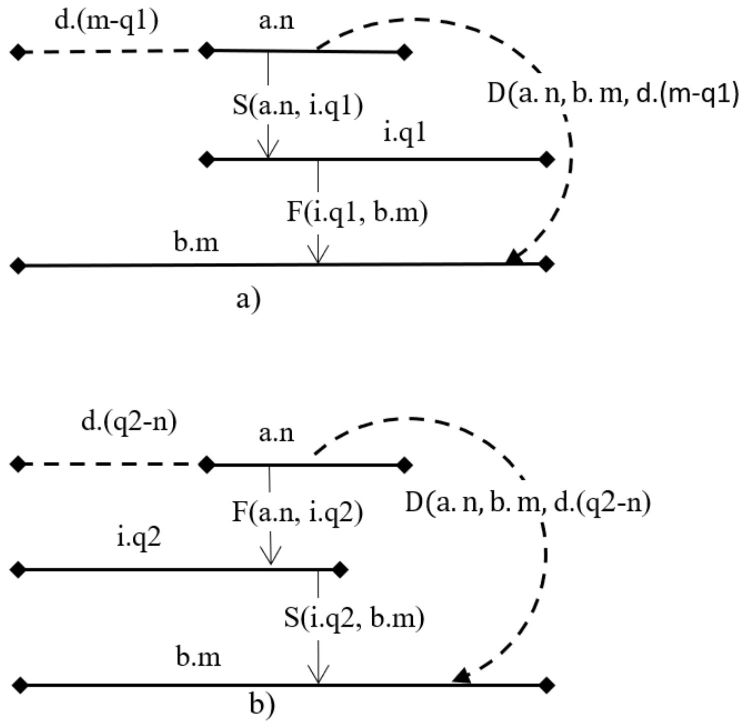

Figure 4. Illustration of the *Swap-Rule* conversion problem.

Next we apply *G-Rule*:

$$P_x = F(a.n, i.(m\text{-}q1\text{+}n) \circ (S(i.(m\text{-}q1\text{+}n), b.m) \circ S(b.m, c.z)) \quad (83)$$

This is followed by the application of *A-Rule* MIRT.1.1 which is defined as

$$S(a.n, i.q) \circ S(i.q, b.m) = S(a.n, b.m) \quad (84)$$

with the following results:

$$P_x = F(a.n, i.(m\text{-}q1\text{+}n) \circ (S(i.(m\text{-}q1\text{+}n), b.m) \circ S(b.m, c.z)) = F(a.n, i.(m\text{-}q1\text{+}n) \circ S(i.(m\text{-}q1\text{+}n), c.z) \quad (85)$$

Next we apply to (85) *D-rule* using MIRT.2.1 which is defined as

$$F(a.n, i.q) \circ S(i.q, b.m) = D(a.n, b.m, d.(q\text{-}n)). \quad (86)$$

During this process we convert (86) to terms of (85) through the following substitutions: *q*/*(m-q1+n)*, *b*/*c*, *m*/*z*. We will get

$$F(a.n, i.\,(m\text{-}q1\text{+}n)) \circ S(i.\,(m\text{-}q1\text{+}n), c.z) = D(a.n, c.z, d.(\,(m\text{-}q1\text{+}n)\text{-}n)). \quad (87)$$

Simplifying (87) we arrive at

$$F(a.n, i.\,(m\text{-}q1\text{+}n)) \circ S(i.\,(m\text{-}q1\text{+}n), c.z) = D(a.n, c.z, d.(\,(m\text{-}q1). \quad (88)$$

Applying (88) to (85) we get the following conclusion:

$$P_x = D(a.n, c.z, d.(m - q1)). \quad (89)$$

6. Conclusions

Now we're getting to the heart of the matter: Meaning Computation AI (MCAI). Its aim is to extend horizons in symbolic AI. The key to doing so is to establish a scientific basis for knowledge foundations. This will let us better understand knowledge, its role in intelligence, the essence of meaning and its computation. The relational view of knowledge introduced by the paper is the continuation of Codd's relational view of data [12]. This shows that the concept of relation plays a fundamental role in data- and knowledge-based systems. As the building blocks of intelligence solutions, knowledge structures form the core of MCAI. This paper focuses on one of these structures, the semantic triple chain.

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How did we come to the Meaning Computation AI? The first question we asked ourselves was what is intelligence? When we found out that knowledge plays a pivotal role in intelligence, the next question became: what is knowledge? We found out that knowledge is relational by nature and can be expressed through implicit and semantic relations. Next, a question arises: what should be the mathematical apparatus for knowledge research? The traditional approach is based on first-order predicate logic (FOPL). However, despite its ease of use for creating declarative knowledge, it has limited procedural abilities. Therefore, instead, we base our mathematical apparatus on a subset of FOPL with algebraical capabilities.

To efficiently apply knowledge structures to the development of intelligent applications, MCAI contains a development framework consisting of four components. The first component stems from our discovery that knowledge is relational in nature. It contains an algebraic apparatus designed to represent and compute meaning carried out by relational knowledge. It can be referred to as knowledge mathematics. Axiomatic models of relational domain ontologies comprise the second component, while domain-oriented semantic reasoning models encompass the third. The fourth component contains domain-oriented semantic reasoning engines developed based on related semantic reasoning models.

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